Finite Volume Effects on QCD Phase Diagram Using NJL Model

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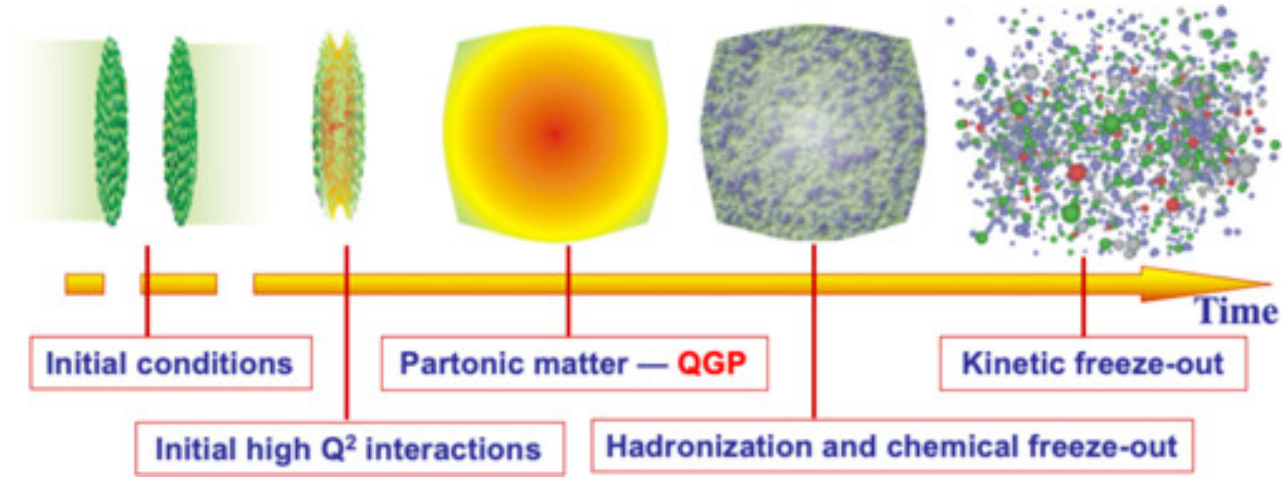
Introduction and Motivation



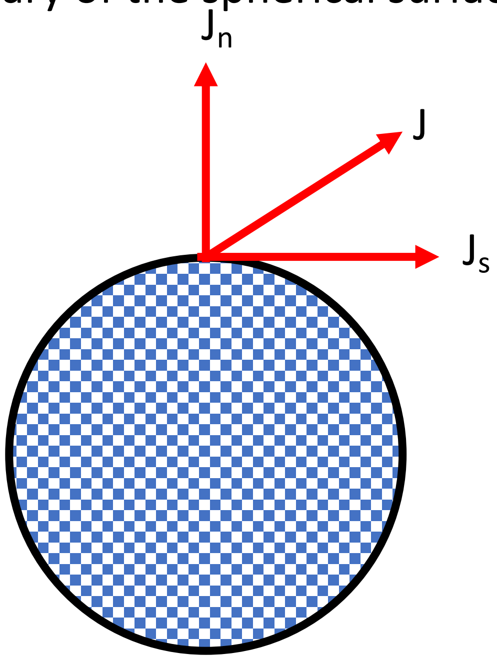
- Quark-Gluon Plasma (QGP) is a state of matter where quarks and gluons are no more confined inside hadrons.
- It exists at very temp (T > 170 MeV) or high baryon densities.
- QGP is produced in heavy-ions collision experiments (RHIC, LHC) where heavy ion like Au-Au, Cu-Cu etc collide at relativistic velocities to produce QGP.
- Size of QGP is ~ fm comparable to the typical strong interaction scale.
- QGP is rapidly evolving and small in size where the geometry and boundary conditions are important for theoretical understanding and realistic analysis.
- We study the effect of finite size of QGP on the QCD phase diagram using NJL model in spherical geometry and MIT boundary condition.

MIT boundary condition

- Finite volume system requires a shape geometry and proper boundary condition.
- Complicated dynamics: QGP medium evolution is time dependent and shape differs event by event.
- Simplification: Spherical geometry containing deconfined QGP and MIT boundary condition on the surface.
- MIT boundary condition: Normal component ($\hat{n}$) of quark current vanishes at the boundary of the spherical surface. This mimics the confinement of quarks inside the sphere.

$$J_n = n_\mu \bar{\psi} \gamma^\mu \psi = 0$$

$$(1 - i\hat{n} \cdot \vec{\gamma})\, \psi(t, r, \theta, \phi)\Big|_{r=R} = 0$$



- Allowed momentum modes obtained by solving Dirac equation.

$$[i\gamma^\mu \partial_\mu - \mathrm{M}]\psi_j = 0$$

$$j_{l_k}(pR) = -sign(k)\left(\frac{p}{E+M}\right) j_{l_{\bar{k}}}(pR)$$

- Previous studies:
  Chernodub et al., JHEP 01, 136 (2017) (fermions in rotating cylinder with MIT b.c)
  Zhang et al., Phys. Rev. D 101, 043006 (2020) (NJL Model in a sphere with MIT b.c)
  Wang et al, Mod. Phys. Lett A 39,1850232 (2018) (NJL Model with cubic geometry and periodic, anti periodic, swc b.c

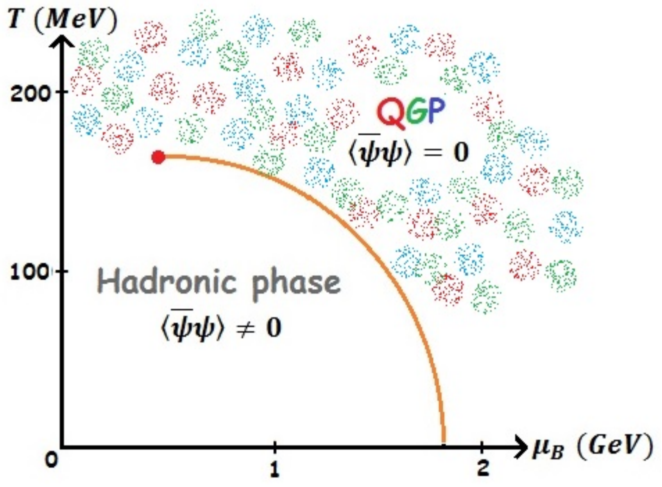
NJL model

- Nambu–Jona-Lasinio (NJL): Effective model to study spontaneous breaking of chiral symmetry in QCD
  ($SU(2)_V \times SU(2)_A \rightarrow SU(2)_V$)

$$\mathcal{L}_{NJL} = \bar{\psi}(i\gamma_\mu \partial^\mu - \hat{m})\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\bar{\tau}\psi)^2\right] \qquad (N_f = 2, N_c = 3)$$

- In the mean-field approximation, the four-fermion interaction term generates the dressed quark mass: M = m + σ where σ = −2G$\langle\bar{\psi}\psi\rangle$

$$\langle\bar{\psi}\psi\rangle = -2\, N_c\, N_f \int \frac{d^3p}{(2\pi)^3}\frac{M}{E}\left(1 - \frac{1}{1+e^{(E-\mu)/T}} - \frac{1}{1+e^{(E+\mu)/T}}\right)$$



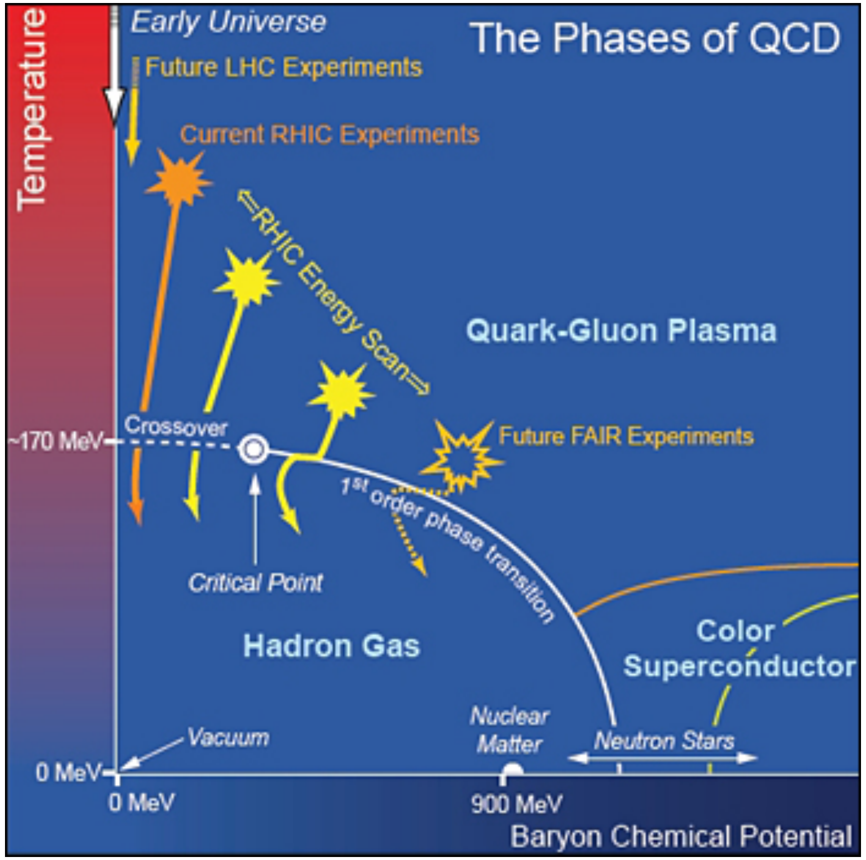
- vacuum term is regularised by proper time regularisation method

- Effective potential :

$$\Omega(\sigma) = \frac{\sigma^2}{4G} - 2\, N_c\, N_f \int_{-\infty}^{\infty} \frac{d^3p}{(2\pi)^3}\left[E + T\log\left(1 + e^{-(E-\mu)/T}\right) + T\log\left(1 + e^{-(E+\mu)/T}\right)\right]$$

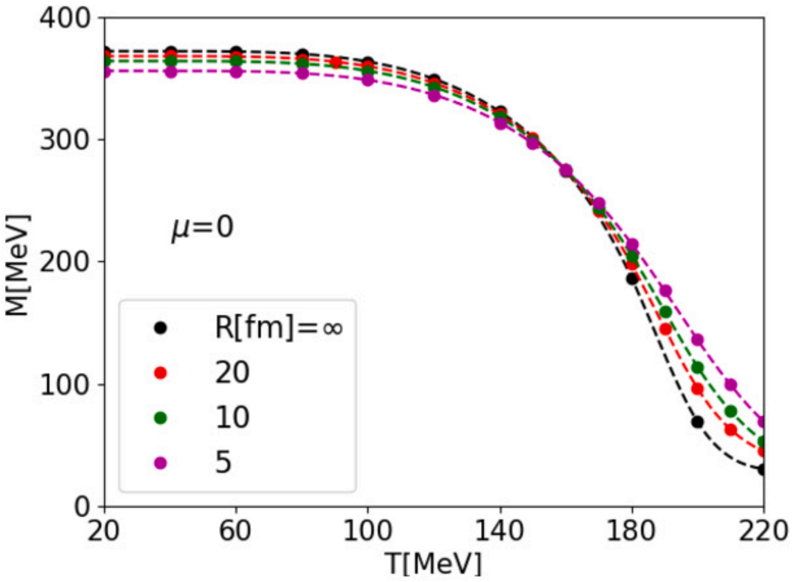
- Minimize effective potential to get M: $\frac{\partial\Omega(\sigma)}{\partial\sigma} = 0$

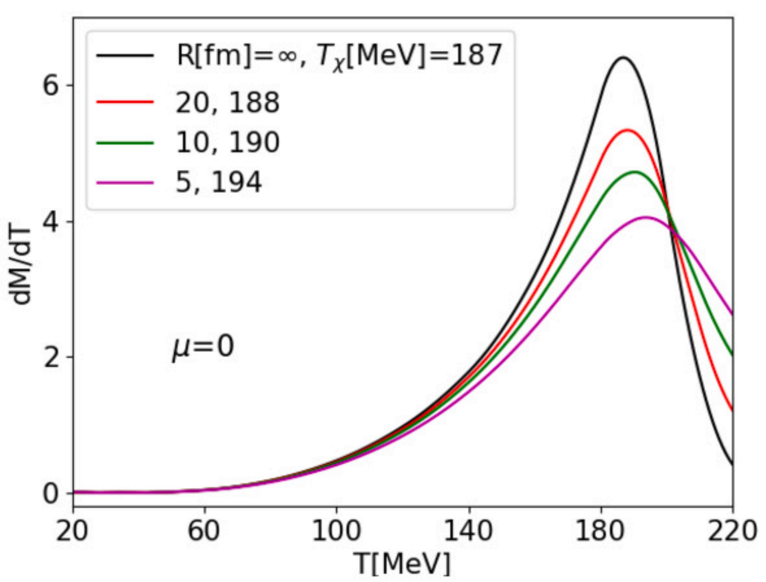
QCD phase diagram



- Lattice QCD - At low μ_B , QCD transition from confined to deconfined phase is a cross over.
- At high μ_B , QCD phase transition is a first order phase transition (Effective Models)
- The search for QCD critical point is still actively going on in heavy ion collision experiments

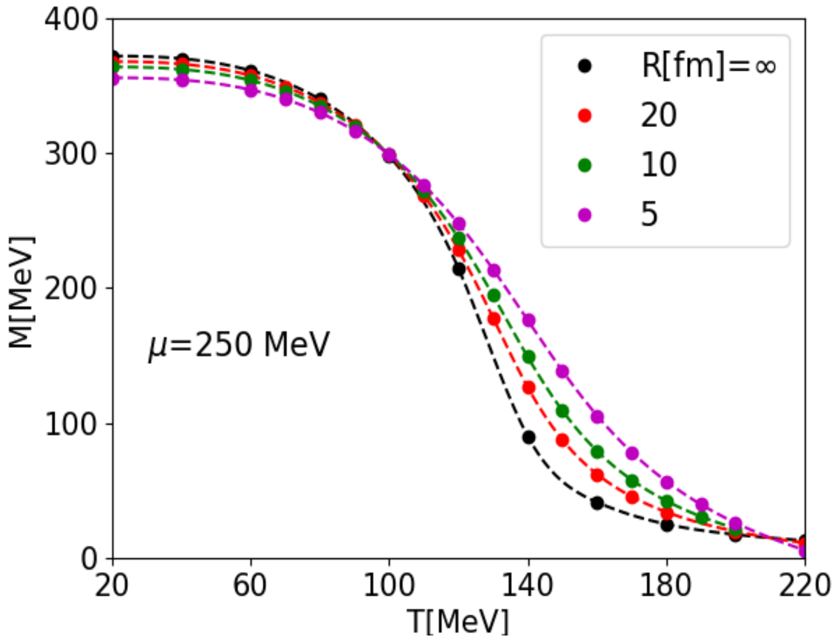
Results: Chiral transition at μ=0

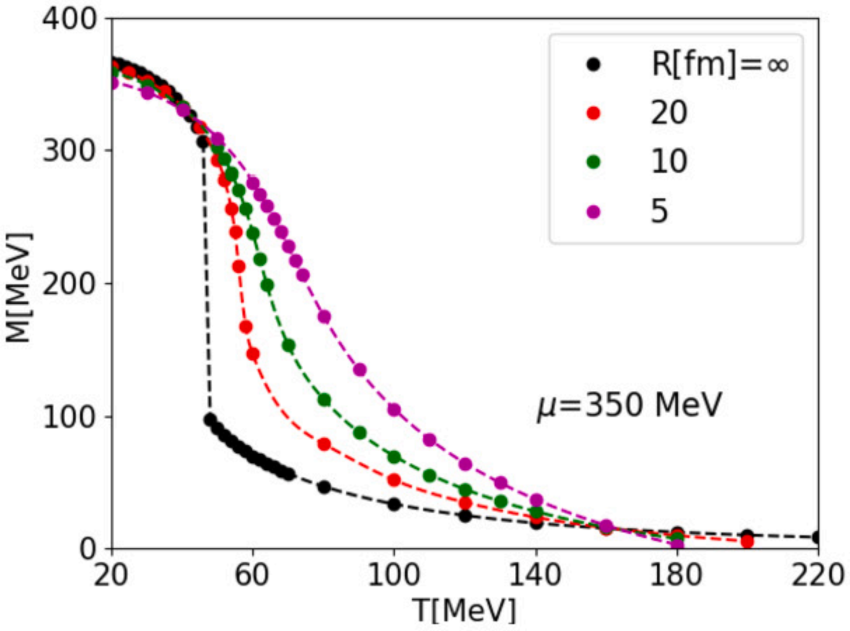




- At μ = 0, chiral symmetry is broken at low temperature (M large) and the chiral symmetry is restored at high temperature M -> 0.
- M decreases slightly due to finite volume effect at low temp compared to infinite volume. But M increases as volume of the system decreases at high temp.
- At μ = 0, chiral symmetry restoration is a crossover from confined phase to deconfined phase.
- Chiral crossover temperature T_χ estimated from the peak position of the derivative dM/dT. T_χ increases slightly when the volume of the system decreases.

Chiral transition at finite μ

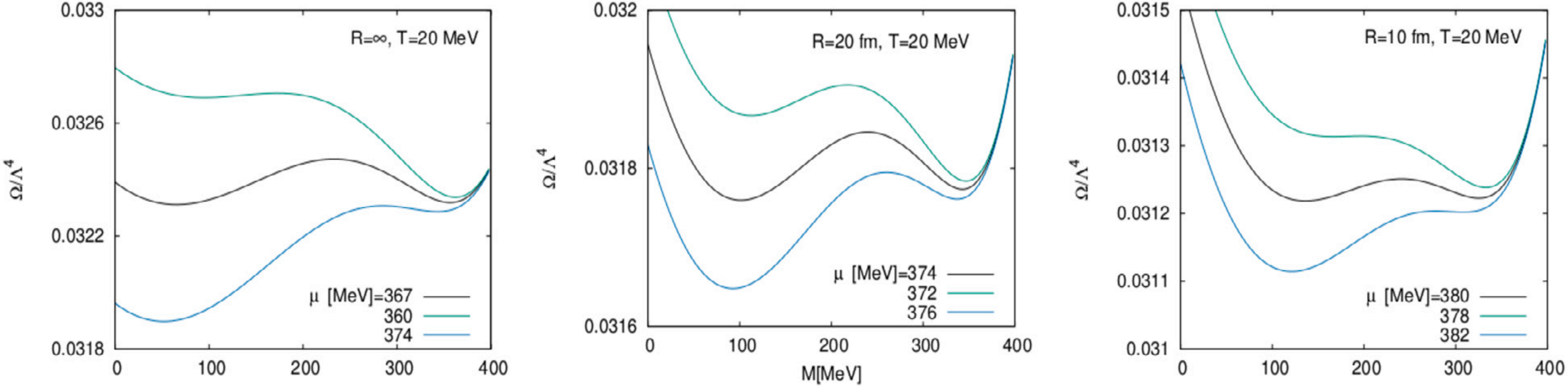




- As μ increases, the crossover shifts towards small temp.
- At μ = 250 MeV, the chiral transition is still a crossover.
- For large μ, the crossover transition become stiffer for larger volumes.
  At μ = 350 MeV, discontinuity in M says that it is a first order phase transition for infinite volume. But it is still crossover for finite volume.

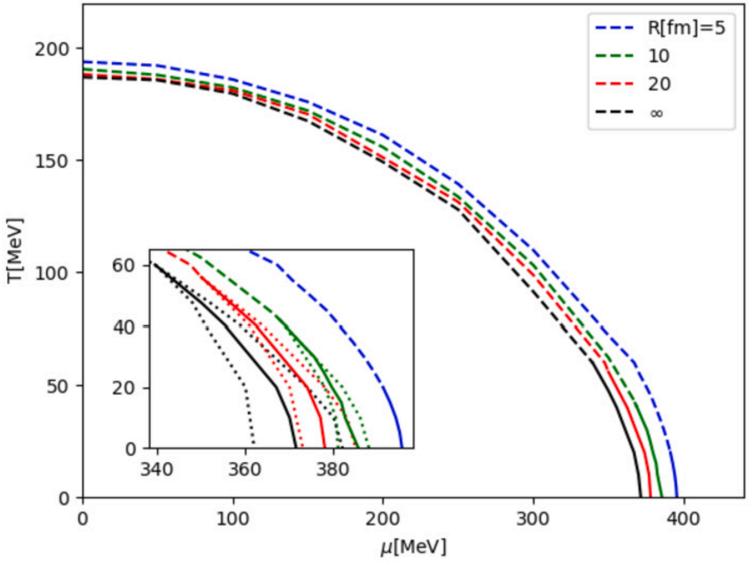
Effective potential for first-order phase transition:

- The nature of phase transition can be also be studied from the shape of effective potential.
- Two local minima structures with equal depths signal a first order phase transition.



- As volume decreases, the chiral transition occurs at higher chemical potential for the same temp.

QCD phase diagram with finite volume



- At very low temp, phase transition is first order for all system sizes.
- Shift of the phase transition (solid lines) towards larger μ is significant as volume is reduced.
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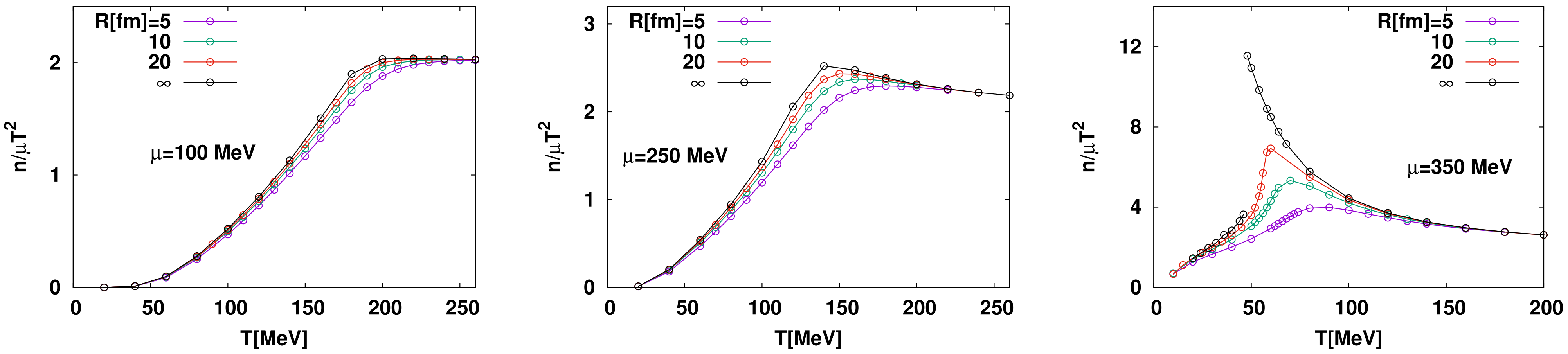
- Shift of the crossover (dashed lines) for small μ is relatively milder.
- Coexistence region shrinks for smaller volumes and shifted towards lower T
  (dotted lines indicate boundary of coexistence regions).

Net quark number density



- The baryon number density (n_B) and its susceptibilities are related to experimentally determined quantities (e.g., cumulants ratio), especially important at high μ_B near the first-order line and its end-point.
- Quark number density ($n = 3n_B$) shows volume dependence in the crossover/transition region.

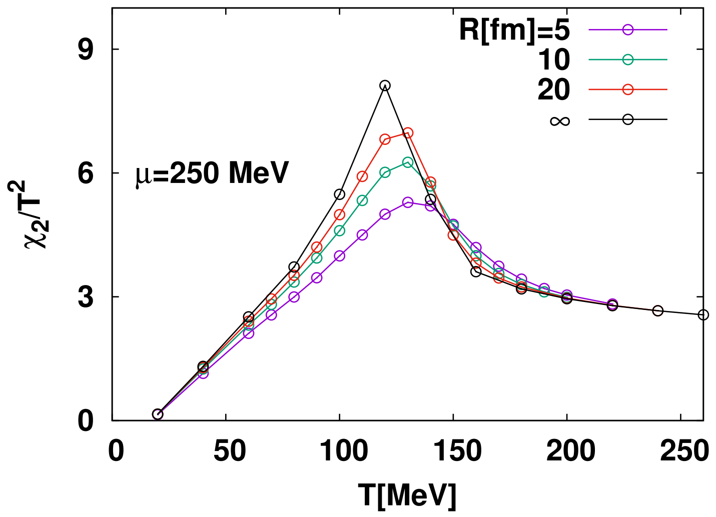
$$n = \frac{\partial P}{\partial \mu} = 2N_c N_f \int \frac{d^3p}{(2\pi)^3} \left(\frac{1}{1+e^{(E-\mu)/T}} - \frac{1}{1+e^{(E+\mu)/T}} \right)$$

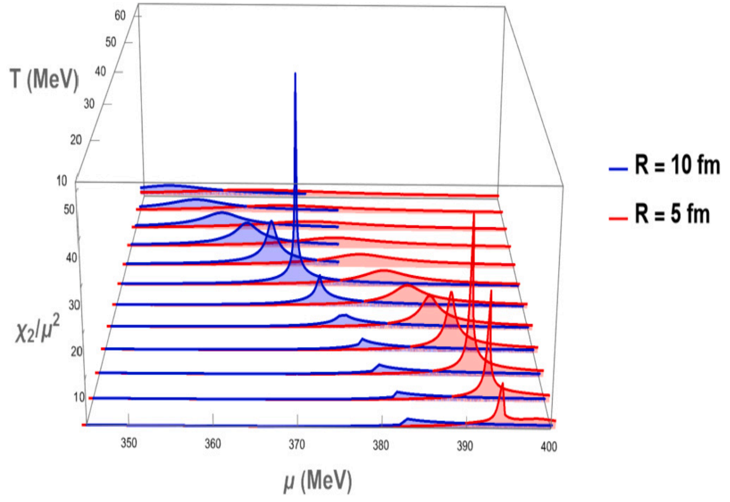
- At low $\mu \leq 100$ MeV , the volume dependence of n is small except in the crossover region (150 MeV ≤ T ≤ 200 MeV).
- n increases as temperature increases and reaches its asymptotic behaviour ~ μT^2 at high temperature.
- Crossover is sharper for high μ and larger volume. At μ = 350 MeV, infinite volume system shows discontinuous first order phase transition.
- But finite volume system still shows a sharp crossover
- For high μ, asymptotic limit ~ μT^2 reached at low T

Quark number susceptibilities

- Non-linear susceptibilities of conserved quantities (e.g., baryon number) are sensitive for critical point search.
- Susceptibilities of kth order are defined as

$$\chi_k = \frac{\partial^k P}{\partial \mu^k} = \frac{\partial^{k-1} n}{\partial \mu^{k-1}}$$

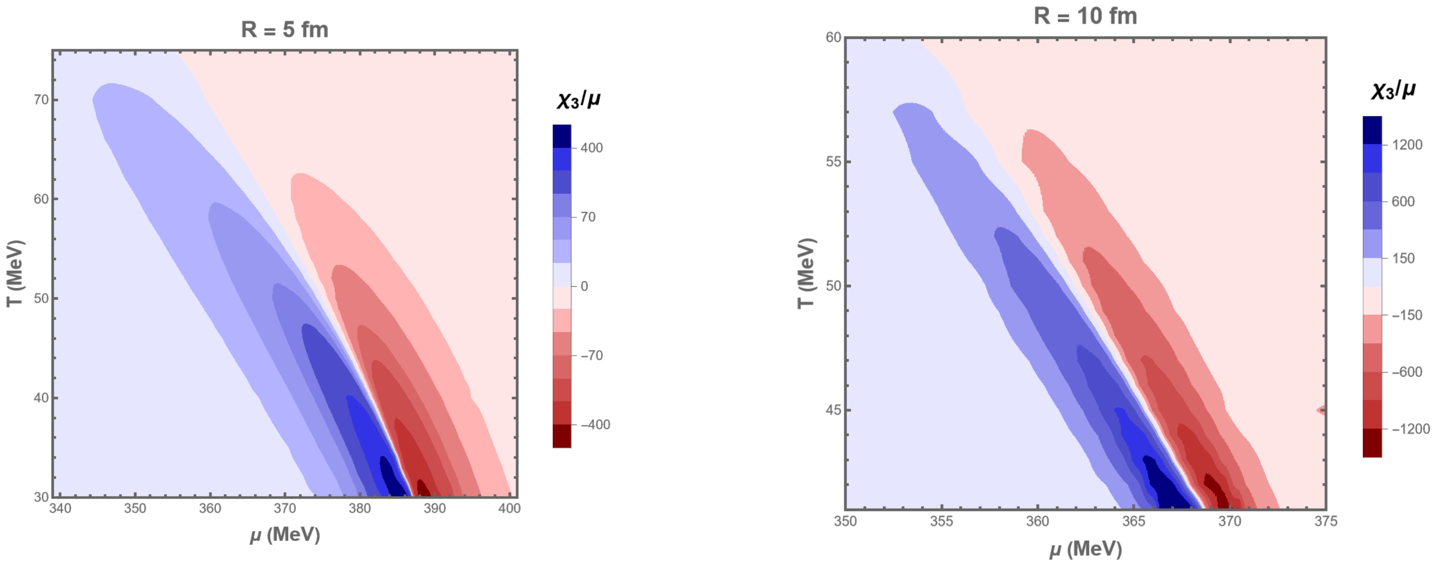




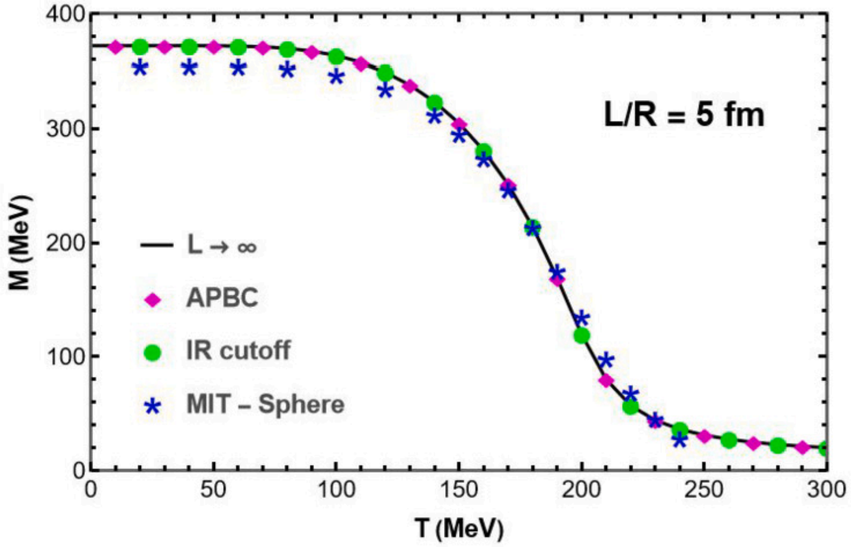
- Peak height decreases with decreasing volume.
- Peak is also shifted slightly to higher temperature for smaller volumes (smoother crossover for smaller volumes).
- The critical point is taken where χ_2 peaks [$T_c \approx 42$ MeV (10 fm), $T_c \approx 20$ MeV (5 fm)], T_c decreases as volume decreases.

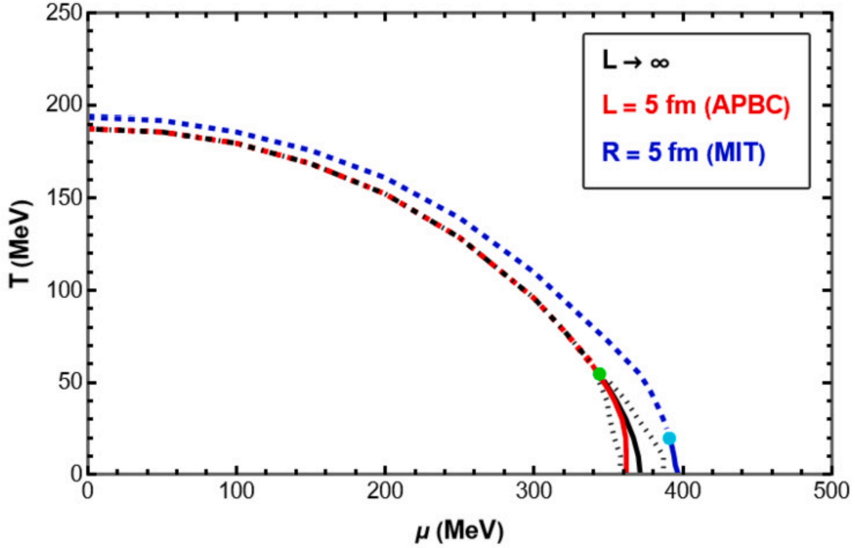
Quark number susceptibilities (χ_3)

- Higher order susceptibilities are crucial probes for the critical point.
- Third order susceptibility changes sign along the transition/crossover line, peaks and dips become sharper near the critical point.
- Difference in fluctuations (peak/dip for χ_3) compared to χ_2 peak is greater as volume is increased from R = 5 fm to R = 10 fm.
- Peak structure modifies due to shift in the first order endpoint with volume.



Comparison with different Boundary conditions





APBC b.c with cubic geometry : Allowed momentum modes (2n+1)$\boldsymbol{\pi}$/L

IR cutoff : infrared cutoff $\boldsymbol{\pi}$/L on momentum

Finite volume effect is significant for MIT b.c with sphere geometry compared to other b.c in chiral symmetry broken phase.

The boundary effect is small at high temp symmetry restored phase for all b.c.

The crossover line is shifted to higher chemical potential for MIT b.c as compared to APBC b.c.

Conclusion

- Finite volume effects on the QCD chiral phase transition is studied using spherical QGP with MIT b.c. in the mean-field NJL model framework.

- The effect of the finite size of the QGP on the experimental observables is very crucial for low energy (small QGP system) in future experiments - especially looking for a critical point and first-order transition.

- The phase transition line (and critical point) shows a substantial shift with volume. Reduction in the co-existence region is observed.

- Number density and its derivatives (susceptibilities) are sensitive to QGP system size.

THANK YOU