

Overview of Neutrino Mass Models After LHC Run-II Data

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Neutrino Mass Models

The neutrino oscillation experiments provided evidences for neutrino flavour oscillations, and hence nonzero neutrino masses and mixings.

The usual Higgs mechanism: $N_R (1,1,0)$

$$-\mathcal{L}_Y = Y_N^{i\alpha} \bar{L}_i \tilde{H} N_{R\alpha} + \text{H.c.}$$

Dirac neutrino mass: $M_D \sim \frac{Y_N v}{\sqrt{2}}$

- ① $m_\nu \sim 0.1\text{eV}$, $Y_N \sim 10^{-12}$: philosophically displeasing
- ② Conservation of Lepton Number

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$$-\mathcal{L}_Y = Y_N^{i\alpha} \bar{L}_i \tilde{H} N_{R\alpha} + \frac{1}{2} M_N^{\alpha\beta} \bar{N}_{R\alpha}^C N_{R\beta} + \text{H.c.}$$

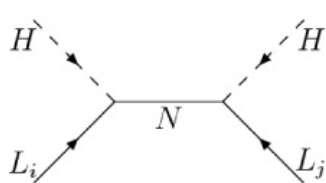
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② Conservation of Lepton Number

"Of the supposedly exact conservation laws of physics, two are especially questionable: the conservation of baryon number and lepton number."

Weinberg, 1979



Type-I Seesaw

$$\mathcal{M}_\nu = \begin{pmatrix} \mathbf{0} & M_D \\ M_D^T & M_N \end{pmatrix}$$

$$M_\nu \simeq -M_D M_N^{-1} M_D^T$$

$$V_{\ell N_\alpha} \sim M_D M_N^{-1}$$

$$\mathcal{L}_{d=5} = \frac{1}{\Lambda} LLHH$$

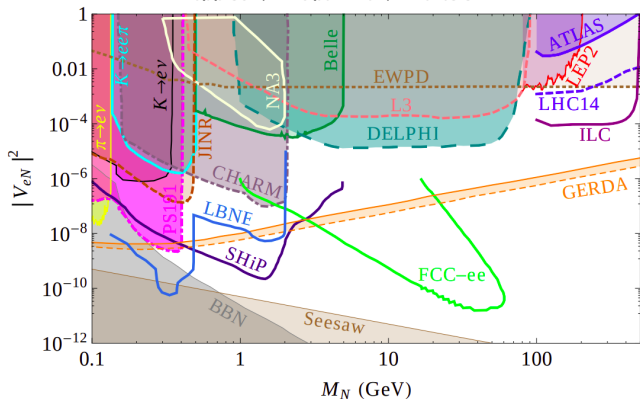
Majorana mass

$$m_\nu \propto \frac{v^2}{\Lambda}$$

The seesaw mechanism

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Deppisch, Bhupal Dev, Pilaftsis



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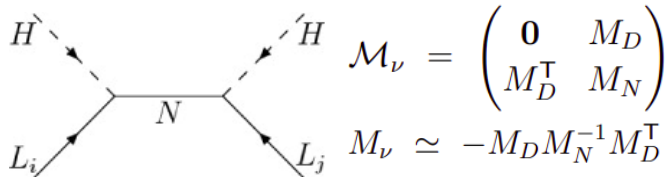
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$$\mathcal{L}_{d=5} = \frac{1}{\Lambda} LLHH$$

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The seesaw mechanism



Type-III Seesaw

Lagrangian generating

ν -masses

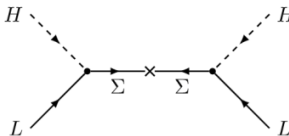
Seesaw formula

$$M_{\Sigma} \bar{\Sigma}_R \Sigma_R + \sqrt{2} Y_{\Sigma} \tilde{H}^{\dagger} \bar{\Sigma}_R L + \text{h.c.}$$

$$m_{\nu} \approx -\frac{v^2}{2} Y_{\Sigma}^T M_{\Sigma}^{-1} Y_{\Sigma}$$

$SU(2)_L$ Triplet fermion

$$\begin{pmatrix} \Sigma_R^0/\sqrt{2} & \Sigma_R^+ \\ \Sigma_R^- & -\Sigma_R^0/\sqrt{2} \end{pmatrix}$$



$$U^T m_{\nu} U = D_{m_{\nu}} = \text{diag}(m_1, m_2, m_3)$$

Most general texture of Y_{Σ}

$$Y_{\Sigma} = i \frac{\sqrt{2}}{v} \sqrt{M_{\Sigma} R} \sqrt{D_{m_{\nu}}} U^{\dagger}$$

Production:

- Drell-Yan processes: $q \bar{q}' \rightarrow \gamma/Z \rightarrow \Sigma^+ \Sigma^-$, $q \bar{q}' \rightarrow W^{\pm} \rightarrow \Sigma^{\pm} \Sigma^0$

Decays:

- Heavy state transitions: $\Sigma^{\pm} \rightarrow \Sigma^0 \pi^{\pm}$, $\Sigma^{\pm} \rightarrow \Sigma^0 \ell^{\pm} \nu_{\ell}$
- SM 2-body decays:
 - $\Sigma^0 \rightarrow \nu h$, $\Sigma^0 \rightarrow \nu Z$, $\Sigma^0 \rightarrow \ell^{\mp} W^{\pm}$.
 - $\Sigma^{\pm} \rightarrow \ell^{\pm} h$, $\Sigma^{\pm} \rightarrow \ell^{\pm} Z$, $\Sigma^{\pm} \rightarrow \nu W^{\pm}$.

Multilepton Final States Search:

- A recent CMS multilepton search (137.1 fb^{-1}) excluded triplet fermions below 880 GeV at 95% CL in flavour democratic scenario. [JHEP 03 \(2020\) 051](#)
- This CMS limit is not straightforwardly applicable for a realistic type-III seesaw model.

Bounds on realistic Type-III Seesaw: Implications of R -matrix

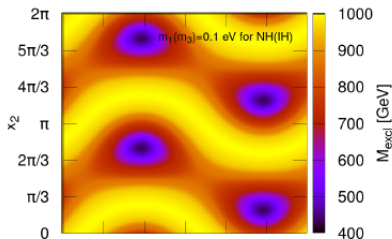
3RHN: $\theta_{12} = x_3 + iy_3$, $\theta_{23} = x_1 + iy_1$ and $\theta_{13} = x_2 + iy_2$ with $x_{1,2,3}, y_{1,2,3} \in \mathbb{R}$:

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

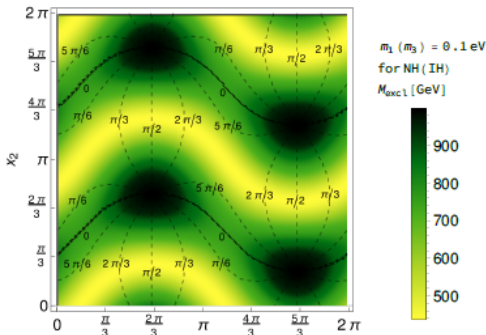
2RHN

$$R = \begin{cases} \begin{pmatrix} 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{pmatrix} & \text{for NH} \\ \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \end{pmatrix} & \text{for IH} \end{cases}$$

Only Σ_1 is produced



Only Σ_2 is produced



LHC limits are derived for a simplified type-III seesaw model, not applicable for a realistic model accounting for the neutrino oscillation data.

Proposal: Look for τ -rich final states

A. Tumasyan et al. [CMS], Phys. Rev. D **105** (2022)

Type-II Seesaw

$SU(2)_L$ Triplet Scalar

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$$

Triplet VEV after the EWSB

$$\langle \Delta \rangle = \begin{pmatrix} 0 & 0 \\ v_t/\sqrt{2} & 0 \end{pmatrix},$$

$$v_t \approx \frac{\mu v_d^2}{\sqrt{2} m_\Delta^2}$$

Phenomenology: $m_{H^\pm}, v_t$ and

$$\Delta m = m_{H^\pm} - m_{H^\pm}$$

Constraint on v_t and Δm

1. Oblique parameters:

$$|\Delta m| \leq 40 \text{ GeV}$$

2. ρ parameter:

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2 \theta_w} = \frac{v_d^2 + 2v_t^2}{v_d^2 + 4v_t^2}$$

3. LFV decays:

$$l_\alpha \rightarrow l_\beta \gamma @ 1\text{-loop and}$$

$$l_\alpha \rightarrow l_\beta l_\gamma l_\delta @ \text{tree-level.}$$

Scalar potential

$$-m_\Phi^2 \Phi^\dagger \Phi + \frac{\lambda}{4} (\Phi^\dagger \Phi)^2 + m_\Delta^2 \text{Tr}(\Delta^\dagger \Delta) + [\mu (\Phi^T i \sigma^2 \Delta^\dagger \Phi) + \text{h.c.}] +$$

$$\lambda_1 (\Phi^\dagger \Phi) \text{Tr}(\Delta^\dagger \Delta) + \lambda_2 [\text{Tr}(\Delta^\dagger \Delta)]^2 + \lambda_3 [\text{Tr}(\Delta^\dagger \Delta)^2] + \lambda_4 \Phi^\dagger \Delta \Delta^\dagger \Phi$$

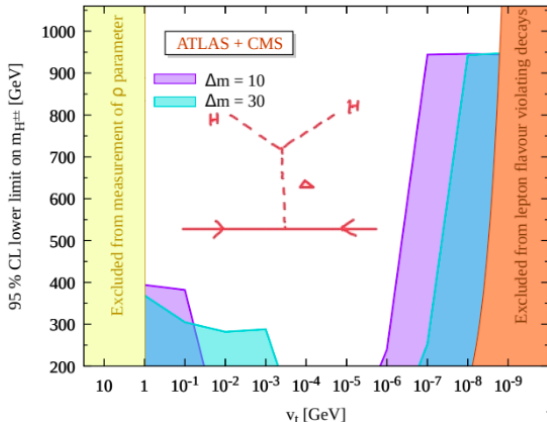
Triplet Yukawa

$$-\mathcal{L}_\nu = Y_{ij}^\nu L_i^T C i \sigma^2 \Delta L_j + \text{h.c.}$$

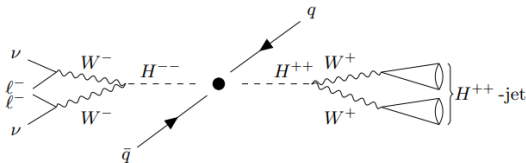
$\xrightarrow{\text{EWSB}}$

$$m_\nu = \sqrt{2} Y^\nu v_t$$

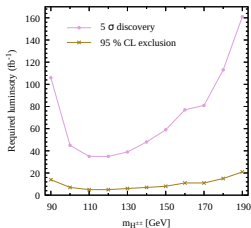
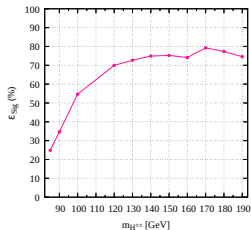
Ashanujjaman, KG, JHEP 03 (2022) 195



Search for light (within 84–200 GeV) $H^{\pm\pm} \rightarrow W^{\pm}W^{\pm(*)}$



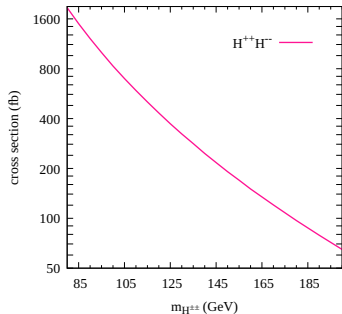
BDT classifier for tagging boosted hadronically decaying $H^{\pm\pm}$



[Ashanujjaman, Ghosh, Sahu, PRD 2023]

Motivation

Such a low-mass triplet $H^{\pm\pm}$, slightly heavier singly-charged and neutral components can explain the CDF measurement of the W -boson mass. [Kanemura and Yagyu; Heeck; Bahl, Chiu, Gao, Wang, and Zhong; Cheng, He, Huang, Sun, and Xing]



- 1 Large Prod. Cross section
- 2 Soft leptons and jets in the final state

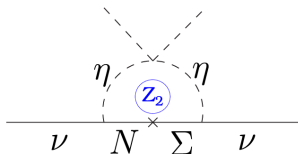
Weinberg operator(s)

① $\mathcal{L}_5 \propto \frac{1}{\Lambda} LLHH \Rightarrow m_\nu \propto \frac{v^2}{\Lambda}$

② $\mathcal{L}_{5+2n} \propto \frac{1}{\Lambda^{2n+1}} LLHH (H^\dagger H)^n$

$\Rightarrow m_\nu \propto \epsilon \left(\frac{1}{16\pi^2} \right)^\# \left(\frac{v}{\Lambda} \right)^{d-5} \frac{v^2}{\Lambda}$

$\mathcal{L}_5$ @loop-level



- Without additional symmetries:
Interesting Collider Signatures
- With additional symmetries:
Candidate for dark matter.

① $\mathcal{L}_5$ at tree level $\rightarrow$ classical seesaws

- for $y \sim \mathcal{O}(1)$, $\Lambda \sim (10^{14} - 10^{15})$ GeV
 $\Rightarrow$ direct tests impossible.
- for $\Lambda \sim \mathcal{O}(1)$ TeV, $y \sim \mathcal{O}(10^{-12})$
 $\Rightarrow$ philosophically displeasing.

② $\mathcal{L}_9$ at tree level

- $y \sim \mathcal{O}(0.1)$ as well as $\Lambda \sim \mathcal{O}(1)$ TeV

$\Rightarrow$ philosophically pleasing & collider testable.

$\mathcal{L}_5$ @tree-level:

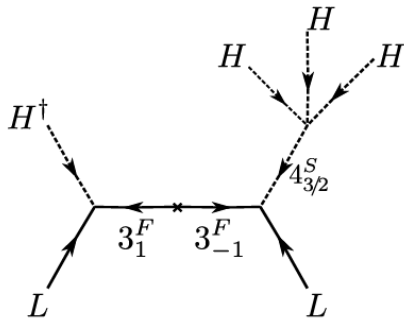
Type I, Type II and Type III seesaw

$$\mathcal{L} = \mathcal{L}_{SM} + \underbrace{\mathcal{L}_{d=5}}_{\text{dominant}} + \underbrace{\mathcal{L}_{d=7}}_{\text{subdominant}} + \underbrace{\mathcal{L}_{d=9}}_{\text{subsubdominant}}$$

How to make $\mathcal{L}_9$ contribution to m_ν dominant?

- Introduce a discrete symmetry
- Choose the particle content such a way that it doesn't allow to complete lower order operator $\Rightarrow$ genuine models

Higher Dimensional Neutrino Mass Models

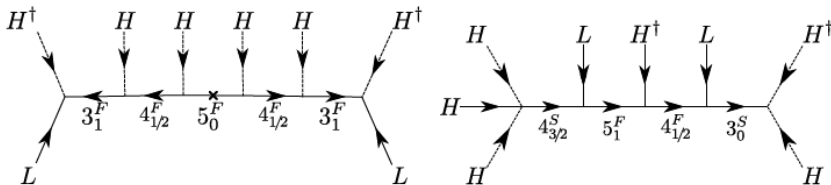


UV completion of generalized
Weinberg operator at **dimension-7**

Babu, Nandi, Tavartkiladze, Phys. Rev. D
80, 071702 (2009)

Dimension-9

Anamiati, Castillo-Felisola, Fonseca, Helo, Hirsch, JHEP 12 (2018) 066



A Genuine Quintuplet (Type-V) Seesaw

Exotic fields:

$$\Sigma_{L,R} = (++, +, 0)_{L,R} \sim (1, 3, 1)$$

$$\Delta_{L,R} = (++, +, 0, -)_{L,R} \sim (1, 4, \frac{1}{2})$$

$$\Phi_R = (++, +, 0, -, --)_R \sim (1, 5, 0)$$

Relevant Lagrangian:

$$\begin{aligned}\mathcal{L}_Y &= Y_{23} \bar{\Sigma}_L H \tilde{L} + Y_{34} \bar{\Delta}_R \Sigma_L H^\dagger + Y'_{34} \bar{\Delta}_L \Sigma_R H^\dagger \\ &\quad + Y_{45} \bar{\Delta}_L \Phi_R H + Y'_{45} \bar{\Delta}_R \tilde{\Phi}_R H \\ \mathcal{L}_M &= M_\Sigma \bar{\Sigma}_R \Sigma_L + M_\Delta \bar{\Delta}_R \Delta_L + \frac{1}{2} M_\Phi \bar{\Phi}_R \Phi_R\end{aligned}$$

Light neutrino mass:

$$\begin{aligned}m_\nu &\approx \frac{v^6}{48} Y_{23}^\dagger M_\Sigma^{-1} Y_{34}'^\dagger M_\Delta^{-1} Y_{45}' M_\Phi^{-1} Y_{45}'^T M_\Delta^{-1} Y_{34}'^* M_\Sigma^{-1} Y_{23}^* \\ &= \frac{v^2}{2} Y_{23}^\dagger \mathcal{M}^{-1} Y_{23}^*\end{aligned}$$

Casas-Ibarra parametrisation:

$$Y_{23}^* = \frac{\sqrt{2}}{v} U_{\mathcal{M}} \sqrt{\hat{\mathcal{M}}} R \sqrt{\hat{m}_\nu} U_{\text{PMNS}}^\dagger \quad (R^T R = 1)$$

A Simplified Model:

$$M_\Sigma = \text{diag}(m_\Sigma, m_\Sigma, m_\Sigma)$$

$$M_\Delta = \text{diag}(m_\Delta, m_\Delta, m_\Delta)$$

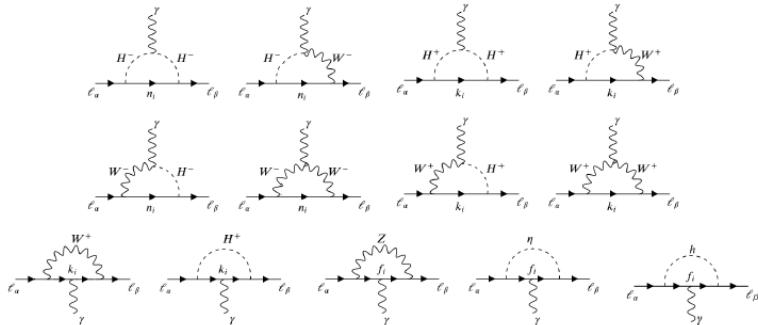
$$M_\Phi = \text{diag}(m_\Phi, m_\Phi, m_\Phi)$$

$$Y_{34}' = \text{diag}(y_{34}, y_{34}, y_{34})$$

$$Y_{45}' = \text{diag}(y_{45}, y_{45}, y_{45})$$

$$R = \mathbf{I}_{3 \times 3}$$

Bounds: Lepton Flavour Violating Decays: $l_\alpha \rightarrow l_\beta \gamma$



Plus 14
self-
energy
diagrams

In the limit $M_\Sigma \gg m_{W,Z,h}$,

$$\text{Br}(l_\alpha \rightarrow l_\beta \gamma) \approx \frac{3\alpha}{2\pi} \left| \frac{51 + 16 \sin^2 \theta_w}{12} \lambda_{\beta\alpha} \right|^2 \times \text{Br}(l_\alpha \rightarrow l_\beta \bar{\nu}_\beta \nu_\alpha)$$

Bounds from MEG
collaboration

$$\text{BR}(\mu \rightarrow e \gamma) < 4.2 \times 10^{-13}$$

$$|\lambda_{e\mu(e\tau)[\mu\tau]}| = \left| \left(\frac{v^2}{8} Y_{23}^T M_\Sigma^{-2} Y_{23}^* \right)_{e\mu(e\tau)[\mu\tau]} \right| \leq 2.3 \times 10^{-6} (1.5 \times 10^{-3}) [1.8 \times 10^{-3}]$$

- **Drell-Yan:** $q \bar{q}' \rightarrow \gamma/Z \rightarrow \chi^{++}\chi^{--}/\chi^+\chi^-/\chi^0\chi^0$, $q\bar{q}' \rightarrow W^\pm \rightarrow \chi^{\pm\pm}\chi^\mp/\chi^\pm\chi^0$
- **Photon-photon fusion:** $\gamma\gamma \rightarrow \chi^{++}\chi^{--}/\chi^+\chi^-$
- **Heavy state transition:** $\chi^Q \rightarrow |\chi^{Q-1}\pi^+, \chi^{Q-1}K^+, \chi^{Q-1}\ell^+\nu, \chi^{Q-1}\pi^+\rho$
- **SM 2-body decays:**

$$\chi_i^{++} \rightarrow \ell_j^+ W^+$$

$$\chi_i^+ \rightarrow \ell_j^+ Z/h, \nu W^+$$

$$\chi_i^0 \rightarrow \ell_j^\pm W^\mp, \nu Z/h$$

	$\chi^{++} \rightarrow \ell^+ W^+$	$\chi^+ \rightarrow \nu W^+, \ell^+ Z, \ell^+ h$			$\chi^0 \rightarrow \ell^\pm W^\mp, \nu Z, \nu h$		
$\chi^{--} \rightarrow \ell^- W^-$	$\ell^- \ell^+ W^- W^+$	$\ell^- \nu W^- W^+$	$\ell^- \ell^+ W^- Z$	$\ell^- \ell^+ W^- h$	-	-	-
$\chi^- \rightarrow \nu W^-$	$\nu \ell^+ W^- W^+$	$\nu \nu W^- W^+$	$\nu \ell^+ W^- Z$	$\nu \ell^+ W^- h$	$\nu \ell^\pm W^- W^\mp$	$\nu \nu W^- Z$	$\nu \nu W^- h$
$\chi^- \rightarrow \ell^- Z$	$\ell^- \ell^+ Z W^+$	$\ell^- \nu Z W^+$	$\ell^- \ell^+ Z Z$	$\ell^- \ell^+ Z h$	$\ell^- \ell^\pm Z W^\mp$	$\ell^- \nu Z Z$	$\ell^- \nu Z h$
$\chi^- \rightarrow \ell^- h$	$\ell^- \ell^+ h W^+$	$\ell^- \nu h W^+$	$\ell^- \ell^+ h Z$	$\ell^- \ell^+ h h$	$\ell^- \ell^\pm h W^\mp$	$\ell^- \nu h Z$	$\ell^- \nu h h$
$\chi^0 \rightarrow \ell^\pm W^\mp$	-	$\ell^\pm \nu W^\mp W^\pm$	$\ell^\pm \ell^+ W^\mp Z$	$\ell^\pm \ell^+ W^\mp h$	$\ell^\pm \ell^\pm W^\mp W^\mp (\pm)$	$\ell^\pm \nu W^\mp Z$	$\ell^\pm \nu W^\mp h$
$\chi^0 \rightarrow \nu Z$	-	$\nu \nu Z W^+$	$\nu \ell^+ Z Z$	$\nu \ell^+ Z h$	$\nu \ell^\pm Z W^\mp$	$\nu \nu Z Z$	$\nu \nu Z h$
$\chi^0 \rightarrow \nu h$	-	$\nu \nu h W^+$	$\nu \ell^+ h Z$	$\nu \ell^+ h h$	$\nu \ell^\pm h W^\mp$	$\nu \nu h Z$	$\nu \nu h h$

- Masses below 720, 970, and 1200 GeV are excluded for triplet, quadruplet and quintuplet, respectively from CMS 3/4 lepton final state search [JHEP 03 (2020) 051].
- Mutilepton + fat-jets final states@LHC
- Displaced vertices, LLPs, vanishing charge track signature, etc. may result for smaller values of the lightest neutrino mass i.e. $m_1(m_3) \rightarrow 0$ for NH(IH).

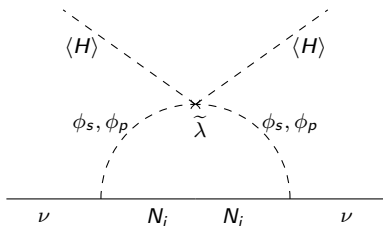
	$3^{++}(4^{++})\{5^{++}\}[3^+]$	$4^+(5^+)$	$4^-/3^0/4^0/5^0$
$c\tau_{\max}$ (in mm)	0.04 (0.22) {0.43} [1.0]	3.6 (16.0)	arbitrarily large
where to look for	LHeC, FCC-he	HL-LHC, LHeC, FCC-he	MATHUSLA

Radiative Neutrino Mass Generation

Symmetry	The SM fields						Exotics	
	Q_L^α	u_R^α	d_R^α	L_L^α	e_R^α	H	N_R^i	Φ
$SU(3)_C$	3	3	3	1	1	1	1	1
$SU(2)_L$	2	1	1	2	1	2	1	1
$U(1)_Y$	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$
Z_2	+	+	+	+	+	+	-	-

The model features a dark matter candidate, but its collider detection remains challenging.

$$\mathcal{L} \ni -\mu_\phi^2 \Phi^\dagger \Phi - \lambda_0 (\Phi^\dagger \Phi)^2 - \lambda_1 (H^\dagger H) (\Phi^\dagger \Phi) - \lambda_2 |H^\dagger \Phi|^2 - \frac{\tilde{\lambda}}{2} [(H^\dagger \Phi)^2 + \text{h.c.}] + \frac{1}{2} M_{N_i} \overline{N_i^C} N_i - [Y^{\alpha i} \overline{L_L^\alpha} \tilde{\Phi} N_R^i + \text{h.c.}]$$



$$M_\nu \simeq \frac{\tilde{\lambda} v^2}{32\pi^2} \left(Y \mathcal{M}^{-1} Y^T \right)$$

$$G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

Fermions: $Q_{\alpha L} = \begin{pmatrix} u_\alpha \\ d_\alpha \end{pmatrix}_L \sim (3, 2, \frac{1}{6}), \quad L_{\alpha L} = \begin{pmatrix} \nu_\alpha \\ e_\alpha \end{pmatrix}_L \sim (1, 2, -\frac{1}{2})$

$$u_{\alpha R} \sim (3, 1, \frac{2}{3}), \quad d_{\alpha R} \sim (3, 1, -\frac{1}{3}), \quad e_{\alpha R} \sim (1, 1, -1)$$

$$E_{\alpha L(R)}^{++} \sim (1, 1, 2)$$

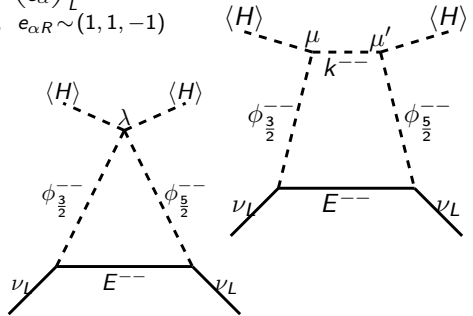
Scalars: $H = \begin{pmatrix} h^+ \\ \frac{h+i\eta}{\sqrt{2}} \end{pmatrix} \sim (1, 2, \frac{1}{2})$

$$\Phi_{\frac{3}{2}} = \begin{pmatrix} \phi_{\frac{3}{2}}^{++} \\ \phi_{\frac{3}{2}}^{++} \\ \phi_{\frac{3}{2}}^{++} \end{pmatrix} \sim (1, 2, \frac{3}{2})$$

$$\Phi_{\frac{5}{2}} = \begin{pmatrix} \phi_{\frac{5}{2}}^{+++} \\ \phi_{\frac{5}{2}}^{++} \\ \phi_{\frac{5}{2}}^{++} \end{pmatrix} \sim (1, 2, \frac{5}{2})$$

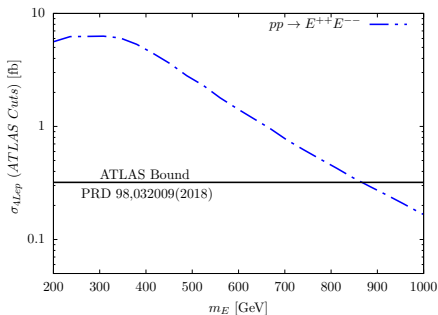
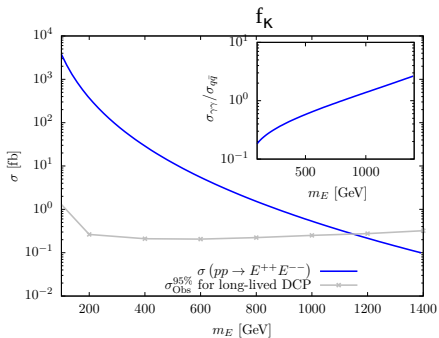
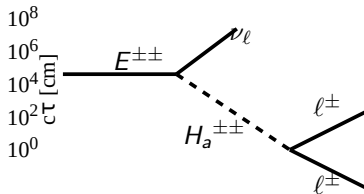
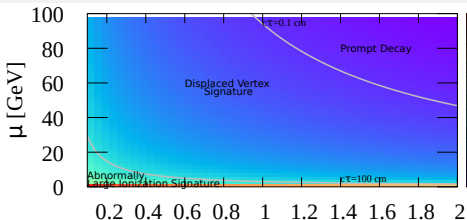
$$k^{++} \sim (1, 1, 2)$$

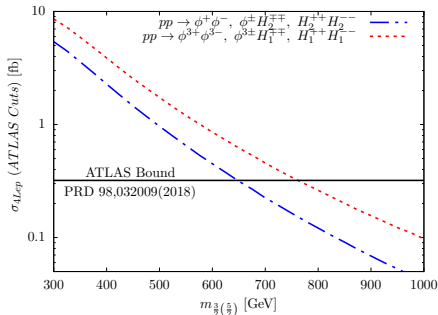
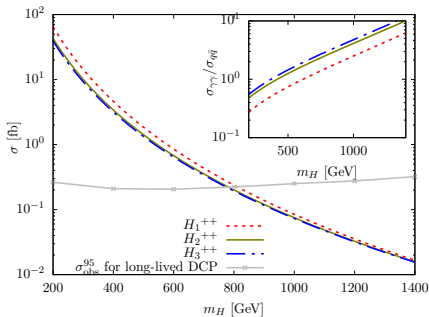
Loop-induced Weinberg operator



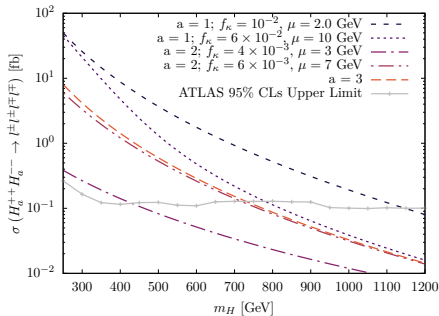
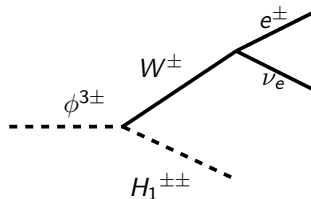
$$\mathcal{L} \supset m_E^{\alpha\beta} \overline{E_\alpha^{++}} E_\beta^{++} + y_{\frac{3}{2}}^{\alpha\beta} \overline{L_{\alpha L}} \Phi_{\frac{3}{2}} (E_{\beta L}^{++})^c + y_{\frac{5}{2}}^{\alpha\beta} \overline{L_{\alpha L}} i\sigma_2 \Phi_{\frac{5}{2}}^* E_{\beta R}^{++} + y_k^{\alpha\beta} \overline{e_{\alpha R}} k^{--} (e_{\beta R})^c + \mu (H^T i\sigma_2 \Phi_{\frac{3}{2}}) k^{--} + \mu' (H^\dagger \Phi_{\frac{5}{2}}) k^{--} + \lambda (H^T i\sigma_2 \Phi_{\frac{3}{2}}) (H^T \Phi_{\frac{5}{2}}^*) + \text{c.c.}$$

Collider Signatures (Exotic Fermion)





Exotic Scalars



Summary

- **Tree-level realizations of the Weinberg operator:**

The effective dimension-5 operator $\mathcal{L}_{d=5} = \frac{1}{\Lambda}(LH)(LH)$ can arise at tree level through three possible seesaw mechanisms:

- ① **Type-I seesaw:** exchange of heavy Majorana singlet fermions (N_R)
- ② **Type-II seesaw:** exchange of scalar $SU(2)_L$ triplet (Δ)
- ③ **Type-III seesaw:** exchange of fermionic $SU(2)_L$ triplet (Σ)

These generate small Majorana neutrino masses $m_\nu \simeq v^2/\Lambda$ after electroweak symmetry breaking.

- **Higher-dimensional tree-level operators:**

Extensions of the Weinberg framework involve dimension-7, 9, or higher operators such as $(LH)(LH)(H^\dagger H)/\Lambda^3$, which yield further suppression of neutrino masses and often predict rich scalar or gauge structures.

- **Loop-induced Weinberg operator:**

In radiative neutrino mass models, the Weinberg operator is realized at one or more loop levels. New scalar and/or fermion fields running in the loop generate

$$m_\nu \sim \frac{1}{(16\pi^2)^n} \frac{v^2}{\Lambda},$$

naturally explaining the smallness of neutrino masses and often introducing a stable **dark matter candidate** protected by a discrete symmetry.

Thank You