Collective modes in chiral QCD plasma

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Outline

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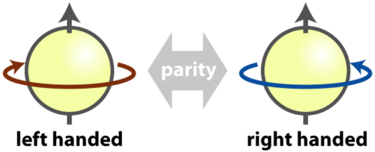
Motivation

- QCD is comprised of infinite number of energy-degenerate vacua characterized by an integer winding number and separated by potential barrier. ☞ M. A. Shifman

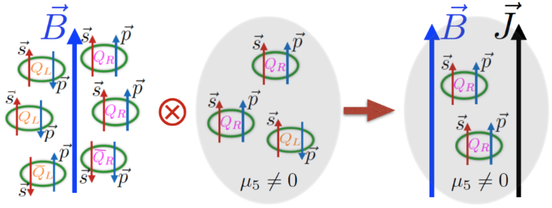
- These vacua can be probed by non-trivial topological gauge configurations which can switch the helicities of quarks. ☞ E. Shuryak , Schäfer et. al. , L. D. Mclerran

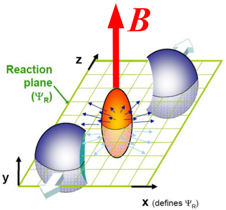
- Local breaking of P and CP symmetries via the axial anomaly of QCD $\longrightarrow$ an asymmetry between left and right handed quarks. ☞ D. E. Kharzeev

- This locally induced chirality imbalance ($n_R \neq n_L$) is characterised by means of a chiral chemical potential (CCP).



Motivation





Chiral Magnetic Effect : $\mathbf{J} = N_c \sum_f \frac{{q_f}^2}{2\pi^2} \mu_5 \, \mathbf{B}$ $\qquad eB \simeq {m_\pi}^2 \sim 10^{18}$ G

- Dedicated efforts are currently undergoing at RHIC for detection of CME.
- We will consider chirally imbalanced quark-gluon plasma with $\mu_5 = \frac{1}{2}(\mu_\mathrm{R} - \mu_\mathrm{L}) > 0$.

Outline

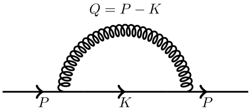
Collective oscillations

❀ In-medium properties of elementary particles are modified due to interactions.

❀ Propagation may involve emergence of collective modes or quasi-particles.

❀ The collective excitations are characterized by dispersion relations, significantly different from vacuum counterparts. Often leads to novel dispersive modes.

❀ Quasi-particle dispersion from poles of effective fermion propagator $\mathcal{S}(P)$



$$\mathcal{S}(P) = \frac{1}{\not{P} - \Sigma(P)}; \qquad \Sigma(P) = g^2 C_F \sum\!\!\!\!\!\!\int_{\{K\}} \gamma_\mu S(K) \gamma^\mu \frac{1}{Q^2}$$

❀ Here $\Sigma(P)$ is 1-loop self energy for fermion in chirally imbalanced medium.

❀ Bare propagator $S(K)$ is modified due to presence of hot chiral medium.

General structure of quark self-energy

- Fermion self-energy $\Sigma(P) \in$ Dirac space and it is a Lorentz scalar.
- Any 4×4 matrix can be expressed in terms of sixteen basis matrices: $\{\mathbb{1}, \gamma_5, \gamma^\mu, \gamma^\mu\gamma_5, \sigma^{\mu\nu}\}$ where $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ and $\sigma^{\mu\nu} = i/2\,[\gamma^\mu, \gamma^\nu]$.
- At $T, \mu_5 \neq 0$, it is a function of external four-momentum P and medium four-velocity U.
- General covariant structure of $\Sigma(P)$ in chirally imbalanced media

$$\Sigma(P) = -a\not{P} - b\not{U} - a'\gamma_5\not{P} - b'\gamma_5\not{U} \, .$$

- a and b are present even in a parity-symmetric thermal medium.
- Additional functions a' and b' arise solely due to chiral imbalance.

General structure of quark self-energy

- Structure factors are given by

$$a = \frac{1}{4p^2}\left(\mathrm{Tr}\left[\not{P}\Sigma\right] - (P\cdot U)\,\mathrm{Tr}\left[\not{U}\Sigma\right]\right),$$

$$b = \frac{1}{4p^2}\left(-(P\cdot U)\,\mathrm{Tr}\left[\not{P}\Sigma\right] + P^2\mathrm{Tr}\left[\not{U}\Sigma\right]\right),$$

$$a' = -\frac{1}{4p^2}\left(\mathrm{Tr}\left[\gamma_5\not{P}\Sigma\right] - (P\cdot U)\,\mathrm{Tr}\left[\gamma_5\not{u}\Sigma\right]\right),$$

$$b' = -\frac{1}{4p^2}\left(-(P\cdot U)\,\mathrm{Tr}\left[\gamma_5\not{P}\Sigma\right] + P^2\mathrm{Tr}\left[\gamma_5\not{U}\Sigma\right]\right).$$

- Fermion propagator in a chirally imbalancd medium is given by

$$S(K) = \mathbb{P}_+\frac{\not{K}_+}{K_+^2} + \mathbb{P}_-\frac{\not{K}_-}{K_-^2}$$

- with $\mathbb{P}_\pm = \frac{1}{2}(1\pm\gamma_5)$ and $K_\pm^\mu = \left(k_0 + \mu \mp \mu_5, \vec{k}\right)$.

- We use HTL (Hard Thermal Loop) approximation

Evaluation of structure factors

✧ loop momentum K is hard, i.e. $K \sim T$ external momentum $P \sim gT \ll T$ is soft.

$$q = \left|\vec{p} - \vec{k}\right| = k - p\cos\theta = k - \vec{p}\cdot\hat{k}$$

$$n_B(k - \vec{p}\cdot\hat{k}) \simeq n_B(k) - \vec{p}\cdot\hat{k}\frac{dn_B(k)}{dk}$$

Employing these approximations

$$a = -\frac{M^2}{p^2}Q_1(p_0,p), \quad a' = \frac{\delta M^2}{p^2}Q_1(p_0,p)$$

$$b = \frac{M^2}{p}\left[\frac{p_0}{p}Q_1(p_0,p) - Q_0(p_0,p)\right]$$

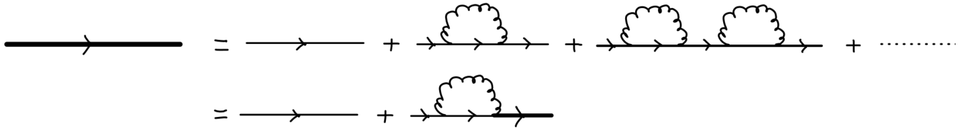
$$b' = \frac{\delta M^2}{p}\left[\frac{p_0}{p}Q_1(p_0,p) - Q_0(p_0,p)\right]$$

✧ Here $M^2 = \frac{g^2 C_F}{8}\left(T^2 + \frac{\mu^2}{\pi^2} + \frac{\mu_5^2}{\pi^2}\right)$ $\qquad \delta M^2 = \frac{g^2 C_F}{4\pi^2}\mu\mu_5$

✧ $Q_0(p_0,p) = \frac{1}{2}\ln\left[\frac{p_0+p}{p_0-p}\right]$, $Q_1(p_0,p) = \frac{p_0}{p}Q_0(p_0,p) - 1$ are Legendre functions.

Collective excitation of quarks

- Employing Dyson-Schwinger equation $\mathcal{S}^{-1}(P) = \not{P} - \Sigma(P)$



- Chiral invariance — $\mathcal{S}^{-1}(P)$ anti-commutes with γ_5 by construction.

- $\mathcal{S}^{-1}(P)$ anti-commutes with the helicty operator $\frac{\gamma_0\vec{\gamma}\cdot\vec{p}}{p}$.

- Interplay between helicity and chirality is evident if one write

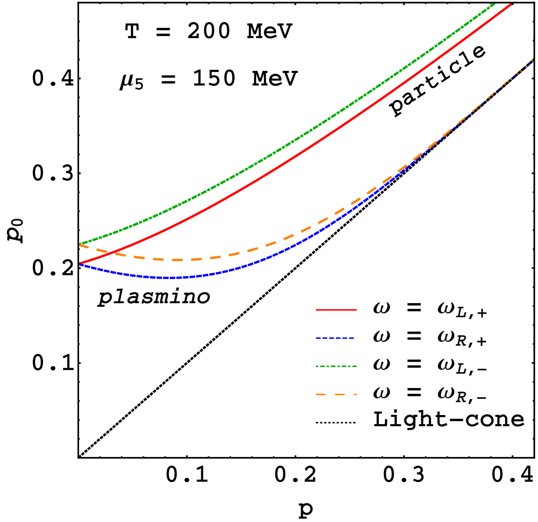
$$\mathcal{S}(P) = \mathbb{P}_+ \left(\frac{\gamma_0\Lambda_+}{A_0^L - A_s^L} + \frac{\gamma_0\Lambda_-}{A_0^L + A_s^L} \right) \mathbb{P}_- + \mathbb{P}_- \left(\frac{\gamma_0\Lambda_+}{A_0^R - A_s^R} + \frac{\gamma_0\Lambda_-}{A_0^R + A_s^R} \right) \mathbb{P}_+$$

with $A_0^{L,R} = (1 + a \pm a')\, p_0 + (b \pm b')$, $A_s^{L,R} = (1 + a \pm a')\, p$ and $\Lambda_\pm = \frac{1}{2}\left(1 \mp \gamma_0 \frac{\vec{\gamma}\cdot\vec{p}}{p}\right)$.

- $\Lambda_\pm$ projects out spinors whose chirality is equal or opposite to the helicity.

☞ arXiv 2509.07491

Collective excitation of quarks



- Particle mode disperse with effective mass $\sim M_{\pm}$ for large momenta.
- The plasmino branches exhibit a non-monotonic dispersion.
- At small momentum the dispersion of plasmino branches is similar to a negative-energy mode — energy decreases with momentum.
- Presence of a minimum at finite momentum — distinctive feature of the plasmino branch
- At large p plasmino branch approaches the light cone.

☞ arXiv 2509.07491

Outline

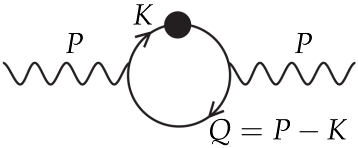
Set up

- The emission rate of real photons with energy E and momentum $\vec{p}$

$$E\frac{dR}{d^3p} = \frac{2}{(2\pi)^3}\,\mathrm{Im}\,{}^{\mathrm{ret}}\Pi^{\mu}_{\mu}(E,\vec{p})\,\frac{1}{e^{\beta E}-1}$$

- Since the energy of the produced photon is hard ($E \gg T$) there is only one dressed propagator (represented by blob) which can be present kinematically.

- Photon carries a large momentum, it effectively probes the internal vertex structure eliminating the need for vertex corrections.



- Retarded photon self-energy (p_0 has a small positive imaginary part)

$$^{\mathrm{ret}}\Pi^{\mu}_{\mu}(P) = -\frac{5}{3}e^2 \sum\!\!\!\!\!\!\int_{\{K\}} \mathrm{Tr}\left[\mathcal{S}(K)\gamma^{\mu}S(Q=P-K)\gamma_{\mu}\right]$$

Spectral representation

- Introduce a spectral representation for convenience

$$F(k_0) = \int_{-\infty}^{\infty} d\omega \frac{\rho(\omega)}{k_0 - \omega + i\epsilon}$$

- Spectral density function can be determined by inverting above relation

$$\rho(k_0) = -\frac{1}{\pi}\,\text{Im}\,F(k_0) = -\frac{1}{2\pi i}\,\text{Disc}\,F(k_0)\,.$$

- $\rho_{L,\pm}$ corresponding to the dressed propagators $1/\left(A_0^L \mp A_s^L\right)$.

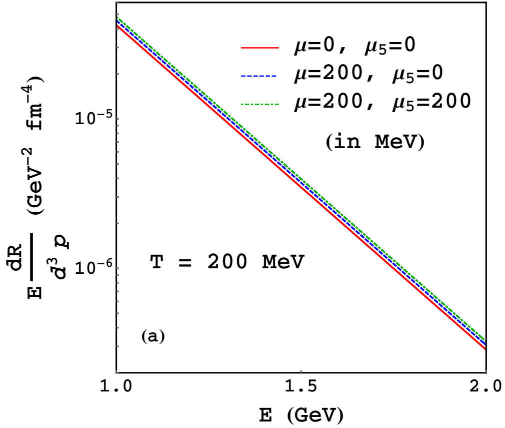
- External photon is hard i.e. $p_0 \gg T$ and exchange quark is soft i.e. $\omega \ll T$

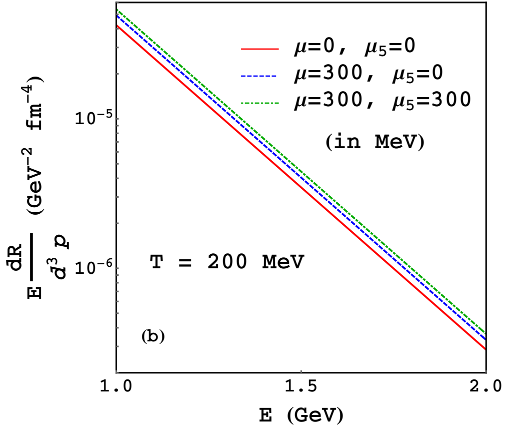
$$n_F^{L/R,+}(\omega) \approx \frac{1}{2} \qquad \text{and} \qquad n_F^{L/R,-}(p_0 - \omega) \approx n_F^{L/R,-}(p_0) \approx e^{-\beta p_0}\,.$$

Photon production rate

- Soft contribution to the thermal photon production rate from chirally imbalanced matter

$$E\frac{dR}{d^3p} = \frac{5}{9}\frac{\alpha\alpha_s}{2\pi^2}e^{-\beta E}\left(T^2 + \frac{\mu^2}{\pi^2} + \frac{\mu_5^2}{\pi^2}\right)\left[\ln\frac{k_c^2}{2M^2} + \frac{\delta M^2}{M^2}\ln\frac{M_R}{M_L}\right] .$$

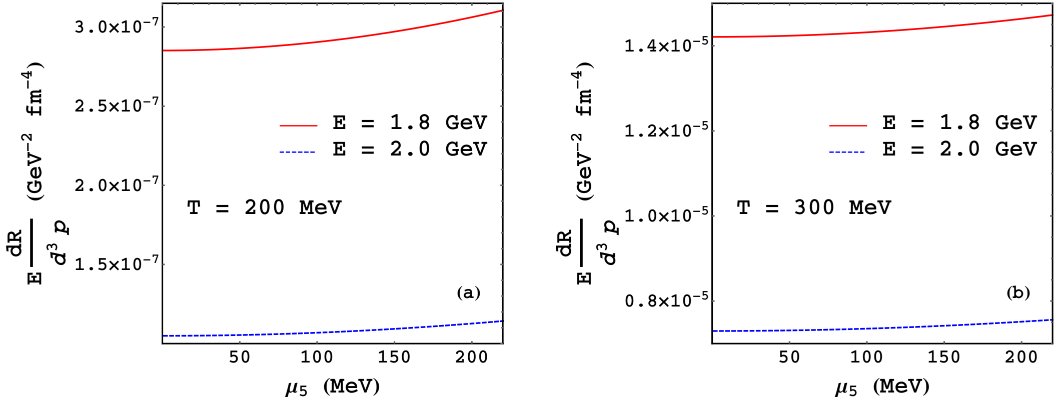




☞ arXiv 2511.04034

Photon production rate

- Photon production rate as a function of μ_5 for different photon energy



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Outline

Summary

- ✔ Quark self-energy evaluated shows two additional structure functions compared to the case when $\mu_5 = 0$
- ✔ Along with the thermal mass M, an additional mass scale δM arises due to the presence of chiral asymmetry.
- ✔ We observe distinct collective modes for left and right handed quark quasi-particles and plasmino.
- ✔ Enhancement to thermal photon emission in the presence of chiral imbalance.
- ✔ With increase in temperature the production rate of increases owing to the availability of a larger thermal phase space.

Team

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Thank You for your attention

Back up

Need for re-summation

It turns out that $\Pi_{L/T}(Q) \sim g^2T^2$, so that

– For hard gluon momenta ($Q \sim T$) the self-energy provides a small correction to the tree-level propagator, which can be accounted for perturbatively, e.g.

$$\Delta_L = \frac{-1}{q^2} + \frac{-1}{q^2}\Pi_L\frac{-1}{q^2} + \dots$$

– For soft gluon momenta ($Q \sim gT$) all the terms in the above expansion would be of the same order

$$\Delta_L \sim \frac{1}{g^2T^2} + \frac{1}{g^2T^2}(g^2T^2)\frac{1}{g^2T^2} + \dots$$

and one has to keep the resummed expressions for the propagators, *no matter how small the coupling is.*

☞ Slide by Andrea Beraudo

A simple QM model

Let us consider the hamiltonian of a *perturbed* harmonic oscillator:

$$H = \frac{p^2}{2} + \frac{1}{2}\,\omega_k^2\,x^2 + \frac{\lambda}{4!}\,\omega_k^3\,x^4 \equiv H_0 + H_1$$

One introduces as usual the raising/lowering operators $a^\dagger$ and a, so that

$$H_o = \omega_k\left(a^\dagger a + \frac{1}{2}\right) \quad \text{with} \quad x \equiv \frac{1}{\sqrt{2\omega_k}}\left(a + a^\dagger\right) \quad \text{and} \quad [a, a^\dagger] = 1.$$

The thermal average in the unperturbed system is of course:

$$\langle \ldots \rangle_0 \equiv \frac{\mathrm{Tr}\left[\mathrm{e}^{-\beta \mathrm{H}_0} \ldots\right]}{\mathrm{Tr} e^{-\beta H_0}}.$$

The fluctuation of the "field" x are of particular interest:

$$\langle x^2 \rangle_0 = \frac{1}{2\omega_k}\langle \underbrace{aa + a^\dagger a^\dagger}_{=0} + \underbrace{aa^\dagger + a^\dagger a}_{1+2a^\dagger a} \rangle \quad \Rightarrow \quad \langle x^2 \rangle_0 = \frac{1 + 2N_k}{2\omega_k},$$

where $N_k \equiv \langle a^\dagger a \rangle_0 = 1/[\exp(\beta\omega_k) - 1]$.

☞ Slide by Andrea Beraudo

A simple QM model

The expectation value of H_1 is given by (the weight is gaussian!)

$$\langle H_1\rangle_0 = \frac{\lambda}{4!}\,\omega_k^3\,\langle x^4\rangle_0 = \frac{\lambda}{8}\omega_k^3(\langle x^2\rangle_0)^2 = \frac{\lambda}{8}\,\omega_k^3\left(\frac{1+2N_k}{2\omega_k}\right)^2$$

to be compared with the e.v. of the unperturbed hamiltonian:

$$\langle H_0\rangle_0 = \omega_k\left(N_k+\frac{1}{2}\right) = \omega_k^2\left(\frac{1+2N_k}{2\omega_k}\right)$$

At $T=0$ one has:

$$\langle H_1\rangle^{T=0} = \frac{\lambda}{32}\omega_k, \quad \langle H_0\rangle^{T=0} = \frac{\omega_k}{2} \quad \longrightarrow \quad \langle H_1\rangle^{T=0} \underset{\lambda\ll 1}{\ll} \langle H_0\rangle^{T=0}$$

and if $\lambda \ll 1$ the system is *always* perturbative.
However at large T ($T \gg \omega_k$) the e.v. of H_1 can become larger then H_0:

$$\frac{\lambda}{8}\,\omega_k\left(\frac{1+2N_k}{2\omega_k}\right) \sim \frac{\lambda}{8}\frac{T}{\omega_k} > 1$$

Hence *no matter how small the coupling is* for modes with $\omega_k \lesssim \lambda T$ the system is *strongly coupled*!

☞ Slide by Andrea Beraudo

Collective modes

* Denominators of the L and R-modes

$$A_0^L \mp A_s^L = k_0 \mp k - \frac{M_L^2}{k}\left[\frac{1}{2}\left(1 \mp \frac{k_0}{k}\right)\ln\frac{k_0+k}{k_0-k} \pm 1\right]$$

$$A_0^R \mp A_s^R = k_0 \mp k - \frac{M_R^2}{k}\left[\frac{1}{2}\left(1 \mp \frac{k_0}{k}\right)\ln\frac{k_0+k}{k_0-k} \pm 1\right]$$

$$M_{L/R}^2 = M^2 \pm \delta M^2 = \frac{g^2 C_F}{8}\left(T^2 + \frac{\mu^2}{\pi^2} + \frac{\mu_5^2}{\pi^2}\right) \pm \frac{g^2 C_F}{4\pi^2}\mu\mu_5 \,.$$

* Approximate solutions in the limit $p \ll M_L$ are given by

$$\omega_{L,+} \simeq M_L + \frac{1}{3}p + \frac{1}{3M_L}p^2 + \cdots \qquad \omega_{L,-} \simeq M_L - \frac{1}{3}p + \frac{1}{3M_L}p^2 + \cdots$$

* Particle and hole-like solutions at large momenta $(M_\pm \ll p \ll T)$

$$\omega_{L,+} \simeq p + \frac{M_L^2}{p} + \frac{M_L^4}{2p^3}\ln\frac{M_L^2}{2p^2} + \cdots \qquad \omega_{L,-} \simeq p + 2p\exp\left[-\frac{M_L^2+2p^2}{M_L^2}\right] \,.$$

μ_5

- QCD Lagrangian has the following form

$$\mathscr{L}_{QCD} = \overline{\psi}\left(i\not{D} - \hat{m}\right)\psi - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \qquad D^\mu = \partial^\mu - igT_a A^\mu_a \qquad \left[T^a, T^b\right] = i f^{abc} T^c$$

- To mimic the local P and CP violation and the topological fluctuation, we assume that the $\theta = \theta(x,t)$ so the Lagrangian $\mathscr{L}_\theta = \frac{g^2}{32\pi^2}\theta(x,t)F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$

- Axial $U(1)$ transformation $\psi_f \to \exp\left[i\frac{\theta\gamma_5}{2N_f}\right]\psi_f$ can cancel θ-term.

- Due to the chiral anomaly, $\partial_\mu j^\mu_5 = -\frac{g^2 N_f N}{16\pi^2}\theta(x,t)F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$ the Lagrangian changes as follows:

$$\mathscr{L} = \mathscr{L}_{QCD} + \frac{g^2}{32\pi^2}\theta(x,t)F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a \to \mathscr{L}_{QCD} + \frac{1}{2N_f}\partial_\mu\theta\,\overline{\psi}\gamma^\mu\gamma_5\psi$$

☞ Yang et. al. Symmetry **12** 2095 (2020).

μ_5

- Thus, the local θ-term is equivalent to a fermionic contribution.

- If we further assume that the θ angle is only dependent on time, i.e.,$|\nabla\theta|^2 \ll \dot{\theta}$ we can define a chiral chemical potential μ_5 as $\mu_5 = \partial_0\theta/(2N_f)$. Then the Lagrangian can be written as

$$\mathscr{L} = \mathscr{L}_{QCD} + \mu_5\theta\,\overline{\psi}\gamma^0\gamma_5\psi$$

- The chiral chemical potential μ_5 is coupling to the chiral charge density operator $\overline{\psi}\gamma^0\gamma_5\psi$.

- Just like the chemical potential being able to reflect the quark number density, the chiral chemical potential can also mimic the chiral charge density.

- However, the chiral charge is not conserved because of the chiral anomaly and we cannot treat μ_5 as a true chemical potential.

- The chiral chemical potential is nothing but an indication of the magnitude of the chiral imbalance.

☞ Yang et. al. Symmetry **12** 2095 (2020).

Spectral representation

- Introduce a spectral representation for convenience

$$F(k_0) = \int_{-\infty}^{\infty} d\omega \frac{\rho(\omega)}{k_0 - \omega + i\epsilon}$$

- Spectral density function can be determined by inverting above relation

$$\rho(k_0) = -\frac{1}{\pi}\,\mathrm{Im}\,F(k_0) = -\frac{1}{2\pi i}\,\mathrm{Disc}\,F(k_0)\,.$$

- $\rho_{L,\pm}$ corresponding to the dressed propagators $1/\left(A_0^L \mp A_s^L\right)$ as

$$\rho_{L,\pm}(k_0^+,k) = \frac{{k_0^+}^2 - k^2}{2M_L^2}\left[\delta\left(k_0^+ - \omega_{L,\pm}\right) + \delta\left(k_0^+ + \omega_{L,\mp}\right)\right] + \beta_{L,\pm}(k_0^+,k)\theta\left(k^2 - {k_0^+}^2\right)$$

$$\beta_{L,\pm}(k_0,k) = \frac{M_L^2\left(1 \mp k_0/k\right)/2k}{\left\{k_0 \mp k - \frac{M_L^2}{k}\left[\frac{1}{2}\left(1 \mp \frac{k_0}{k}\right)\ln\left|\frac{k_0+k}{k_0-k}\right| \pm 1\right]\right\}^2 + \frac{\pi^2 M_L^4}{4p^2}\left(1 \mp \frac{k_0}{k}\right)^2}$$

$$M_{L/R}^2 = M^2 \pm \delta M^2$$

Polarization function

- Discontinuity from imaginary part of a product of two complex functions

$$\mathrm{Im}\ T\sum_{k_0} F_1(k_0)F_2(p_0-k_0) = \pi\left(e^{\beta p_0}-1\right)\int d\omega d\omega' \rho_1(\omega)\rho_2(\omega')n_F^+(\omega)n_F^-(\omega')\delta\left(p_0-\omega-\omega'\right)\ .$$

- The exchanged quark propagator must be dressed and satisfies

$$-k_c^2 \le \omega^2 - k^2 \le 0 \quad \Longrightarrow \quad 0 \le k^2-\omega^2 \le k_c^2,$$

where k_c^2 denotes the cutoff in the four-momentum transfer t (and u).

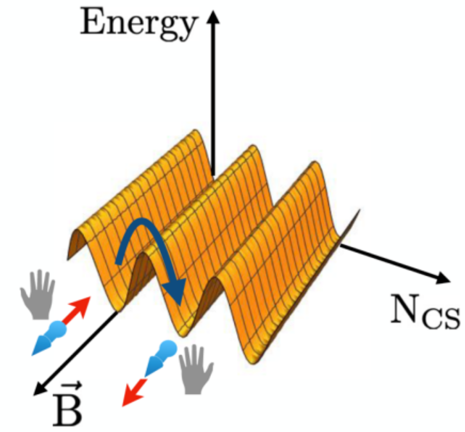
- External photon is hard i.e. $p_0 \gg T$ and exchange quark is soft i.e. $\omega \ll T$ we can write

$$n_F^{L/R,+}(\omega) \approx \frac{1}{2} \qquad \text{and} \qquad n_F^{L/R,-}(p_0-\omega) \approx n_F^{L/R,-}(p_0) \approx e^{-\beta p_0}\ .$$

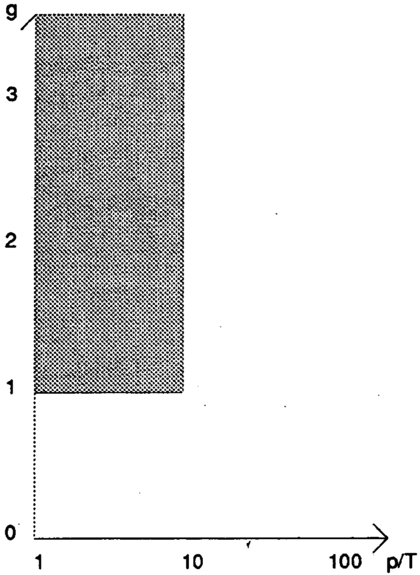
- Thus finally we arrive at

$$\mathrm{Im}\ {}^{\mathrm{ret}}\Pi^\mu_\mu(P) = \frac{5e^2}{12\pi}\left(e^{\beta p_0}-1\right)\frac{e^{-\beta p_0}}{2}\left[M_L^2 \ln\frac{k_c^2}{2M_L^2} + M_R^2\ln\frac{k_c^2}{2M_R^2}\right]$$
$$\approx \frac{5e^2}{12\pi}\left(e^{\beta p_0}-1\right)e^{-\beta p_0}M^2\left[\ln\frac{k_c^2}{2M^2} + \frac{\delta M^2}{M^2}\ln\frac{M_R}{M_L}\right]$$

qcd vacuum



HTL g



- Finite and gauge independent in naive perturbation theory – high temperature limit of quark or gluon self-energy. They should hold as long as α_s is small.
- Logarithmically infrared divergent in naive perturbation theory but finite using the effective one – transport rates and energy loss. Can be calculated for $g \sim 1$ and $p \gg T$.
- Quadratically infrared divergent in naive and logarithimically divergent in the effective perturbation theory – ordinary interaction rate and debye mass beyond leading order. Can not be calculated unambiguously.

Thoma et. al. PRD 49 451