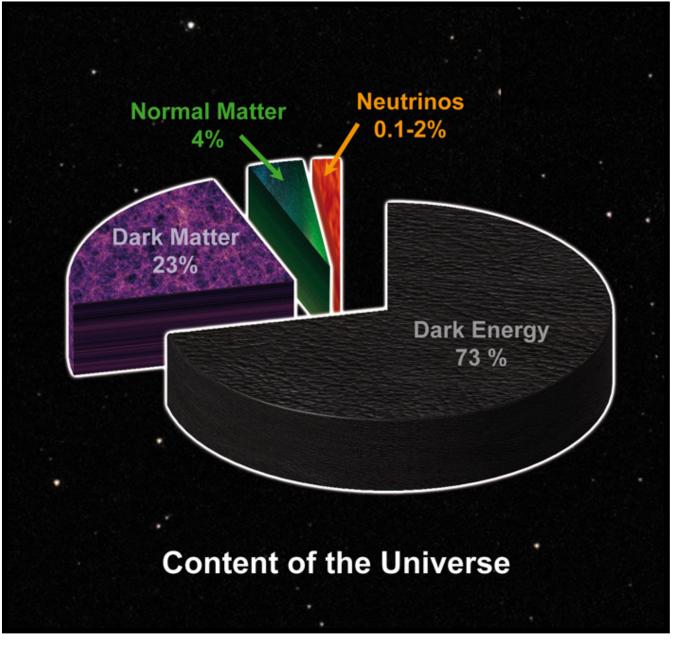
Neutrino mass variables in 3 active and 2 sterile neutrino model

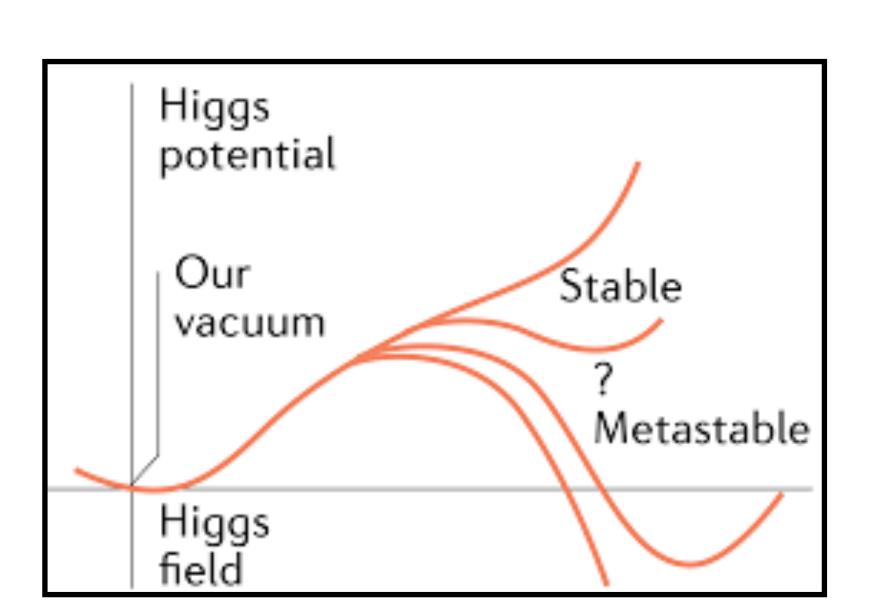
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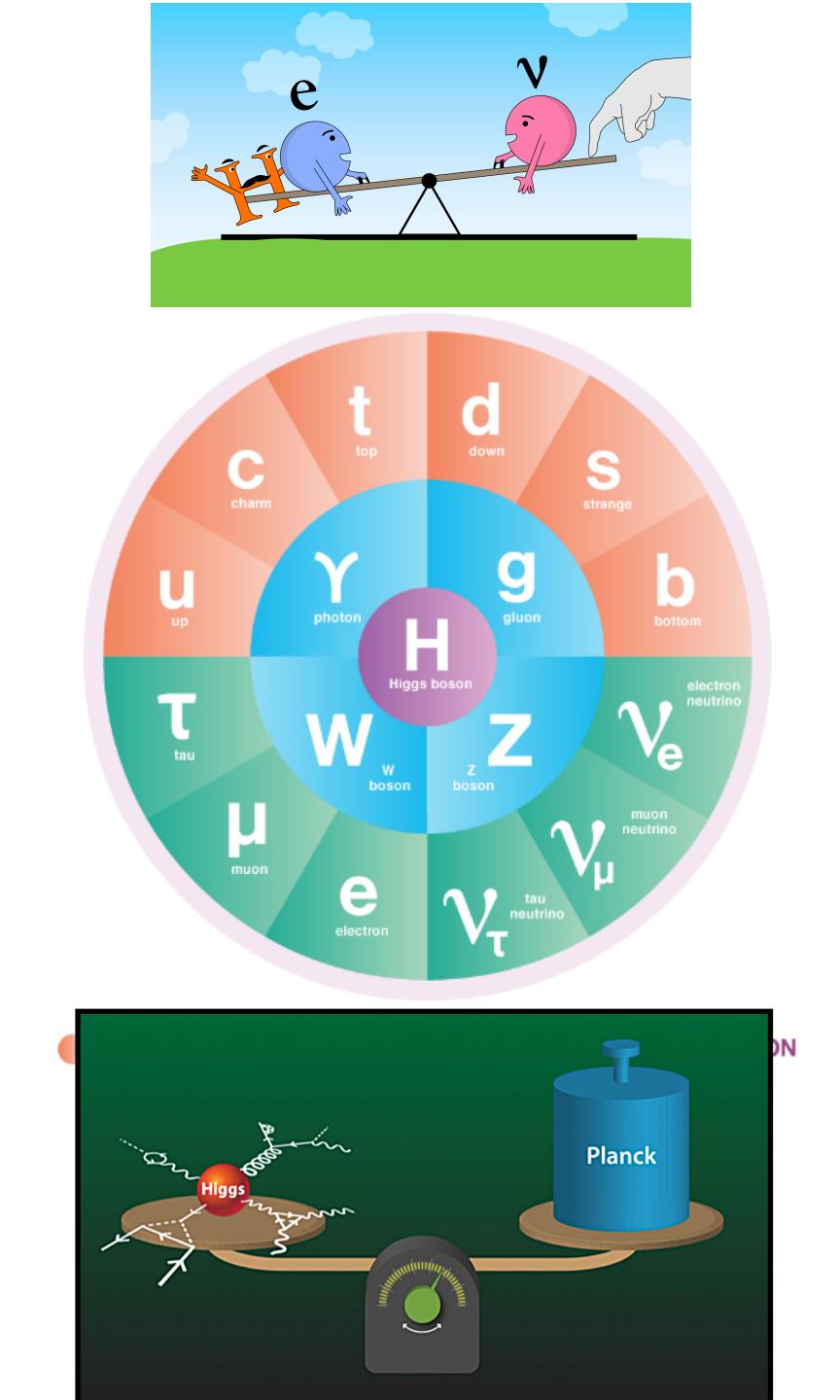


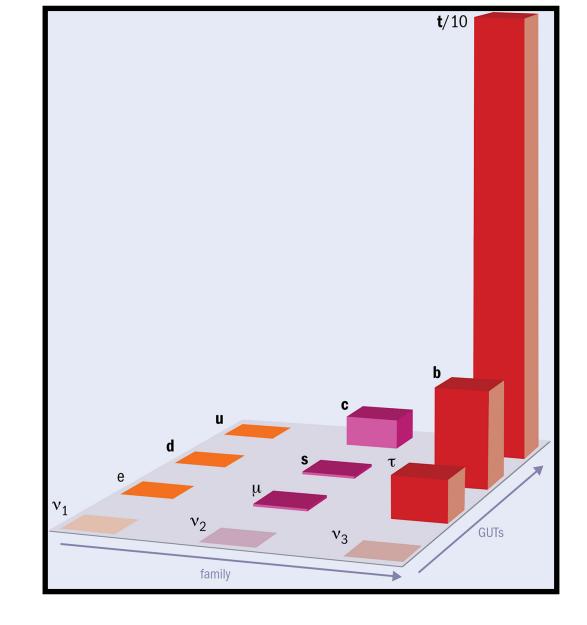
Outline

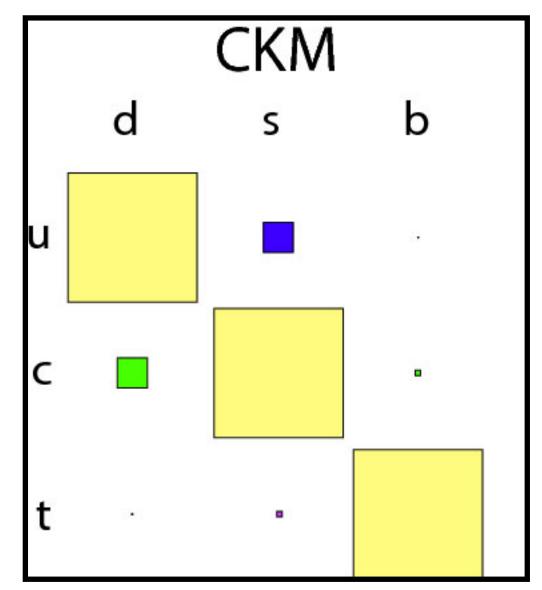
- SM and its Limitations
- Neutrinos and their Anomalies
- Sterile Neutrinos
- Classification of mass ordering in 3 + 2 sterile neutrino framework
- Mass Constraints on 3+2 Sterile Neutrinos
- Summary



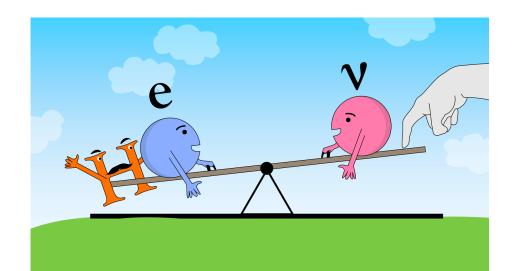








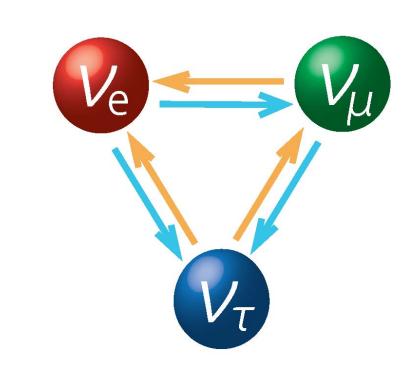
Neutrinos!

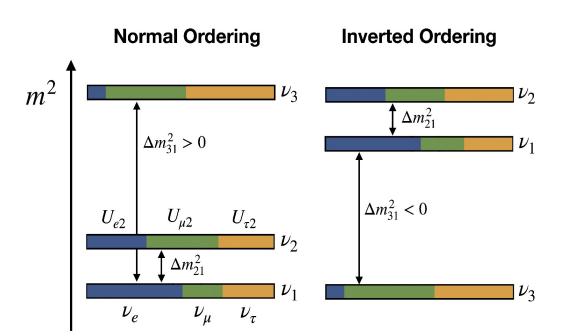


• Within the SM, neutrinos were originally postulated as exactly massless particles, appearing in three copies (ν_e, ν_u, ν_τ) corresponding to each of the charged leptons.



- The discovery of **neutrino oscillations** demonstrated that neutrinos have tiny but **nonzero** masses and that lepton flavor is **not** a conserved quantity.
- The first evidence came from atmospheric neutrinos observed at Super-Kamiokande and solar neutrinos studied by SNO.
- Followed by confirmation from reactor (KamLAND, Daya Bay, RENO, Double Chooz) and accelerator experiments (K2K, MINOS, T2K, NOvA).
- These observations firmly establish that the three active neutrinos mix via the PMNS matrix, characterized by two mass-squared splittings ($\Delta m_{21}^2, \Delta m_{31}^2$), three mixing angles, and possibly CP-violating phases.
- While the three-flavor oscillation paradigm successfully explains the bulk of experimental data, several anomalies persist which cannot be accommodated within this framework





Known in Standard Picture

$$\Delta m_{21}^2 = (6.82 - 8.04) \times 10^{-5} \, eV^2$$

$$|\Delta m_{3l}^2| = (2.42 - 2.59) \times 10^{-2} \, eV^2$$

$$\sin^2 \theta_{12} = (0.275 - 0.344)$$

$$\sin^2 \theta_{13} = (0.023 - 0.024)$$

$$\sin^2 \theta_{23} = (0.407 - 0.620)$$

Unknown in Standard Picture

 θ_{23} octant : $\theta_{23} > 45^{\circ} / \theta_{23} < 45^{\circ}$ Mass ordering : $\Delta m_{31}^2 > 0 / \Delta m_{31}^2 < 0$ Value of CP phase $(\delta_{CP}) = \delta_{13}$ Absolute mass scale

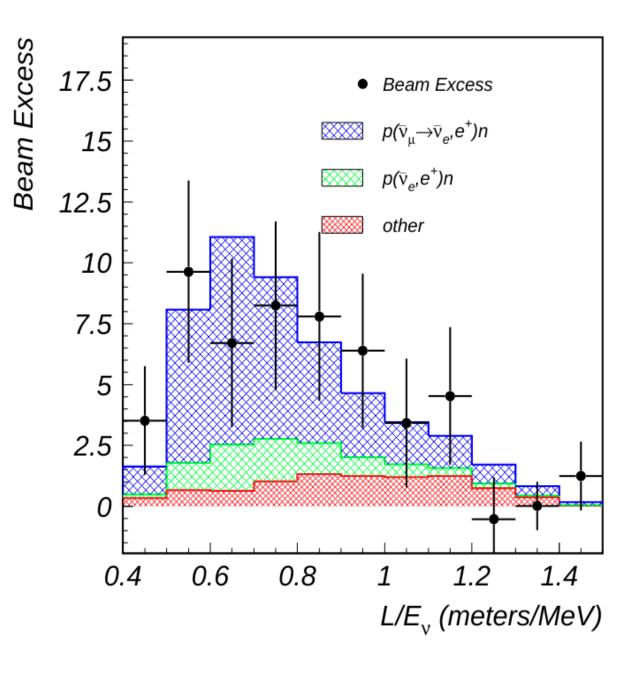
Dirac/Majorana

Neutrino Anomalies.!

LSND

LSND Anomaly

- LSND stands for Liquid Scintillator Neutrino Detector at Los Alamos National Laboratory, USA (1993–1998). This base has a length of ~30m and a neutrino energy of ~30 MeV.
- LSND used $ar{
 u}_{\mu}$ produced from the muon decay at rest $\mu^+ o e^+
 u_e ar{
 u}_{\mu}$
- Looked for $\bar{\nu}_e$ appearance via inverse beta decay $\bar{\nu}_e + p \to e^+ n$
- Found an **excess of** $\bar{\nu}_e$ events over the expected background with statistical significance 3.8σ .
- This suggests oscillation $\bar{\nu}_{\mu} \to \bar{\nu}_e$ with $\Delta m^2 \sim 1~{\rm eV}^2$ for larger than solar (7.4×10^{-5}) or atmospheric (2.5×10^{-3}) mass-squared splittings.
- Requires **an additional neutrino state**, since three neutrinos can only provide two independent Δm^2 .



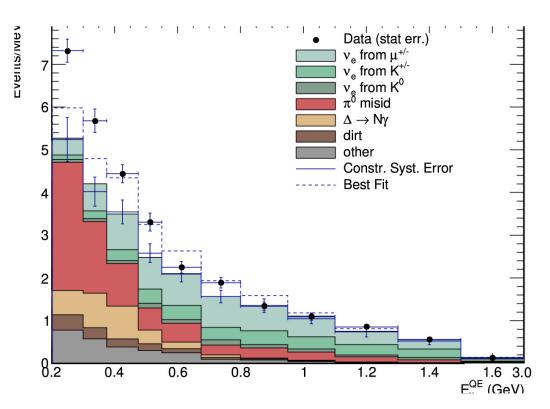
 $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}$ at 3.8σ (C.

Athanassopoulos et al, PRL 1995

MiniBooNE Anomaly

- MiniBooNE stands for Mini Booster Neutrino Experiment at Fermilab, USA (2002–2019): baseline: 541 m; Neutrino energy: 200–1250 MeV.

 MiniBooNE
- The goal was to test LSND anomaly at higher energy and longer baseline independently.
- Setup: Neutrino beam from pion decay-in-flight, primarily u_{μ}
- MiniBooNE detected an excess of electron-like events in both u and $\bar{\nu}$ modes.
- Combined excess significance: 4.8σ (2020).
- Consistent with additional neutrino oscillations $\nu_{\mu}
 ightarrow \nu_e$ at $\Delta m^2 \sim 1~{\rm eV}^2$



 $\nu_{\mu} \rightarrow \nu_{e}$ at 4.8 σ (Aguilar-Arevalo et al.,PRL,2009)

- However, MiniBooNE can't distinguish electrons from single photons so **photon-like backgrounds (e.g.** π ° **misidentification)** might mimic a signal.
- MicroBooNE (2021, LArTPC detector) observed no evidence for excess ν_e events, casting doubt on the oscillation interpretation though it doesn't fully explain MiniBooNE's anomaly.
- The puzzle remains unresolved.

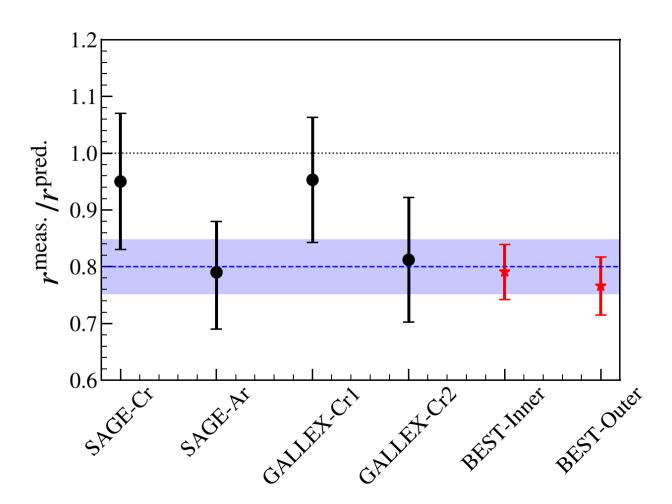
Reactor Antineutrino Anomaly

- Nuclear reactors produce huge fluxes of $ar{
 u}_e$ from eta-decays of fission fragments.
- Detected via inverse beta decay (IBD).
- Around **2011**, updated theoretical models (Huber and Mueller) predicted slightly higher $\bar{\nu}_e$ fluxes than older models.
- When compared to data from ~20 short-baseline reactor experiments (L < 100 m), a deficit of ~6% was observed. Leading to the statistical significance: 2.8σ .
- Could indicate oscillations into **an additional neutrino state** with $\Delta m^2 \sim 1\,\mathrm{eV}^2$ and small mixing angle. However, recent data (DANSS, NEOS, STEREO, PROSPECT) suggest that the anomaly may instead arise from inaccurate reactor flux predictions, especially from U^{235} and Pu^{239} .
- Still debated. The flux-shape distortion ("5 MeV bump") complicates the interpretation.

Gallium Anomaly

Gallium (Source) Anomaly

- GALLEX and SAGE solar neutrino detectors calibrated using intense radioactive neutrino sources (eg. Cr^{51} and Ar^{37}) with baseline: ~1–2 m; Energy: ~1 MeV.
- Measured event rates were 10-20% lower than predicted. Combined significance: ~2.8 σ deficit.



Deficit in ν_e at GALLEX, SAGE, BEST (Barinov et al.,2021)

- This may be due to the short-baseline oscillation $\nu_e o \nu_s$ with $\Delta m^2 \sim 1~{\rm eV}^2$.
- Same mass range as LSND and MiniBooNE hints potentially a common additional neutrino origin
- Recently, **BEST (Baksan Experiment on Sterile Transitions)** (2022) confirmed the deficit (~20%), strengthening the sterile neutrino interpretation.

T2K and NOvA Tensions

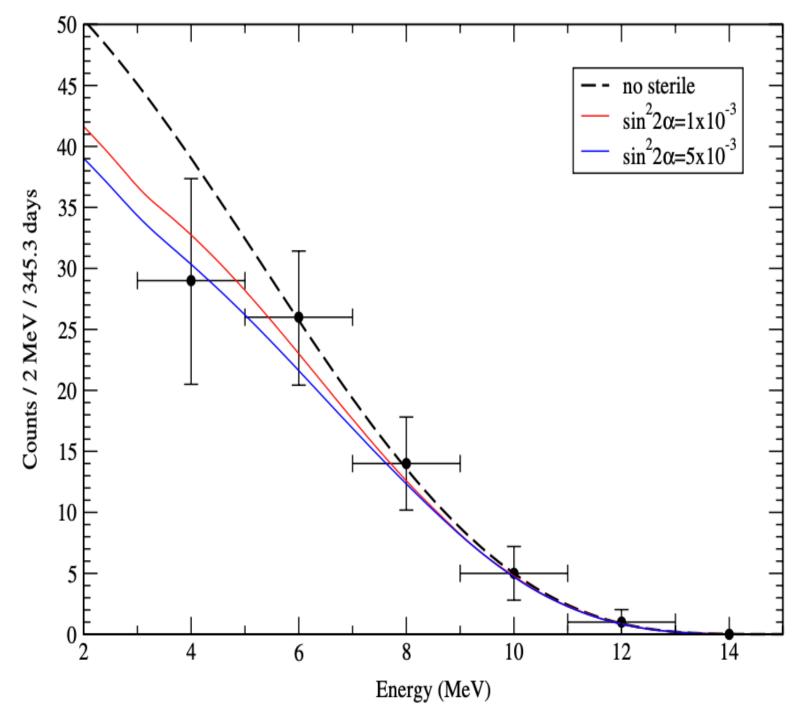
- T2K (Japan) and NOvA (USA) are long-baseline accelerator neutrino experiments (study $\nu_{\mu}
 ightarrow \nu_{e}$ appearance).
- Goal: Measure CP violation (δ_{CP}), mass ordering, and θ_{23} in the three-flavor framework.
- T2K favors large CP violation, NO and maximal $\theta_{23} \approx 45^{\circ}$: it observes more ν_e appearance events than NOvA.
- NOvA also prefers NO but δ_{CP} lies near 0 or π and a non-maximal θ_{23} : it observed ν_e appearance is **less** than T2K.
- When both datasets are combined under the **3-flavor framework**, there's a **~2–2.5\sigma tension** in the preferred values of δ_{CP} and θ_{23} .
- This may be interpreted as either statistical fluctuations or systematic uncertainty.
- If the δ_{CP} or θ_{23} tensions remain even after improved systematics, it could point to: NP light **additional** neutrino states ($\Delta m_{n1}^2 \leq 10^{-2}\,\mathrm{eV}^2$)

Solar upturn problem

• Solar MSW-LMA predicts a **rise (upturn)** in the electron-neutrinc survival probability $P_{ee}(E)$ at low energies (few MeV \rightarrow sub-MeV), but current data (SNO, Super-K, Borexino) show a **weaker or absent upturn** than predicted.

- Introducing a very light **additional neutrino state** $\Delta m_{01}^2 \sim 10^{-5}$ adds an extra oscillation frequency with a long wavelength comparable to the Sun-Earth scale.
- This modifies the low-energy shape of $P_{ee}(E)$ and can suppress the expected upturn.

[PhysRevD. 83,113011]



Sterile Neutrinos?









- Sterile neutrinos are hypothetical neutral fermions that do not interact via the SM weak force only through mixing with the active neutrinos (ν_e, ν_μ, ν_τ).
- Why: Since the invisible width of the Z boson measured at LEP constrains the number of light active neutrinos to be three, any additional light states must be "sterile," i.e., singlets under the SM gauge interactions.
- Additionally, these sterile states must be **non-thermal**: The early Universe's expansion rate and structure formation depend on the **number of relativistic species** during recombination, quantified by N_{eff} .
- The Standard Model predicts $N_{
 m eff}pprox 3.046$. A fully thermalized sterile neutrino would increase $N_{
 m eff}$ to pprox 4.
- Planck (CMB) + BAO data: $N_{\rm eff}=2.99\pm0.17$, disfavoring such extra species. Also, the total neutrino mass bound $\sum m_{\nu} < 0.12$ eV is inconsistent with an eV-scale sterile neutrino.
- This interprets: If sterile neutrinos exist, they must be **non-thermal**, **weakly coupled**, or **interacting with a dark sector**, preventing full thermalization.

Sterile Neutrinos?







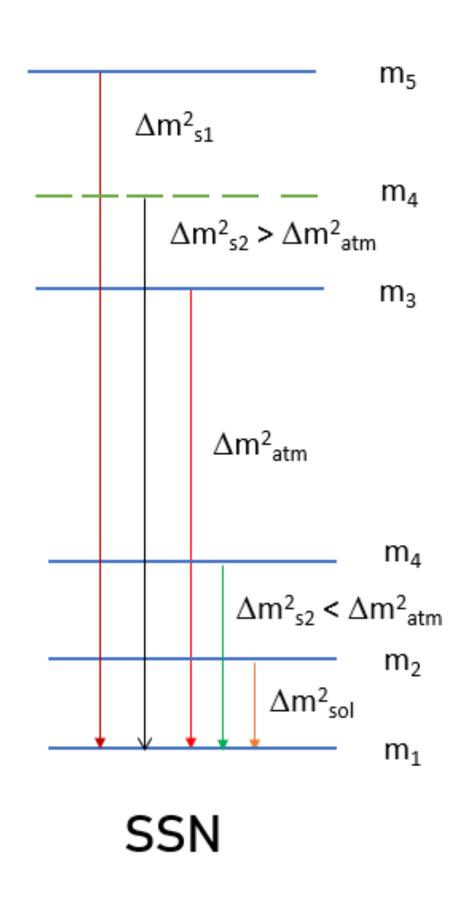


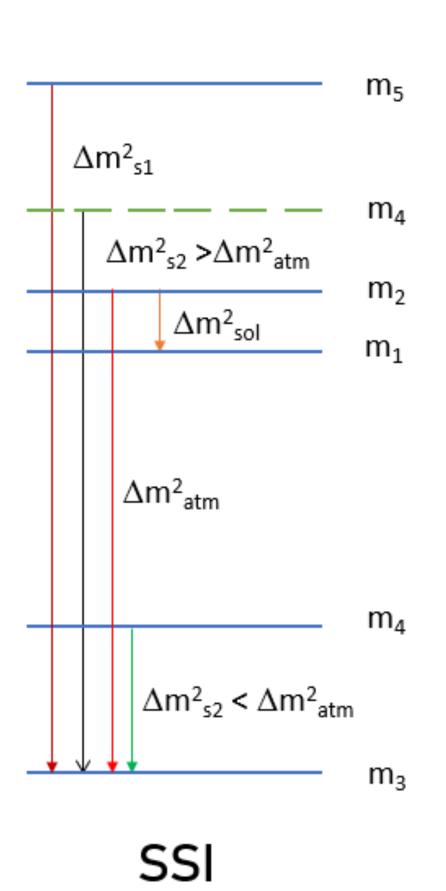
- Naturally appear in many extensions of the SM, e.g. **Type-I Seesaw models** (as right-handed neutrinos).
- Proposed to explain neutrino anomalies such as LSND, MiniBooNE, reactor and Gallium deficits.
 Also, help with tensions in long-baseline data (T2K-NOvA).
- They appear in Oscillations just by expanding the flavor states to include one (or more) sterile states:

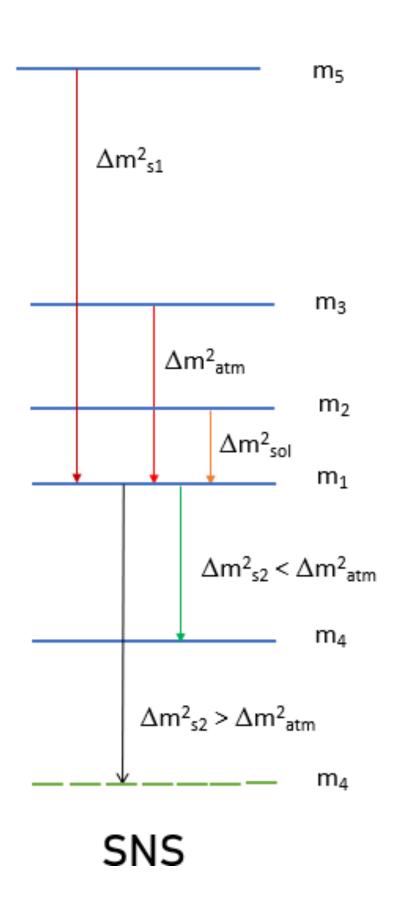
$$u_lpha = \sum_{i=1}^{3+N_s} U_{lpha i} \,
u_i, \quad lpha = e, \mu, au, s$$

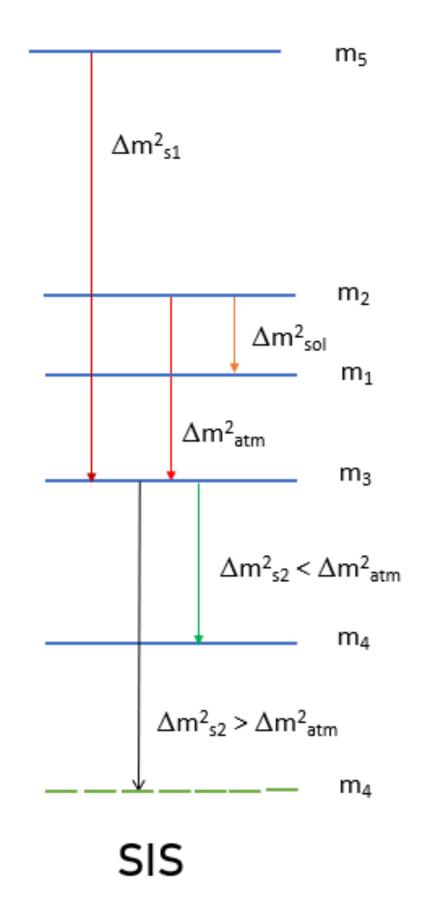
- This led to new mass-squared splittings and mixing angles.
- The extra oscillation frequency can cause **short-baseline appearance/disappearance** or **interference effects** in long-baseline experiments.
- Sterile neutrinos can exist in several mass scales: GeV-TeV (Seesaw), KeV (Warm dark matter), eV (LSND, MiniBooNE anomalies) and sub-eV (T2K-NOvA tension)

Classification of mass ordering in 3 + 2 sterile neutrino framework









Mass Constraints on 3+2 Sterile Neutrino model

Sum of neutrinos Σ

• Direct probes of the absolute neutrino mass scale arise from cosmological observations, which are sensitive to the sum of all neutrino mass eigenvalues,

$$\Sigma = \sum_i m_i.$$

• In the minimal three-neutrino framework, this quantity is determined by the lightest neutrino mass $m_{
m lightest}$ together with the precisely measured mass-squared differences from oscillation experiments.

$$m_1 = m_{ ext{lightest}}, \ m_2 = \sqrt{m_{ ext{lightest}}^2 + \Delta m_{21}^2}, \ m_3 = \sqrt{m_{ ext{lightest}}^2 + \Delta m_{31}^2},$$

$$m_3 = m_{\text{lightest}}, \ m_2 = \sqrt{m_{\text{lightest}}^2 + |\Delta m_{32}|^2}, \ m_1 = \sqrt{m_{\text{lightest}}^2 + |\Delta m_{32}|^2 - \Delta m_{21}^2}.$$

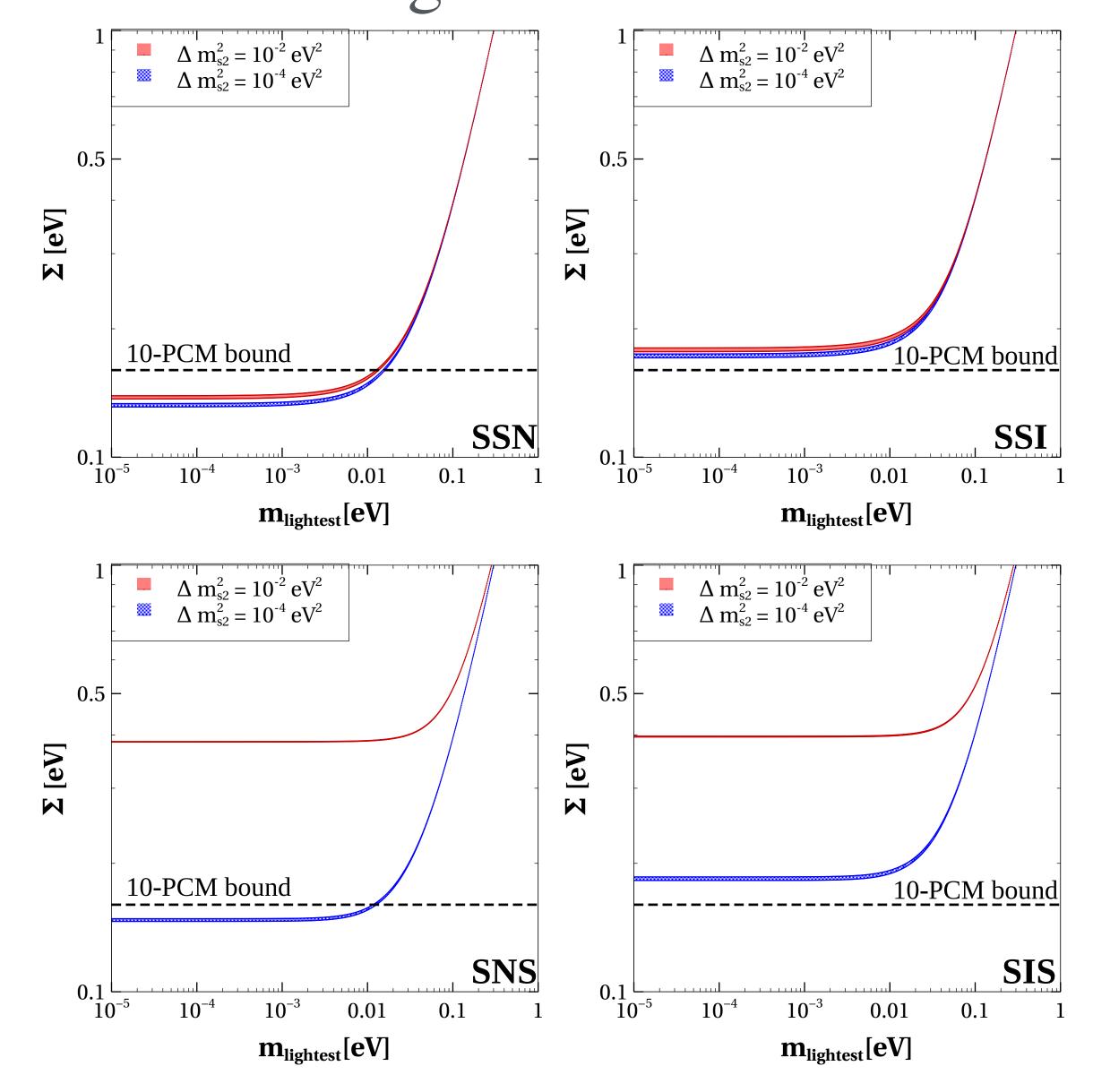
- The Planck 2018 data in combination with BAO measurements impose an upper bound $\Sigma \lesssim 0.12$ eV (95% C.L.)
- In the 3+2 framework, we have two additional mass eigenstates m_4 and m_5 which enter the sum.

Cosmological constraint on Σ

- If the active-sterile mixing is large, sterile neutrinos become fully thermalised in the early Universe, following the same Fermi-Dirac distribution as active flavors.
- As a result, they contribute equally to the effective number of relativistic degrees of freedom, $N_{
 m eff}$, and to the total mass sum, which is enhanced relative to the three-flavor case.
- The precise measurement of CMB by the Planck collaboration constrains the $N_{\rm eff}=2.99^{+0.34}_{-0.33}$ (95 % *CL*) which ruled out the possibility of an extra sterile state.
- This tension can only be alleviated if the sterile neutrinos are not fully thermalized in the early Universe.
- In such cases, physical masses of the sterile neutrinos do not contribute to Σ .

$$\Sigma^{3+2} = m_1 + m_2 + m_3 + \Delta N_{\text{eff}} (m_4 + m_5).$$

Σ vs $m_{ ext{lightest}}$ in different mass schemes



- SSN scenario is favored by 10-PCM bound up to $m_{\rm lightest}\sim 0.015$ eV for both the mass squared differences, i.e $\Delta m_{\rm s2}^2=0.01~\&~10^{-4}~\rm eV^2$.
- SSI scenario is disfavored by the 10-PCM bound for both the mass squared differences.
- SNS scenarios is disfavored for $\Delta m_{s2}^2=0.01\,\mathrm{eV^2}$ however, it is allowed for $\Delta m_{s2}^2=10^{-4}\,\mathrm{eV^2}$ up to $m_{\mathrm{lightest}}\sim0.010\,\mathrm{eV}$ as of 10-PCM bound is concerned.
- On the other hand, the SIS scenario is completely disfavor by the 10-PCM bound.

Mass ordering	$\Delta m_{s1}^2 = 1.3 \text{ eV}^2$		
	$\Delta m_{s2}^2 = 0.01 \text{ eV}^2$	$\Delta m_{s2}^2 = 10^{-4} \text{ eV}^2$	
SSN	< 0.011	< 0.014	
SSI	Disallowed	Disallowed	
SNS	Disallowed	< 0.010	
SIS	Disallowed	Disallowed	

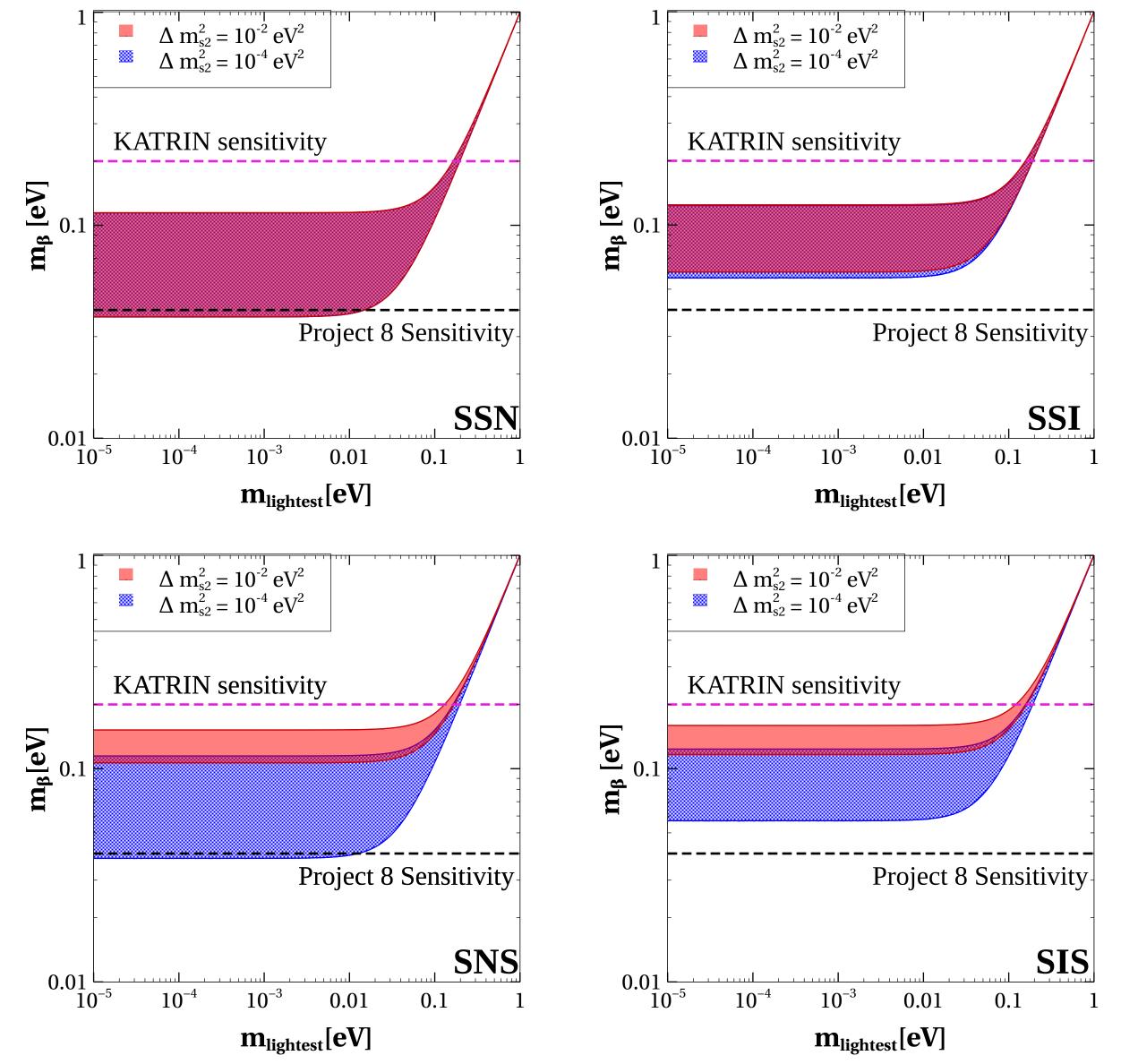
Effective Electron Neutrino Mass in Beta Decay

• The effective electron-neutrino mass measured in β -decay experiments is given by $m_{\beta} = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$,

$$m_{eta} = \sqrt{\sum_i |U_{ei}|^2 m_i^2},$$

- This quantity reflects the incoherent sum of neutrino mass contributions to the electron spectrum.
- In tritium β -decay: ${}^3{
 m H} o {}^3{
 m He}^+ + e^- + \bar{\nu}_e$ the **endpoint energy** of the emitted electron depends on the neutrino mass.
- If neutrinos are massive, the electron energy spectrum near the endpoint is slightly distorted the maximum kinetic energy of the electron is reduced by m_{eta} .
- Thus, by precisely measuring this endpoint shape, one can put a bound on the neutrino mass, independent of any assumptions about whether neutrinos are Majorana or Dirac.
- KATRIN (2024) put the most stringent direct bound on the effective neutrino mass to date $m_{\beta} < 0.8$ eV (90% CL). Project 8 (atomic tritium): aims for $m_{\beta} \sim 40$ meV.
- When a sterile neutrino mixes with $\nu_{e'}$ the β -spectrum gains an **additional component**: it leaves a measurable imprint on the β -spectrum.

m_{eta} vs $m_{ ext{lightest}}$ in different mass schemes



- There is an overlap of both contributions $\Delta m_{s2}^2 = 0.01~\&~10^{-4}~\rm eV^2~in~SSN~and~SSI.~This~is~not~the~case~with~SNS~and~SIS.$
- Although the current and projected KATRIN sensitivity $(m_{\beta} < 0.2 \ \text{eV}) \ \text{allows all the mass-ordering schemes}$ considered, they are disfavored once the more stringent Project 8 sensitivity is taken into account.
- In the case of normal ordering, the SSN scenario is consistent with KATRIN up to $m_{\rm lightest} \sim 0.015$ eV, while the SNS scheme is allowed up to $m_{\rm lightest} \sim 0.01$ eV.
- $\Delta m_{s2}^2=10^{-4}\,\mathrm{eV}^2$ contribution can meet Project 8 sensitivity in SNS upon considering the 3 σ uncertainties.
- However, the 10-PCM bound on the lightest neutrino mass strongly excludes both inverted ordering schemes (SSI and SIS).

Effective Majorana Mass in Neutrinoless Double Beta Decay

• The effective Majorana mass,

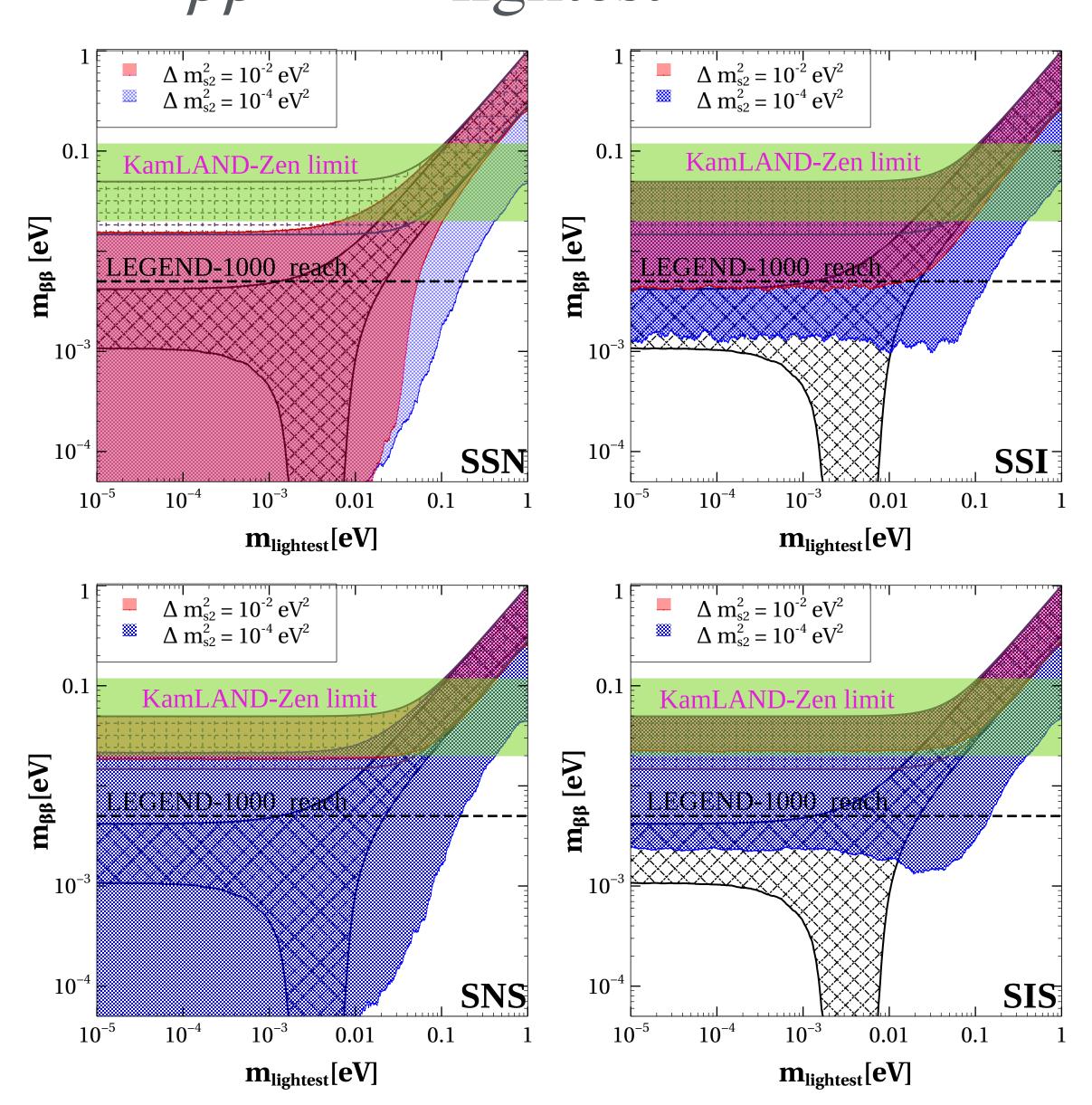
$$m_{etaeta} = \left|\sum_{i=1}^3 U_{ei}^2 \, m_i \, e^{ilpha_i} \right|,$$

- Probed in neutrinoless double- β decay (Ov $\beta\beta$) sensitive only if neutrinos are Majorana.
- Sensitive to phases and cancellations among mass terms: Because $m_{\beta\beta}$ involves coherent summation, the Majorana phases can lead to constructive or destructive interference
- Current limits (KamLAND-Zen, GERDA, CUORE):

$$m_{\beta\beta} \lesssim (0.028 - 0.122) \,\mathrm{eV}.$$

- 3+2 sterile neutrinos **can significantly alter** both the magnitude and the interpretation of the $Ov\beta\beta$ bounds.
- Next-generation detectors: nEXO, LEGEND-1000, CUPID, KamLAND-Zen 800, NEXT with target sensitivity $m_{\beta\beta}\sim 10$ meV.

$m_{\beta\beta}$ vs $m_{ ext{lightest}}$ in different mass schemes



SSN

Region	$m_{etaeta}^{ m NO}~({ m eV})$	$ t_{14}^2 m_4 \text{ (eV)}$	$ t_{14}^2 m_4 \text{ (eV)}$	$ t_{15}^2 m_5 $ (eV)
		$(\Delta m_{s2}^2 = 10^{-4})$	$(\Delta m_{s2}^2 = 10^{-2})$	$(\Delta m_{s1}^2 = 1.3)$
$m_1 \approx 0$	0.001 - 0.004	$1.0 \times 10^{-3} - 2.0 \times 10^{-3}$	$5.0 \times 10^{-6} - 5.0 \times 10^{-5}$	$1.14 \times 10^{-3} - 1.14 \times 10^{-2}$
$m_1 pprox \sqrt{\Delta m_{ m sol}^2}$	0.0018 - 0.018	$1.319 \times 10^{-3} - 2.638 \times 10^{-3}$	$5.0 \times 10^{-6} - 5.0 \times 10^{-5}$	$1.14 \times 10^{-3} - 1.14 \times 10^{-2}$
$m_1 \approx 0.1 \text{ eV}$	0.02 - 0.10	$1.005 \times 10^{-2} - 2.010 \times 10^{-2}$	$7.07 \times 10^{-6} - 7.07 \times 10^{-5}$	$1.145 \times 10^{-3} - 1.145 \times 10^{-2}$

SSI

Region	$m_{etaeta}^{ m IO}~({ m eV})$	$ t_{14}^2 m_4 \text{ (eV)}$	$ t_{14}^2 m_4 \text{ (eV)}$	$ t_{15}^2 m_5 \text{ (eV)}$
		$(\Delta m_{s2}^2 = 10^{-4})$	$(\Delta m_{s2}^2 = 10^{-2})$	$(\Delta m_{s1}^2 = 1.3)$
$m_3 \approx 0$	0.015 - 0.050	$1.00 \times 10^{-3} - 2.00 \times 10^{-3}$	$5.00 \times 10^{-6} - 5.00 \times 10^{-5}$	$1.140 \times 10^{-3} - 1.140 \times 10^{-2}$
$m_3 pprox \sqrt{\Delta m_{ m sol}^2}$	0.0018 - 0.018	$1.319 \times 10^{-3} - 2.638 \times 10^{-3}$	$5.02 \times 10^{-6} - 5.02 \times 10^{-5}$	$1.140 \times 10^{-3} - 1.140 \times 10^{-2}$
$m_3 \approx 0.1 \text{ eV}$	0.02 - 0.10	$1.005 \times 10^{-2} - 2.010 \times 10^{-2}$	$7.07 \times 10^{-6} - 7.07 \times 10^{-5}$	$1.145 \times 10^{-3} - 1.145 \times 10^{-2}$

SNS & SIS

Region	m_{etaeta} (eV)		$ t_{14}^2 m_4 \; ({ m eV})$	$ t_{14}^2 m_4 \; ({ m eV})$	$ t_{15}^2 m_5 $ (eV)
	NO	IO	$(\Delta m_{s2}^2 = 10^{-4})$	$(\Delta m_{s2}^2 = 10^{-2})$	$(\Delta m_{s1}^2 = 1.3)$
$m_4 \approx 0$	0.001 - 0.004	0.015 - 0.050	0	0	$1.14 \times 10^{-3} - 1.14 \times 10^{-2}$
$m_4 \approx 0.01 \text{ eV}$	0.017 - 0.055	0.0018 - 0.018	$1.00 \times 10^{-3} - 2.00 \times 10^{-3}$	$5.00 \times 10^{-7} - 5.00 \times 10^{-6}$	$1.140 \times 10^{-3} - 1.140 \times 10^{-2}$
$m_4 \approx 0.1 \text{ eV}$	0.05 - 0.15	0.02 - 0.10	$1.00 \times 10^{-2} - 2.00 \times 10^{-2}$	$5.00 \times 10^{-6} - 5.00 \times 10^{-5}$	$1.145 \times 10^{-3} - 1.145 \times 10^{-2}$

Summary

- Although the three-flavor oscillation paradigm successfully explains several experimental data, anomalies persist which cannot be accommodated within this framework.
- LSND, MiniBooNE, etc, short-baseline-expt. suggest the need for eV-scale sterile neutrinos.
- T2K and NOvA (long-baseline-expt.) suggest the need for sub-eV scale sterile neutrinos.
- By combining oscillation data, constraints from neutrinoless double beta decay, cosmological considerations and tritium β decay, we aim to delineate the viable parameter space for the 3+2 model.
- 10-PCM bound allows SSN and SNS. KATRIN bound on m_{β} allows all 4 mass schemes, SSN and SNS show better for the future Project-8 sensitivity on m_{β} .
- For $m_{\beta\beta}$, eV-scale sterile is dominating, sub-eV sterile for 10^{-4} term becomes comparable to the active contribution, sub-eV sterile for 10^{-2} is subleading. Cancellation is possible depending on the phase.