Insights into Gravitational Redshift in the Context of Isolated Neutron Stars

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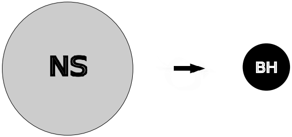
Universal Relations

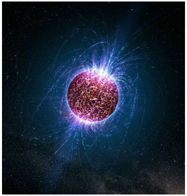
- As per the 'No-Hair' theorem, Isolated non-charged Black Holes can be entirely described by their mass and spin angular momentum.
- The material properties of any object become unmeasurable (hence unknowable) as the object collapses into a black hole.



- Astrophysical objects other than black holes are not expected to share the same type of universality.
- However, Neutron Stars and Quark Stars have recently been found to present certain universality.
- These relations can provide a greater understanding of the structural properties of these compact astrophysical objects as they are independent of the equation of state of matter.

Introducing Neutron Stars

- It's a collapsed core of a massive astrophysical object.
- The fate of the remnant depends on the mass of the progenitor.
  - $M < 8\ M_\odot$: White Dwarf
  - $8\ M_\odot < M < 20\ M_\odot$: Neutron Star
  - $M > 20\ M_\odot$: Black Hole

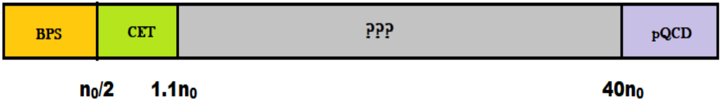


- The NSs are modeled using the following equations of hydrostatic equilibrium:

$$\begin{aligned} \frac{dP}{dr} &= -(\mathcal{E} + P)\frac{m(r) + 4\pi r^3 P}{r(r - 2m(r))}, \\ \frac{dm}{dr} &= 4\pi \mathcal{E} r^2. \end{aligned}$$

Equation of state

- Here, we try to construct a generic family of EoSs that follow both low density (CET) and high density (pQCD) limits.



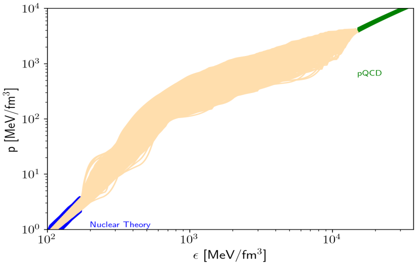
- We interpolate between a CET EoS below saturation density and pQCD result at high densities.
- The adiabatic speed of sound ($c_s = \sqrt{\frac{\partial p}{\partial \epsilon}}$) has been used to obtain the EoS by randomising it arbitrarily in the intermediate densities.
- The piecewise linear segments for the sound speed of the following form:

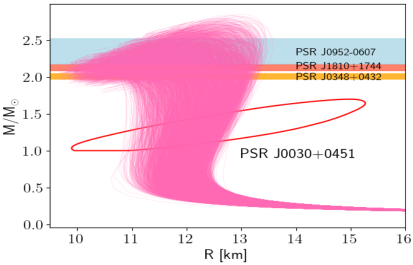
$$c_s^2(\mu) = \frac{(\mu_{i+1} - \mu)\, c_{s,i}^2 + (\mu - \mu_i)\, c_{s,i+1}^2}{\mu_{i+1} - \mu_i}\,, \tag{1}$$

- In the next step, we have imposed the observational constraints.

Equation of State

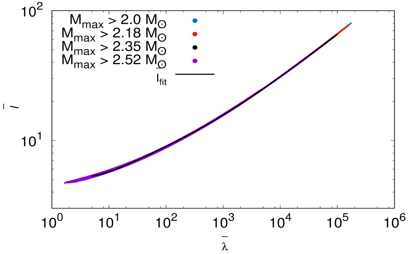
- The family of EoSs we get after imposing the observational constraints:

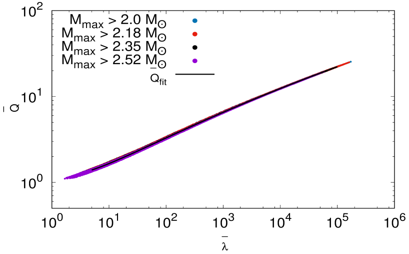




- We have also highlighted the latest maximum mass constraint.
- Once we construct the EoSs, we solve the TOV equation to generate the mass-radius sequence of the NSs.

Universality



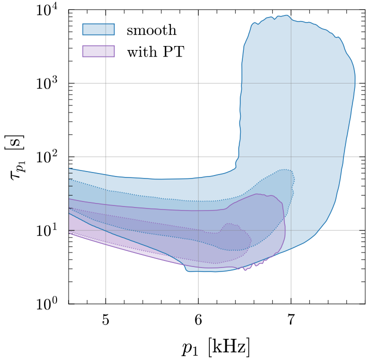


K. K. Nath, R. Mallick, S. Chatterjee, MNRAS, 524, 1438 (2023).

- $\bar{I}$ and $\bar{Q}$ have considerable spread as a function of gravitational mass.
- However, the picture changes drastically when $\bar{I}$ and $\bar{Q}$ are plotted as a function of love number.
- They follow a curve with minimal spread.

Limitations of URs

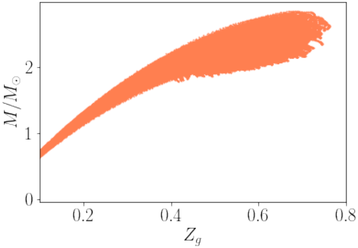
- URs can largely cater to understanding macroscopic variables, but fail to capture the microphysical elements of NSs.
- The comparison of URs for smooth and PT EOSs shows that the exclusive region has no implications on the URs.



P. Thakur, S. Chatterjee, K. K. Nath and R. Mallick, *Phys. Rev. D*, 110, 10, 103045 (2024)

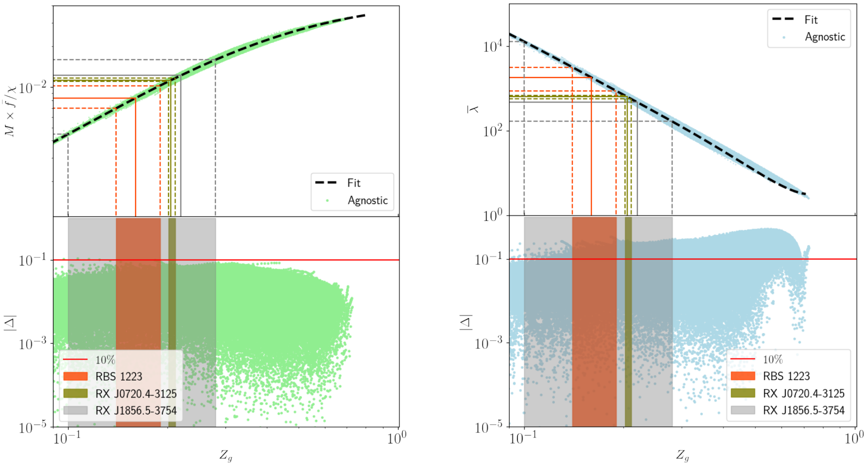
Gravitational Redshift

- Gravitational redshift is defined as $Z_g = 1/\sqrt{1 - 2GM/Rc^2} - 1$, and recent observations have motivated us to analyze the URs in the context of Z_g.



- As the EoSs follow the astrophysical constraints, they provide us with an upper limit on the value of redshift, $Z_g(max) \leq 0.763$ which further constrains the previous maximum estimates of ≤ 2.
- The range of values of Z_g for a $1.4M_\odot$ NS can also be seen to agree with the limiting values provided by Lindblom et al. (2014), i.e. $0.854 \geq Z_g \geq 0.184$.

URs Related to Gravitational Redshift



$$\log y = \Sigma_{i=0}^{4} a_{yi} \, \log(Z_g)^i, |\Delta| = |\frac{V_y - V_{fit}}{V_{fit}}|$$

Theoretical estimates of the parameters using URs

	C	$\bar{\lambda}$	$M \times \bar{f}/\chi$
RBS 1223 ($Z_g = 0.16^{+0.03}_{-0.02}$)	$0.128^{+0.019}_{-0.013}$	1787^{-923}_{+1350}	$0.008^{+0.002}_{-0.001}$
RX J0720.4-3125 ($Z_g = 0.205^{+0.006}_{-0.003}$)	$0.156^{+0.003}_{-0.002}$	626^{-72}_{+40}	$0.011^{+0.0004}_{-0.0002}$
RX J1856.5-3754 ($Zg = 0.22^{+0.06}_{-0.12}$)	$0.164^{+0.031}_{-0.077}$	464^{-299}_{+12194}	$0.012^{+0.004}_{-0.008}$

S. Chatterjee & K. K. Nath, *EPJC*, 85, 862 (2025).

Comparison of $\bar{\lambda}$ estimates with Luo et al. 2022

	Luo et al.	This work
RBS 1223	420^{+3260}_{-370}	1787^{+1350}_{-923}
RX J0720.4-3125	641^{+56}_{-48}	626^{+40}_{-72}
RX J1856.5-3754	1460^{+890}_{-980}	464^{+12194}_{-299}

- The $\bar{\lambda}$ estimate of RX J0720.4-3125 closely agrees with the Bayesian estimate since the uncertainty in measurement is less.
- It shows that theoretical estimates are very accurate for observations with less uncertainty and can be used as an alternative for statistical analysis.

References

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2. Prospect of unraveling the first-order phase transition in neutron stars with f and p1 modes
   Thakur P., Chatterjee S., Nath K. K., Mallick R., PhRvD, 110, 103045 (2024)
3. I-Love-Q relations for a generic family of neutron star equations of state, Nath K. K., Mallick R., Chatterjee S., MNRAS, 524, 1438 (2023).

Thank You