Probing quantum entanglement at lepton-hadron colliders

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in collaboration with Y. Hatta, Y. Guo and B. Xiao,

[M.F., Hatta, PRD 113, L031504 (2026)],

[M.F., Hatta, Xiao, PRD 114, 034031 (2026)]

[M.F., Guo, Hatta, arXiv: 2609.32800]

Precision Hour seminar series, 30 October 2026

Quantum entanglement

Einstein-Podolsky-Rosen paradox

- Assuming **locality** and their criterion of **physical reality**, the wave function cannot provide a complete description

Quantum Entanglement

- The quintessential phenomenon of QM; introduced by Schrödinger (in response to the EPR paper)

Quantum Indeterminacy

- **Local Realism:** Hidden variables
- **Copenhagen:** Indeterminacy is intrinsic
- **Agnosticism:** Only the predictive power of the theory matters

Bell Inequality: Observable difference for Realism vs Copenhagen

Aspect, Clauser, and Zeilinger closed the debate [Nobel Prize (2022)]



Einstein-Podolsky-Rosen

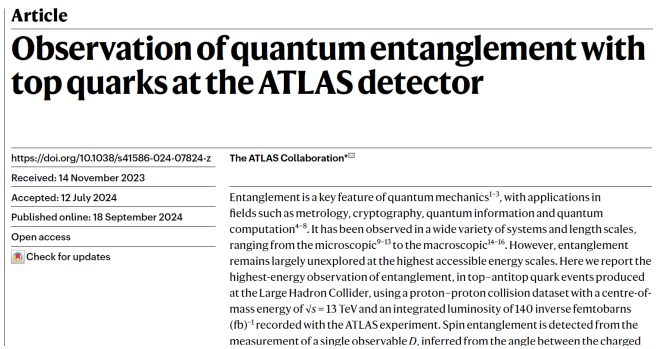
"Spooky action at a distance"



Schrödinger, Bohr and Einstein

Quantum entanglement at colliders

- In QCD, entanglement has a broad meaning (e.g., entanglement entropy)
  [Kharzeev, Levin (2017)] [Hentschinski, Kharzeev, Kutak, Tu (2023)]
- **Spin entanglement** in $t\bar{t}$-production at LHC [Atlas (2024)] [CMS (2024)]



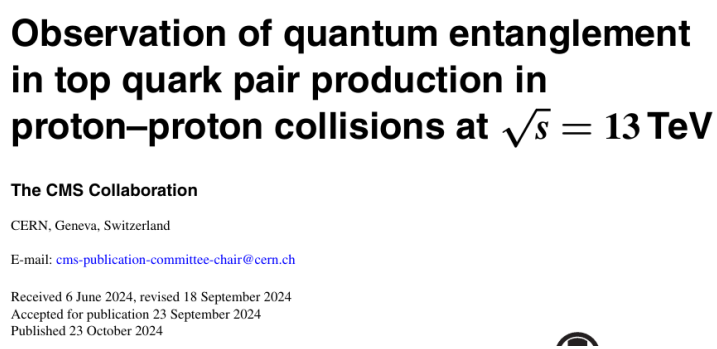
Article

Observation of quantum entanglement with top quarks at the ATLAS detector

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Open access
Check for updates

The ATLAS Collaboration

Entanglement is a key feature of quantum mechanics[1–3], with applications in fields such as metrology, cryptography, quantum information and quantum computation[4–8]. It has been observed in a wide variety of systems and length scales, ranging from the microscopic[9–13] to the macroscopic[14–16]. However, entanglement remains largely unexplored at the highest accessible energy scales. Here we report the highest-energy observation of entanglement, in top–antitop quark events produced at the Large Hadron Collider, using a proton–proton collision dataset with a centre-of-mass energy of $\sqrt{s} = 13$ TeV and an integrated luminosity of 140 inverse femtobarns $(fb)^{-1}$ recorded with the ATLAS experiment. Spin entanglement is detected from the measurement of a single observable D, inferred from the angle between the charged



Observation of quantum entanglement in top quark pair production in proton–proton collisions at $\sqrt{s} = 13$ TeV

The CMS Collaboration

CERN, Geneva, Switzerland

E-mail: cms-publication-committee-chair@cern.ch

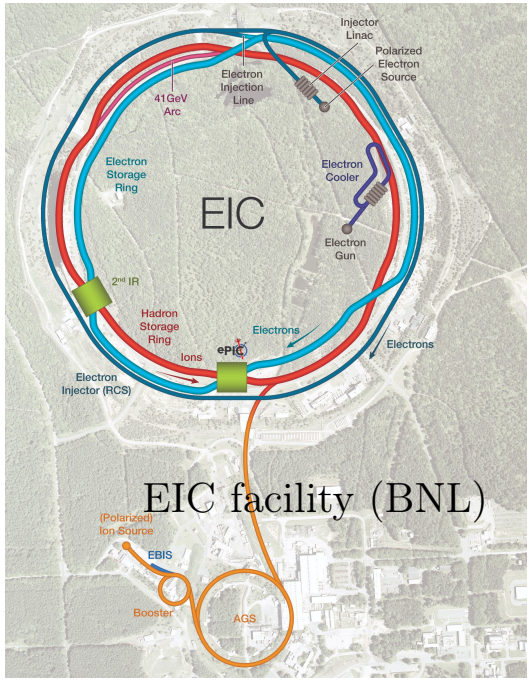
Received 6 June 2024, revised 18 September 2024
Accepted for publication 23 September 2024
Published 23 October 2024

- Set of **new observables** to test the Standard Model
- Possibility to investigate features of quantum mechanics at the highest accessible energy scales (e.g., entanglement, Bell nonlocality, ...)
- Quantum entanglement at the **Electron-Ion Collider**?
  [Qi, Guo, Xiao (2025)][Cheng, Han, Trifinopoulos (2025)] ...
  [M.F., Hatta (2025)] [M.F., Hatta, Xiao (2026)] [M.F., Guo, Hatta, (2026)]
- Results also usable for Ultra-Peripheral Collisions (UPCs) at LHC

The Electron-Ion Collider (EIC)

First generation electron-ion collider

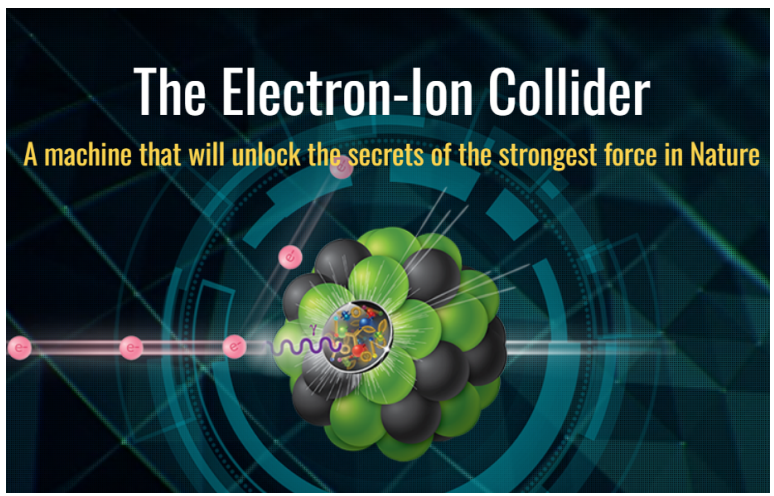
- Forthcoming collider (starts collecting data in 10 years)
- Electron-proton and electron-ion collisions
- $\sqrt{s} \sim 20$–140 GeV
- High luminosity and polarization
- Imaging the **quark-gluon structure** of matter



EIC facility (BNL)

Physics goals

- Gluon saturation
- Multi-dimensional structure of nucleons
- Spin decomposition of the proton
- Nuclear structure and QCD in nuclei



Density matrix and entanglement

- General two-qubit density matrix:

$$\rho = \frac{1}{4}\left(\mathbb{I}\otimes\mathbb{I} + B_a\,\sigma^a\otimes\mathbb{I} + \bar{B}_b\,\mathbb{I}\otimes\sigma^b + C_{ab}\,\sigma^a\otimes\sigma^b\right)$$

- Consider $B_a = \bar{B}_b = 0$ (all statements below assume this)
- **Product state** (no correlations): $\rho = \rho_A \otimes \rho_B$
- **Separable state** (correlations but no entanglement): $\rho = \sum_i p_i\ \rho_A^{(i)} \otimes \rho_B^{(i)}$
- **Entangled state**: $\rho \neq \sum_i p_i\ \rho_A^{(i)} \otimes \rho_B^{(i)}$
- Separable or Entangled $\Longrightarrow$ **Peres-Horodecki criterion**

$$\Delta_1 = \sqrt{(C_{rr} - C_{kk})^2 + (C_{rk} + C_{kr})^2} - 1 + C_{nn},$$

$$\Delta_2 = \sqrt{(C_{rr} + C_{kk})^2 + (C_{rk} - C_{kr})^2} - 1 - C_{nn}$$

$$\textbf{Entangled} \iff \Delta_1 > 0 \ \text{ or } \ \Delta_2 > 0$$

- Quantification via **Concurrence**:

$$\mathcal{C}[\rho] = \max\left\{\frac{\Delta_1}{2}, \frac{\Delta_2}{2}, 0\right\} \Rightarrow 0\ \text{(separable)} \quad \text{to} \quad 1\ \text{(maximally entangled)}$$

Bell-CHSH inequality

- Even stronger correlations $\longrightarrow$ **Bell non-locality**
- In a classical theory

$$\textbf{Bell-CHSH inequality}: \qquad S = \langle AB + A'B + AB' - A'B' \rangle, \quad |S| \leq 2$$

[Bell (1964)][Clauser, Horne, Shimony, Holt (1969)]

Its violation ruled out local hidden variable theories $\longrightarrow$ 2022 Nobel Prize

- Quantum mechanics allows violation: $|\langle \cdots \rangle| \leq 2\sqrt{2}$
- Saturation of the bound: $2\sqrt{2}$

$$\textbf{Bell state}: \qquad |\Phi^+\rangle = \frac{1}{\sqrt{2}} \left(|00\rangle + |11\rangle \right)$$

- Maximal violation depends on the maximal eigenvalues of $C^{\mathcal{T}}_{ab} C_{ab}$:

$$\textbf{CHSH violation} \iff 1 < \mu_1 + \mu_2 \leq 2$$

- Other intriguing QI related quantity: e.g. **magic**
- Quantified via stabilizer Rényi entropy

$$S_{\mathrm{R}}^{\mathrm{stab}} = -\ln \left(\frac{1 + C_{nn}^4 + C_{rr}^4 + C_{kk}^4 + C_{kr}^4 + C_{rk}^4}{1 + C_{nn}^2 + C_{rr}^2 + C_{kk}^2 + C_{kr}^2 + C_{rk}^2} \right)$$

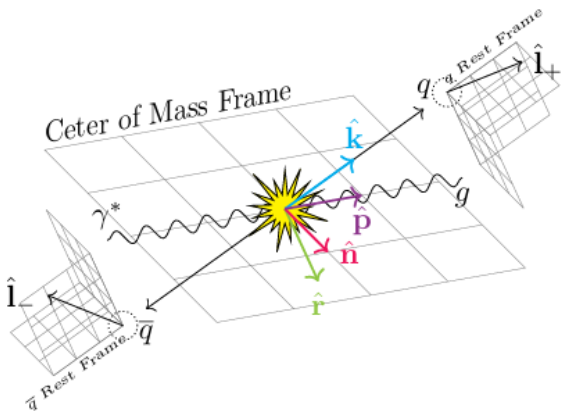
From cross section to spin density matrix

Strategy to compute the spin density matrix

- Start from the **center-of-mass frame** of the $q\bar{q}$-pair

$$S_{\alpha\alpha'\beta\beta'} = \bar{v}_{\beta'}(k')\Gamma u_{\beta}(k)\bar{u}_{\alpha}(k)\bar{\Gamma}v_{\alpha'}(k') \qquad \bar{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0$$

- Do not sum over the helicity indices
- Move to a new **rotated frame** $\{\hat{n}, \hat{r}, \hat{k}\}$ (Picture from [Qi, Guo, Xiao (2025)])



- In this frame

$$u_\alpha(k) = \begin{pmatrix} \sqrt{k\cdot\sigma}\xi_\alpha \\ \sqrt{k\cdot\bar{\sigma}}\xi_\alpha \end{pmatrix} \qquad v_{\alpha'}(k') = \begin{pmatrix} \sqrt{k\cdot\bar{\sigma}}\tilde{\eta}_{-\alpha'} \\ -\sqrt{k\cdot\sigma}\tilde{\eta}_{-\alpha'} \end{pmatrix} \qquad \tilde{\eta}_{-\alpha'} = -i\sigma^2(\eta_{\alpha'})^*$$

- The spin density matrix can be written

$$S_{\alpha\alpha'\beta\beta'} = \frac{A}{4}\left(\delta_{\alpha\beta}\delta_{\alpha'\beta'} + C_{ab}\xi^\dagger_\alpha\sigma^a\xi_\beta\eta^\dagger_{\alpha'}\sigma^b\eta_{\beta'}\right)$$

High-energy QCD in the Regge limit

Collinear factorization vs High-Energy factorization

- Collinear factorization in hadron-hadron collision

$$\sigma = \sum_{ij} \int dx_1 \int dx_2 f_i^{H_1}(x_1, Q^2) f_j^{H_2}(x_1, Q^2) \hat{\sigma}_{ij}(x_1 p_1, x_2 p_2, Q^2)$$

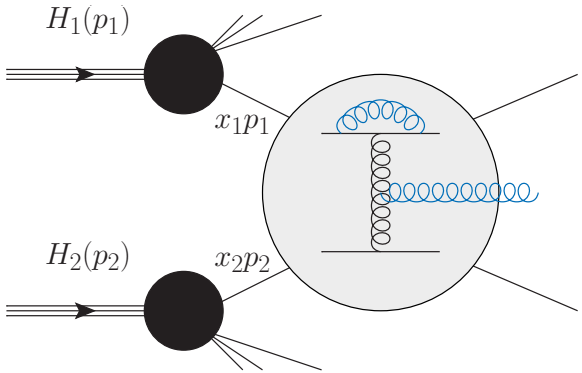
Parton distribution function (PDF)

Renormalization group eq. in Q^2

[Dokshitzer, Gribov, Lipatov, Altarelli, Parisi]

Partonic cross section

$$\hat{\sigma}_{ij} = \hat{\sigma}_{ij}^{(0)} + \alpha_s \hat{\sigma}_{ij}^{(1)} + \alpha_s^2 \hat{\sigma}_{ij}^{(2)} + ...$$



Collinear factorization vs High-Energy factorization

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$$\sigma = \sum_{ij} \int dx_1 \int dx_2 f_i^{H_1}(x_1, Q^2) f_j^{H_2}(x_1, Q^2) \hat{\sigma}_{ij}(x_1 p_1, x_2 p_2, Q^2)$$

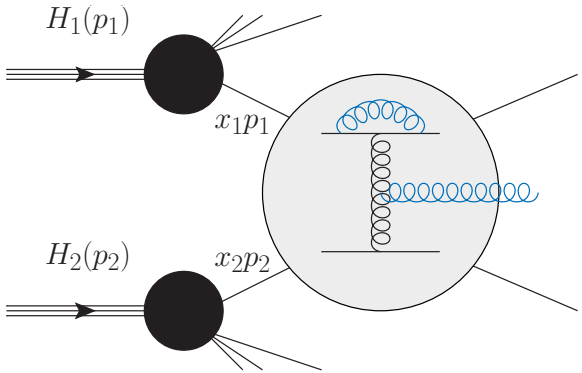
Parton distribution function (PDF)

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Partonic cross section

$$\hat{\sigma}_{ij} = \hat{\sigma}_{ij}^{(0)} + \alpha_s \hat{\sigma}_{ij}^{(1)} + \alpha_s^2 \hat{\sigma}_{ij}^{(2)} + \ldots$$



- High-energy limit of QCD: $s \gg Q^2 \gg \Lambda^2_{\rm QCD} \implies \alpha_s \ln(s/Q^2) \sim 1$

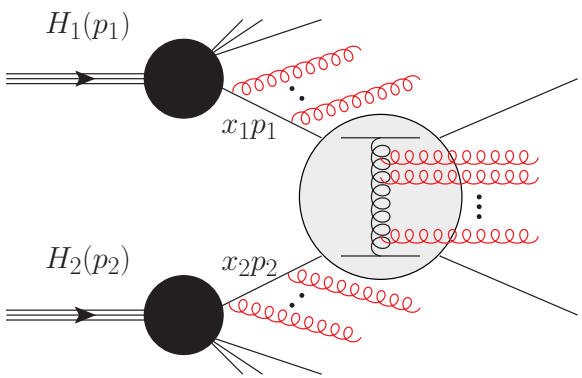
$$\sigma_0 = \sum_n \left[A_n \, \alpha_s^n(Q^2) \ln^n(s/Q^2) + B_n \, \alpha_s^{n+1}(Q^2) \ln^n(s/Q^2) + \ldots \right]$$

Leading logarithmic approximation (LL)

[Balitsky, Fadin, Kuraev, Lipatov]

Next-to-LL (NLL)

(state of the art)



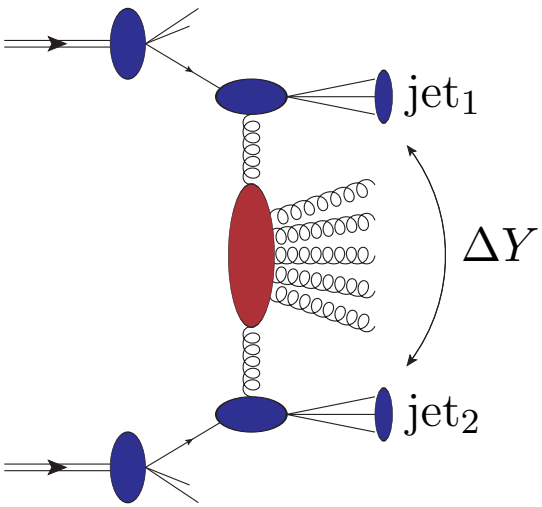
High-energy resummation at colliders

- Proton-proton collision at center-of-mass energy $S = (P_1 + P_2)^2$

$$\ln\left(\frac{S}{Q^2}\right) = \ln\left(\frac{x_1 x_2 S}{Q^2}\right) + \ln\left(\frac{1}{x_2}\right) + \ln\left(\frac{1}{x_1}\right)$$

Forward/backward

(e.g. Mueller-Navelet jets)



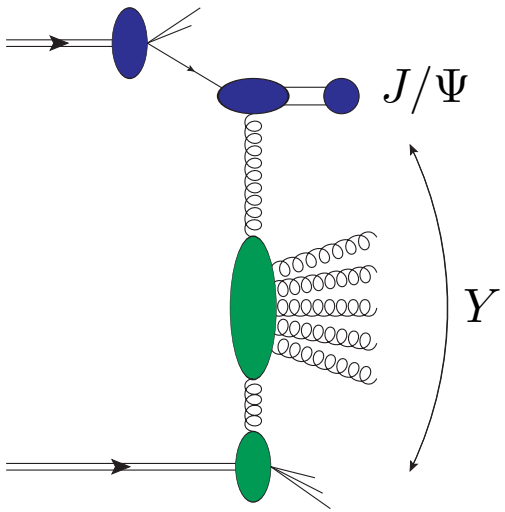
$$\sigma = \Phi_1 \otimes G_{\text{BFKL}} \otimes \Phi_2$$

$$\Phi_{\text{i}}(x_i) \otimes_{x_i} f(x_i)$$

$$\alpha_s \ln\left(\frac{x_1 x_2 S}{|p_{1\perp}||p_{2\perp}|}\right) = \alpha_s \Delta Y \sim 1$$

Single forward

(e.g. forward J/Ψ production)



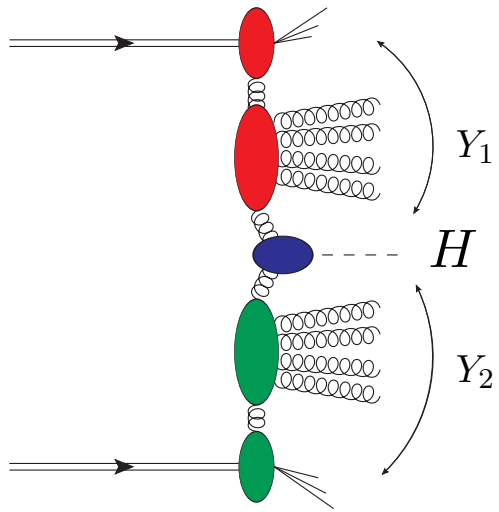
$$\sigma = \Phi \otimes \mathcal{U}_{\text{BFKL}}$$

$$\mathcal{U}_{\text{BFKL}} = \Phi_{\text{prot}} \otimes G_{\text{BFKL}}$$

$$\alpha_s \ln(1/x_2) = \alpha_s Y \sim 1$$

Central

(e.g. central Higgs production)



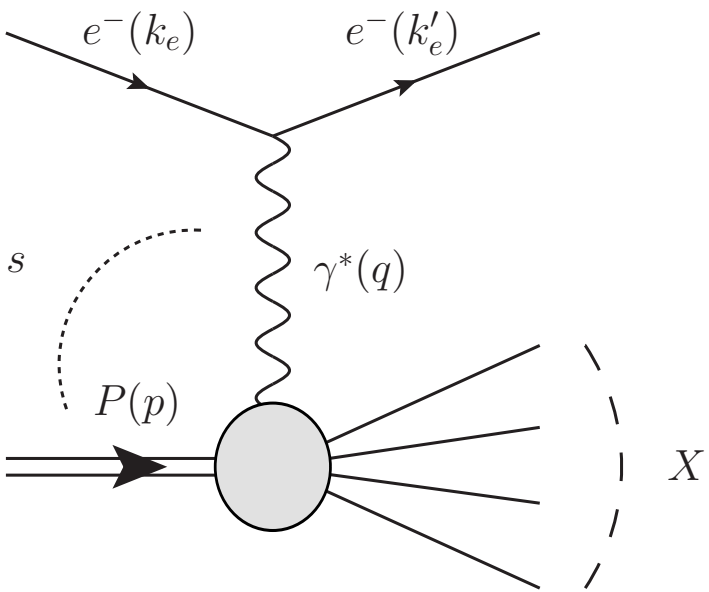
$$\sigma = \mathcal{U}_{\text{BFKL}} \otimes \Xi_{\text{centr.}} \otimes \mathcal{U}_{\text{BFKL}}$$

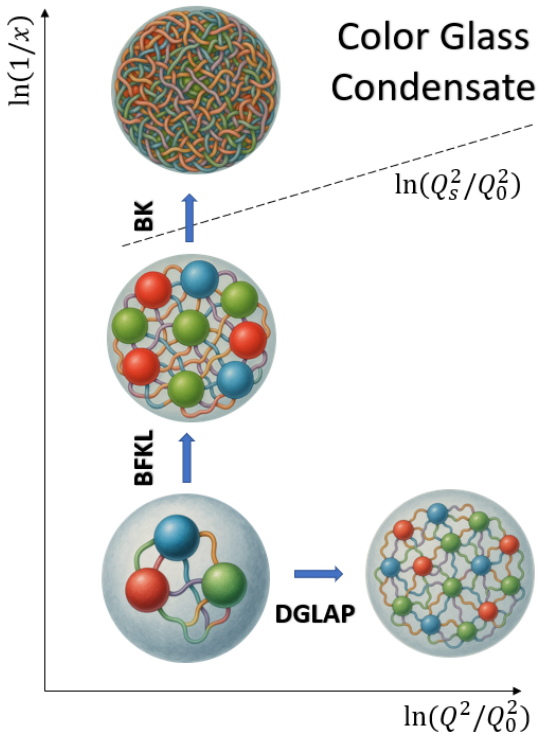
$$\alpha_s \ln(1/x_2) = \alpha_s Y_2 \sim 1$$

$$\alpha_s \ln(1/x_1) = \alpha_s Y_1 \sim 1$$

Deep Inelastic Scattering

- **Deep Inelastic Scattering (DIS)**

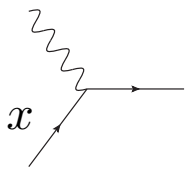




- DIS variables

$$Q = \sqrt{-q^2} \longrightarrow \text{related to the transverse resolution } \Delta x_\perp = \frac{1}{Q}$$

$$x = \frac{Q^2}{2p \cdot q} = \frac{Q^2}{Q^2 + s}$$



High energy $\longleftrightarrow$ Small x

Saturation physics

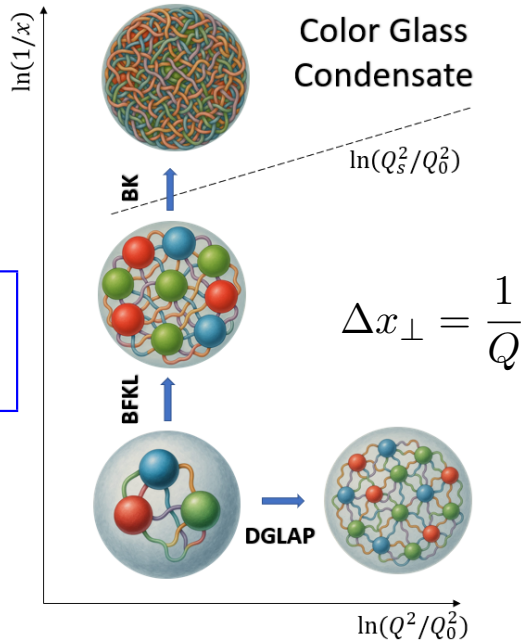
- DIS total cross-section (BFKL)

$$\sigma_{\gamma^* P}(x) = \Phi_{\gamma^*\gamma^*}(\vec{k}\,) \otimes_{\vec{k}} \mathcal{F}_g(x, \vec{k})$$

$$\downarrow$$

$$\sigma_{\gamma^* P}(x) \sim \left(\frac{s}{Q^2}\right)^{\omega_0} = \left(\frac{1}{x}\right)^{\omega_0}$$

One of the pillars of the EIC physics program

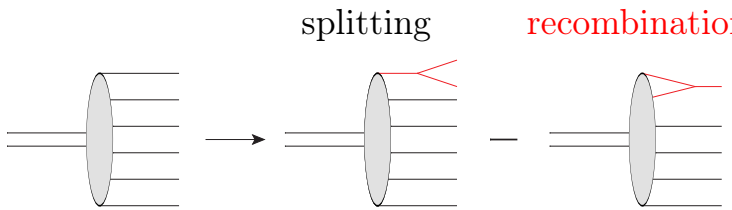


$$\Delta x_\perp = \frac{1}{Q}$$

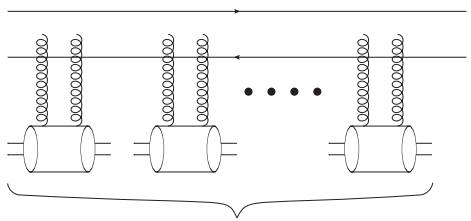
A too fast growth of the **gluon** distribution at small-x

$$\downarrow$$

Saturation effects to stop the growth



Multiple rescattering



Nucleus

- Semi-classical small-x effective theory [McLerran, Venugopalan]

Color

Color degrees of freedom

Glass

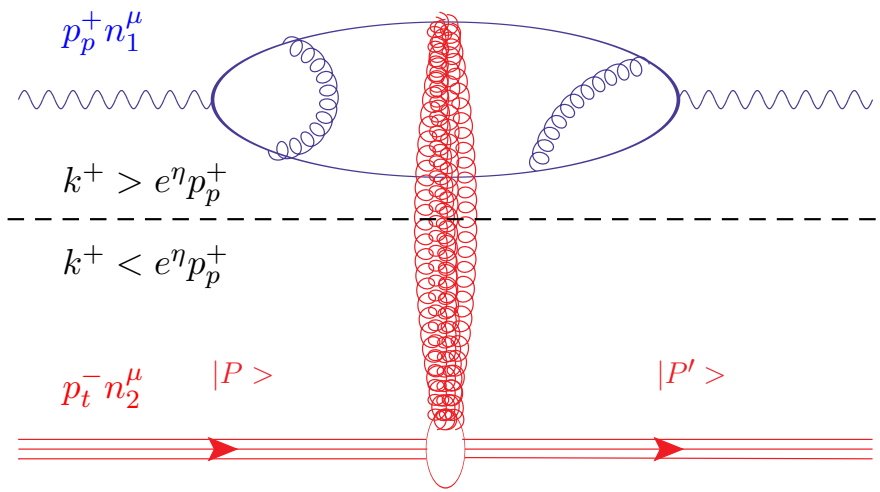
Separation of time scales and disordered system

Condensate

System as dense as possible

Shockwave approach to high-energy QCD

- High-energy approximation $s = (p_p + p_t)^2 \gg \{Q^2\}$



$$p_p = p_p^+ n_1 - \frac{Q^2}{2p_p^+} n_2$$

$$p_t = \frac{m_t^2}{2p_t^-} n_1 + p_t^- n_2$$

$$p_p^+ \sim p_t^- \sim \sqrt{\frac{s}{2}}$$

$$e^\eta \ll 1$$

- Separation of the gluon field into "fast" (quantum) part and "slow" (classical) part through a rapidity parameter $\eta < 0$

[McLerran, Venugopalan (1994)] [Balitsky (1996-2001)]

$$\mathcal{A}^\mu(k^+, k^-, \vec{k}) = A^\mu(k^+ > e^\eta p_p^+, k^-, \vec{k}) + b^\mu(k^+ < e^\eta p_p^+, k^-, \vec{k})$$

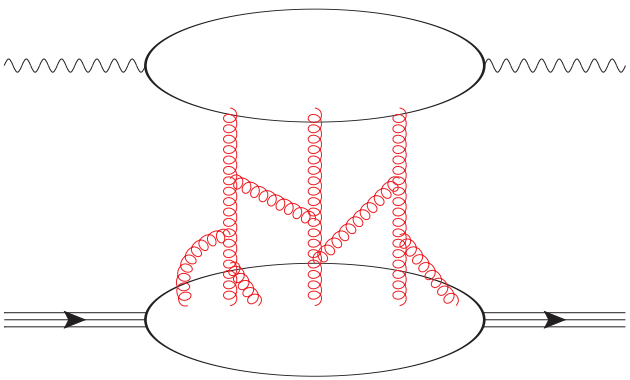
- **Eikonal** interaction between projectile and target
- High occupation number (small-x and/or large nuclei)

$\Longrightarrow$ Semi-classical description of the target

Shockwave approach to high-energy QCD

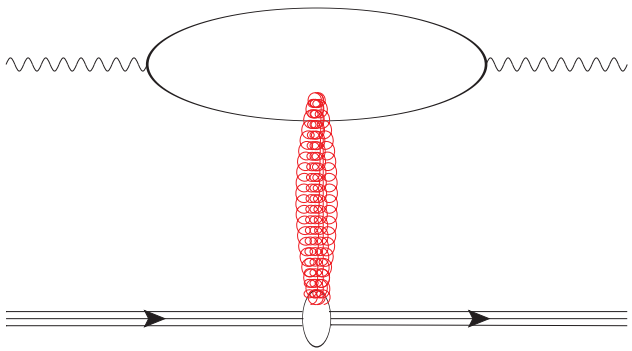
- Large longitudinal boost: $\Lambda = \sqrt{\frac{1+\beta}{1-\beta}} \sim \frac{\sqrt{s}}{m_t}$

$$\begin{cases} b^+(x^+,x^-,\vec{x}) & = \Lambda^{-1} b_0^+(\Lambda x^+, \Lambda^{-1}x^-, \vec{x}) \\ b^-(x^+,x^-,\vec{x}) & = \Lambda b_0^-(\Lambda x^+, \Lambda^{-1}x^-, \vec{x}) \\ b^i(x^+,x^-,\vec{x}) & = b_0^i(\Lambda x^+, \Lambda^{-1}x^-, \vec{x}) \end{cases}$$



$b_0^\mu(x)$

boost $\longrightarrow$



$b^\mu(x^+,x^-,\vec{x}) = \delta(x^+)\mathbf{B}(\vec{x})n_2^\mu + \mathcal{O}\left(\Lambda^{-1}\right)$

Shockwave approximation

- Light-cone gauge $A \cdot n_2 = 0$

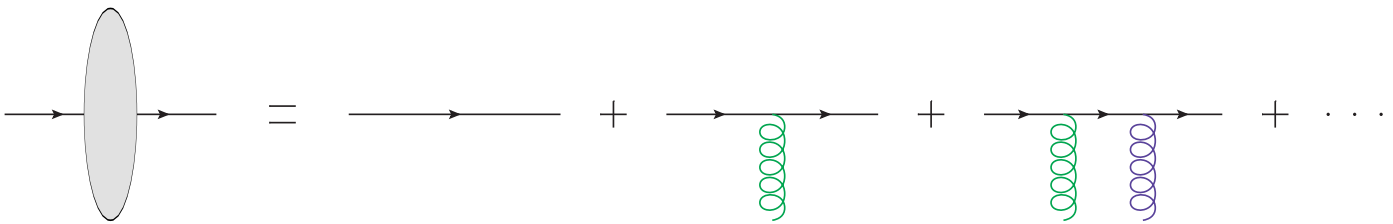
$$A \cdot b = 0 \implies \textit{Simple effective Lagrangian}$$

Shockwave approach to high-energy QCD

- Saturation effects appears for $Q^2 \lesssim Q_s^2(x) \sim A^{1/3} \left(\frac{1}{x}\right)^{\omega_0}$

- Wilson-line Shockwave approach (semi-classical effective theory)

[McLerran, Venugopalan] [Balitsky]



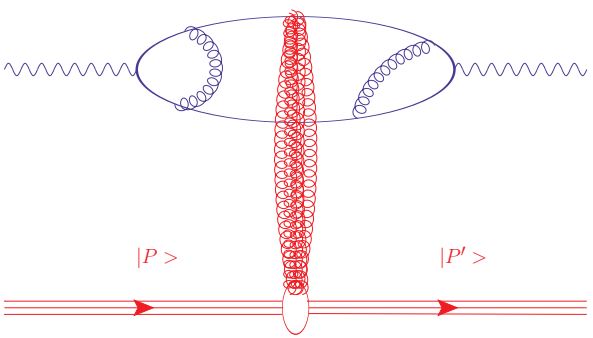
$$V_{\vec{z}}^{\eta} = \mathcal{P} \exp\left[ig \int_{-\infty}^{+\infty} dz_i^+ b_\eta^- \left(z_i^+, \vec{z}\right)\right]$$

- Example of factorization in the Shockwave approximation

$$\mathcal{M}^{\eta} = N_c \int d^d z_{1\perp} d^d z_{2\perp} \Phi^{\eta}(z_{1\perp}, z_{2\perp}) \left\langle P' \left| \left[\frac{1}{N_c} \mathrm{Tr}\left(V_{\vec{z}_1}^{\eta} V_{\vec{z}_2}^{\eta\dagger}\right) - 1\right](\vec{z}_1, \vec{z}_2) \right| P \right\rangle$$

- **Dipole operator**

$$\mathcal{U}_{ij}^{\eta} = \frac{1}{N_c} \mathrm{Tr}\left(V_{\vec{z}_i}^{\eta} V_{\vec{z}_j}^{\eta\dagger}\right) - 1$$



Balitsky-JIMWLK evolution equations

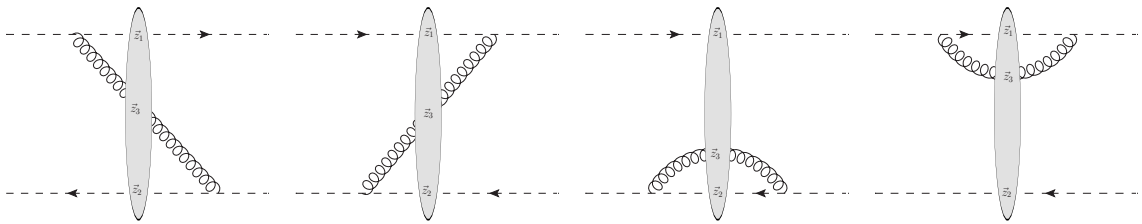
- **Balitsky-JIMWLK evolution equations** for the dipole

  [Balitsky — Jalilian-Marian, Iancu, McLerran, Weigert, Kovner, Leonidov]

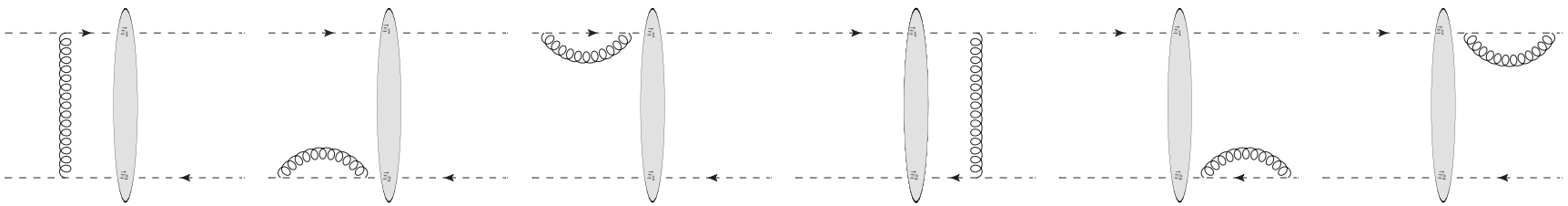
$$\frac{\partial \mathcal{U}^{\eta}_{12}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d^2\vec{z}_3 \left(\frac{\vec{z}^{\,2}_{12}}{\vec{z}^{\,2}_{23}\vec{z}^{\,2}_{31}} \right) \Big[\underbrace{\mathcal{U}^{\eta}_{13} + \mathcal{U}^{\eta}_{32} - \mathcal{U}^{\eta}_{12}}_{\text{BFKL}} - \mathcal{U}^{\eta}_{13}\mathcal{U}^{\eta}_{32} \Big]$$

$$\frac{\partial \mathcal{U}^{\eta}_{13}\mathcal{U}^{\eta}_{32}}{\partial \eta} = \cdots \qquad \longleftarrow \quad \textit{Balitsky hierarchy}$$

- Double dipole contribution and Dipole contribution



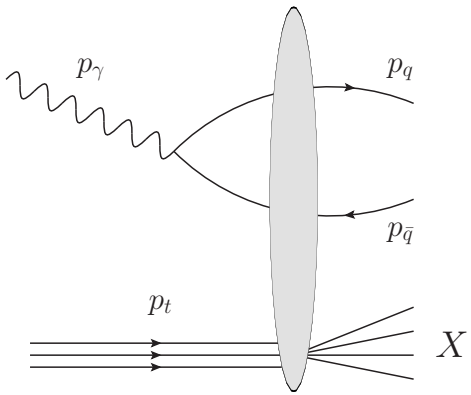
- Dipole contribution



- Gluon with rapidity slightly above the cut-off: $\eta + \Delta\eta$

Heavy-quark pair production in DIS

Inclusive heavy-quark pair production in DIS



- Higher number of events
- LO Collinear factorization [Qi, Guo, Xiao (2025)]
- Small-x TMD treatment [M.F., Hatta, Xiao (2026)]

- CGC dijet cross section [Dominguez, Marquet, Xiao, Yuan (2011)]

$$\frac{d\sigma^{L/T}_{\alpha\alpha'\beta\beta'}}{dz d^2p_{q\perp} d^2p_{\bar{q}\perp}} \sim \int \frac{d^2x_{1\perp}}{(2\pi)^2} \int \frac{d^2x_{2\perp}}{(2\pi)^2} \int \frac{d^2x_{1'\perp}}{(2\pi)^2} \int \frac{d^2x_{2'\perp}}{(2\pi)^2} e^{-ip_{q\perp}\cdot x_{11'\perp}} e^{-ip_{\bar{q}\perp}\cdot x_{22'\perp}}$$
$$\times \left(1 + \mathcal{U}^{(4)}(x_{1\perp}, x_{2\perp}; x_{2'\perp}, x_{1'\perp}) - \mathcal{U}^{(2)}(x_{1\perp}, x_{2\perp}) - \mathcal{U}^{(2)}(x_{2'\perp}, x_{1'\perp})\right)$$
$$\times \psi^{L/T(\lambda)*}_{\beta\beta'}(x_{1'2'\perp}) \psi^{L/T(\lambda)}_{\alpha\alpha'}(x_{12\perp})$$

- **Correlation limit** [Dominguez, Marquet, Xiao, Yuan (2011)]

$$|P_\perp| \simeq |p_{q\perp}| \simeq |p_{\bar{q}\perp}| \gg |p_{q\perp} + p_{\bar{q}\perp}| \simeq |q_\perp| \quad \Rightarrow \quad \text{back-to-back jets}$$

- **Factorization**

$$\text{Hard part}(P_\perp) \otimes \text{Gluon TMD } (q_\perp)$$

General structure of the spin density matrix

- Result

$$\frac{d\sigma^{L/T}_{\alpha\alpha'\beta\beta'}}{dz d^2P_\perp d^2q_\perp} \sim \frac{\partial}{\partial P'^i_\perp}\frac{\partial}{\partial P^j_\perp}\psi^{L/T(\lambda)*}_{\beta\beta'}(P'_\perp)\psi^{L/T(\lambda)}_{\alpha\alpha'}(P_\perp)\Big|_{P'_\perp=P_\perp}$$
$$\times\left[\frac{\delta_{ij}}{2}G_0(q_\perp)+\left(\frac{q^i_\perp q^j_\perp}{q^2_\perp}-\frac{\delta_{ij}}{2}\right)G_2(q_\perp)\right]$$

- Target: G_0: unpolarized W.W. G_2: linearly polarized W.W.

- Hard factor structure: $\partial_i\psi^*\,\partial_j\psi \;\Rightarrow\; \boxed{\mathbb{I}\otimes\mathbb{I}+C_{ab}\sigma^a\otimes\sigma^b}$

- Total DIS spin density matrix

$$\frac{d\sigma_{\alpha\alpha'\beta\beta'}}{dz d^2P_\perp d^2q_\perp} \sim \mathcal{A}\xi^\dagger_\alpha\eta^\dagger_{\alpha'}\left(\mathbb{I}\otimes\mathbb{I}+\mathcal{C}_{ab}\sigma^a\otimes\sigma^b\right)\xi_\beta\eta_{\beta'} \qquad \xi^\dagger\eta^\dagger(\sigma^a\otimes\sigma^b)\xi\eta\equiv\xi^\dagger\sigma^a\xi\eta^\dagger\sigma^b\eta$$

$$\mathcal{A}=G_0\left(A_T^{(0)}+\varepsilon A_L^{(0)}\right)+\cos(2\phi_{P,q})G_2\left(A_T^{(2)}+\varepsilon A_L^{(2)}\right)$$

$$\mathcal{C}_{ab}=\frac{1}{\mathcal{A}}\left[G_0\left(\tilde{C}_T^{(0)}+\varepsilon\tilde{C}_L^{(0)}\right)+\cos(2\phi_{P,q})G_2\left(\tilde{C}_T^{(2)}+\varepsilon\tilde{C}_L^{(2)}\right)\right]_{ab}$$

Spin density matrix in dijet production

- $A^{(0)}_{T/L}$ and $A^{(2)}_{T/L}$ reproduce the known results for the total cross sections ✓

  [Dominguez, Marquet, Xiao, Yuan (2011)]
- Averaging over the angle $\phi_{P,q}$ we recover the collinear-factorization result ✓

  [Qi, Guo, Xiao (2025)]
- Differential with respect to $\phi_{P,q}$ $\Longrightarrow$ Target description comes into play
- Large nuclei and/or small-x $\rightarrow$ **MV description** [McLerran, Venugopalan]
- G_0 and G_2 given in [Dumitru, Skokov (2016)]

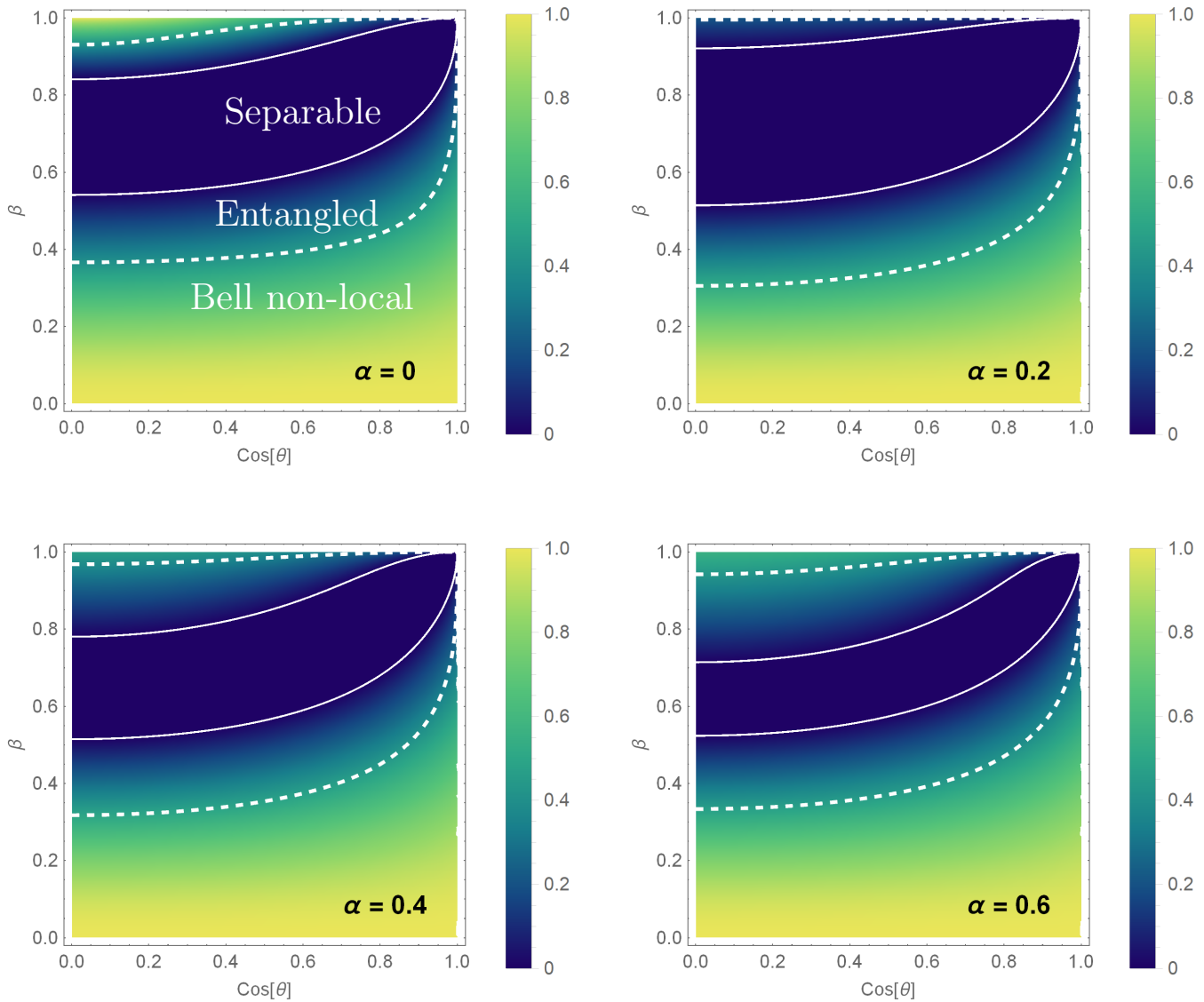
$$G_0(q_\perp) = 8\pi S_\perp \frac{C_F}{N_c} \int_0^\infty d|r_\perp| \frac{J_0(|q_\perp||r_\perp|)}{|r_\perp|} \left[1 - \exp\left(-\frac{Q_s^2|r_\perp|^2}{4} \ln \frac{1}{|r_\perp|^2\Lambda_{\rm IR}^2}\right)\right]$$

$$G_2(q_\perp) = 8\pi S_\perp \frac{C_F}{N_c} \int_0^\infty d|r_\perp| \frac{J_2(|q_\perp||r_\perp|)}{|r_\perp| \ln \frac{1}{|r_\perp|^2\Lambda_{\rm IR}^2}} \left[1 - \exp\left(-\frac{Q_s^2|r_\perp|^2}{4} \ln \frac{1}{|r_\perp|^2\Lambda_{\rm IR}^2}\right)\right]$$

- $G_0 = G_2$ in the dilute (BFKL) limit $\longrightarrow$ reproduced for $|q_\perp| \gg Q_s$

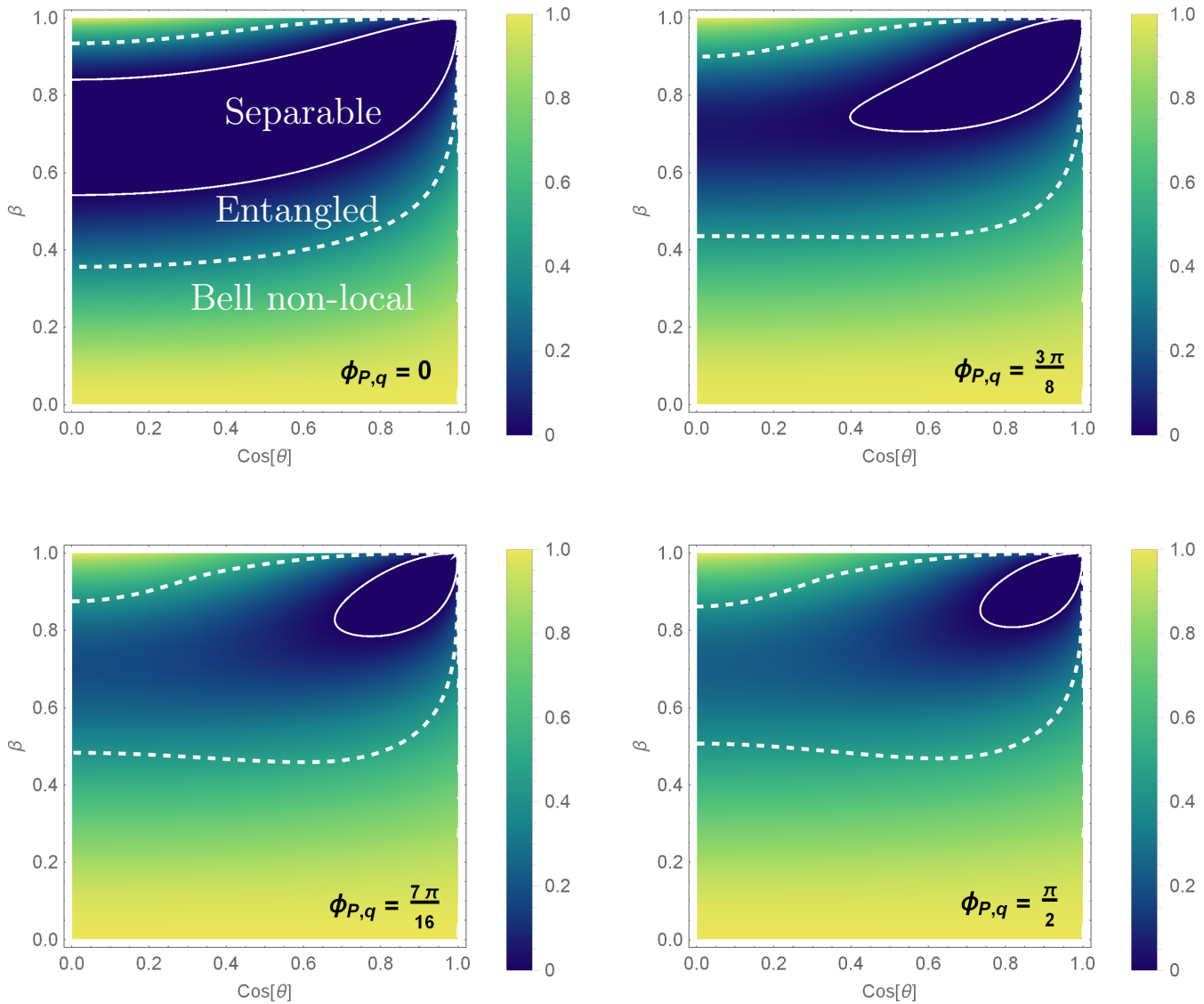
Concurrence in DIS

- **Concurrence** density plot ($\phi_{P,q}$-averaged and varying $\alpha = Q^2/M^2$)

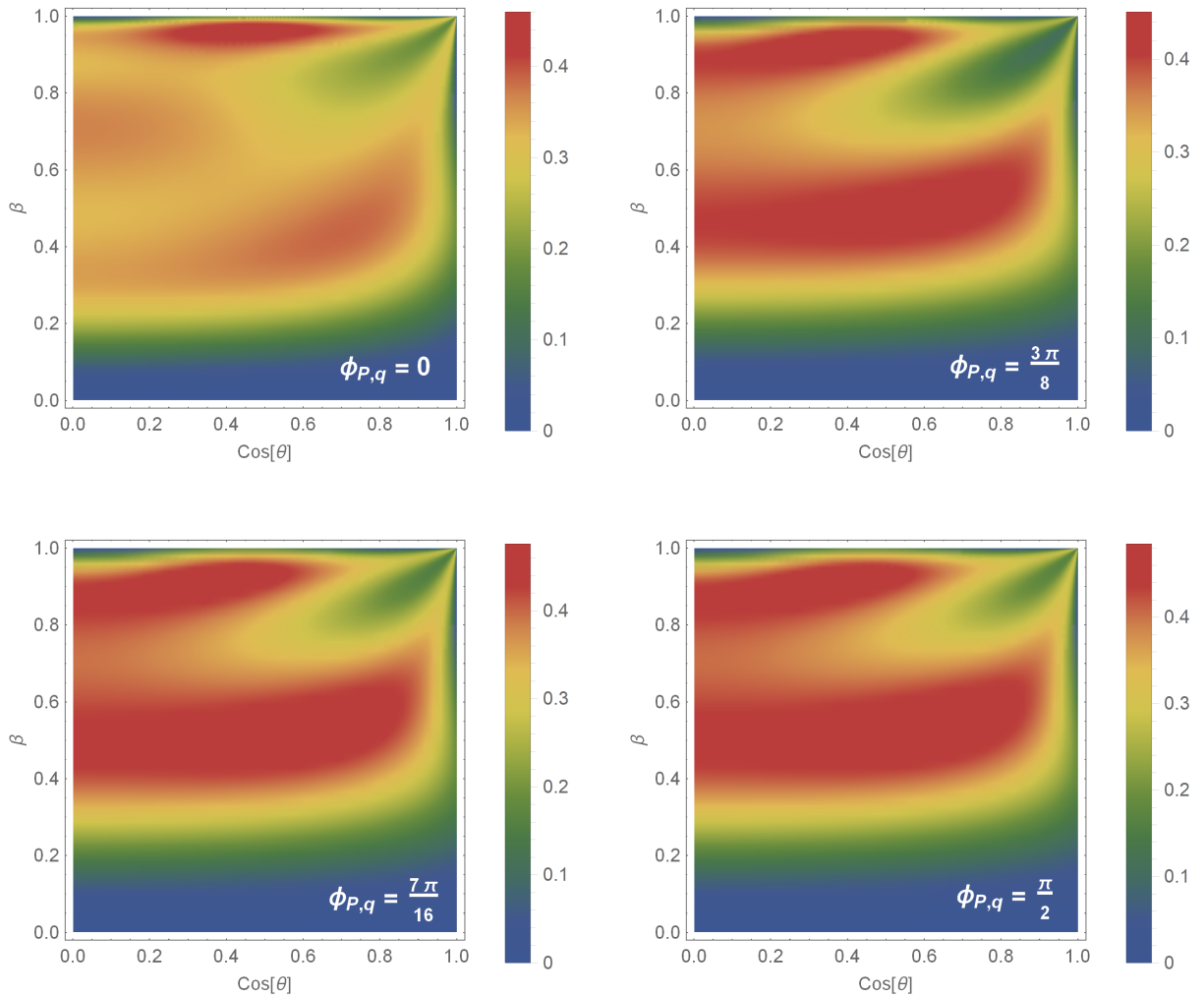


Concurrence in the $\phi_{P,q}$-differential photoproduction

- **Concurrence** density plot ($\alpha = 0$, $q_\perp/Q_s = 3$ and varying $\phi_{P,q}$)



- **Magic** density plot ($\alpha = 0$ and varying $\phi_{P,q}$)

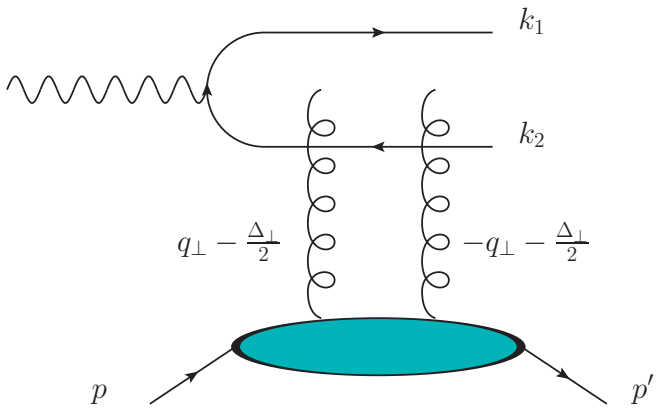


- The upper bound $S_{\mathrm{R}}^{\mathrm{stab}} \leq \ln \frac{9}{5} \simeq 0.588$ is not reached

Diffractive heavy-quark pair production

Diffractive heavy-quark pair production

Diffractive $Q\bar{Q}$-pair production [M.F., Hatta (2025)]



- We consider the **Regge limit**
- Back-to-back configuration $\Delta_\perp = 0$

$$\beta = \sqrt{1 - \frac{4m^2}{M^2}} \qquad \cos\theta = \frac{k^3}{|\vec{k}|}$$

- Shockwave calculation $\xrightarrow{\text{Boost}}$ Center-of-mass frame of the pair
- **Longitudinally polarized case** $\implies$ Simple spin density matrix

$$C^L_{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1-(2-\beta^2)\cos^2\theta}{1-\beta^2\cos^2\theta} & -\frac{\sqrt{1-\beta^2}\sin 2\theta}{1-\beta^2\cos^2\theta} \\ 0 & -\frac{\sqrt{1-\beta^2}\sin 2\theta}{1-\beta^2\cos^2\theta} & \frac{1-(2-\beta^2)\cos^2\theta}{1-\beta^2\cos^2\theta} \end{pmatrix}$$

- $(C^L)^{\mathcal{T}} C^L = \mathrm{diag}(1,1,1)$, $\mu_1 + \mu_2 = 2$ $\implies$ **Maximally entangled state**
- $\beta = 1$ or $\cos\theta = 0$ $\implies$ $\rho^L = |\Phi^+\rangle\langle\Phi^+|$

Diffractive heavy-quark pair production: transverse case

$$\widetilde{C}^T_{nn} = -\frac{4\alpha_{em}e_q^2}{N_c} z\bar{z}k_\perp^2 (T_1+T_2)^2$$

$$\begin{aligned}\widetilde{C}^T_{rr} = \frac{\alpha_{em}e_q^2}{2z\bar{z}N_c}\Big[&2T_1(T_1+T_2)k_\perp m\sqrt{1-\beta^2}\sin\theta(\beta+(1-2z)\cos\theta)\\ &+T_1^2m^2(1-\beta^2)\sin^2\theta\\ &-(T_1+T_2)^2k_\perp^2\Big((1-\beta^2)\sin^2\theta+2z(1-z)\big((2-\beta^2)\cos^2\theta-1\big)\Big)\Big]\end{aligned}$$

$$\begin{aligned}\widetilde{C}^T_{kk} = \frac{\alpha_{em}e_q^2}{2z\bar{z}N_c}\Big[&2T_1(T_1+T_2)k_\perp m\sqrt{1-\beta^2}\sin\theta(\beta-(1-2z)\cos\theta)\\ &+T_1^2m^2(\cos^2\theta-\beta^2)\\ &+(T_1+T_2)^2k_\perp^2\Big(\beta^2-2z(1-z)+\cos^2\theta\big(2z(1-z)(2-\beta^2)-1\big)\Big)\Big]\end{aligned}$$

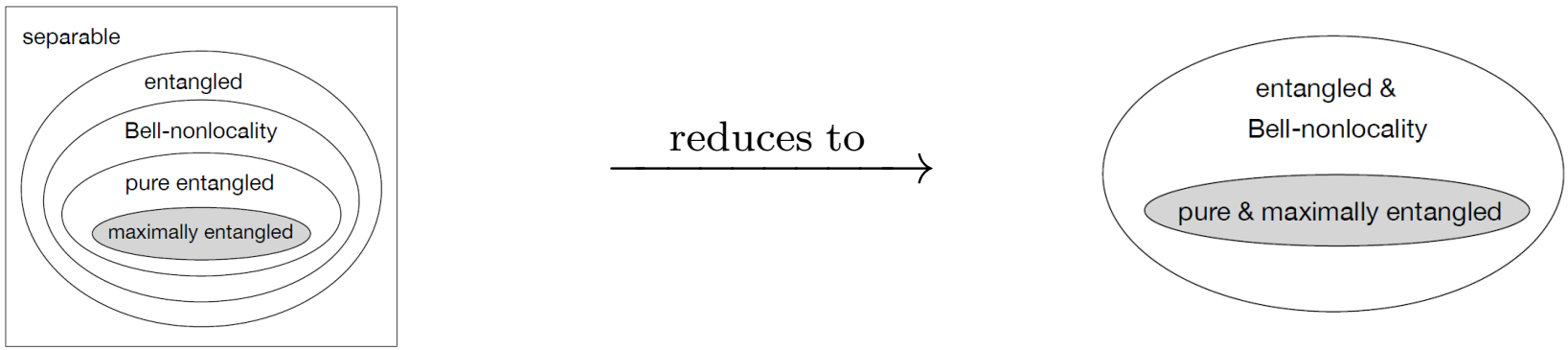
$$\begin{aligned}\widetilde{C}^T_{rk} = \frac{\alpha_{em}e_q^2}{2z\bar{z}N_c}\Big[&2T_1(T_1+T_2)k_\perp m\Big(z-\beta^2+\cos^2\theta\big(1-(2-\beta^2)z\big)\Big)\\ &+\sqrt{1-\beta^2}\sin\theta\Big(T_1^2m^2(\cos\theta-\beta)+(T_1+T_2)^2k_\perp^2\big(\beta-(1-2z)^2\cos\theta\big)\Big)\Big]\end{aligned}$$

$$\begin{aligned}\widetilde{C}^T_{kr} = \frac{\alpha_{em}e_q^2}{2z\bar{z}N_c}\Big[&2T_1(T_1+T_2)k_\perp m\Big(z-(1-\beta^2)\sin^2\theta-(2-\beta^2)z\cos^2\theta\Big)\\ &+\sqrt{1-\beta^2}\sin\theta\Big(T_1^2m^2(\beta+\cos\theta)-(T_1+T_2)^2k_\perp^2\big(\beta+(1-2z)^2\cos\theta\big)\Big)\Big]\end{aligned}$$

Diffractive heavy-quark pair production

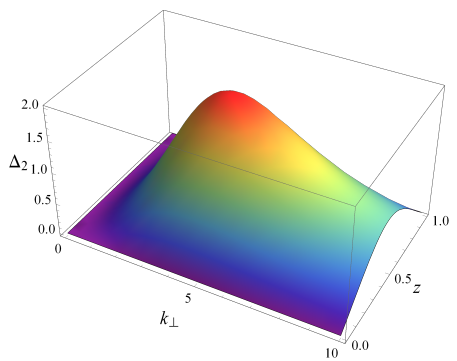
- **Transversally polarized case** $\Longrightarrow$ Very complicated density matrix but:

$$\Delta_2 = -\Delta_1 = -2C_{nn}^T \geq 0$$



- **Gisin's theorem** $\Longrightarrow$ If ρ describes a pure state, entanglement necessarily entails Bell-nonlocality [Gisin (1991)]
- Our case: it holds for a **mixed state** [M.F., Hatta (2025)]

- **Color transparency** implies a point of **maximal entanglement**



- GPDs treatment leads to more complex patterns [Hatta, Schoenleber (2025)]

How we measure spin-spin correlations?

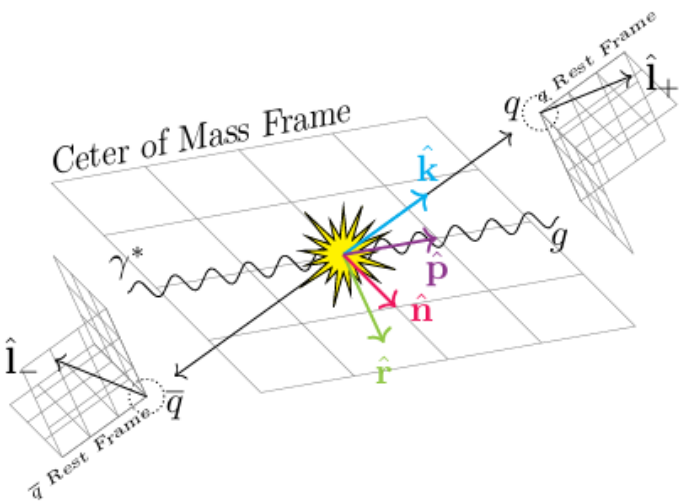
Accessing spin correlations

- **Current strategy**: $Q \to$ decay products (e.g. $t \to b + W^+ \to b + \ell^+ + \nu_\ell$)
- Possible channels for c and b quarks [Kats, Uzan (2024)]

$$b \xrightarrow{7.0\%} \Lambda_b \xrightarrow{11\%} X_c^+ + \mu^- + \bar{\nu}_\mu$$

$$c \xrightarrow{6.4\%} \Lambda_c^+ \xrightarrow{3.5\%} \Lambda + \mu^+ + \nu_\mu$$

- Spin information from angular distributions



$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_+ d\cos\theta_-} = \frac{1 + \alpha_+\alpha_- r_T^2 \cos\theta_+ \cos\theta_- C_{nn}}{4}$$

↑ decay angles ↑ spin correlation

- How does the **quark-level correlation** survive in observables?
- A number of issues: **low branching ratio**, spin transfer in hadronization.
- This problem is particularly severe for diffractive events

Accessing spin correlations at the EIC

- Ways to **reconstruct spin correlations** without resorting to weak decays?

[M.F., Guo, Hatta, arXiv: 2609.32800]

- Given $\mathcal{A} = \mathrm{Tr}[\bar{\Gamma}(\not{k}+m)\Gamma(\not{k}'-m)]$ the normalized spin correlations are

$$C^{ij} = \frac{1}{\mathcal{A}}\mathrm{Tr}\left[\bar{\Gamma}\gamma^i\gamma_5(\not{k}+m)\Gamma\gamma^j\gamma_5(\not{k}'-m)\right] \qquad i,j = n,r$$

$$C^{kk} = -\frac{1}{\mathcal{A}}\mathrm{Tr}\left[\bar{\Gamma}\gamma_5\gamma^0\gamma^3(\not{k}+m)\Gamma\gamma_5\gamma^0\gamma^3(\not{k}'-m)\right]$$

$$C^{ki} = \frac{1}{\mathcal{A}}\mathrm{Tr}\left[\bar{\Gamma}\gamma_5\gamma^0\gamma^3(\not{k}+m)\Gamma\gamma^i\gamma_5(\not{k}'-m)\right]$$

$$C^{ik} = -\frac{1}{\mathcal{A}}\mathrm{Tr}\left[\bar{\Gamma}\gamma^i\gamma_5(\not{k}+m)\Gamma\gamma_5\gamma^0\gamma^3(\not{k}'-m)\right]$$

- The highlighted γ-structures have a precise role in factorization theorem

$$W^{\mu\nu} \sim H^{\mu\nu}_{\alpha\beta;\gamma\delta}(\Delta_1)_{\beta\alpha}(\bar{\Delta}_2)_{\delta\gamma} \qquad \Delta = \frac{1}{4}\sum_A \Gamma_A\,\mathrm{Tr}\left[\Gamma^A\Delta\right],$$

Accessing spin correlations at the EIC

- Consider the FF of a quark into an **unpolarized hadron**

$$\Delta^{h/q}(z, P_{h\perp}) = \frac{1}{z}\int \frac{dx^- d^2x_\perp}{(2\pi)^3} e^{i\frac{P_h^+}{z}x^- - ik_\perp\cdot x_\perp} \sum_X \langle 0|\psi(x^-, x_\perp)|P_h X\rangle\langle P_h X|\bar{\psi}(0)|0\rangle \Big|_{P_{h\perp}=0}$$

- After performing Fierz

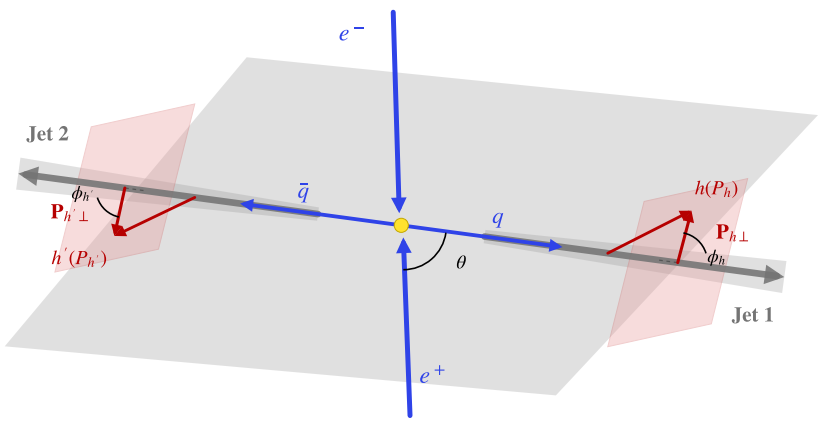
$$k^+\Delta^{h/q} \longrightarrow (\not{k}+m)D_1^{h/q}(z, z^2k_\perp^2) + \gamma^i\gamma_5(\not{k}+m)\frac{\epsilon^{ij}k_\perp^j}{M}H_1^{\perp h/q}(z, z^2k_\perp^2)$$

- $H_1^{\perp h/q}(z, z^2k_\perp^2)$ is the **Collins FF**

  i) Transverse momentum dependent FF

  ii) Unpolarized hadron

  iii) From a transversally polarized parent quark



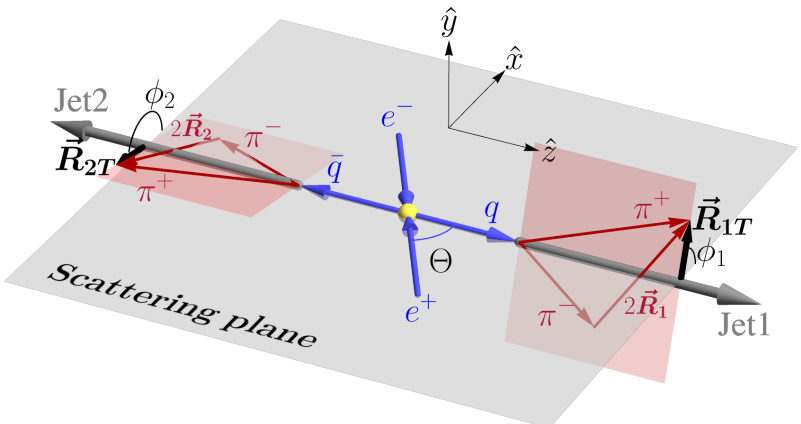
- Diffractive DIS structure

$$d\sigma_{\rm DIS}^{\pi/D} = 1 + A_{12}^{\pi/D}(z, z')\left(\frac{C_{\rm DIS}^{xx} - C_{\rm DIS}^{yy}}{2}\cos(\phi_P + \phi_{P'}) - \frac{C_{\rm DIS}^{xx} + C_{\rm DIS}^{yy}}{2}\cos(\phi_P - \phi_{P'})\right)$$

Accessing spin correlations at the EIC

- The Collins FF is sensitive only to the **transverse polarization** of quarks
- For the **longitudinal polarization** $\longrightarrow$ **Di-hadron FFs**

(explored in e^+e^- into four hadrons) [Cheng, Yan (2025)]



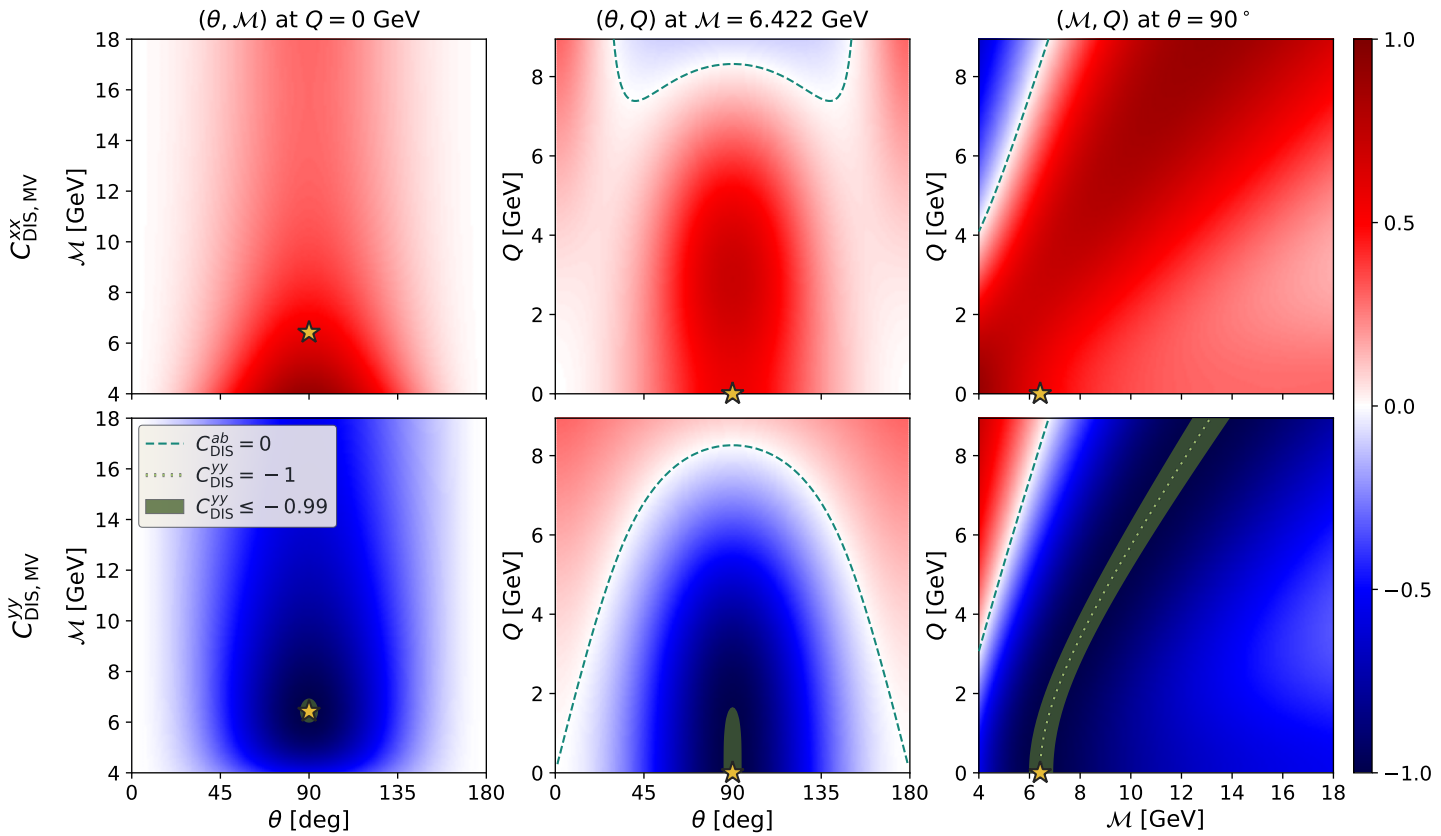
$$k^+\Delta^{h_1h_2/q} \longrightarrow (\not{k}+m)D_1^{h_1h_2/q}(z,\zeta,P_h,R) - \gamma_5\gamma^0\gamma^3(\not{k}+m)\frac{\epsilon^{ij}R_\perp^i k_\perp^j}{M_1M_2}G_1^{\perp h_1h_2/q}(z,\zeta,P_h,R)$$

$$+\gamma^i\gamma_5(\not{k}+m)\left(H_1^{\sphericalangle h_1h_2/q}(z,\zeta,P_h,R)\frac{\epsilon^{ij}R_\perp^j}{M_1+M_2} + H_1^{\perp h_1h_2/q}(z,\zeta,P_h,R)\frac{\epsilon^{ij}k_\perp^j}{M_1+M_2}\right)$$

- Complete **quantum state tomography** in DIS at the EIC

[M.F., Guo, Hatta, arXiv: 2609.32800]

Quantum-state tomography in diffraction



- **Dashed lines**: Correlation and **Collins asymmetry vanish**
- Stars: **Maximally entangled states** ($C^{yy}_{\mathrm{DIS}} = -1$)

$$\cos(\phi_P + \phi_{P'}) \longrightarrow \cos(\phi_P)\cos(\phi_{P'})$$

- Shaded bands represent the near-minimal region ($C^{yy}_{\mathrm{DIS}} \leq -0.99$)

Summary

- **Quantum correlations** are a fundamental feature of QM
- The interface of **Quantum Information** and **QCD/collider physics** provides new perspectives and insights
- We found several interesting theoretical insights about small-x and dense QCD looking at spin correlations
  - **Pomeron** leaves characteristic imprints on the entanglement patterns between $Q\bar{Q}$-pair diffractively produced in DIS
  - We mapped the kinematic regions where the $Q\bar{Q}$ spin state produced in inclusive DIS (or photoproduction) is **separable**, **entangled**, or **Bell non-local**
  - These observables are sensitive to **gluon saturation effects** in protons and nuclei
- We proposed a new way to access the full spin-density matrix in DIS at the EIC by means of the **Collins** and **di-hadron FFs**
- Quantum entanglement at the EIC: novel topic **beyond the Yellow report** (high luminosity and polarization capabilities)

Thank you for your attention!

Backup

EPR incompleteness vs. Bell non-locality

EPR (1935): is the wave function complete?

- **Reality criterion:** if a quantity can be predicted with certainty, without disturbing the system, it corresponds to an element of physical reality.
- **Locality:** a measurement choice on A cannot change the physical reality of a spatially separated system B.
- Entanglement allows Alice to predict either of two non-commuting observables of Bob, e.g. x_B or p_B.

EPR conclusion: ψ is incomplete.

Bell (1964): can a local completion work?

- Local hidden-variable theories obey

$$P(a,b|x,y,\lambda) = P(a|x,\lambda)P(b|y,\lambda).$$

- They imply the CHSH bound $|S| \leq 2$.
- Quantum mechanics allows $|S| \leq 2\sqrt{2}$, and experiments observe $|S| > 2$.

Bell conclusion: no local hidden-variable theory can reproduce all quantum correlations.

Bell violations rule out local realism as a conjunction, not realism alone.

Quantum mechanics violates Bell locality but preserves **no-signalling**.

Product states, classical correlations, and entanglement

- **Product state (no correlations):**

$$\rho = \rho_A \otimes \rho_B$$

Example: $|0\rangle\langle 0| \otimes |0\rangle\langle 0| = (|0\rangle \otimes |0\rangle)(\langle 0| \otimes \langle 0|) = |00\rangle\langle 00|$

$\Rightarrow$ **Independent** measurement outcomes

- **Separable state (classical correlations)**:

$$\rho = \sum_i p_i \; \rho_A^{(i)} \otimes \rho_B^{(i)}$$

Example:

$$\rho = \frac{1}{2}\Big((|0\rangle \otimes |0\rangle)(\langle 0| \otimes \langle 0|) + |1\rangle \otimes |1\rangle\langle 1| \otimes \langle 1|\Big) = \frac{1}{2}\Big(|00\rangle\langle 00| + |11\rangle\langle 11|\Big)$$

$\Rightarrow$ perfectly **correlated** (shared randomness), but no entanglement

- **Entangled state (quantum correlations):**

$$\rho \neq \sum_i p_i \; \rho_A^{(i)} \otimes \rho_B^{(i)}$$

Example (Bell state):

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$\Rightarrow$ **Correlations not reproducible by classical mixtures**

The Color Glass Condensate

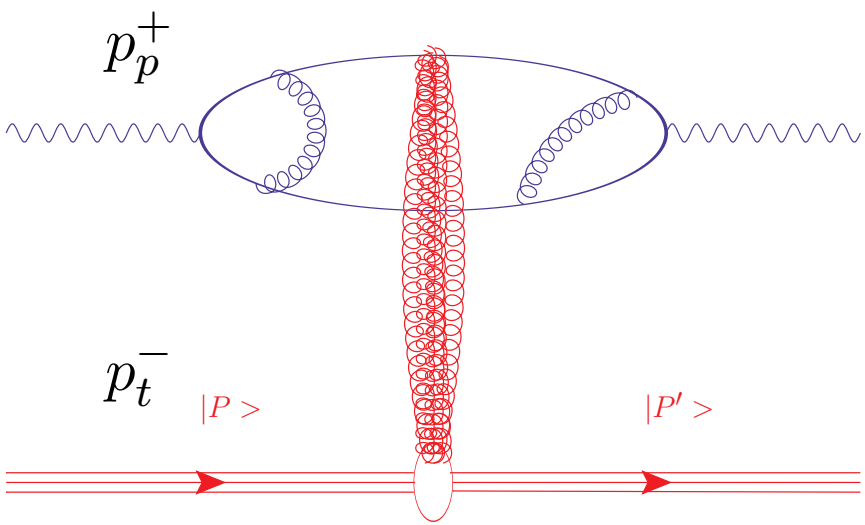
- **High occupation number** $\Longrightarrow$ **Semi-classical** field description

[McLerran, Venugopalan]

$$Q_s^2 \sim \left(\frac{A}{x}\right)^{1/3} \Lambda_{\rm QCD}^2 \qquad \alpha_s(Q_s^2) \ll 1 \qquad A_{\rm classical} \sim \frac{1}{g_s(Q_s^2)}$$

saturation scale

- **"Time" scale separation**



$$p^+ = \frac{p^0 + p^3}{\sqrt{2}} \qquad p^- = \frac{p^0 - p^3}{\sqrt{2}}$$

$$\Delta x^+ = \frac{x^0 + x^3}{\sqrt{2}} \qquad \Delta x^- = \frac{x^0 - x^3}{\sqrt{2}}$$

$$p_p^\mu \simeq (p^0, 0, 0, p^0) \qquad p_t^\mu \simeq (p^0, 0, 0, -p^0)$$

$$p_p^+ \sim \sqrt{s} \qquad p_t^- \sim \sqrt{s}$$

$$\Delta x_p^- \sim 0 \qquad \Delta x_t^+ \sim 0$$

- Fast and slow modes in the target

$$\Delta x^-_{\rm fast} \sim \frac{k^-_{\rm fast}}{\boldsymbol{p^2}} = \frac{x_{\rm fast}\, p_t^-}{\boldsymbol{p^2}} \qquad \Delta x^-_{\rm slow} \sim \frac{k^-_{\rm slow}}{\boldsymbol{p^2}} = \frac{x_{\rm slow}\, p_t^-}{\boldsymbol{p^2}}$$

$$\boxed{\Delta x^-_{\rm fast} \gg \Delta x^-_{\rm slow}}$$

The Color Glass Condensate

- Small-x partons evolve on a very short time w.r.t the large-x ones ($\Delta x^-_{\text{fast}} \gg \Delta x^-_{\text{slow}}$)



Small-x partons

Dynamical degrees of freedom

Obtained by solving the classical Y.M. equations: $[D_{\mu\nu}, F^{\mu}] = J_{\nu}(\rho)$

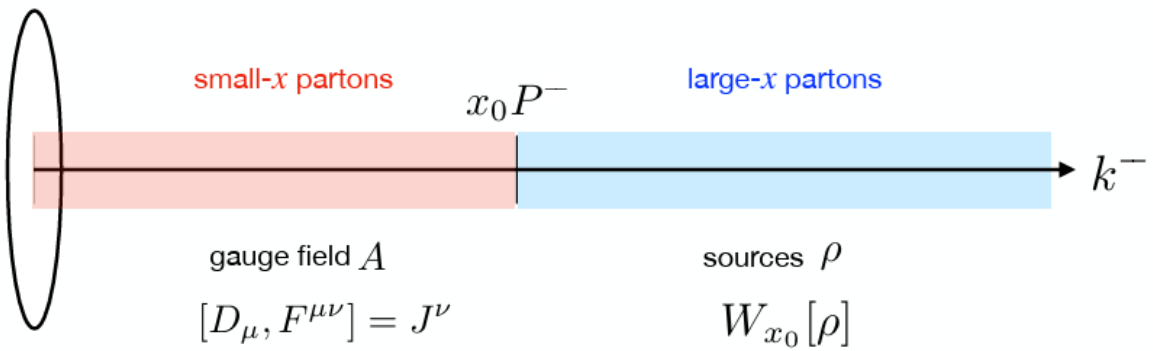
$\leftarrow$

Large-x partons

Static degrees of freedom

Act as a static color charge density ρ





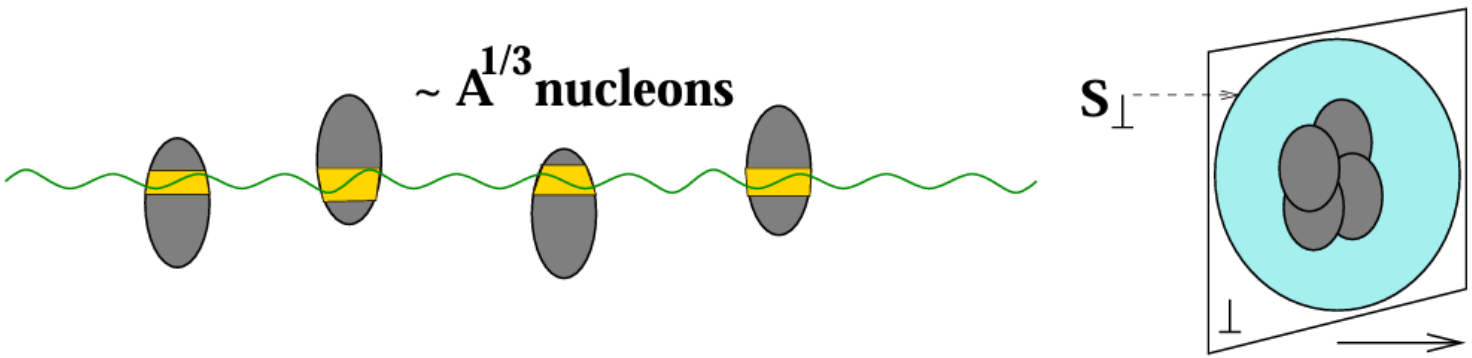
- Sources are **stochastic** $\longrightarrow$ distributed by a gauge invariant weight functional $W_{x_0}[\rho]$

1) Compute $\langle \mathcal{O} \rangle_\rho$ in the presence of sources drawn from $W_{x_0}[\rho]$

2) Average over all possible configurations: $\langle \mathcal{O} \rangle_{x_0} = \int D[\rho] W_{x_0}[\rho] \langle \mathcal{O} \rangle_\rho$

Modelization of the weight functional

- **McLerran-Venugopalan (MV) modelization** of $W_{x_0}[\rho]$
- Individual nucleons are largely **uncorrelated** in a heavy nucleus



- Nucleons are still **point-like** compared to the nucleus: $W[\rho] = \prod_i \rho(z_i)$
- Many uncorrelated nucleons $\longrightarrow$ A **random walk in color space**
- Large number of particles $\longrightarrow$ **Gaussian probability** distribution
- The initial condition resums all classical physics effects ($\alpha_s A^{1/3}$)
- Still **quantum fluctuations** can destroy the unitarity picture, if not taken into account in full glory (i.e. linear BFKL evolution only)

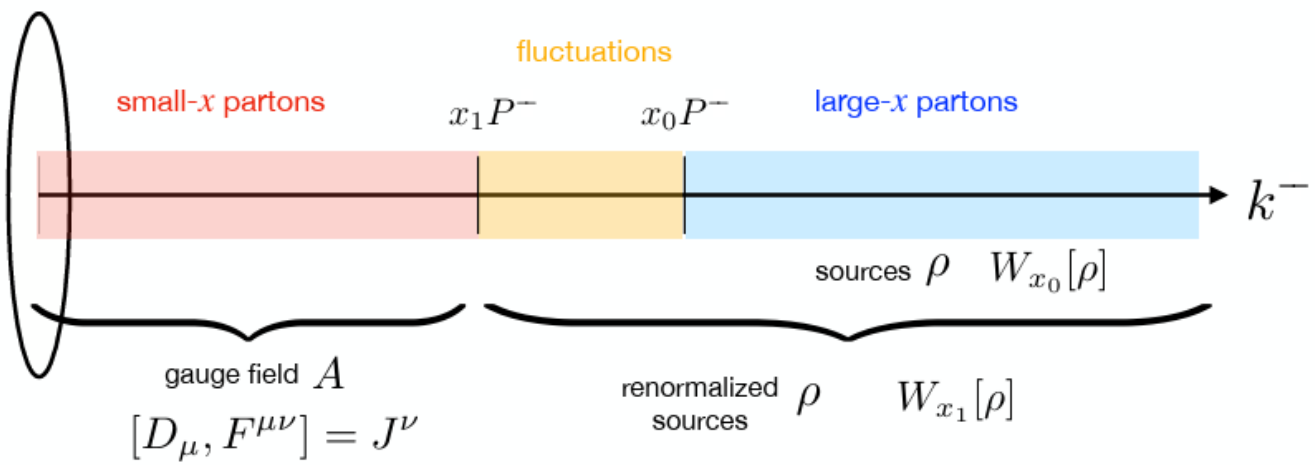
Quantum fluctuations in the CGC

- What about **quantum fluctuations**? $\longrightarrow \alpha_s$ is small
- In QCD quantum fluctuations can be strongly enhanced $\longrightarrow \alpha_s \ln(x/x_0) \sim 1$
- Non-linear generalization of BFKL with an arbitrary number of Reggeons

$$\downarrow$$

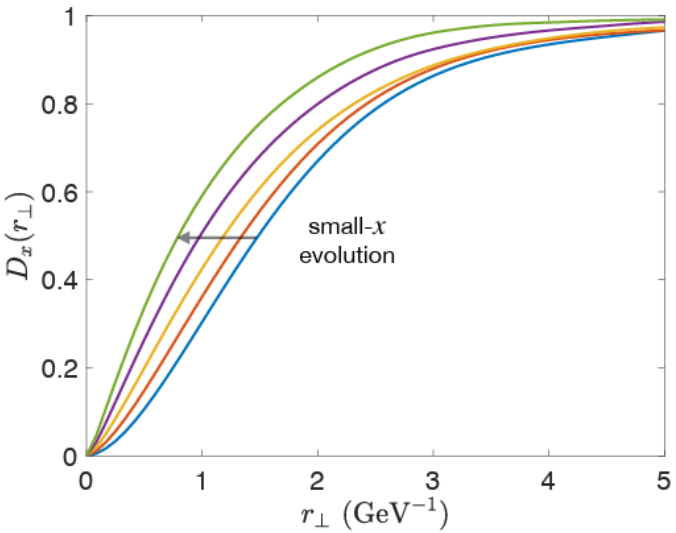
- **Balitsky-Kovchegov equation** (generalized by the Balitsky hierarchy)
- Alternative $\longrightarrow$ **Evolution of the weight functional** from x_0 to x

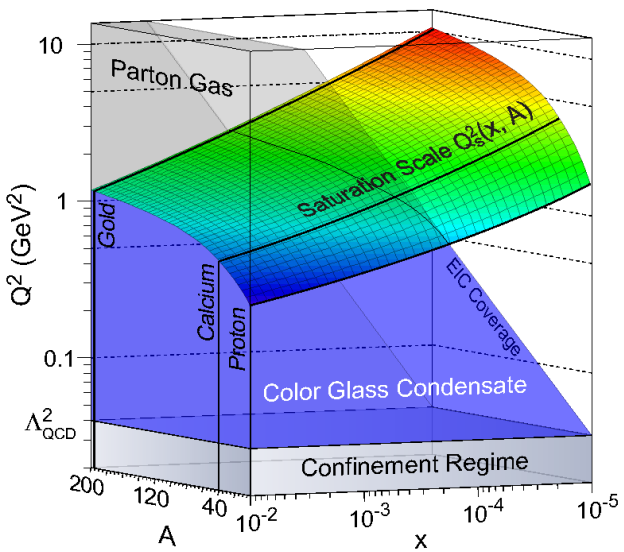
$$\frac{\partial\, W_x[\rho]}{\partial \ln(1/x)} = -\mathcal{H}_{\text{JIMWLK}}\, W_x[\rho]$$



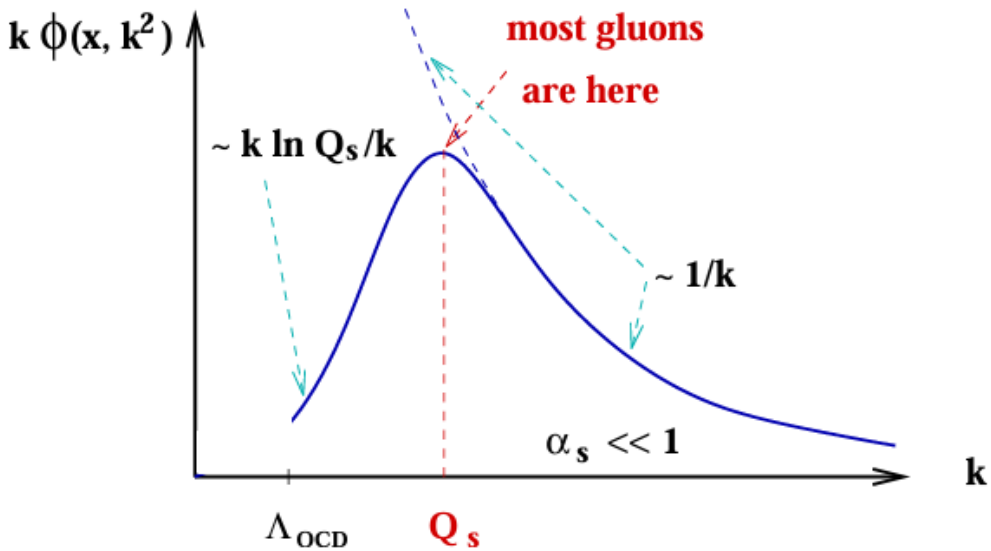
Gluon saturation: some features

- Evolution shift the saturation point in the perturbative region





- Distribution of gluons of large transverse size



- **Saturation scale**

$$Q_s^2(A,x) \sim \left(\frac{A}{x}\right)^{1/3} \Lambda^2_{\rm QCD}$$

- **Perturbative control** on gluon saturation

$$\Lambda^2_{\rm QCD} \ll Q^2 \lesssim Q_s^2$$

Saturation scale

- Cross-section for $gg \to g$

$$\sigma_{gg\to g}\left(x,Q^2\right) \sim \alpha_s \frac{xg\left(x,Q^2\right)}{Q^2}$$

- **Recombination probability**

$$\Gamma(x,Q^2) \sim \alpha_s \frac{xg\left(x,Q^2\right)}{Q^2}\frac{1}{\pi R_p^2}$$

- **Saturation scale** $\to$ Recombination probability of order one

$$\Gamma(x,Q_s^2) \sim 1 \longrightarrow Q_s^2(x) \sim \alpha_s \frac{xg\left(x,Q^2\right)}{\pi R_p^2}$$

- $1/x$-dependence of the gluon distribution coming from LLA BFKL

$$Q_s^2(x) \propto \left(\frac{1}{x}\right)^{\omega_0} \implies \ln Q_s^2\left(Y\right) = \omega_0 Y + K$$

- For a nucleus

$$xg\left(x,Q_s^2\right) \longrightarrow xg_A\left(x,Q_s^2\right) \sim Axg\left(x,Q_s^2\right) \sim A(1/x)^{\omega_0}$$

- **Nucleus saturation scale**

$$Q_s^2(x) \sim A^{1/3}x^{-\omega_0}\Lambda_{\mathrm{QCD}}^2 \sim \left(\frac{A}{x}\right)^{1/3}\Lambda_{\mathrm{QCD}}^2$$

A toy model of saturation

- **BK equation**

$$\frac{\partial\langle\mathcal{U}^{\eta}_{12}\rangle}{\partial\eta} = \frac{\alpha_s N_c}{2\pi^2}\int d^2\vec{z}_3 \left(\frac{\vec{z}^{\,2}_{12}}{\vec{z}^{\,2}_{23}\vec{z}^{\,2}_{31}}\right)\left[\langle\mathcal{U}^{\eta}_{13}\rangle + \langle\mathcal{U}^{\eta}_{32}\rangle - \langle\mathcal{U}^{\eta}_{12}\rangle - \langle\mathcal{U}^{\eta}_{13}\rangle\langle\mathcal{U}^{\eta}_{32}\rangle\right]$$

- **BFKL equation** is the same without the last non-linear term
- **Toy model**

$$\frac{\partial\mathcal{U}(\eta)}{\partial\eta} = A\left(\mathcal{U}(\eta) - \mathcal{U}^2(\eta)\right), \qquad \eta = \ln(1/x)$$

- BFKL solution (only linear term)

$$\mathcal{U}_{\rm BFKL}(\eta) = e^{A\eta} \longrightarrow \mathcal{U}_{\rm BFKL}(\ln(1/x)) = e^{A\ln(1/x)}$$

- BK solution

$$\mathcal{U}_{\rm BK}(\eta) = \frac{e^{A\eta}}{1+e^{A\eta}} \longrightarrow \mathcal{U}_{\rm BK}(\ln(1/x)) = \frac{e^{A\ln(1/x)}}{1+e^{A\ln(1/x)}}$$