A Toy Model Study of Geometric Effects on Topological Transitions in Heavy-Ion Collisions

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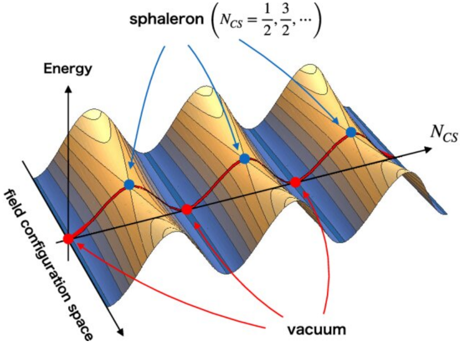
Plan

- Motivation
- Description of toy model
- Toy model estimation of chiral chemical potential

Motivation

- QCD vacuum has non-trivial topological structure — gauge field configurations like instantons and sphalerons.
- These transitions change the Chern–Simons number (N_{CS}), generating axial charge: $\Delta Q_5 = 2N_f \Delta N_{CS}$.
- Understanding the rate of topological transitions is crucial to interpret chirality-related observables in experiments. For example, Local chirality imbalance in presence of strong magnetic field (B) induces the Chiral Magnetic Effect (CME). Image (below) credit: @Phys. Rev. D 101, 096014



Motivation: Geometric evolution in HIC

1. Before collision: two Lorentz-contracted 'pancake-shaped' nuclei approach each other.
2. After collision: hot overlapping zone expands rapidly, becoming approximately spherical.
3. This geometric evolution modifies:
   – Expansion rate
   – Cooling rate and lifetime of the medium
   – Temperature profile $T(\tau)$ which controls transition rates like $\Gamma_s \propto \alpha_s^5 T^4$

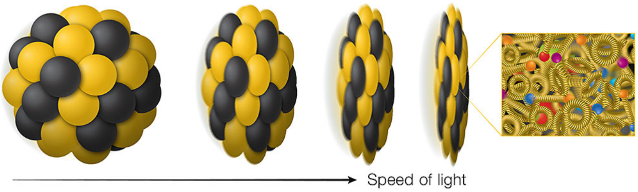


Image Credit: BNL

Motivation: How geometric evolution influences topological transition rates

- Geometry determines expansion and thermalisation dynamics.
- Expansion affects temperature and lifetime — both key factors for topological transitions.
- Therefore, geometric evolution may indirectly control the probability of exciting non-trivial topological configurations.

Key idea

Connect geometry-driven expansion to total integrated sphaleron transitions in a toy model.

Toy model: setup and assumptions

Assumptions:

- Boost-invariant longitudinal expansion; transverse expansion included approximately.
- Temperature profile: $T(\tau) = T_0(\tau_0/\tau)^\alpha$ ($\alpha = \frac{1}{3}$ for Bjorken 1D).
- Initial energy density (Bjorken): $\epsilon_0 = (dE_T/dy)/(A\tau_0)$.
- EoS (toy): $\epsilon = a_{eff}T^4 \rightarrow T_0 = (\epsilon_0/a_{eff})^{1/4}$.
- Sphaleron rate: $\Gamma_{sph}(T) = \kappa\alpha_s^5 T^4$.

Toy model: Analytic expression for integrated sphaleron count

- For sphaleron count we start with:

$$N_{sph} = \int d^4x \, \Gamma_{sph}(T) \approx A \int_{\tau_0}^{\tau_f} d\tau \, \tau \, \Gamma_{sph}(T(\tau))$$

- For Γ_{sph} and $T(\tau)$ we use:

$$\Gamma_{sph} \propto T^4$$

With $T(\tau) = T_0(\tau_0/\tau)^{1/3}$, we express Γ_{sph} as:

$$\tau \, \Gamma_{sph} \propto \tau^{-4/3}.$$

- Integrated number of transitions in one unit rapidity (assuming boost invariance over rapidity chunk) for a transverse area A:

$$N_{sph} = A \, \kappa \, \alpha_s^5 \, T_0^5 \, \tau_0^{4/3} \, \frac{3}{2} \int_{\tau_0}^{\tau_f} d\tau \, \tau^{-1/3}.$$

Toy model: Analytic expression for integrated sphaleron count

- To see geometry dependence explicitly, we substitute

$$T_0^4 = \epsilon_0 / a_{eff} = \frac{1}{a_{eff}} \frac{dE_T/dy}{A\tau_0}.$$

- Finally:

$$N_{sph} = \kappa\, \alpha_s^5 \frac{dE_T/dy}{a_{eff}} \frac{3}{2} \tau_0^{1/3} \left(\tau_f^{2/3} - \tau_0^{2/3}\right).$$

Note:

- Smaller τ_0 translates into larger initial T_0 (due to $\epsilon_0 \propto 1/\tau_0$).
- τ_f increases with smaller τ_0 because T_0 is larger $\left(\tau_f \propto \tau_0 (T_0/T_f)^3\right)$.
- The analytic expression shows competing trends: τ_0 growth vs reduced integration lower limit.

Toy model estimation of Chiral chemical potential

- Each topological transition produces a net axial charge:

$$\Delta Q_5 = 2\, N_f\, \Delta N_{CS}$$

Here, ΔQ_5 is the net axial charge and ΔN_{CS} is the change in the CHern-Simon number.

- The simplest kinetic equation (ignoring diffusion) involving axial charge density:

Kinetic equation

$$\frac{dn_5}{dt} = 2N_f\, \Gamma sph(T(t)) - \Gamma_f\, n_5(t)$$

$n_5 =$ axial charge density

$\Gamma_f =$ Chirality flipping (relaxation) rate.

Toy model estimation of Chiral chemical potential

Relation between axial charge density and chiral chemical potential

For small μ_5 (linear response) the axial charge density is related to chiral chemical potential by the axial susceptibility χ_5:

$$n_5(t) = \chi_5(t)\mu_5(t)$$

Kinetic equation

Using the relation between axial charge and chiral chemical potential, the kinetic can be expressed in the following form:

$$\chi_5 \frac{d\mu_5}{dt} + \frac{d\chi_5}{dt}\mu_5 = 2N_f\,\Gamma_{sph} - \Gamma_f\,\chi_5\,\mu_5$$

Toy model estimation of Chiral chemical potential

- Under adiabatic approximation (ignoring the time derivative of χ_5) can be cast into the following form:

Kinetic equation under adiabatic approximation

$$\frac{d\mu_5}{dt} = \frac{2N_f\,\Gamma_{sph}}{\chi_5} - \Gamma_f\,\mu_5$$

Upon integration, we can have an estimation of the chiral chemical potential.

- Under steady state approximation:

$$\frac{d\mu_5}{dt} = 0$$

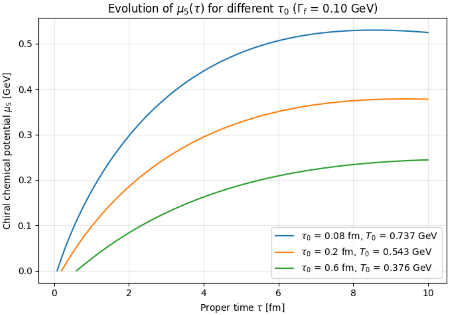
Chiral chemical potential under steady state approximation

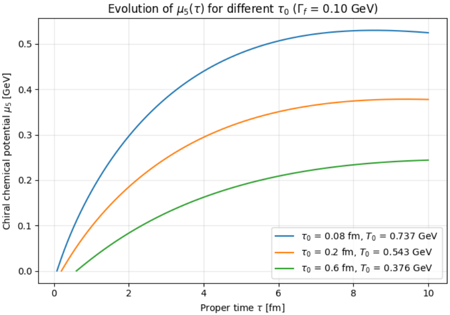
Under steady state approximation, the chiral chemical potential assumes the following simpler form:

$$\mu_5^{ss}(T) \approx \frac{2N_f\,\Gamma_{sph}}{\chi_5\,\Gamma_f}$$

Toy model estimation of chiral chemical potential

μ_5 evolution with example Γ_f values

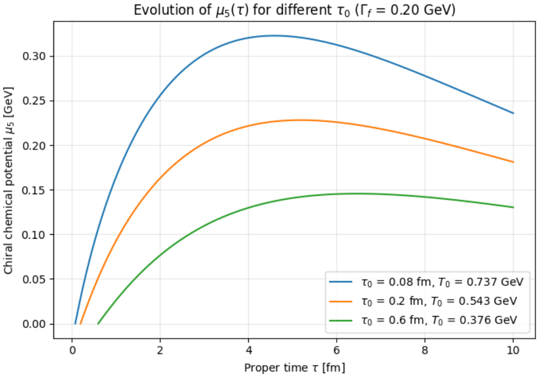




- Initial rise: μ_5 increases rapidly when the temperature is highest and sphaleron transition rate term dominates
- Saturation region: As n_5 builds up, the damping term balances the sphalerons source term. The system achieves a quasi-steady state.

Toy model estimation of chiral chemical potential

μ_5 evolution with example Γ_f values



- Decay phase: As the QGP cools down both Γ_{sph} and χ_5 decreases sharply and the damping term dominates and give rise to slow decay of μ_5.

Thank you

Thank You!

Supplementary