Structure and Decay Properties of the Hoyle State

Abhijit Baishya
SO/D, NAS, BARC

Google Scholar

ResearchGate

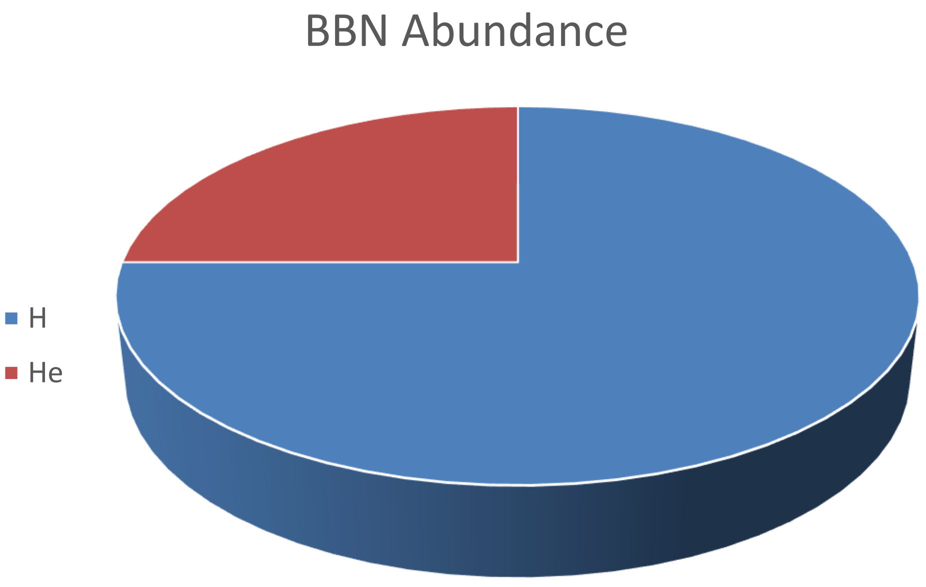
Email: abaishya@barc.gov.in, abhijitb498@gmail.com
Cell: 8250378959

India-JINR Workshop, November 10-12, 2025

Outline

- Introduction
- Upper limit of exotic decay modes of the Hoyle state
- Particle induced deexcitation
- E2 γ-decay of the Hoyle state
- Predicted Efimov like structure near threshold
- Summary

Why study Hoyle state?

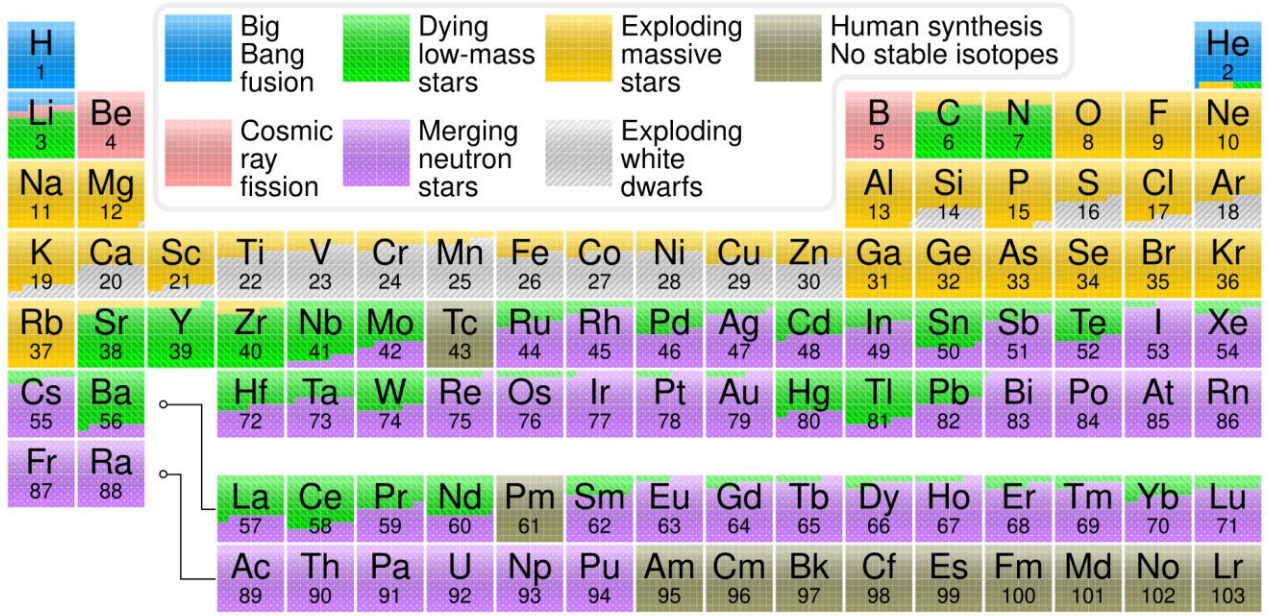


BBN → stars → 12C → life

The Hoyle state, 7.654 MeV, Without this resonance, **no carbon**, no us.

Element cooking in stars: pp chain/CNO → **He**

Elements today



heavier elements need stars

$$^{4}He + n \rightarrow {}^{5}He \;:\; \text{decay } (\tau \sim 7{*}10^{-22}\text{ s})$$

$$^{4}He + {}^{1}H \rightarrow {}^{5}Li \;:\; \text{decay } (\tau \sim 3{*}10^{-22}\text{ s})$$

$$^{4}He + {}^{4}He \rightarrow {}^{8}Be \;:\; \text{decay } (\tau \sim 10^{-17}\text{ s})$$

No stable elements of A = 5, 8! Bottleneck

How to go beyond A = 4?

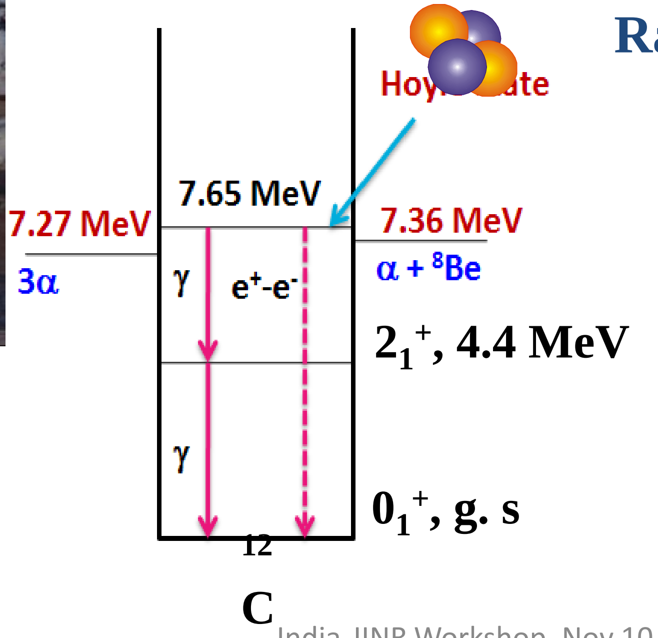
Solution: Direct triple-α

Fusion of three ^{4}He to form ^{12}C directly, bypassing the formation of A = 5, 8 nuclei

^{12}C produced through sequential (two step) process, proposed by Salpeter & Opik [E. J. Opik, Proc. Roy. Irish Acad. A54, 49 (1951), E. E. Salpeter *et al.*, Astrophys. Journals 115, 326 (1952)]

$\alpha+\alpha \rightleftarrows$ 8Be ($\tau \sim 10^{-17}$ s) $\rightarrow +\alpha \rightarrow$ 12C + γ (non-resonant)

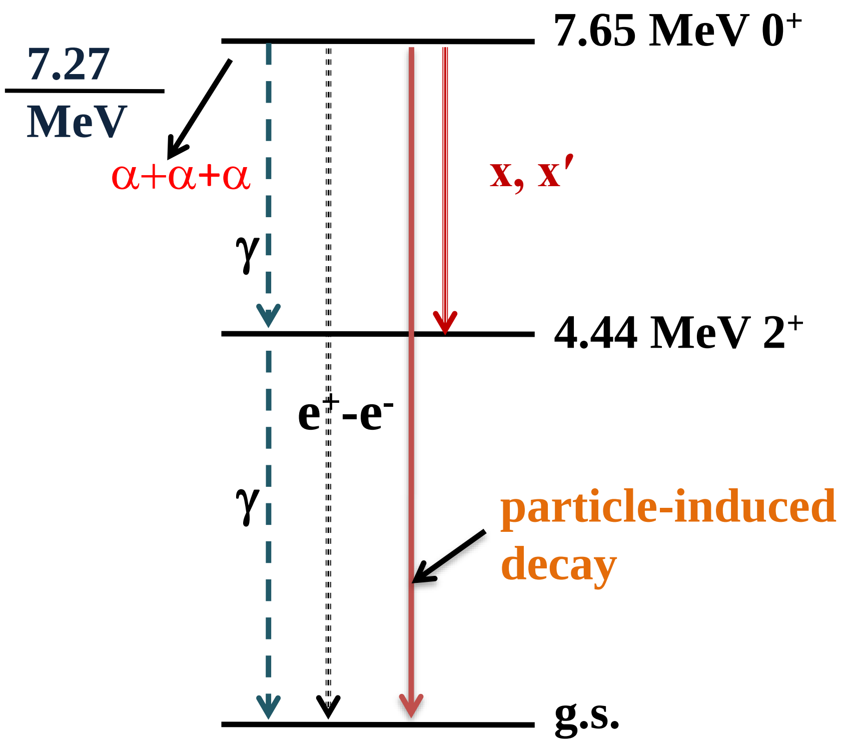
Rate insufficient to explain C abundance!



Hoyle predicts $10^7\times$ rate boost at T≈1.4×10^8 K

F. Hoyle et. al , Astro. J. Suppl. 1 (1954) 121

Decay Channels

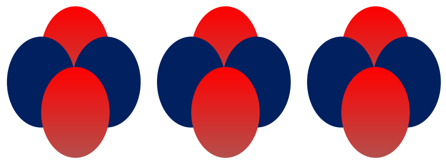


- α-decay: Γ_α = 9.3 eV, i) Sequential, ii) Direct?
- Radiative decay: i) E2 γ-decay, 3.7 meV ii) e^+-e^- pair decay, 62 μeV
- Particle induced decay, p, n, α

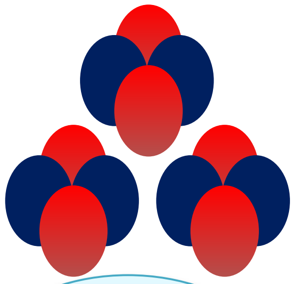
Stable 12C is produced only when the Hoyle state comes to the g.s.

I. Upper limit of exotic decay modes of the Hoyle state

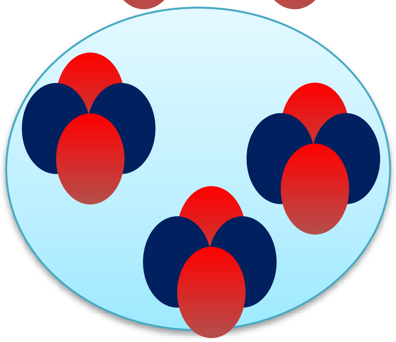
First conjectured : **Linear Chain**
[Phys. Rev. 101, 254 (1956)]



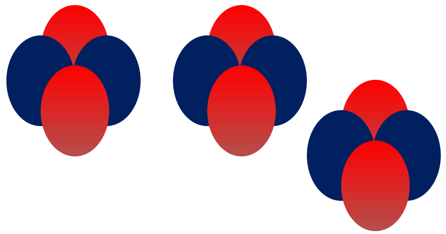
Nuclear Bose-Einstein Condensate
[PRL 87 (2001)192501, PRC 81, 054604 (2010)]



Hoyle state behaves like a *dilute gas* of three α clusters — extended, low density.
[NPA 351, 456 (1981), PRC 67, 051306R (2003)]

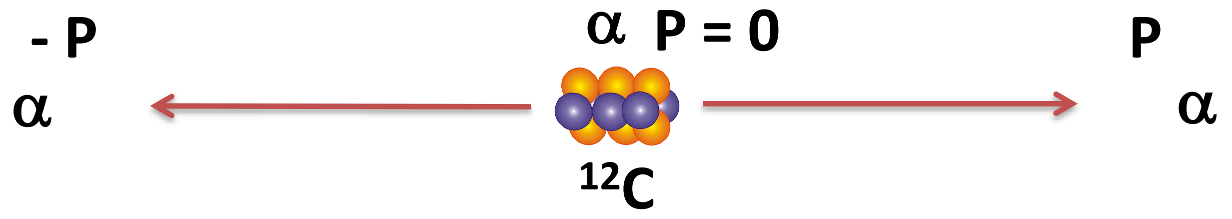


Indication to bent-arm like structure:
[PRL 109, 25201 (2012)]



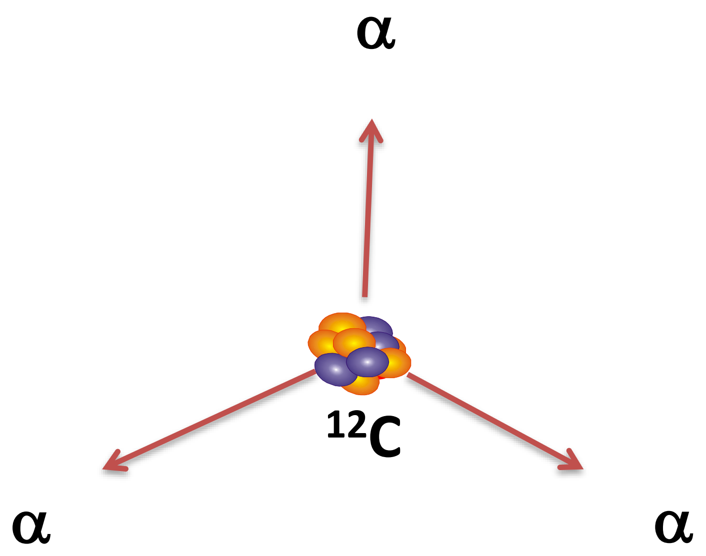
Possible α configurations and how to visualize them?

DDL



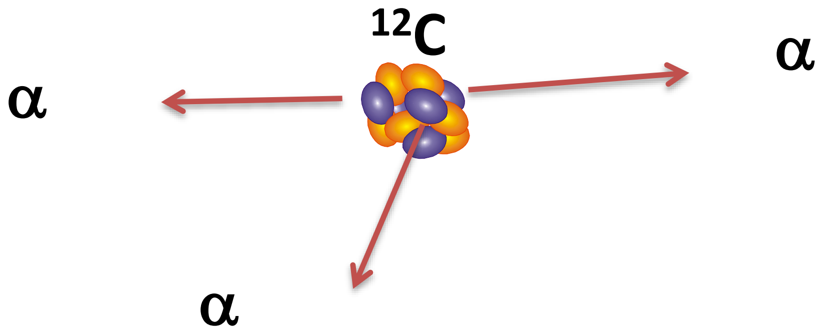
$P_1 = - P_3, P_2 = 0$

DDE



$P_1 = P_2 = P_3$

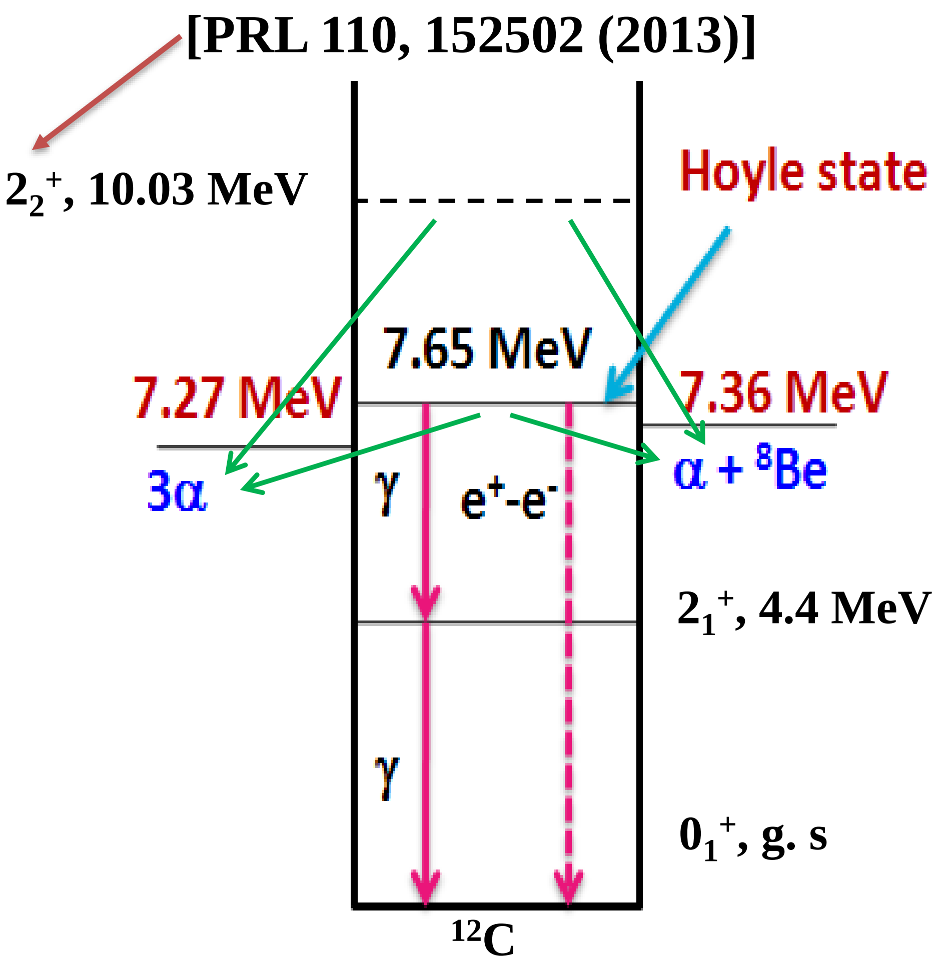
DDΦ



$P_1 + P_2 + P_3 = 0$

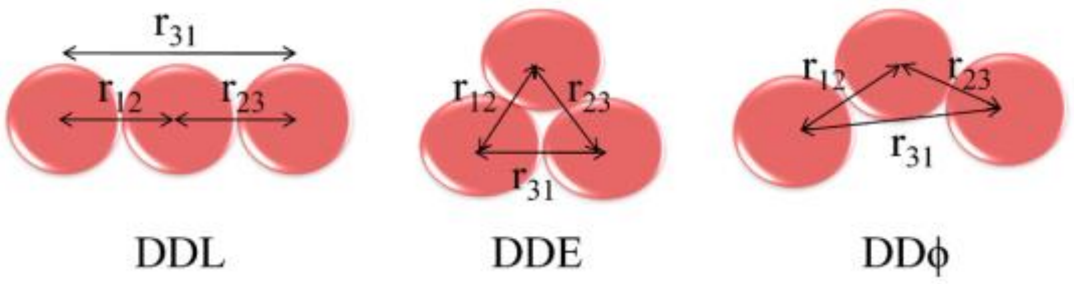
All possible combination

How to put an upper Limit?



$\Gamma_{3\alpha}$ (Hoyle) = $\Gamma_{3\alpha}$ (2^+) × $P_{3\alpha}$(Hoyle)/$P_{3\alpha}$(2^+)

Desired   < 32 keV   From Calculation



WKB $\longrightarrow P \approx \exp(-2S)$, $S = \frac{1}{\hbar}\int_{\rho_0}^{\rho_t} d\rho \sqrt{2m(V(\rho)-E)}$

Calculation Framework:

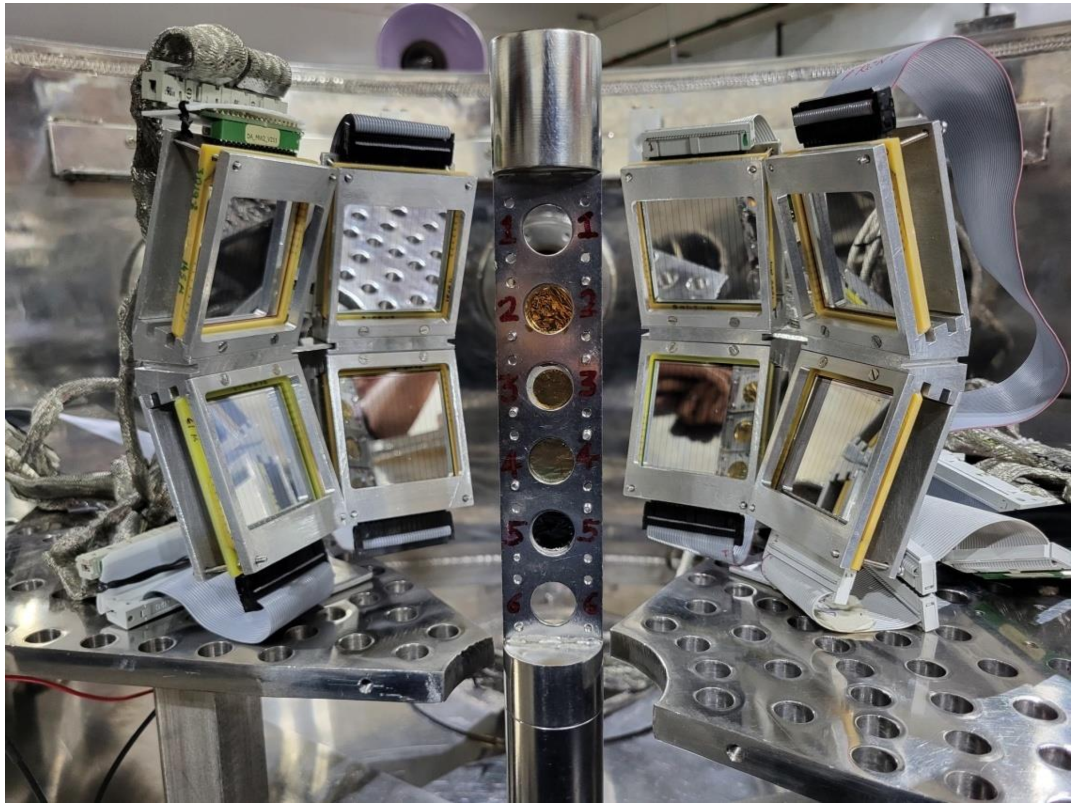
Sample phase space → Compute V(ρ) → Roots ρ_0, ρ_t → Integrate → Average $P_{3\alpha}$(Hoyle)/$P_{3\alpha}$(2^+)

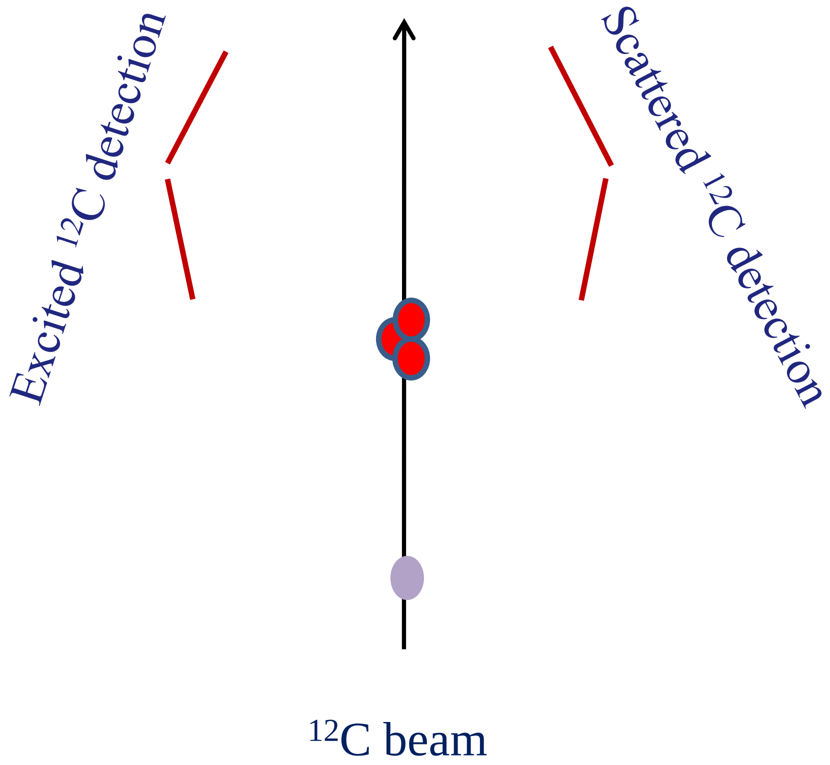
Decay modes	$\frac{P_{DD}(\text{Hoyle})}{P_{DD}(2^+)}$	$\frac{\Gamma_{DD}(\text{Hoyle})}{\Gamma(\text{Hoyle})}$
DDϕ	$(4.8 - 7.3) \times 10^{-10}$	$< 3.1 \times 10^{-6}$
DDL	$(2.9 - 6.2) \times 10^{-11}$	$< 2.6 \times 10^{-7}$
DDE	$(2.1 - 3.6) \times 10^{-9}$	$< 1.5 \times 10^{-5}$

A. Baishya et. al., PRC, 104, 024601 (2021)

Experiment Details

Detector Setup





For Calibration of detectors:

1. Am-Pu α source before the experiment
2. ^{229}Th α source at the end of the experiment.
3. Elastic peaks due to 35 MeV ^{12}C beam on ^{197}Au for high energy points

Details of the experiment:

- 57 MeV ^{12}C pulsed beam
- 25 μg/cm^2 natC target
- Total 8 DSSSDs (Thickness 140 μm to 1.5 mm
- Placed at ±50°
- Cover 23° - 80°
- Coincidence trigger

Data Analysis

Multiplicity = 4

↓

3 from Array-1 & 1 from Array-2 and vice versa

↓

Kinematic Gate

↓

Assume 3 from one array to be αs and the remaining one ^{12}C

↓

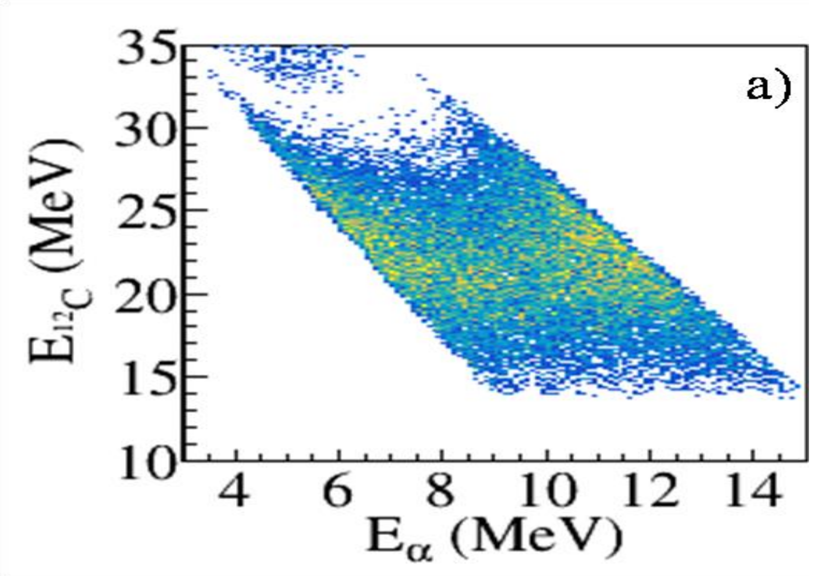
Timing Gate

↓

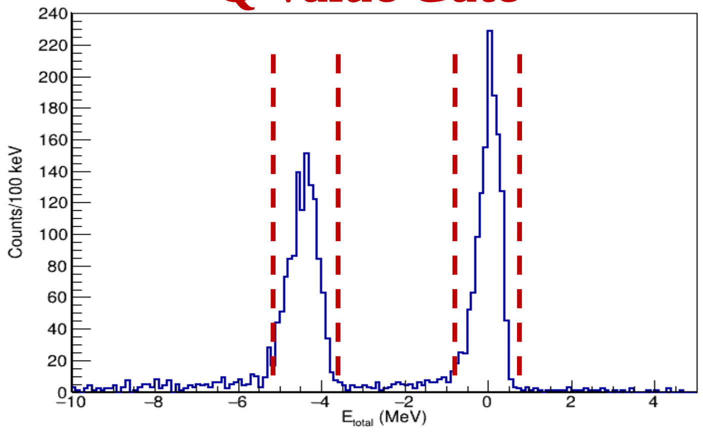
Momentum Gate

↓

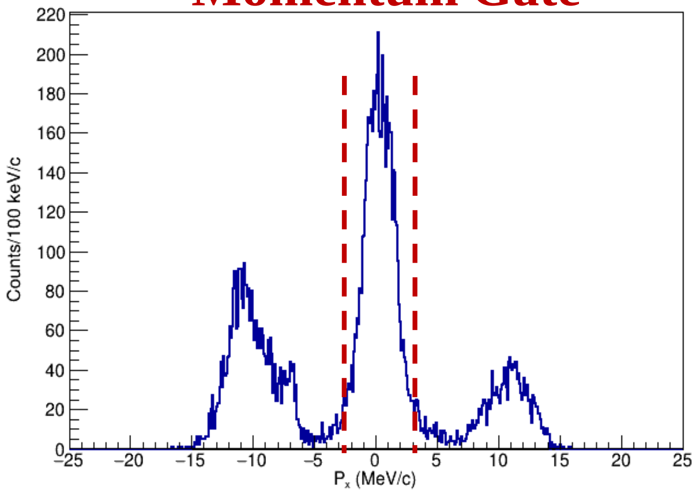
Q-value Gate

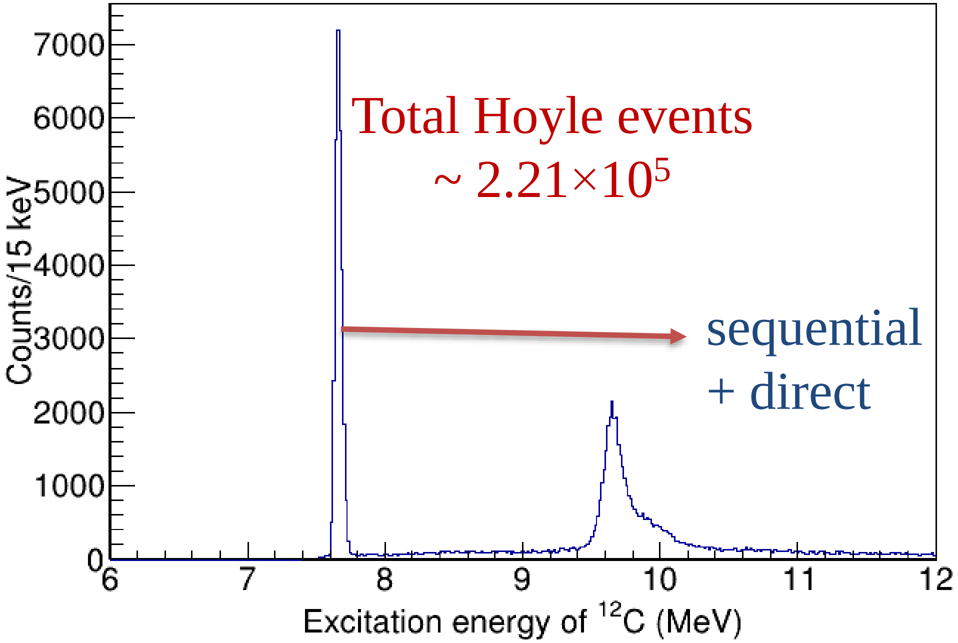


Q-value Gate



Momentum Gate





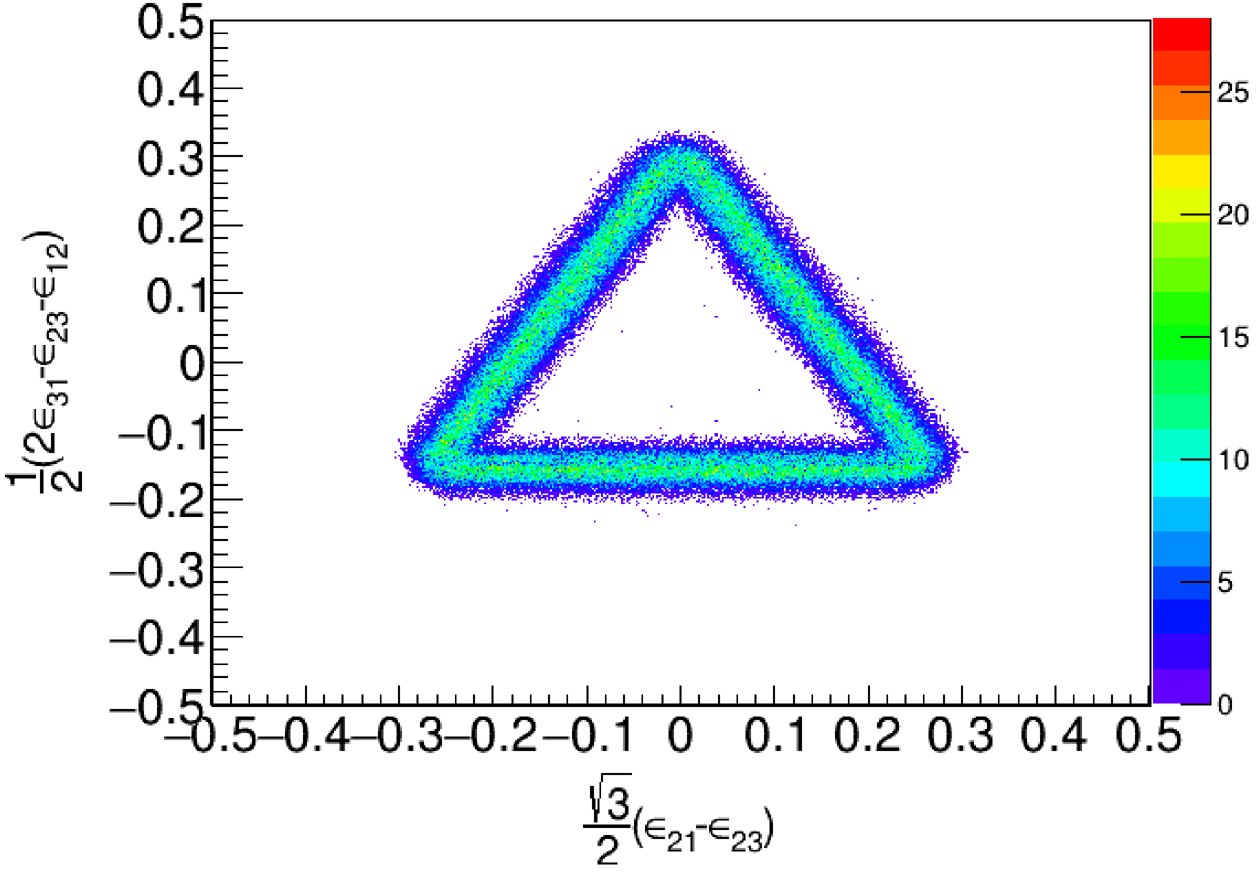
How to separate?

Results: Symmetric Dalitz Plot and its Folded version

$X = \sqrt{3}(T_2 - T_3)/2$

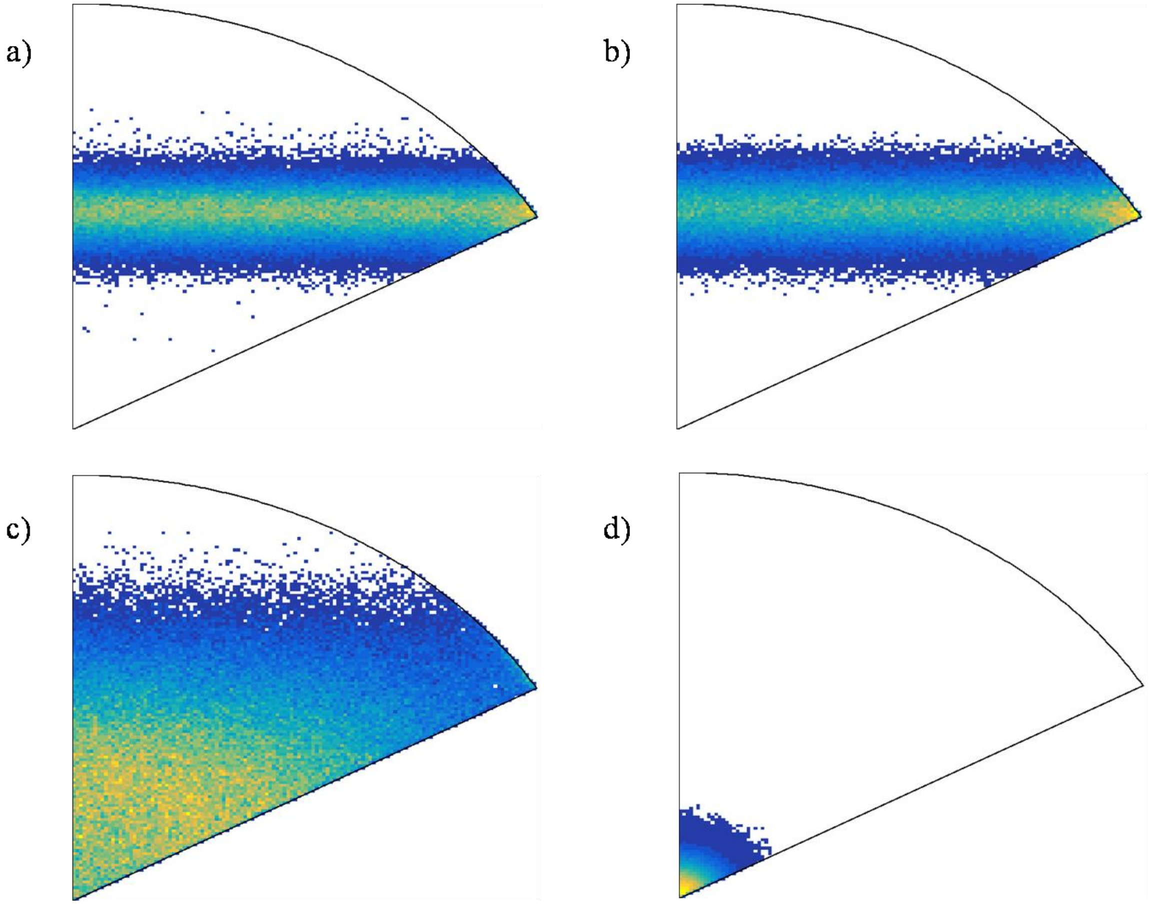
$Y = (2T_1 - T_2 - T_3)/2, \; T_i = T_i / \sum T_i$

Folded: $T_1 > T_2 > T_3$



Experimental Dalitz plot

Experiment SD



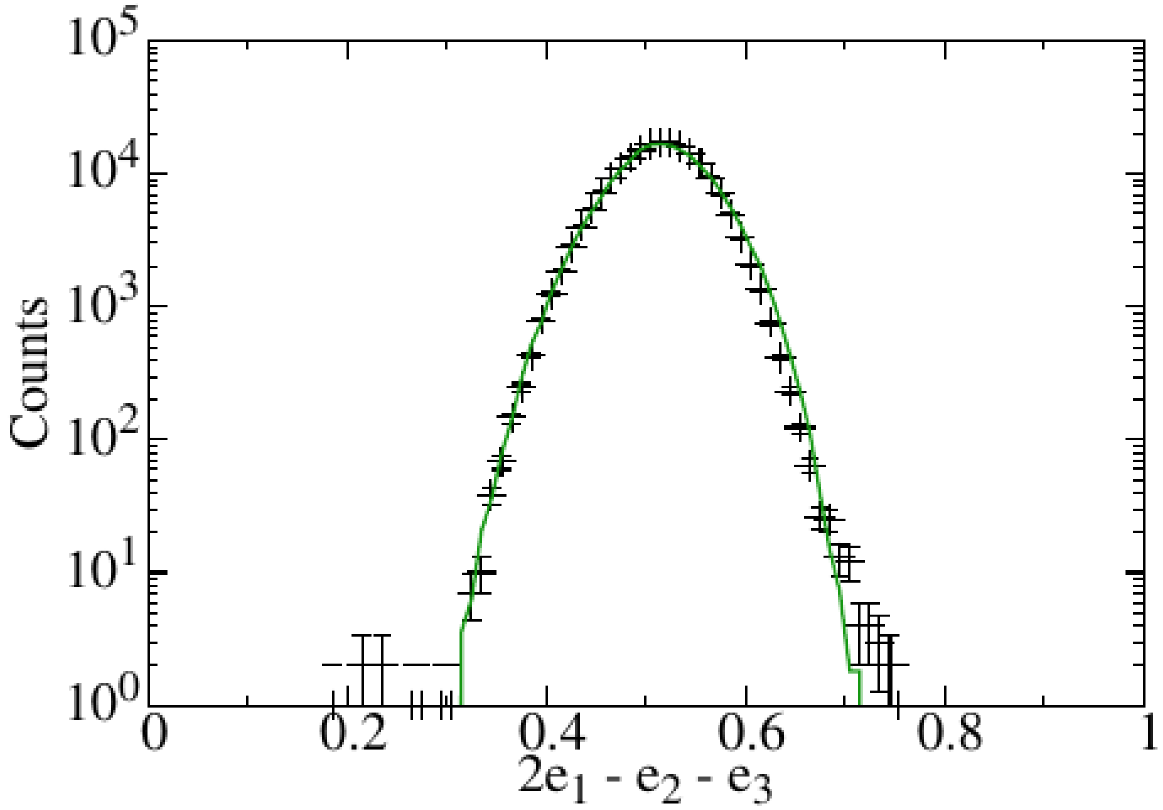
DDϕ DDE

Folded Symmetric Dalitz (FSD) plots

Data aligns with SD; DD contributions tiny

Quantitative DD contribution

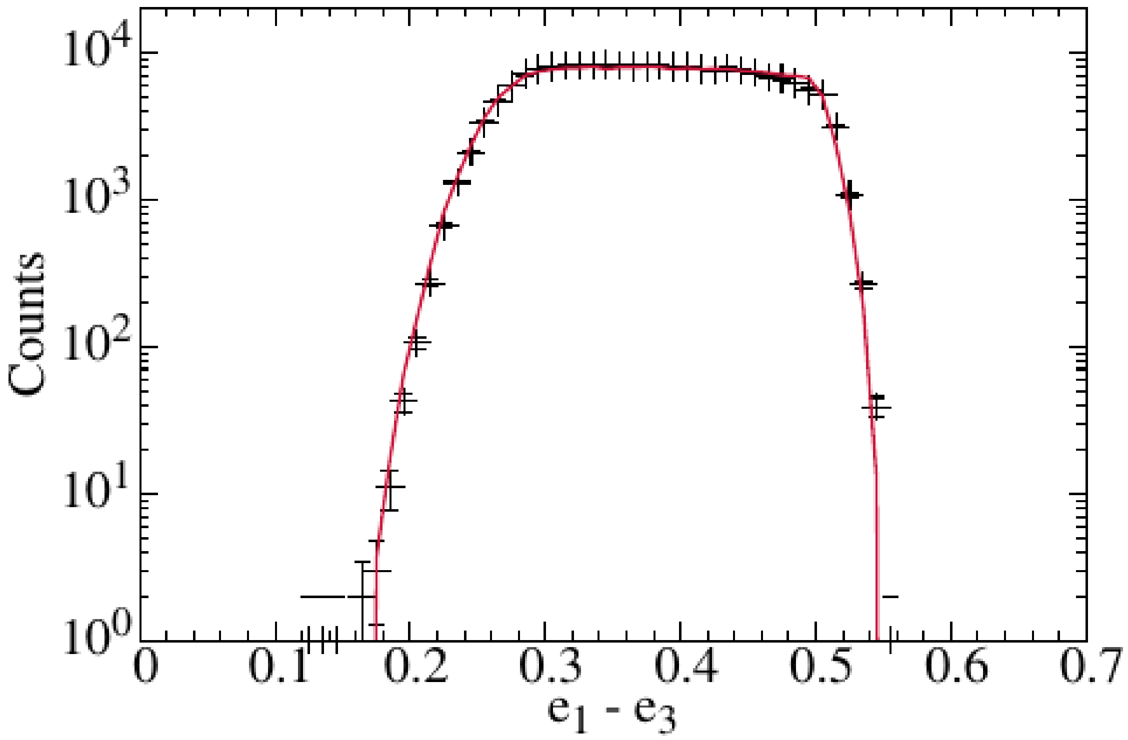
i) FSD y-projection



Obtained upper limits:

1. Method i): 0.019% for DDϕ and 0.002% for DDE

ii) Distribution of "e_α(max) - e_α(min)"



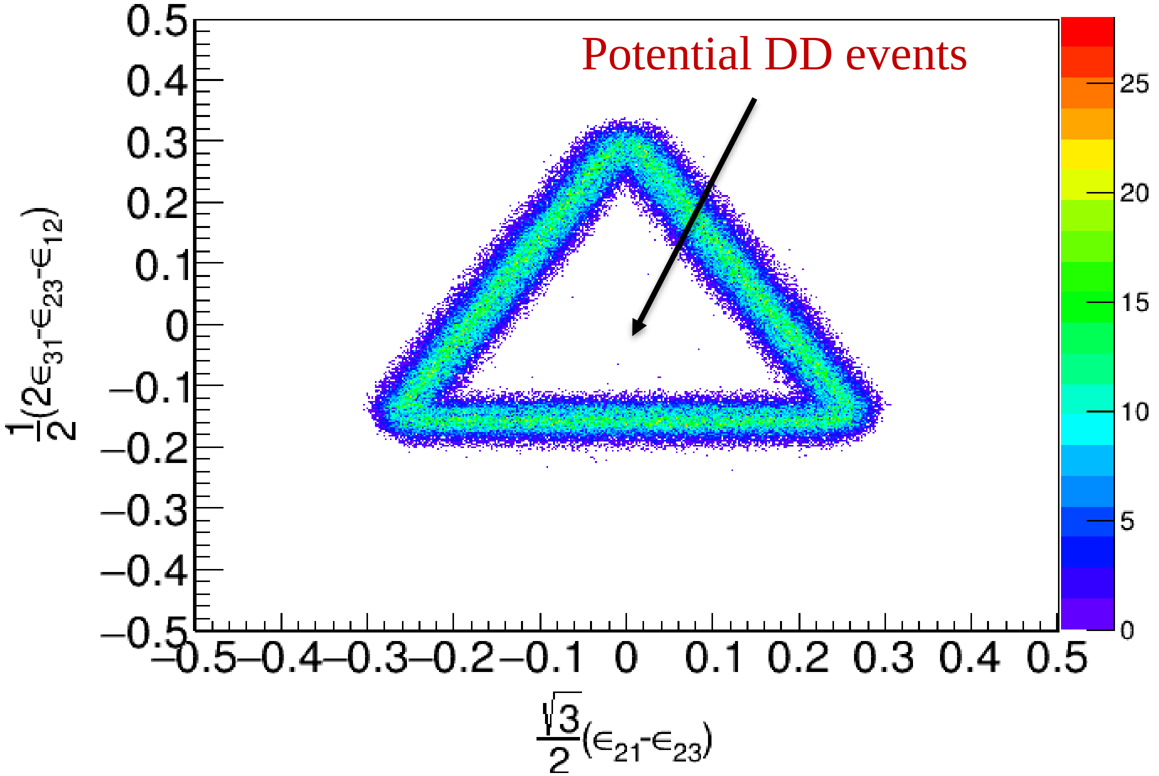
Obtained upper limits:

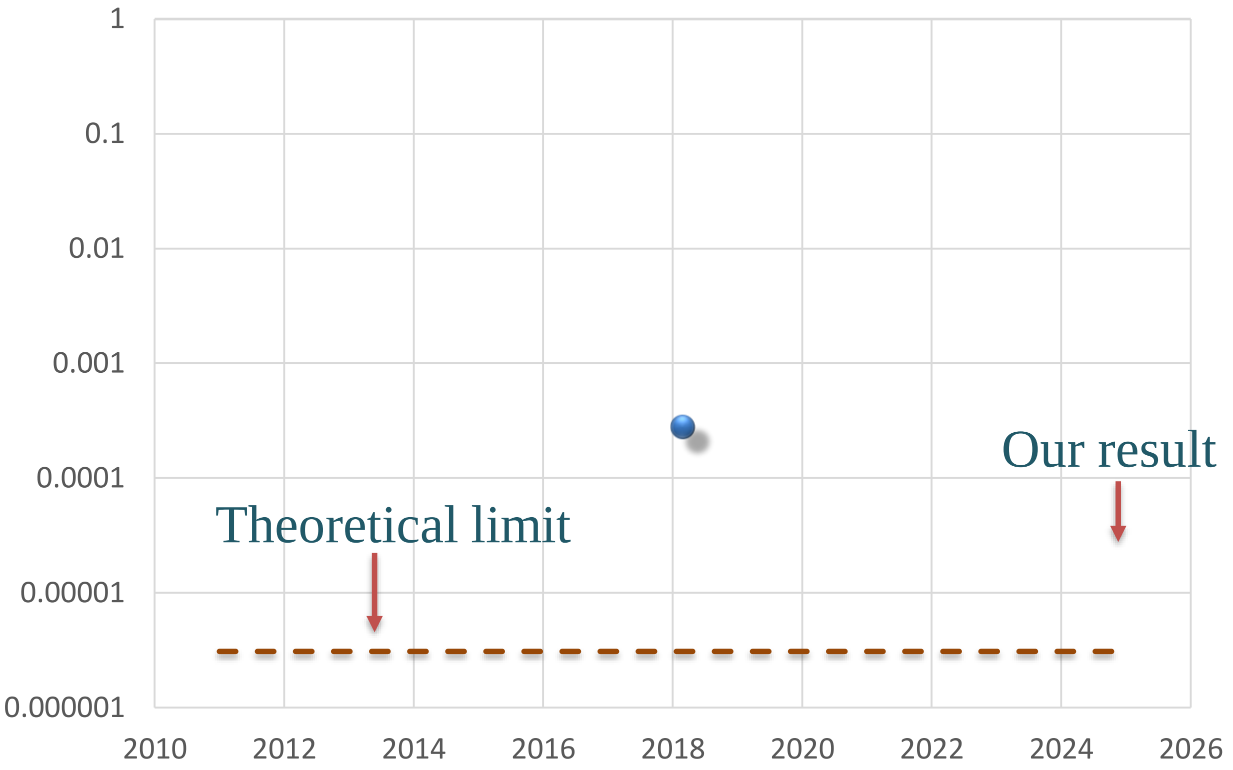
1. Method ii): 0.018% for DDϕ and 0.002% for DDE

Previous upper limits (Rana *et al.*, PLB, 793, 130, 2019):

1. Method ii): 0.019% for DDϕ and 0.012% for DDE

Bayesian Analysis of the experimental data





$$P(DD \mid x) = \frac{f_{DD}(x) \cdot p_{DD}}{f_{DD}(x) \cdot p_{DD} + f_{SD}(x) \cdot (1 - p_{DD})}$$

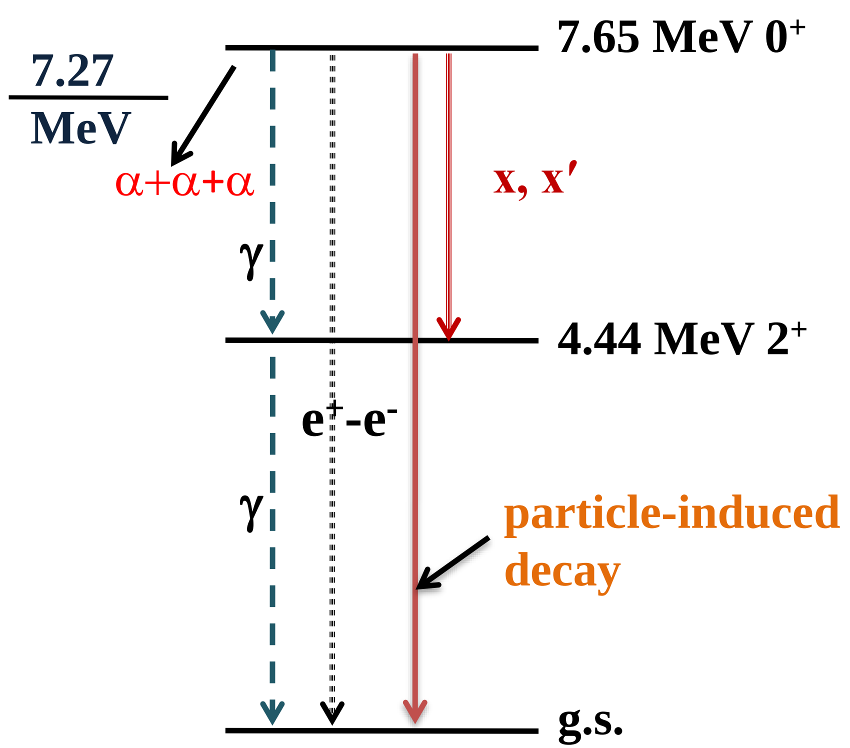
p_{DD} = 0.018%, from likelihood, f_{DD} and f_{SD} from simulation

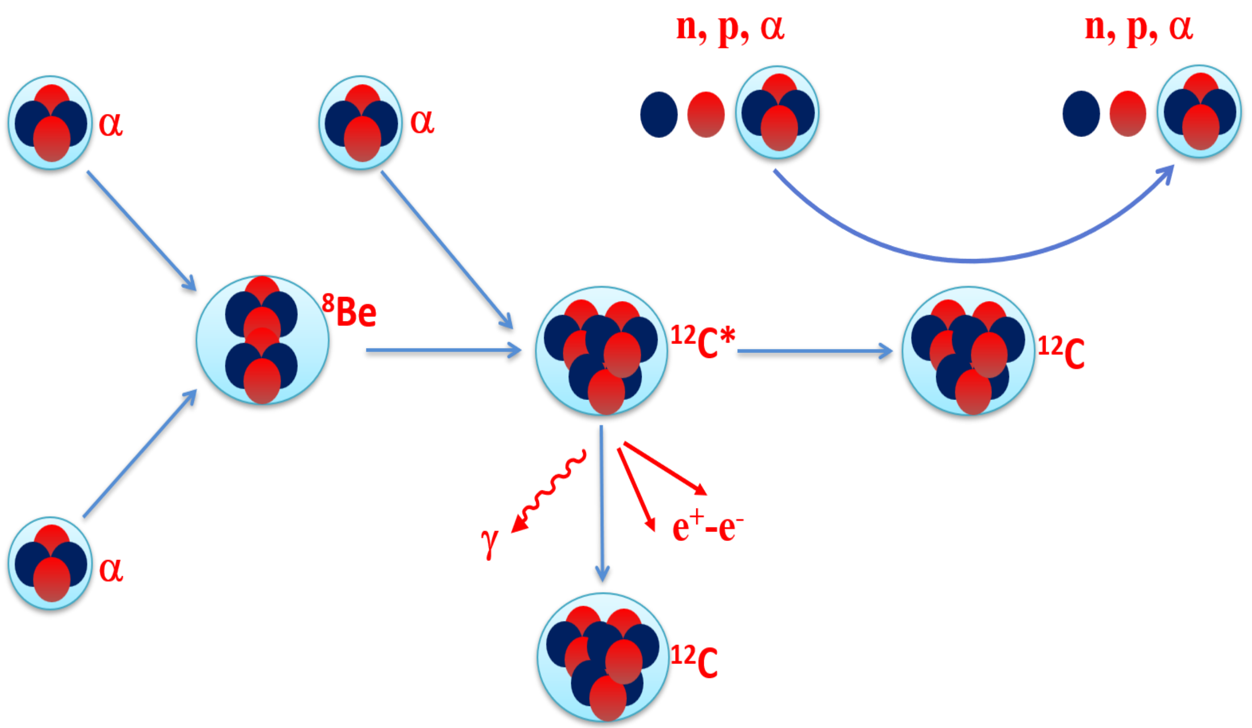
Bayesian soft-assignment scheme (add posterior to counts).

Branching ratio ≈ **0.0018% for DDϕ and 0.00125% for DDE.**

Approaching the theoretical upper limit

II. Particle induced de-excitation





- Scattering nucleus taking away excitation energy from the excited nucleus
- Also known as up-scattering process
- Competes with γ-decay and e^+-e^- pair decay processes
- High temperature and highly dense environments (white dwarf, AGB shells, novae etc.)
- Can impact stellar nucleosynthesis

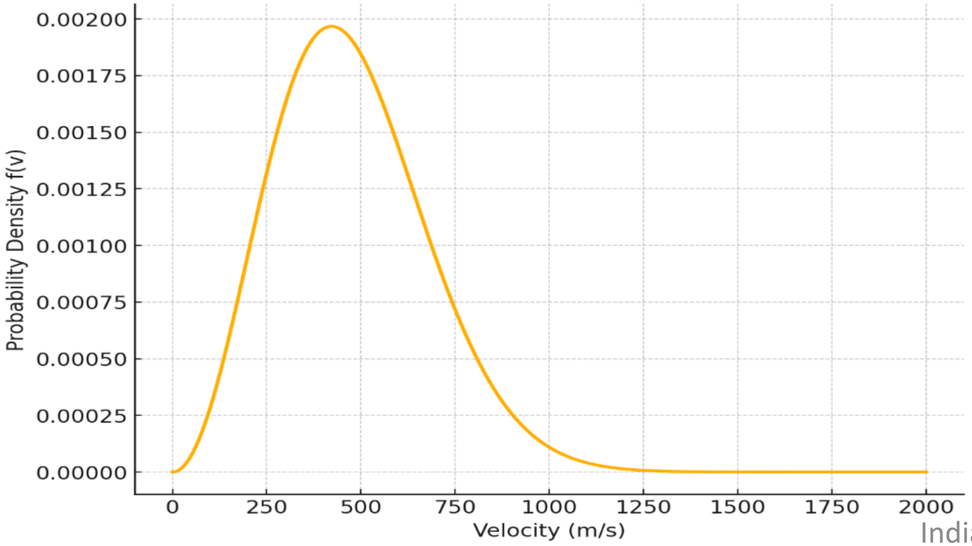
Theoretical Framework

The reaction rate for a particle (say x) induced deexcitation is given by,

$$r_{3\alpha} = \frac{N_{^{12}C}}{\tau_{x'x}(^{12}C^{9.641})} \quad \text{cm}^{-3}\text{sec}^{-1}$$

$$\boxed{\tau_{x'x}(^{12}C^{9.641}) = \frac{1}{N_x \langle \sigma v \rangle_{x'x}} \quad \text{sec}}$$

$$\langle \sigma v \rangle_{xx'} = \left(\frac{8}{\pi\mu}\right)^{1/2} \left(\frac{1}{kT}\right)^{-3/2} \times \int_0^\infty E' \sigma_{xx'}(E') \exp(-E'/kT) dE'$$



Principle of detailed balance

$$\langle \sigma v \rangle_{xx'} \rightarrow \langle \sigma v \rangle_{x'x} = \left(\frac{2I+1}{2I'+1}\right) \exp(-Q/kT) \langle \sigma v \rangle_{xx'}$$

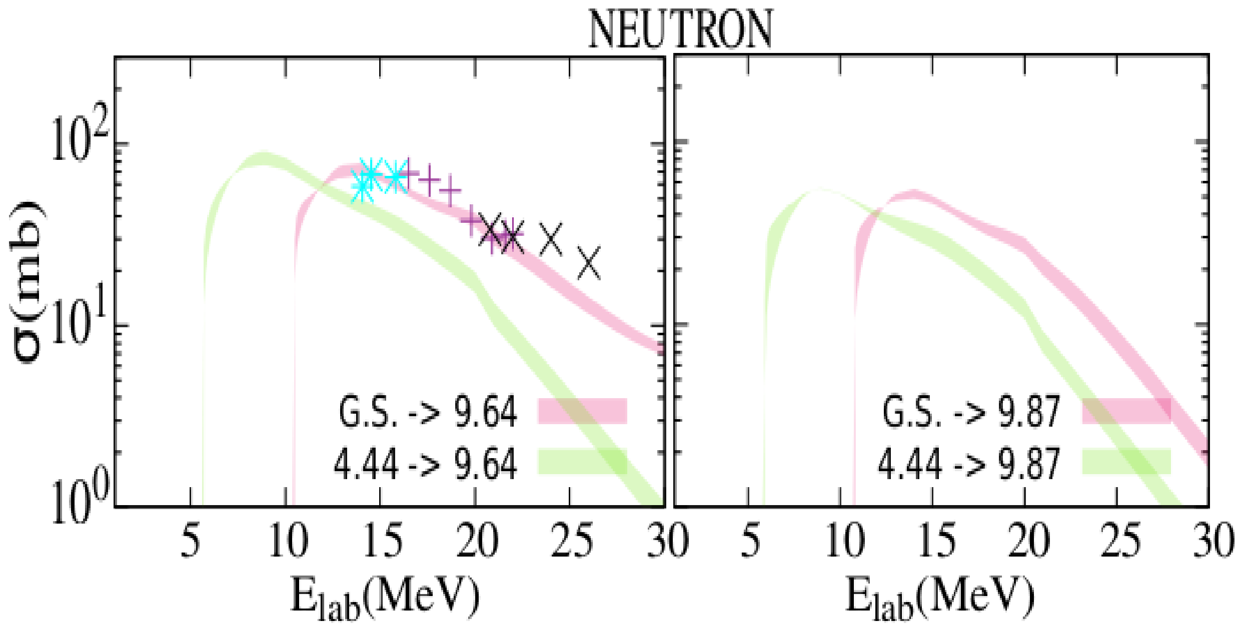
The enhancement in ^{12}C production can be expressed as,

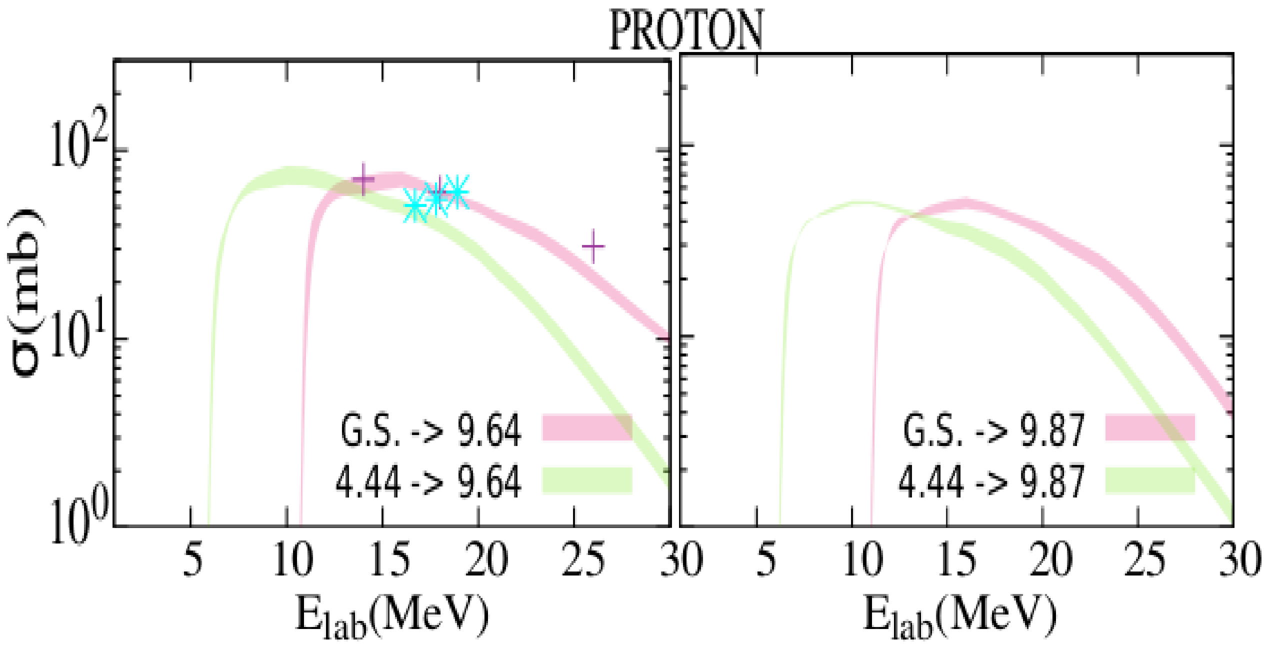
$$R_{xx} = r_{x'x}/r_\gamma = \tau_\gamma/\tau_{x'x} = \tau_\gamma N_x \langle \sigma v \rangle_{x'x}$$

$$\boxed{R_{xx} = k_x \rho_x T_9^{-\frac{3}{2}} f_{spin} \int_0^\infty \sigma_{xx'}(E)(E+E_{th}) \times exp(-11.605E/T_9) dE}$$

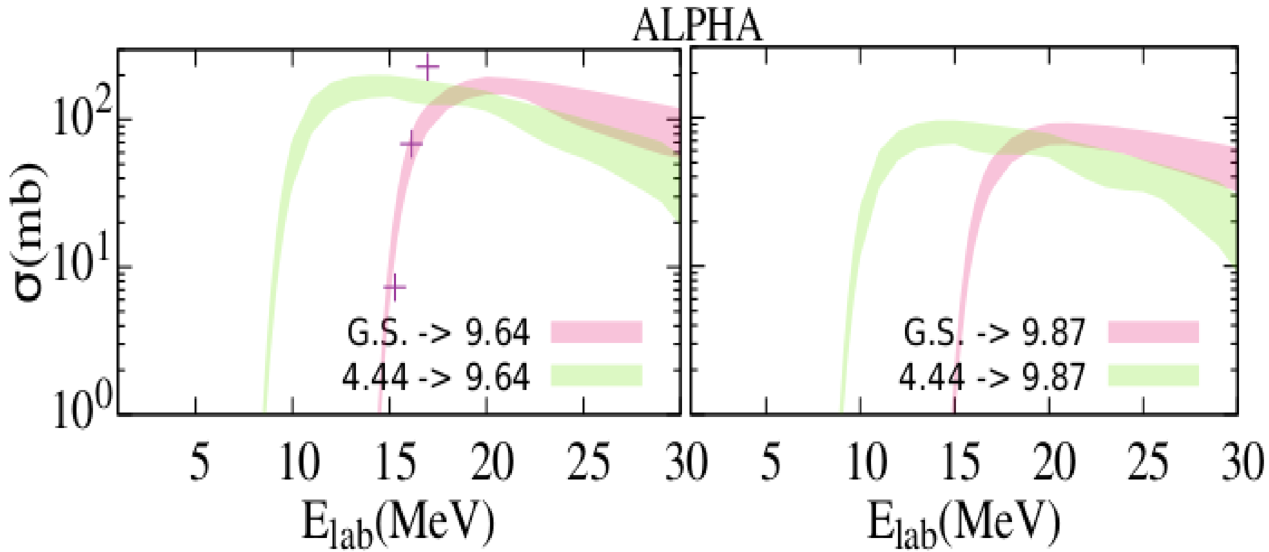
Near-threshold points dominate due to the exp factor

$\sigma_{nn'}$, $\sigma_{pp'}$ and $\sigma_{\alpha\alpha'}$

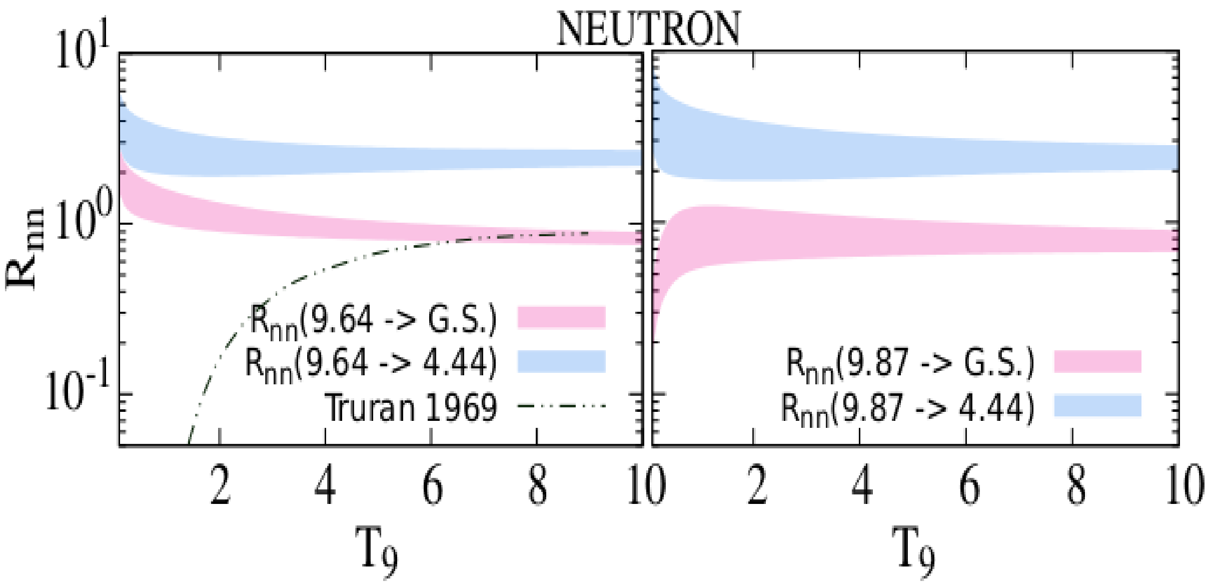


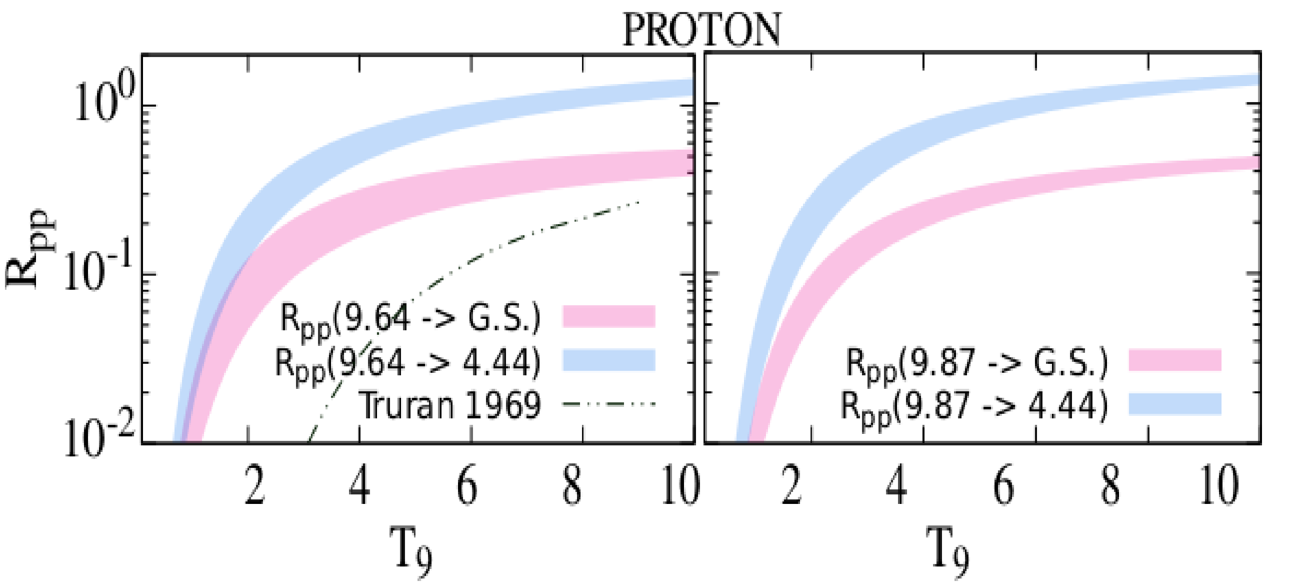


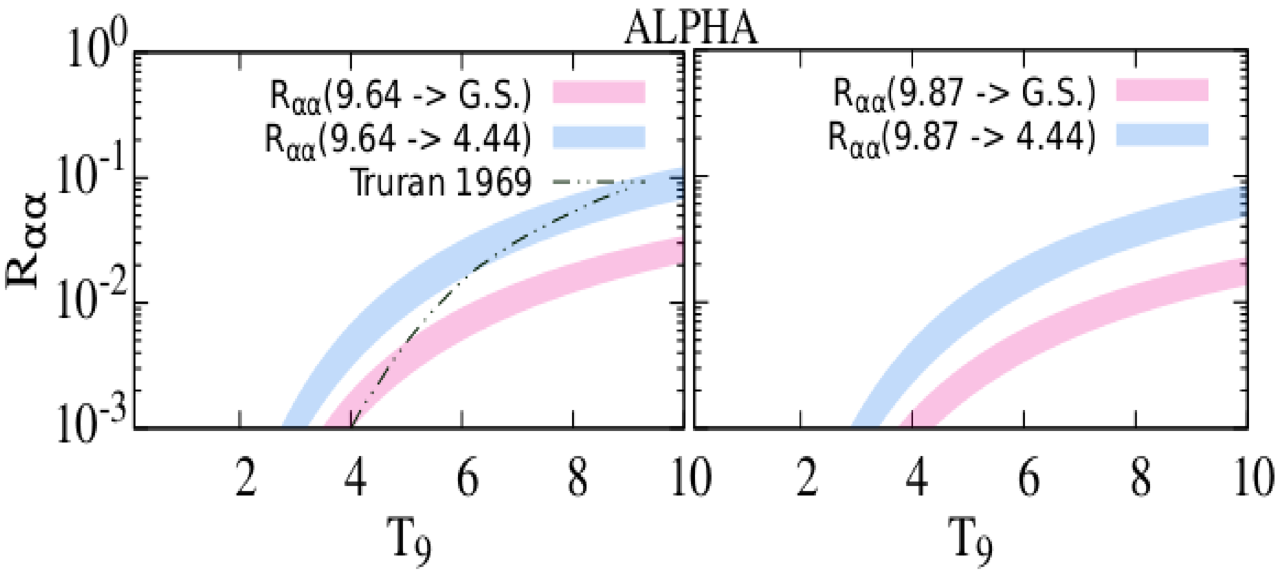
- ✓ $\sigma_{nn'}$, $\sigma_{pp'}$, and $\sigma_{\alpha\alpha'}$ required
- ✓ Only few data points
- ✓ HF reaction code TALYS using the different OMP



Enhancement in ^{12}C production



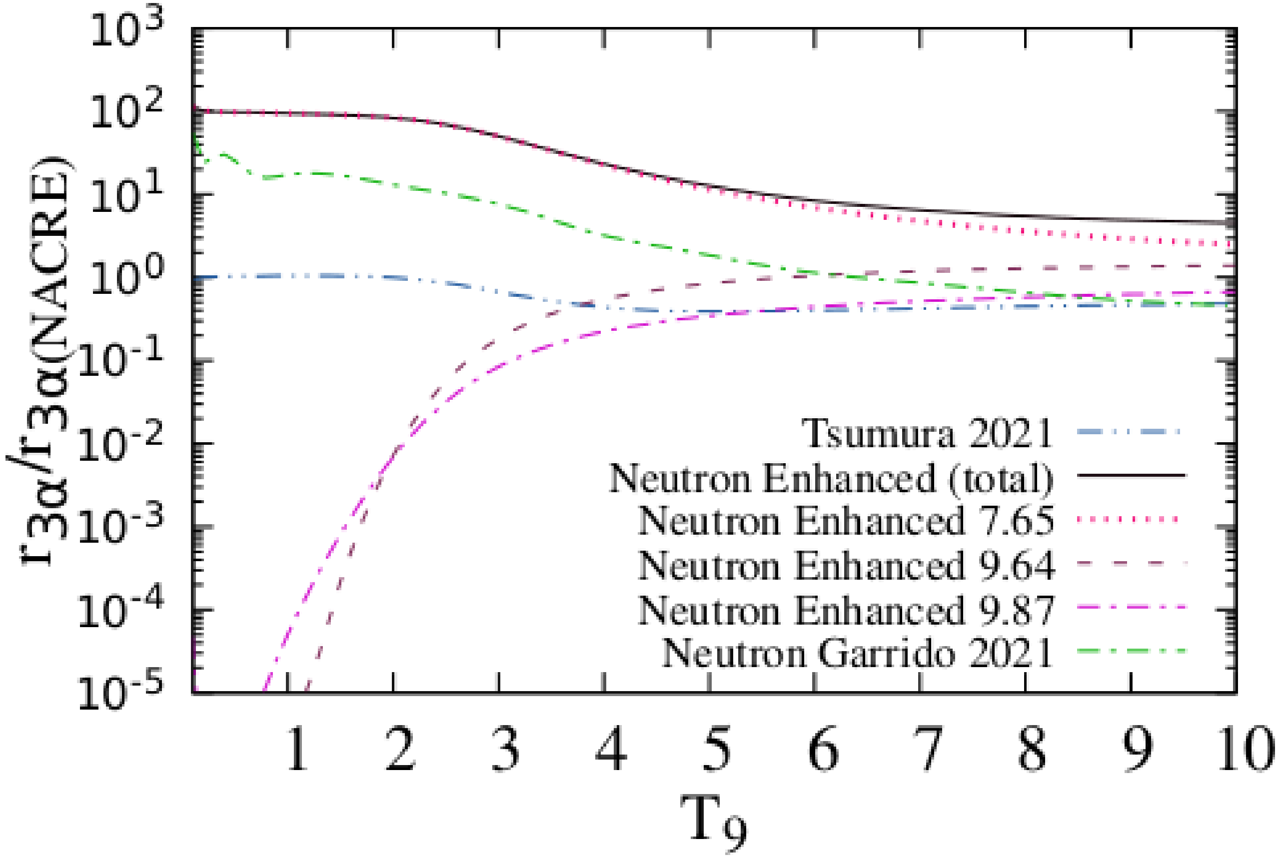




- ✓ R > 1 denotes the induced decay is stronger
- ✓ R_{nn} is the highest and $R_{\alpha\alpha}$ is the lowest.
- ✓ This is due to increasing Coulomb barrier for charged particles.

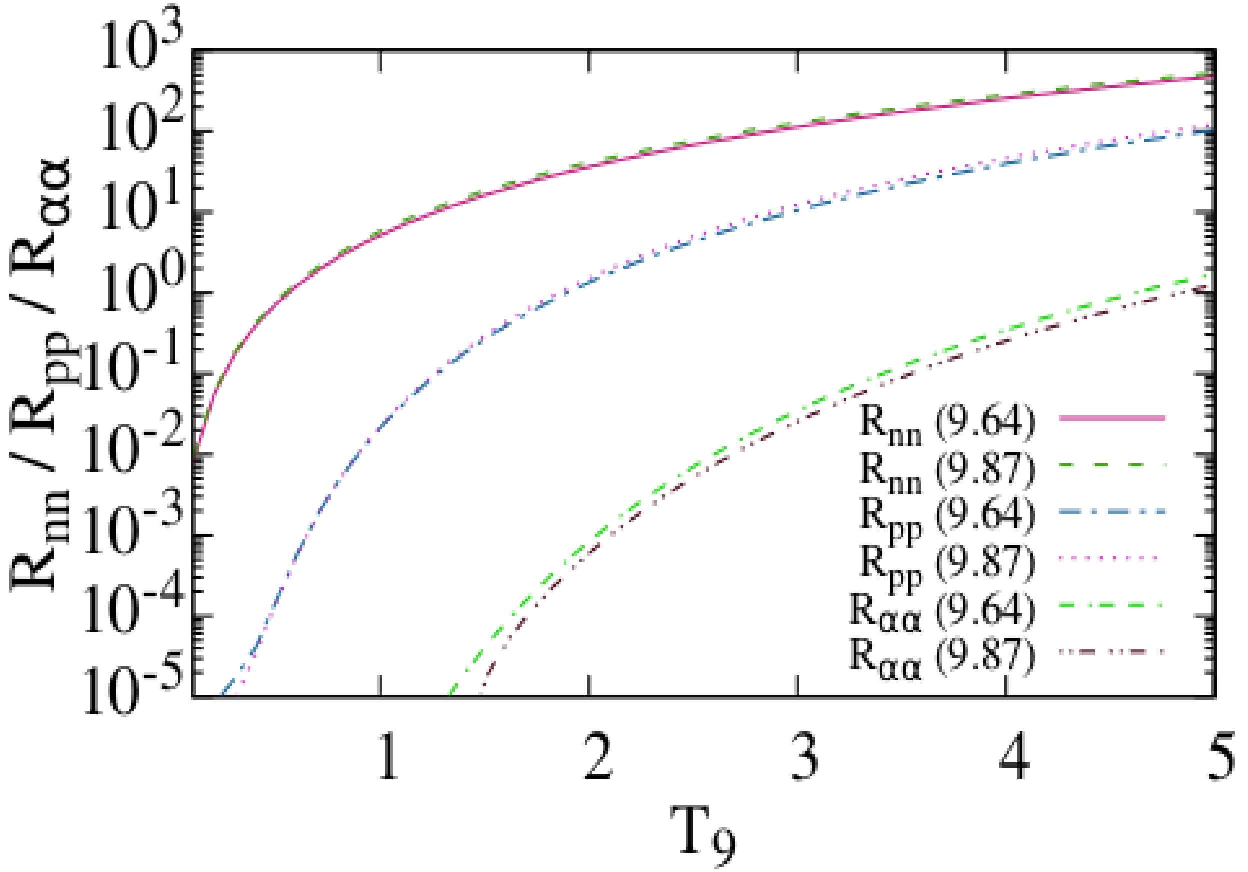
Implications in Nuclear Astrophysics: Enhanced rate compared to NACRE

Triple-α reaction rate for $\rho = 10^6$ g/cm^3



Considerable enhancement in explosive scenarios

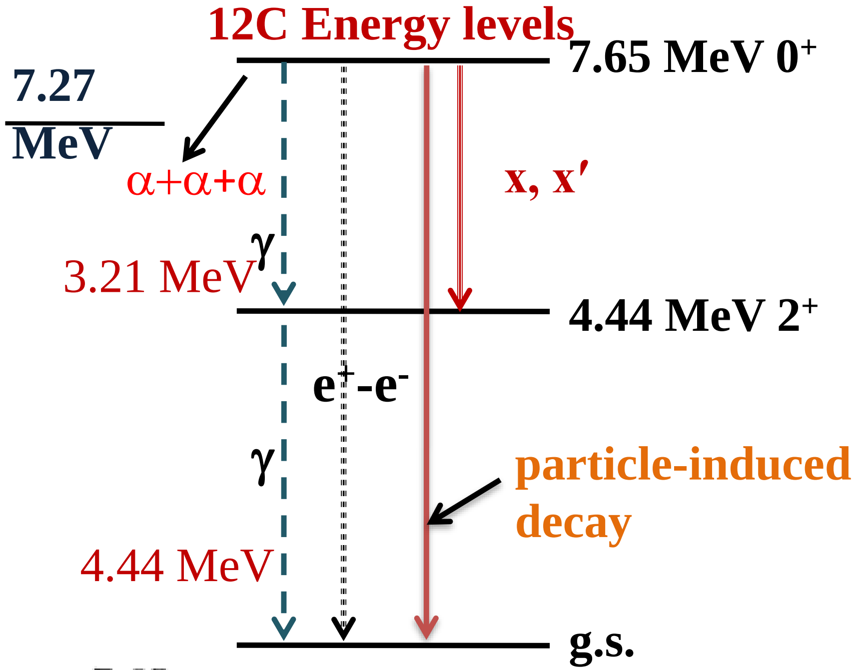
Cowling model star, $\rho \sim 10^6 T_9^3$ valid up to T_9=5



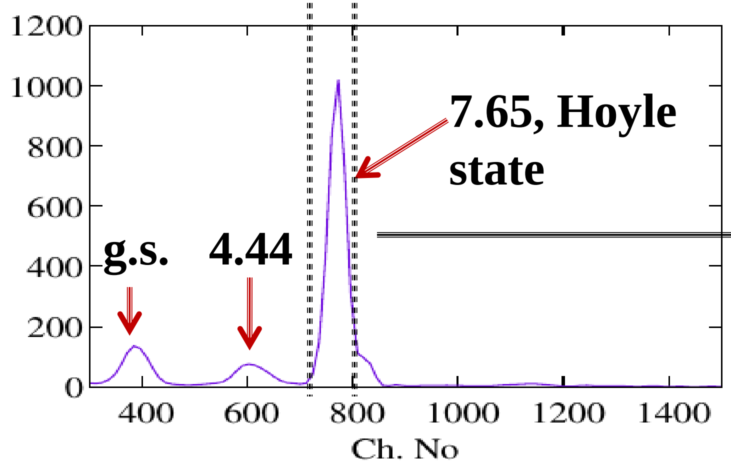
Total enhancement can be 600!

A. Baishya et. al., PRC, 108 (6), 065807 (2023)

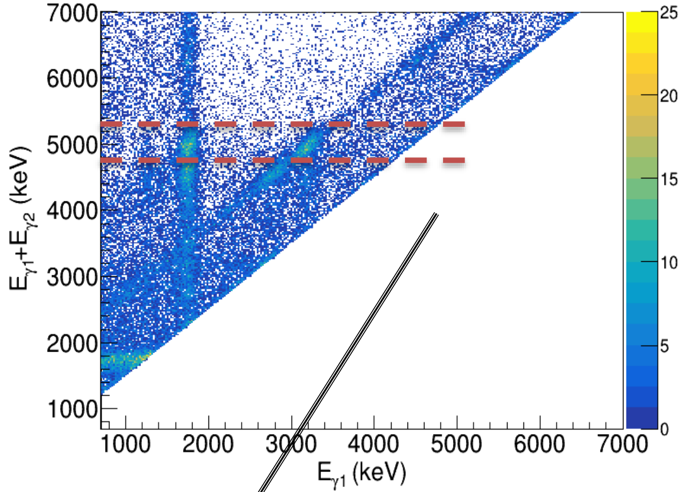
III. Measurement of E2 γ-decay of the Hoyle state

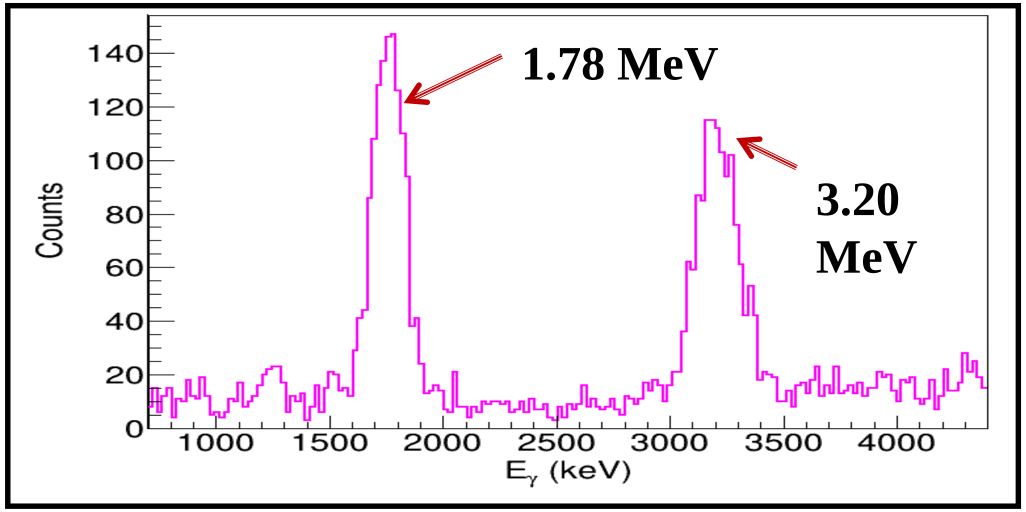


$$\frac{\Gamma_{\gamma}^{7.65}}{\Gamma} = \frac{N_{020}^{7.65}}{N_{\text{singles}}^{7.65} \times \epsilon_{3.21} \times \epsilon_{4.44} \times W_{020}}$$



Energy loss of inelastically scattered protons in ΔE





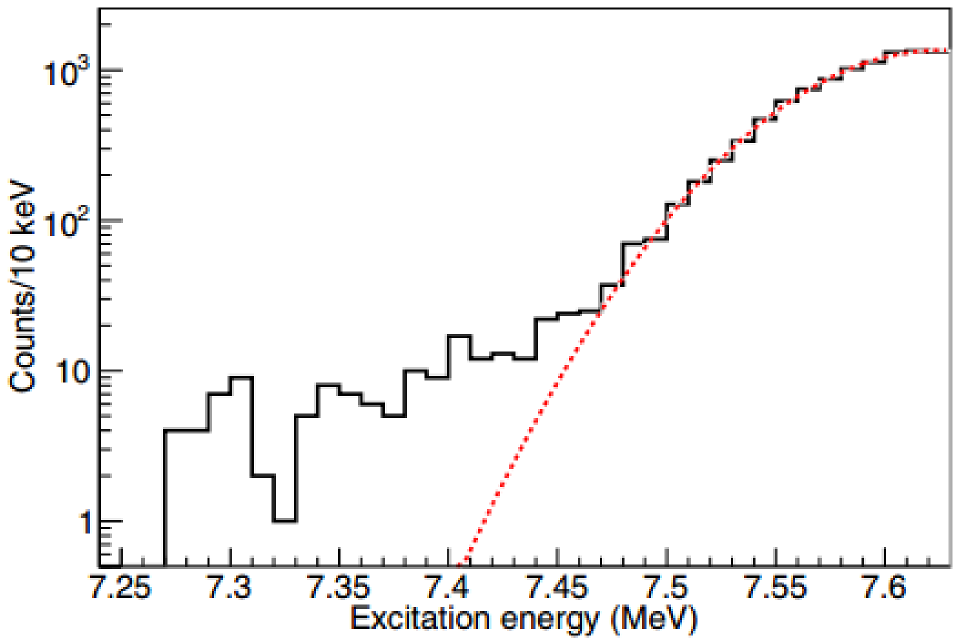
Experimental Setup, i) Charged particle detectors (Si-strips), ii) γ detectors, BGO, total 38

Coincident E2 cascade from 4.98 MeV 0+ state of 28Si, similar expected from 12C

IV. Hypothetical Efimov state above the 3α threshold

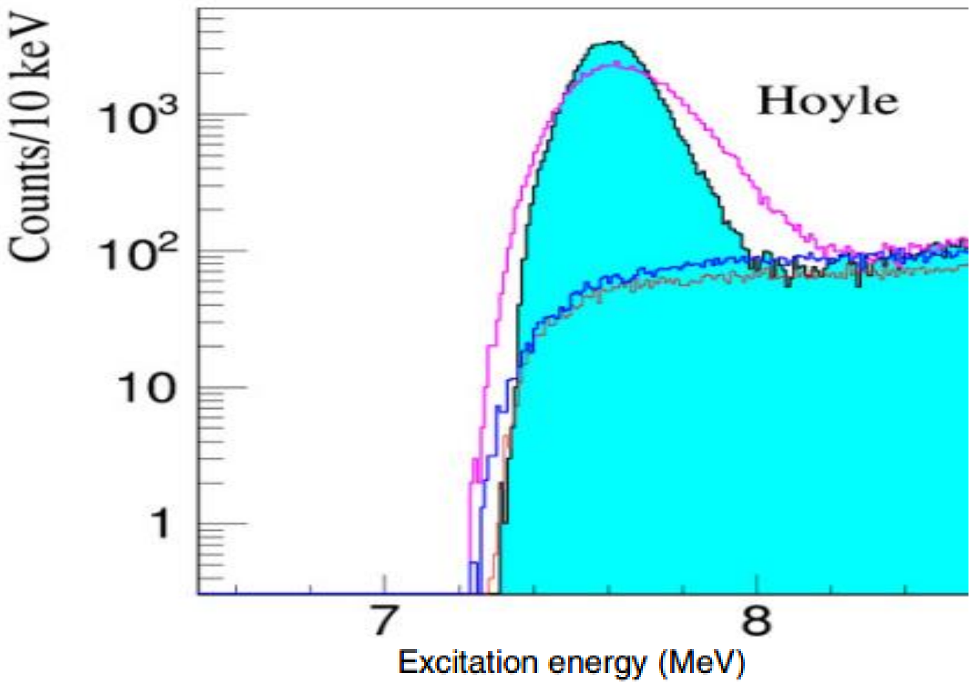
Recently, existence of an Efimov state in ^{12}C at an excitation energy that corresponds to a mutual ^{8}Be(g.s.) resonance for all three α particles was suggested in Ref. (**Phys. Lett. B 779, 460, 2018**). This excitation energy is given by,

$$E_{Efimov} = \frac{2}{3}\sum_{i \neq j}^{3} E_{ij} + E_{th} = \frac{2}{3} \times 0.092 + 7.274 = 7.458$$



Phys. Rev. C 103, L051303 (2021)

Concluded no evidence, with an upper limit of 0.69%

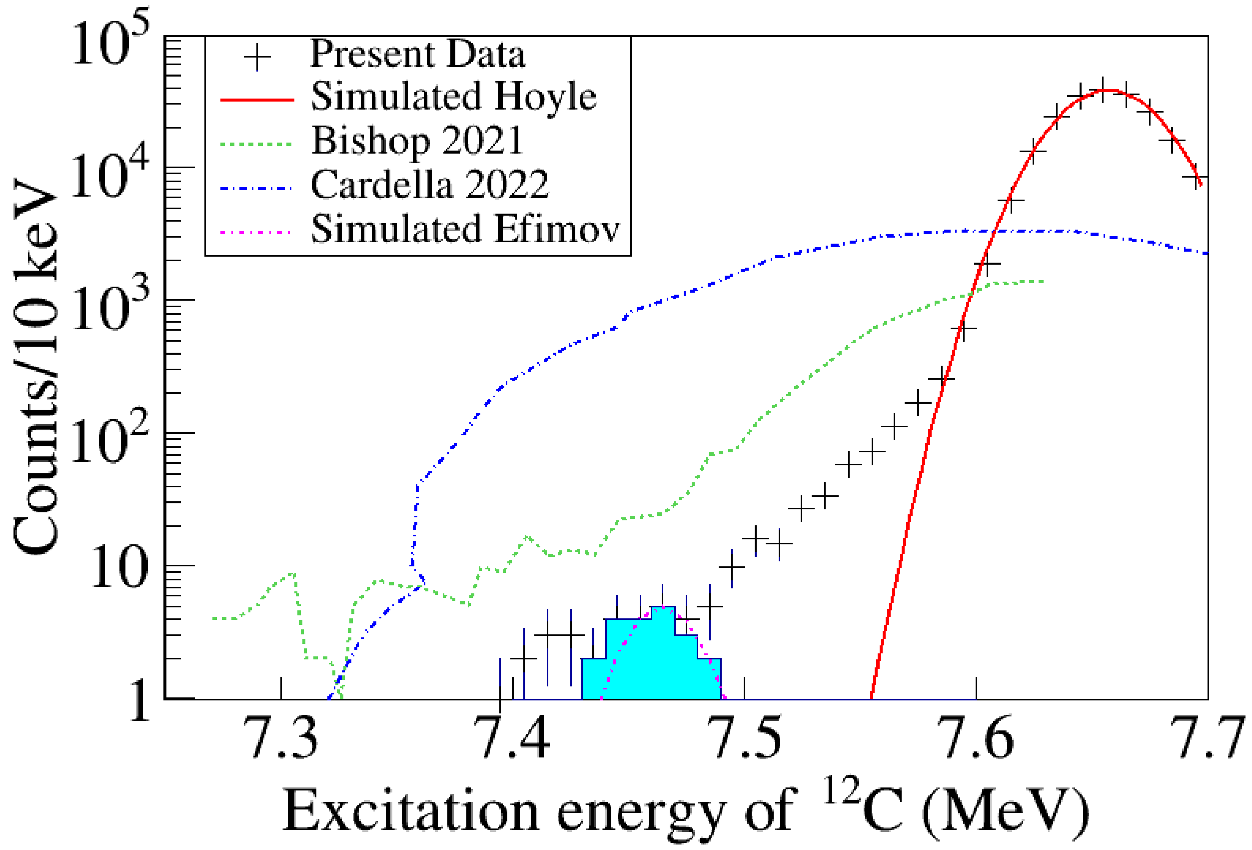


Nuclear Physics A 1020, 122395 (2022).

Concluded an upper limit of 0.2%

Experimental Upper Limit

Excitation energy of ^{12}C



$\Gamma_i = 2P_i\gamma_i^2$,

P_i is the penetrability factor, γ_i^2 is related to the structur

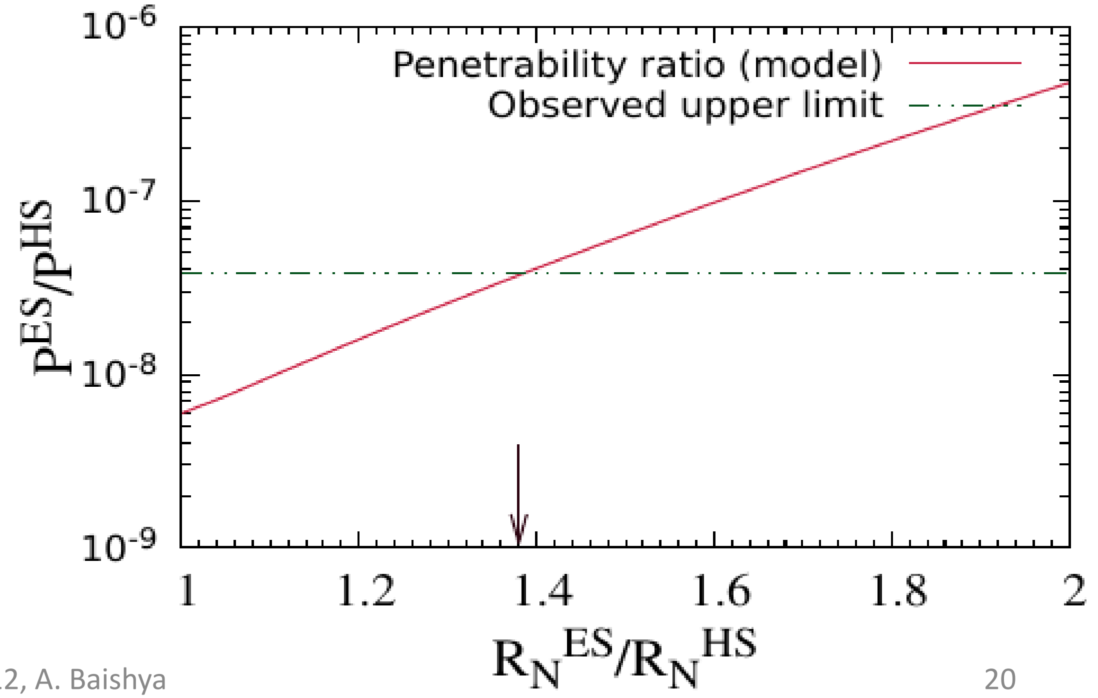
$\gamma^2_{3\alpha}{}^{HS} \approx \gamma^2_{3\alpha}{}^{ES}$ (assumption)

$\Gamma_{3\alpha}{}^{ES} = \Gamma_{3\alpha}{}^{HS} * P_{3\alpha}{}^{ES}/P_{3\alpha}{}^{HS}$

From the WKB theory

$$P_{wkb} = \frac{1}{1 + exp(2S)}$$

$$S = \frac{1}{\hbar}\int_{r_1}^{r_2}\sqrt{2m\left(V_{eff}(r) - E\right)}\,dr$$



Total Hoyle events ~ 2.21×10⁵ → Filter events with mutual 92 keV relative energy → Potential 21 events

$$\frac{\Gamma_\alpha^{ES}}{\Gamma^{ES}} < \frac{\sigma_{HS}}{\sigma_{ES}} \times \frac{N_{ES}}{N_{HS}} \qquad \frac{\Gamma_\alpha^{ES}}{\Gamma^{ES}} \lessapprox \frac{N_{ES}}{N_{HS}}$$

→ Upper limit of 0.014%

Summary

- Analysis for the extraction of direct decay modes of the Hoyle state reveals an upper limit of. 0.0018% for the DDϕ, mode and 0.0003% for the DDE mode using Bayesian analysis, lowest achieved so far.

- Extensive study for the enhancement in extreme stellar situations performed and the enhanced triple-α rate can be much larger than NACRE adopted values at lower temperatures

- Experiment was performed to find out the γ-decay probability of the Hoyle state

- Precision measurement for Efimov state reveals the upper limit of α-decay branching probability to be 0.014%

THANK YOU

Backup slides

Present Status & Motivation

Total events	DD	Reference
~ 20000	$< 9.1\times10^{-3}$	PRC 88 021601 (2013)
~ 21000	$< 2.0\times10^{-3}$	PRL, 113, 102501 (2014)
~ 93000	$< 4.7\times10^{-4}$	PRL, 119, 132501 (2017)
~ 28000	$< 4.3\times10^{-4}$	PRL 119, 132501 (2017)
~ 160000	$< 1.9\times10^{-4}$	PLB, 793, 130 (2019)

Theoretical Upper limit: 1.5×10^{-5}

Data Analysis

Multiplicity = 4

↓

3 from Array-1 & 1 from Array-2 and vice versa

↓

Kinematic Gate

↓

Assume 3 from one array to be αs and the remaining one ^{12}C

↓

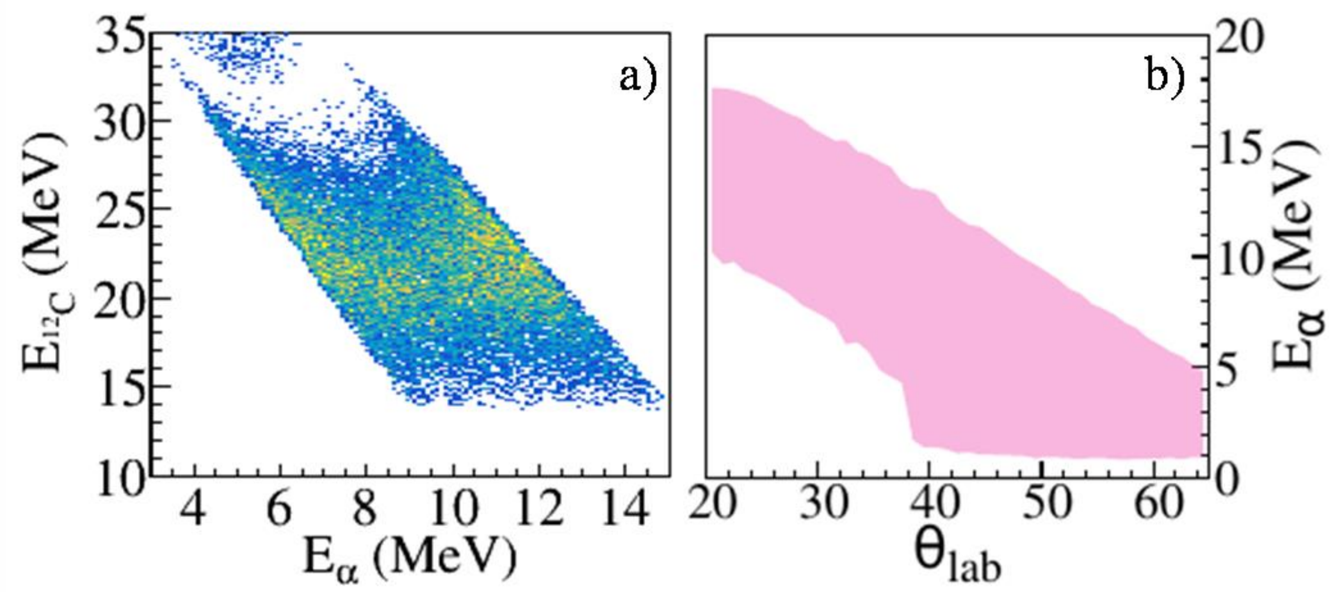
Timing Gate

↓

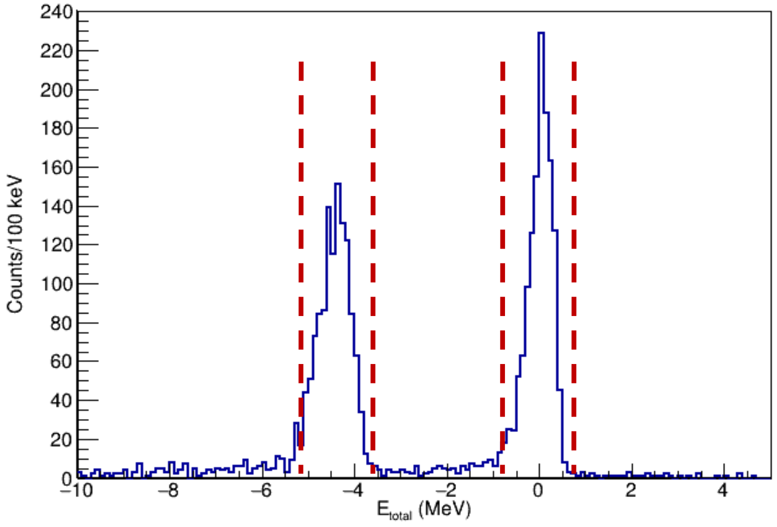
Momentum Gate

↓

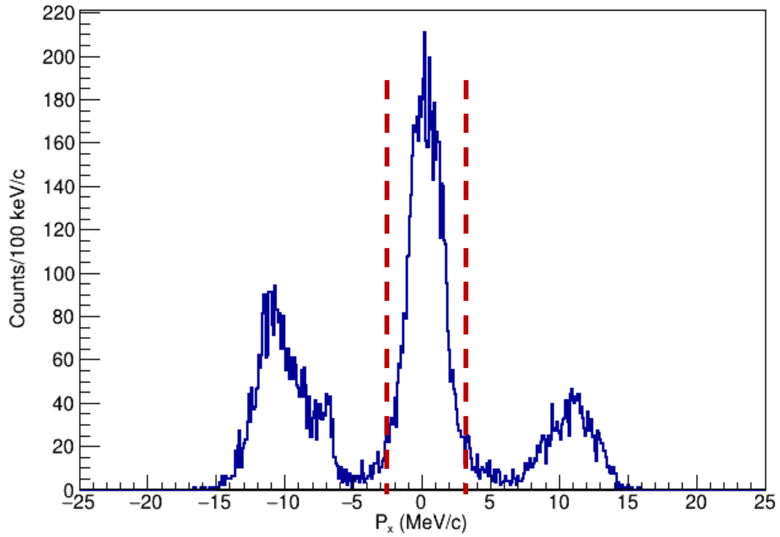
Q-value Gate



Q-value Gate



Momentum Gate



Excitation energy calculation

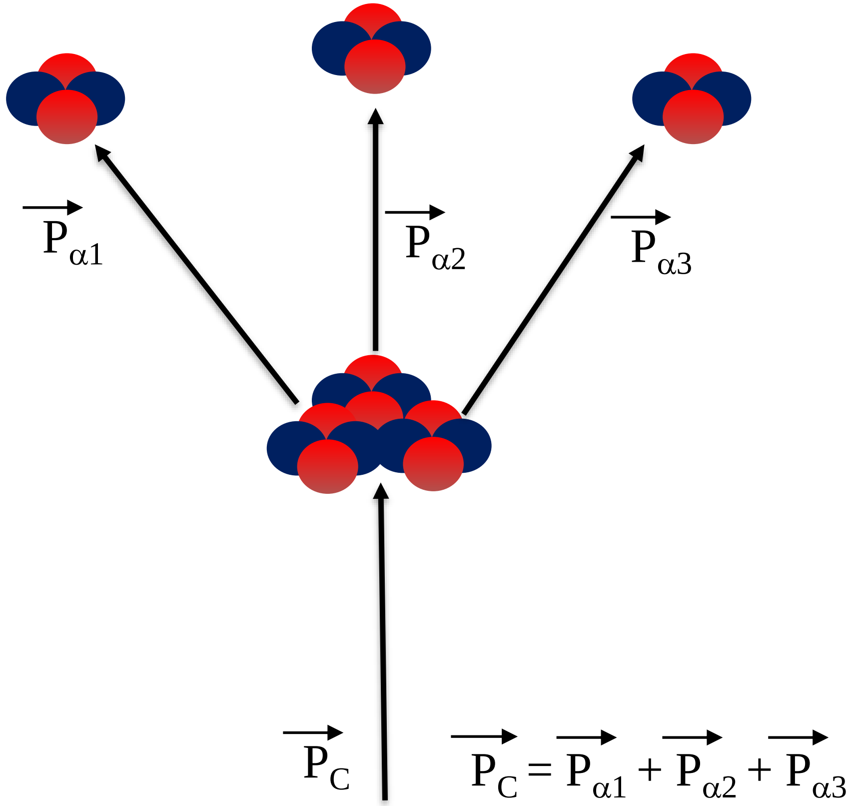
$$\overrightarrow{P_{\alpha_i}} = \sqrt{2E_{\alpha_i} m_{\alpha_i}} \times (sin\theta_{\alpha_i} cos\phi_{\alpha_i} \hat{i} + sin\theta_{\alpha_i} sin\phi_{\alpha_i} \hat{j} + cos\theta_{\alpha_i})$$

$$\overrightarrow{P_C} = \overrightarrow{P_{\alpha 1}} + \overrightarrow{P_{\alpha 2}} + \overrightarrow{P_{\alpha 3}}$$

$$\overrightarrow{V_C} = \overrightarrow{P_C} / m_C$$

$$\overrightarrow{v_{\alpha 1}} = \overrightarrow{P_{\alpha 1}}/m_{\alpha 1}, \ \overrightarrow{v_{\alpha 2}} = \overrightarrow{P_{\alpha 2}}/m_{\alpha 2}, \ \overrightarrow{v_{\alpha 3}} = \overrightarrow{P_{\alpha 3}}/m_{\alpha 2}$$

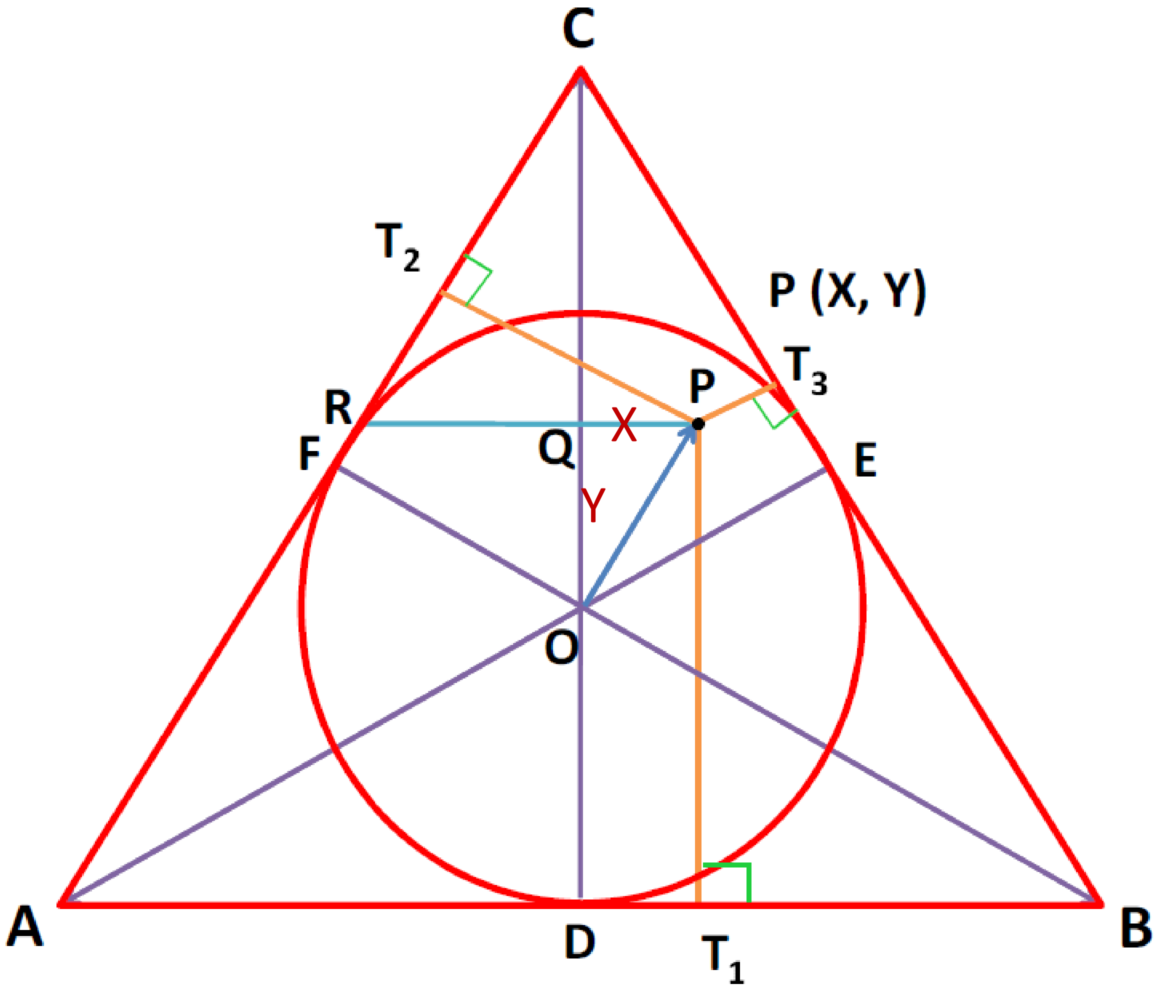
$$\left.\begin{aligned} \overrightarrow{v_{\alpha 1}}' &= \overrightarrow{v_{\alpha 1}} - \overrightarrow{V_C} \\ \overrightarrow{v_{\alpha 2}}' &= \overrightarrow{v_{\alpha 2}} - \overrightarrow{V_C} \\ \overrightarrow{v_{\alpha 3}}' &= \overrightarrow{v_{\alpha 3}} - \overrightarrow{V_C} \end{aligned}\right\} \text{(velocities in } ^{12}\text{C rest frame)}$$



$$Ex = E_{th}\ (7.274\ \text{MeV}) + \tfrac{1}{2} \times m_{\alpha 1} v_{\alpha 1}'^{\,2} + \tfrac{1}{2} \times m_{\alpha 2} v_{\alpha 2}'^{2} + \tfrac{1}{2} \times m_{\alpha 3} v_{\alpha 3}'^{2}$$

Finding Direct Decay Branching Ratio: Techniques

DALITZ plot technique



$$X = \sqrt{3}(T_2 - T_3)$$
$$Y = (2T_1 - T_2 - T_3)$$

- For a 3-body decay with decay particles of same mass, the point P must be inside this circle
- The area of the circle is proportional to the available phase space
- When direct decay happens and any distribution amongst T_i are possible => point P can be anywhere inside the circle
- When the 3-body decay proceeds through a 2-body decay step, the distribution of T_i will be constrained by the intermediate step => restricted locus in plot

Bayesian Analysis of the experimental data

Example: Medical Testing

Suppose a disease affects 1 in 100 people. A test is 99% accurate.

Positive test result?

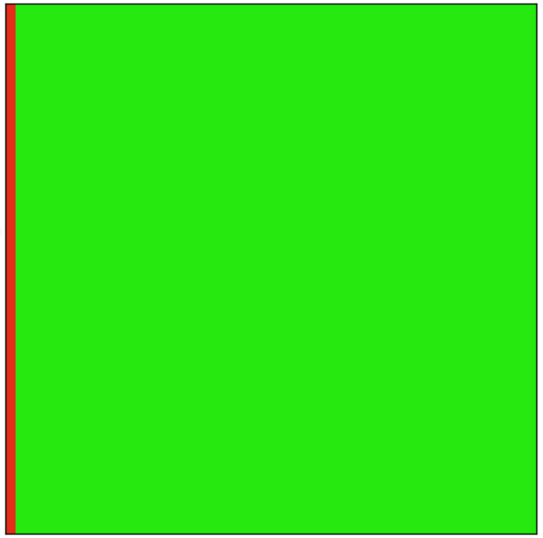
Using Bayes' theorem, the actual chance the person has the disease is **50%**, not 99%.

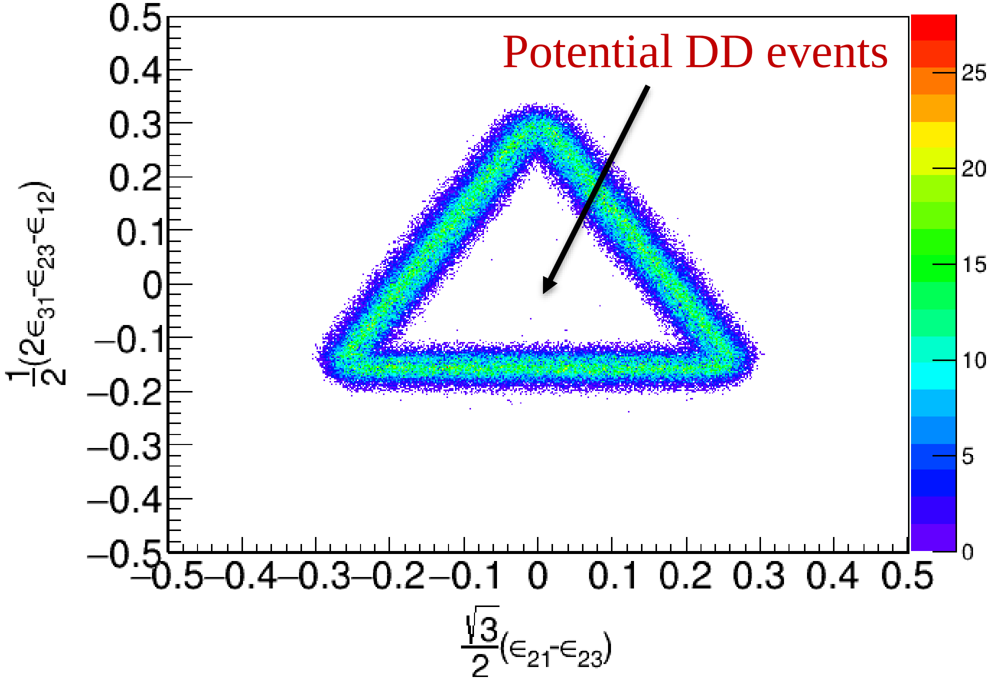
$$P(D|\mathrm{Pos}) = \frac{P(\mathrm{Pos}|D) \cdot P(D)}{P(\mathrm{Pos})}$$

$$P(\mathrm{Pos}) = P(\mathrm{Pos}|D) \cdot P(D) + P(\mathrm{Pos}|\neg D) \cdot P(\neg D)$$

$$= 0.99*0.01 + 0.01*0.99$$

Prior: 0.01





$$P(DD \mid x) = \frac{f_{DD}(x) \cdot p_{DD}}{f_{DD}(x) \cdot p_{DD} + f_{SD}(x) \cdot (1 - p_{DD})}$$

p_{DD} = 0.018%, from likelihood, f_{DD} and f_{SD} from simulation

Bayesian soft-assignment scheme (add posterior to counts).

1. Using this, a realistic branching ratio ≈ **0.0018% for DDϕ and 0.00125% for DDE.**
2. If a moderate background (0.01%) taken, the branching ratio ≈ **0.0013% for DDϕ and 0.0003% for DDE**.

Manuscript prepared, to be submitted

Existing literature

Shaw and Clayton, Phys. Rev. 160, 1193 (1967) → Electromagnetic deexcitation by inelastic scattering

Truran and Kozlovsky, Astrophys. J. 158, 1021 (1969) → Nuclear deexcitation by inelastic scattering, using const. cross section

Davids and Bonner, Astrophys. J. 166, 405 (1971) → Nuclear deexcitation by inelastic scattering, using experimental cross section for the Hoyle state

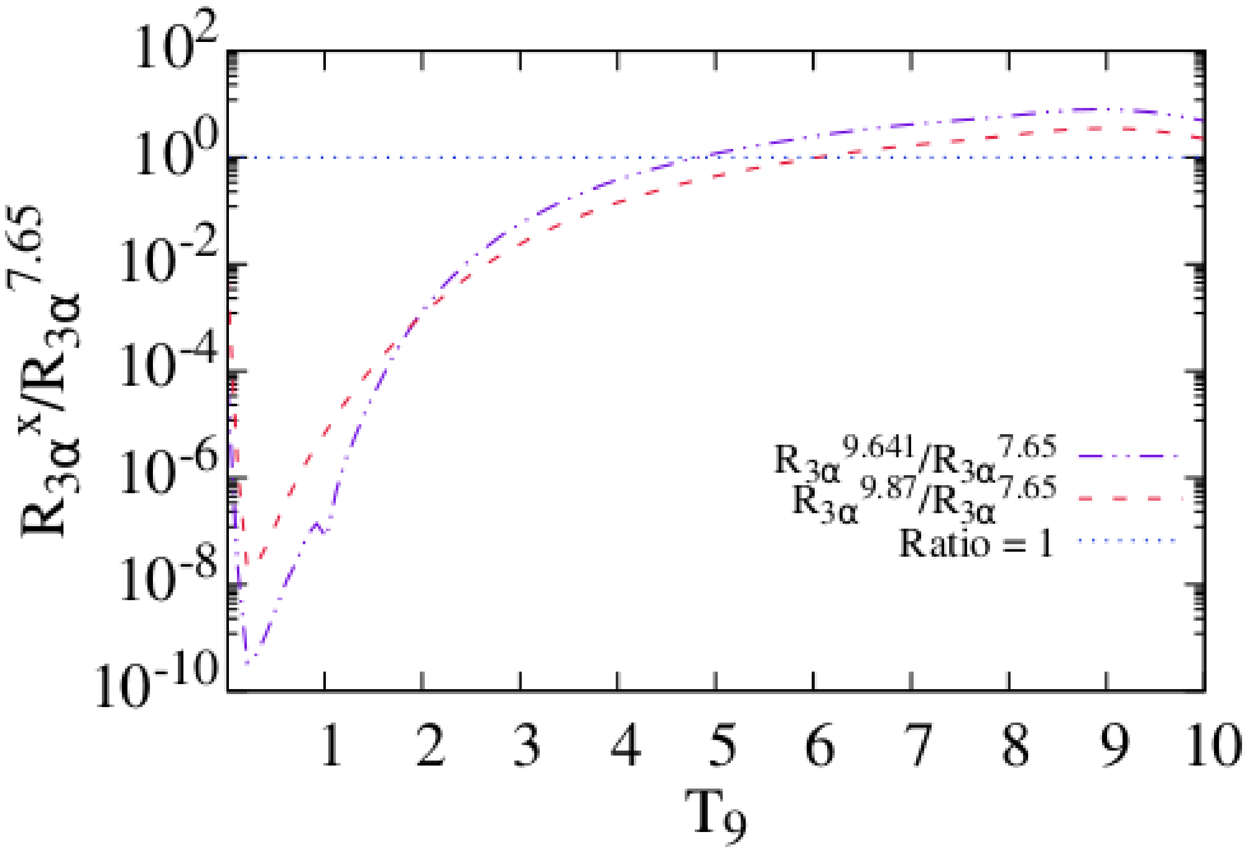
Beard *et al.*, Phys. Rev. Lett. 119, 112701 (2017) → Nuclear deexcitation by inelastic scattering by n, p, and α, using TALYS generated cross section for the Hoyle state

Jin *et al.*, Nature 588, 57-60 (2020) → Effect of proton induced nuclear deexcitation from the Hoyle state on the production of proton rich nuclei

J. Bishop *et al.*, Nat. Commun. 13, 2151 (2022) → Neutron induced nuclear deexcitation, using experimental neutron cross section for the Hoyle state

Our work is on proton, neutron, alpha induced deexcitation from the 9.641 and 9.87 MeV states using TALYS cross sections

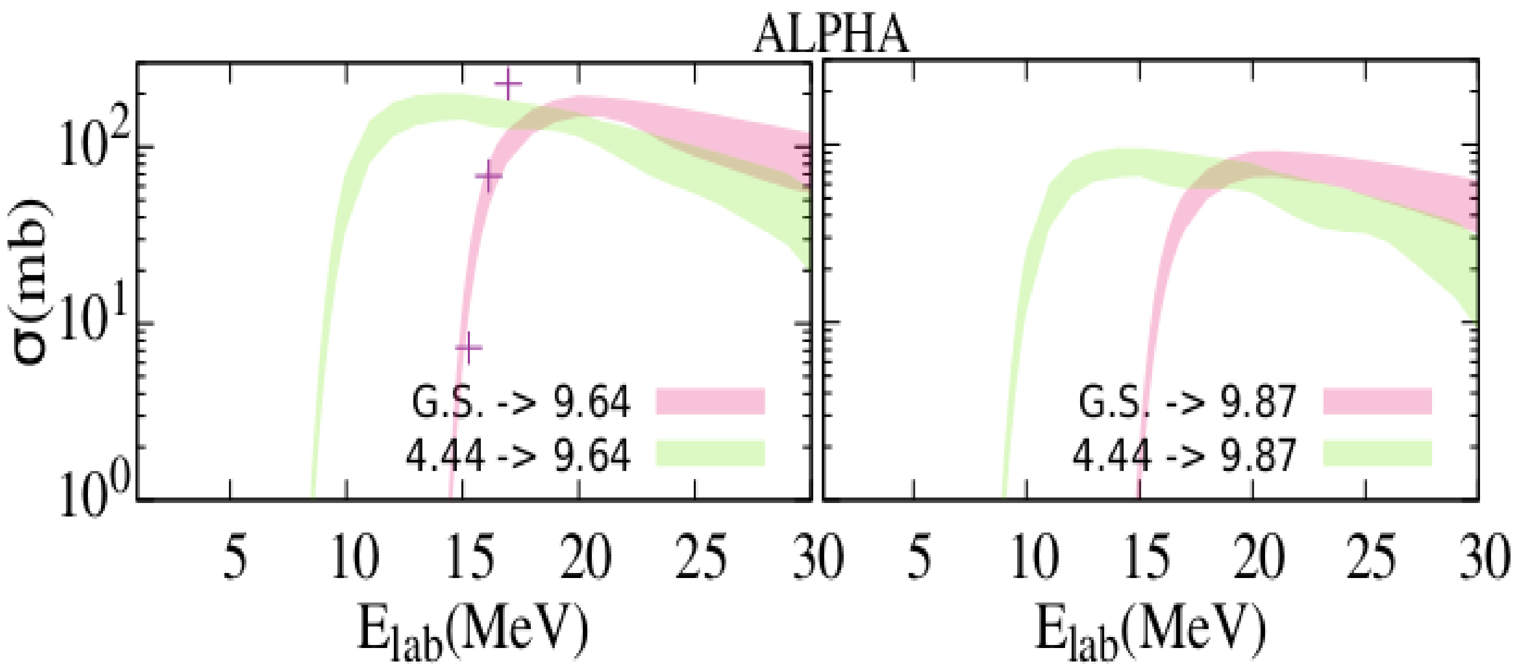
^{12}C production through 7.653 MeV and 9.641 MeV states



- In usual stellar environments, the ^{12}C formation is dominated by the 7.653 MeV Hoyle state
- In explosive stellar environments where temperature can reach ~100 GK, the ^{12}C production can be dominated by the 9.641 MeV state.
- Any enhancement in the stable ^{12}C production due to particle induced deexcitation needs to be addressed
- Enhancement in 3-α reaction rate due to particle induced deexcitation from the Hoyle state has been recently studied by Davids *et al.*, Truran *et al.*, Beard *et al.* etc
- We present the enhancement in decay through 9.641 MeV state

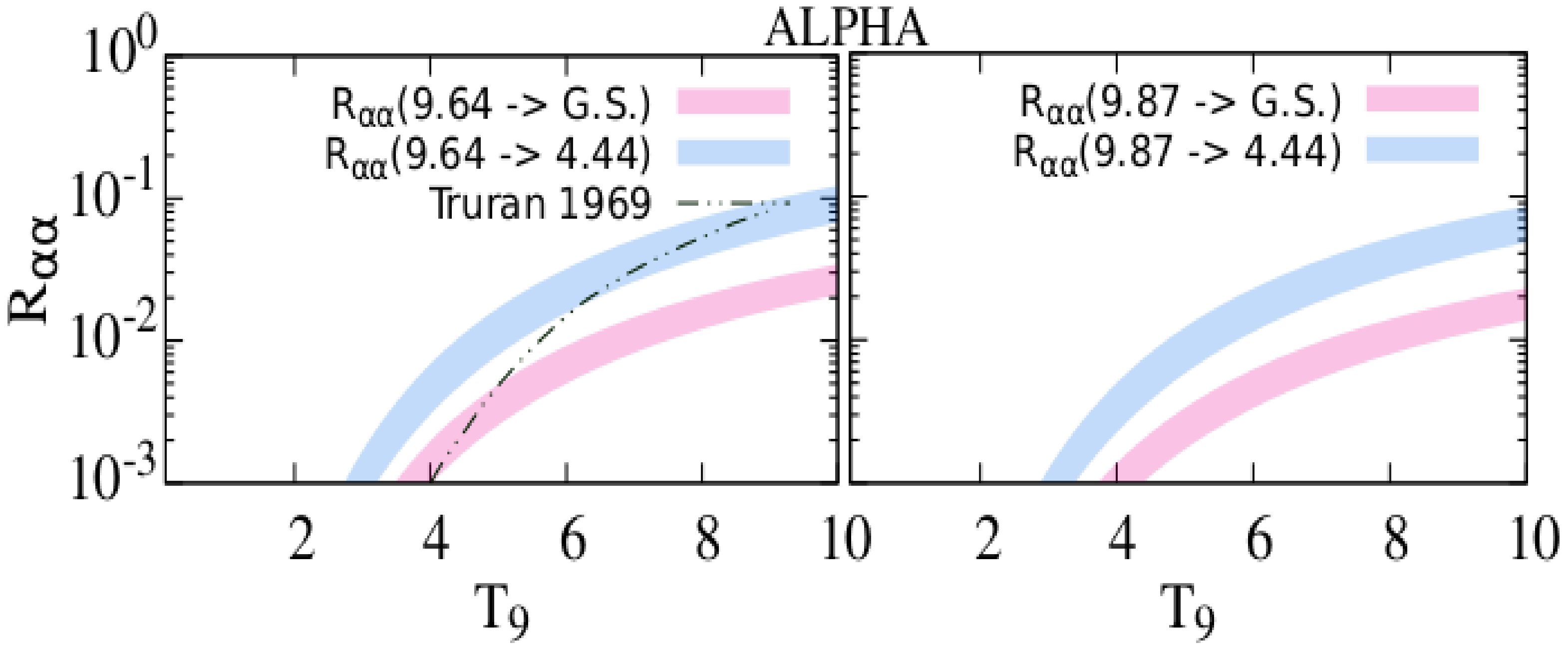
$\sigma_{nn'}$, $\sigma_{pp'}$ and $\sigma_{\alpha\alpha'}$

- ✓ **$\sigma_{nn'}$, $\sigma_{pp'}$, and $\sigma_{\alpha\alpha'}$** either to be known experimentally or theoretically
- ✓ Only few data points above threshold are available
- ✓ Theoretical estimation required
- ✓ Hauser-Feshbach (HF) statistical model can give realistic estimation
- ✓ Cross-sections are generated by HF reaction code TALYS-1.96 using the different OMP



Enhancement in ^{12}C production

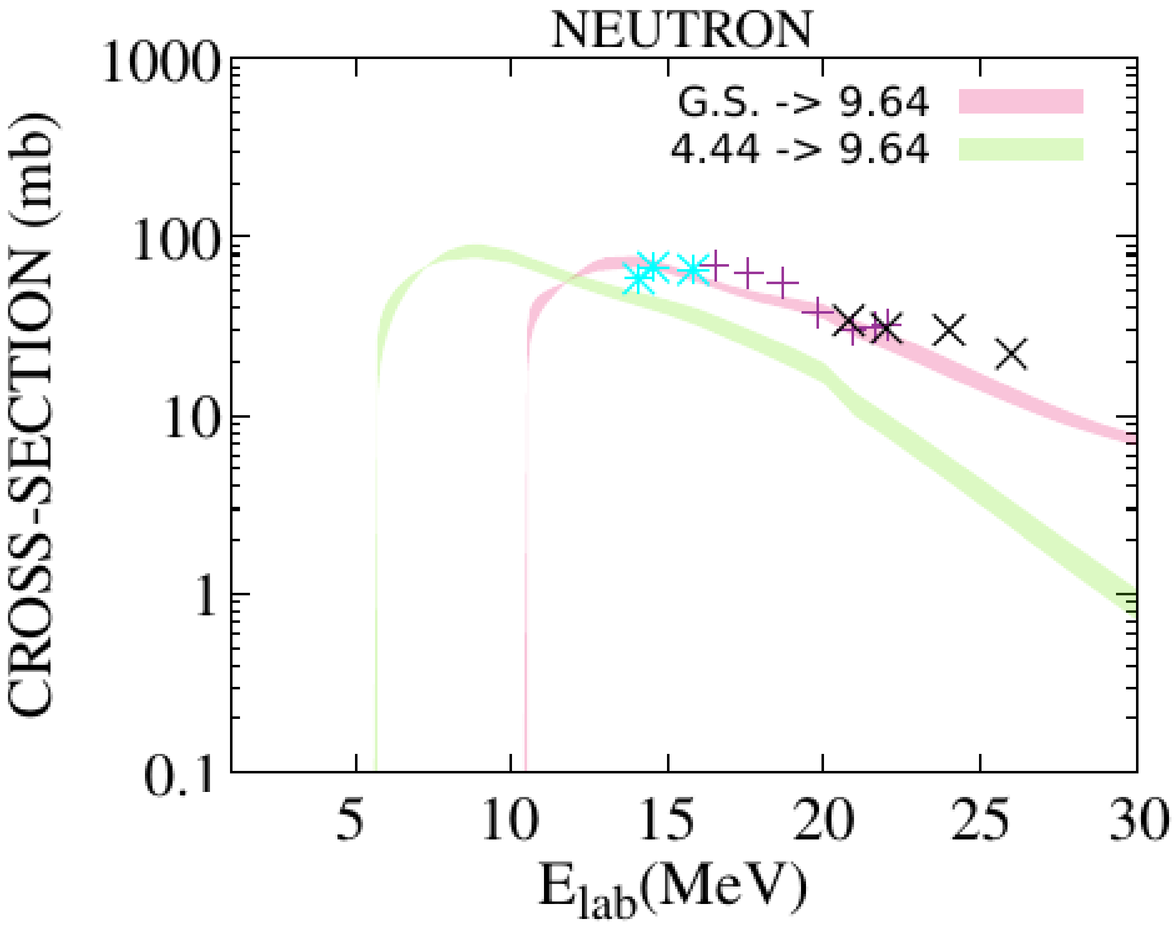
- ✓ Previously defined, R_{xx}, which is the ratio of ^{12}C production due to the particle induced deexcitation process to the ^{12}C production due to spontaneous radiative decay
- ✓ R = 1 means both the processes are equal in strength whereas, R > 1 denotes the induced process is stronger and thus, R is a measure of enhancement
- ✓ R_{nn} is the highest and $R_{\alpha\alpha}$ is the lowest
- ✓ This is due to increasing Coulomb barrier for charged particles



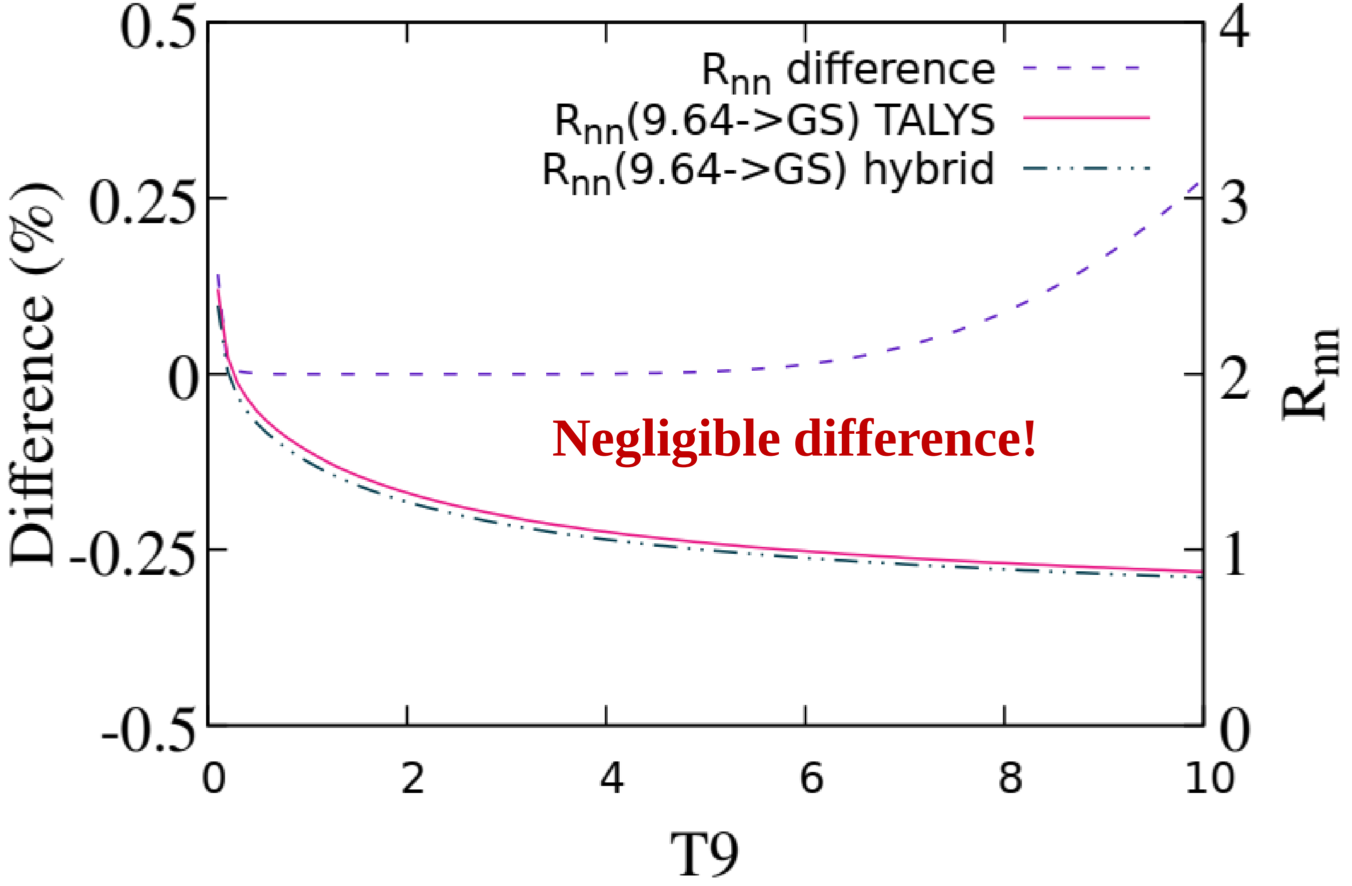
Use of sparse experimental points vs TALYS cross sections

- ✓ Because of the exponential factor, points away from threshold contribute negligibly
- ✓ Hybrid approach taken, use experimental data where available and TALYS otherwise
- ✓ Or use pure TALYS cross sections

$$R_{xx} = k_x \rho_x T_9^{-\frac{3}{2}} f_{spin} \int_0^\infty \sigma_{xx'}(E)(E+E_{th})exp(-11.605E/T_9)dE$$



R_{nn} TALYS × 1.02, R_{nn} Hybrid × 0.98

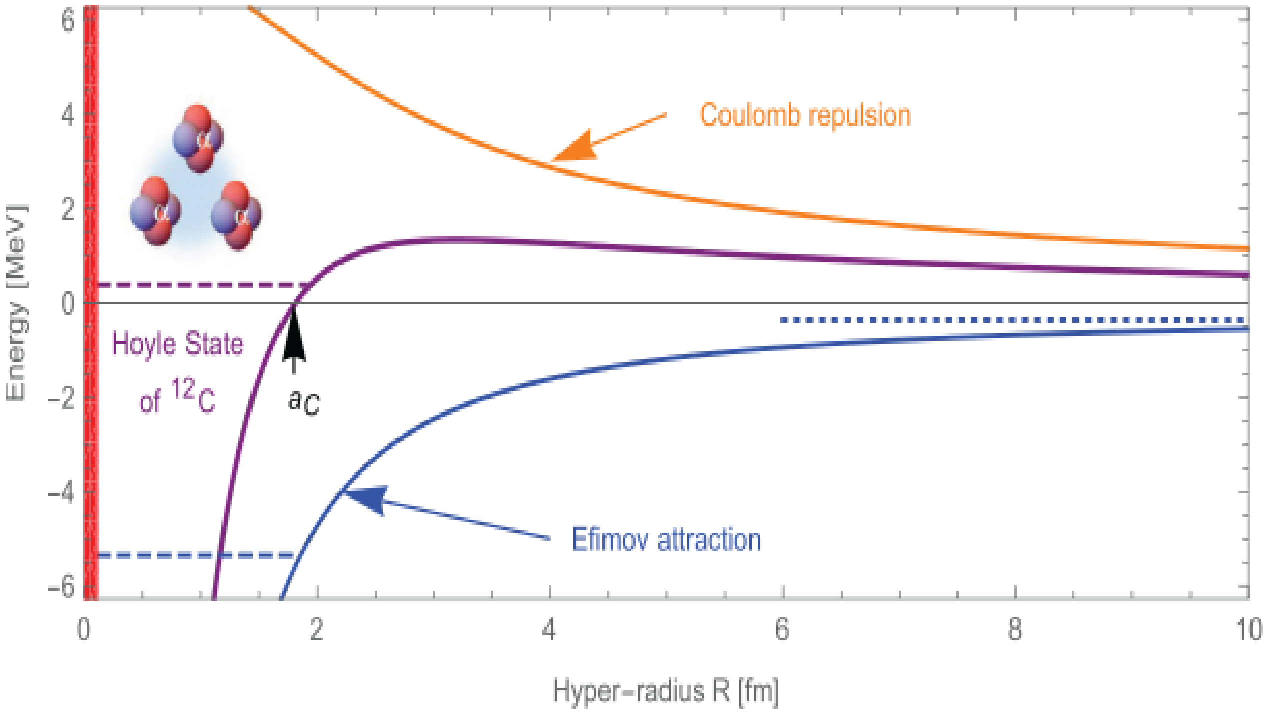


Is the Hoyle state an Efimov trimer?

In his original work, Vitaly Efimov actually suggested that the Hoyle state could be an Efimov state

In principle, a $J^{\pi} = 0^{+}$ (corresponding to L = 0) 3α state in ^{12}C, where the 2α subsystems are unbound but form a long-lived resonant state, can be seen as Efimov trimer.

1. Higa and Hammer (Eur. Phys. J. A, **37** 193–200) conjectured the Hoyle state to be remnant of Efimov spectrum broken by the Coulomb interaction.
2. But, the scattering length of for α-α interaction (a) is about 5 fm, which is similar to the range b and effective range $r_e \approx 3.4$ fm.
3. Even if the α-α interaction is resonant, the Efimov attraction seems too weak to overcome the Coulomb repulsion and support a resonant state at distances larger than the range b.



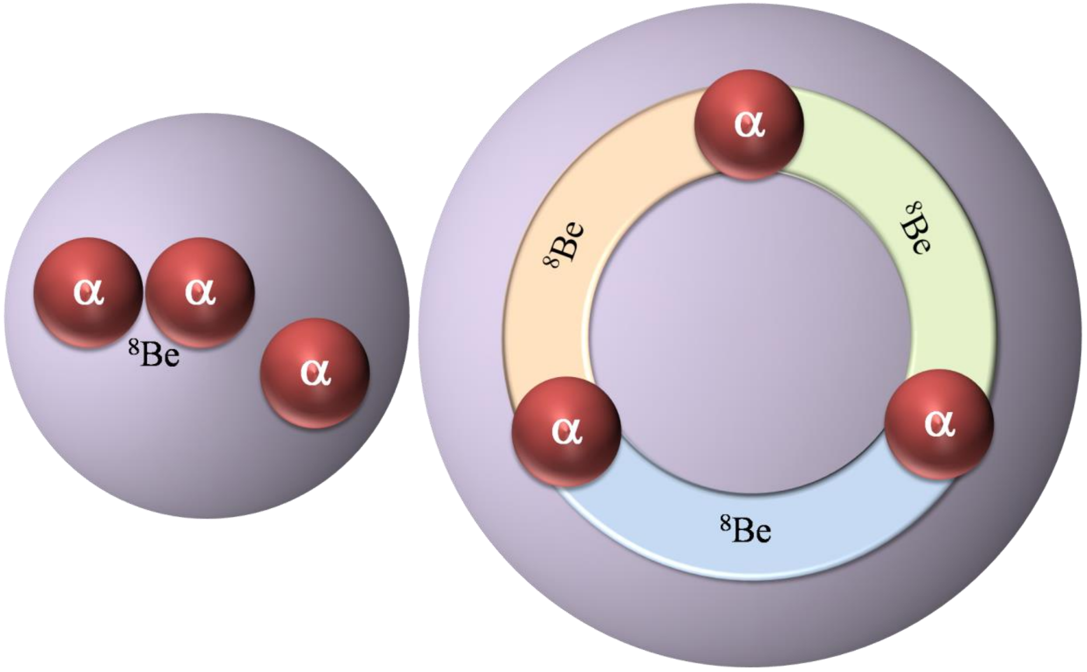
Rep. Prog. Phys. 80 (2017) 056001 (78pp)

What is Efimov effect?

The Efimov effect is a quantum mechanical phenomenon where an infinite series of bound states (Efimov states) appear in a three-body system.

Key Features:

- The effect arises when three particles, such as bosons, interact through short-range forces.
- Despite no two-particle bound states existing, a third particle facilitates binding through a collective three-body interaction.
- Occurs when the two-body scattering length (a) is much larger than the range of the interaction potential (r_0), i.e., $|a| \gg r_0$.
- Predicted by **Vitaly Efimov** in 1970.
- The prediction was experimentally confirmed in 2006, using ultracold atoms like **Cesium** at temperatures close to absolute zero.

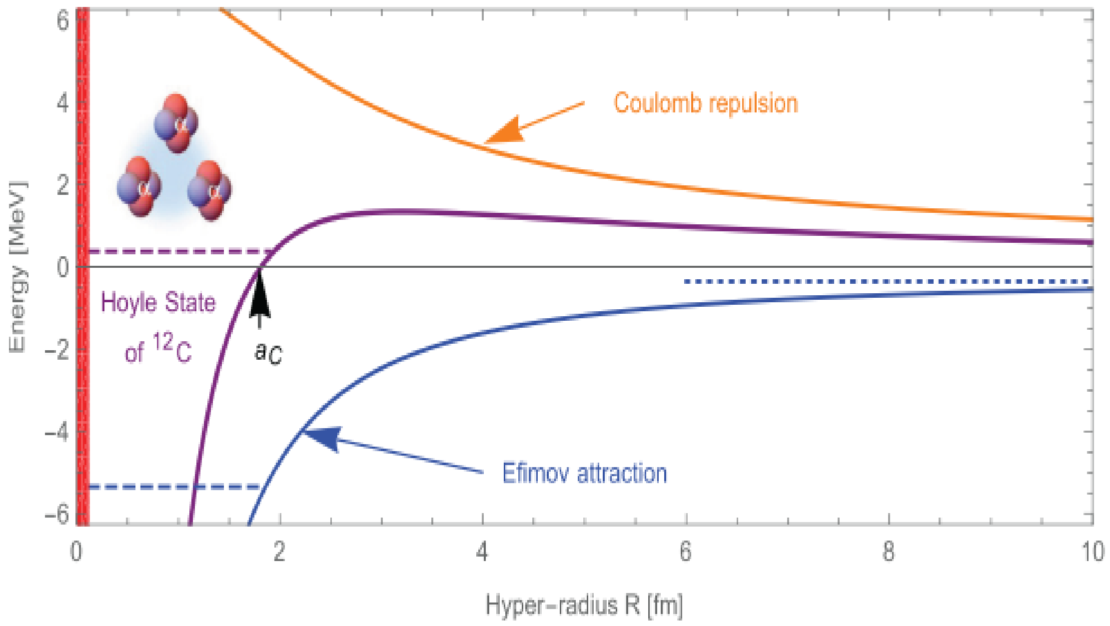


a) Hoyle State b) Efimov State

Is the Hoyle state an Efimov trimer?

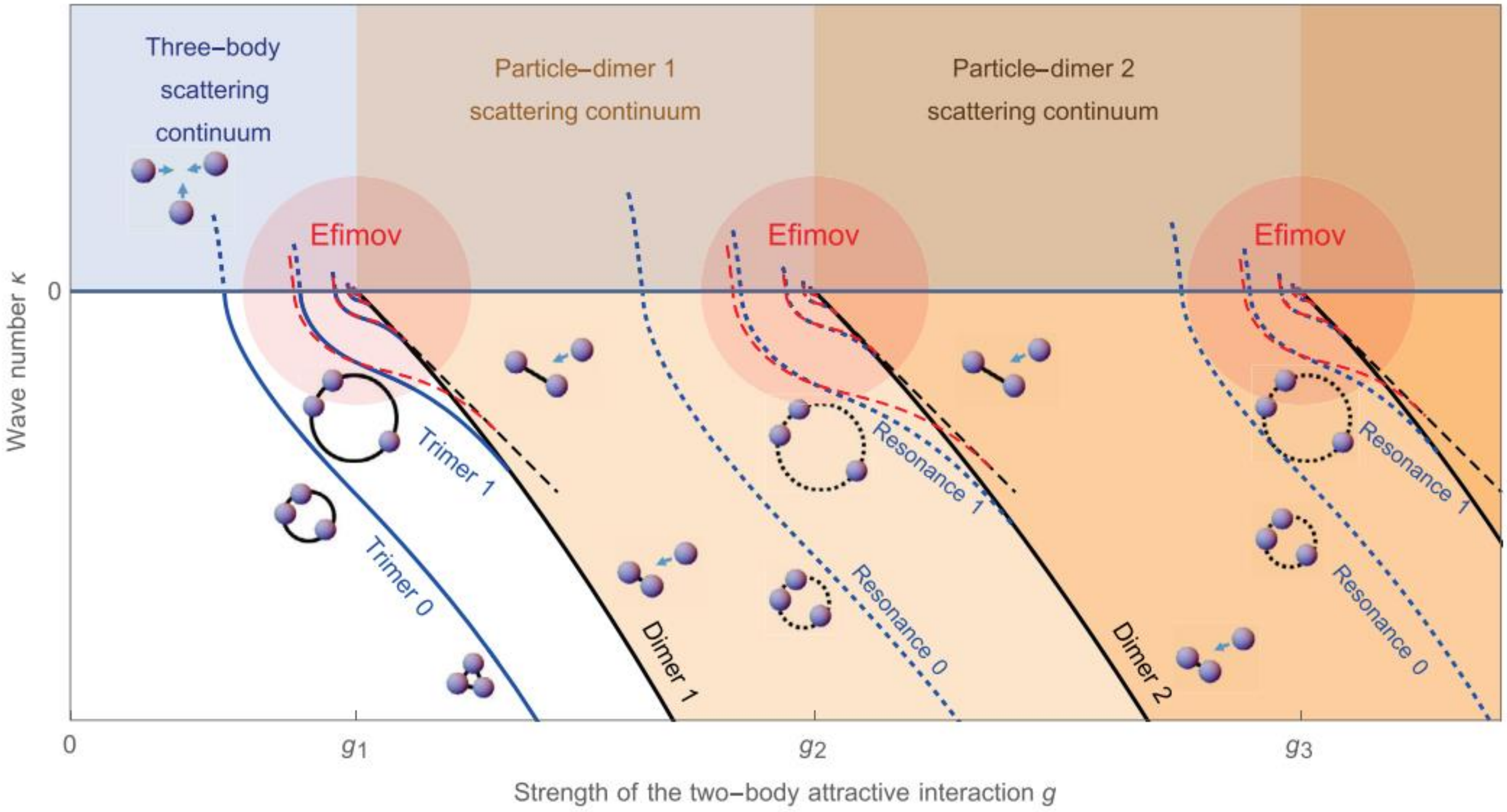
In his original work, Vitaly Efimov actually suggested that the Hoyle state could be an Efimov state

The Hoyle state is an excited resonant state of carbon-12 predicted by Hoyle in 1954. It plays a crucial role in the stellar nucleosynthesis of carbon. In his original papers [1, 2], Vitaly Efimov suggested that the Hoyle state may be viewed as a trimer of alpha particles (i.e. helium nuclei, which are bosons) bound by the Efimov attraction. The works of Higa and Hammer [93, 121] based on effective-field theory looks into the effect of the Coulomb interactions on alpha systems close to unitarity. They conjectured that the Hoyle state is indeed a remnant of the Efimov spectrum broken by the Coulomb interaction, surviving as a resonance above the three-alpha scattering threshold. The corresponding picture of the Hoyle state would be a resonant state resulting from the balance between the Efimov attraction and the Coulomb repulsion. This picture is shown in figure 7. Although this picture of the Hoyle state is quite appealing, there are two points that make it questionable. First, excluding the Coulomb repulsion, the nuclear force between two alpha particles does not seem to be resonant, as the scattering length of the model potentials [122] for the alpha-alpha interaction is about 5 fm, which is similar to the range b and effective range $re \approx 3.4$ fm of these potentials. The resonance condition (2.4) may therefore not be satisfied. This would suggest that the attraction between alpha particles is directly due to the nuclear force rather than the Efimov attraction. Second, even if the alpha-alpha interaction is resonant, the Efimov attraction seems too weak to overcome the Coulomb repulsion and support a resonant state at distances larger than the range b. Indeed, the value of the Bohr radius given by equation (2.45) for alpha particles is $aC \approx 1.8$ fm. The condition $b < aC$ of equation (2.46) therefore does not appear to be satisfied. These conclusions rely on rough estimates, and only a full treatment of the three-body problem with short-range and Coulomb interactions can give a definite answer. The threebody model calculation of the Hoyle state by Suno, Suzuki, and Descouvemont [123] gives a preliminary answer. In their work, they show the contributions from the Coulomb, nuclear and centrifugal (kinetic) energies as a function of the hyperradius. Although an attractive well (presumably due to the Efimov attraction) can be seen in the centrifugal energy, it appears that it is not enough to overcome the Coulomb repulsion, and it is the nuclear force that is responsible for the stability of the Hoyle state in this model. It is therefore likely that the Hoyle state may not be considered as an Efimov state.



Rep. Prog. Phys. 80 (2017) 056001 (78pp)

Efimov spectrum for identical bosons



Rep. Prog. Phys. 80 (2017) 056001 (78pp)

Triple-α Reaction rate

$$r_{3\alpha} = N_A^2 \langle \sigma v \rangle^{\alpha\alpha\alpha} = 3N_A \left(\frac{8\pi\hbar}{\mu_{\alpha\alpha}^2} \right) \left(\frac{\mu_{\alpha\alpha}}{2\pi k_B T} \right)^{3/2} \int_0^\infty \frac{\sigma_{\alpha\alpha}(E)}{\Gamma_\alpha(^8\mathrm{Be}, E)} \exp\left(-\frac{E}{k_B T} \right) N_A \langle \sigma v \rangle^{\alpha ^8\mathrm{Be}} E \, dE$$

$$N_A \langle \sigma v \rangle^{\alpha ^8\mathrm{Be}} = N_A \left(\frac{8\pi\hbar}{\mu_{\alpha ^8\mathrm{Be}}^2} \right) \left(\frac{\mu_{\alpha ^8\mathrm{Be}}}{2\pi k_B T} \right)^{3/2} \int_0^\infty \sigma_{\alpha ^8\mathrm{Be}}(E', E) \exp\left(-\frac{E'}{k_B T} \right) E' \, dE'$$

$$\sigma_{\alpha\alpha}(E) = \frac{2\pi}{\kappa^2} \frac{\Gamma_\alpha(^8\mathrm{Be}, E)^2}{(E - E_r^{8\mathrm{Be}})^2 + \Gamma_\alpha(^8\mathrm{Be}, E)^2/4} \qquad \Gamma_\alpha(^8\mathrm{Be}, E) = \Gamma_\alpha(^8\mathrm{Be}) \frac{P_0(E)}{P_0(E_r^{8\mathrm{Be}})} \qquad P_l(E) = \frac{1}{F_l^2 + G_l^2}$$

$$\sigma_{\alpha ^8\mathrm{Be}}(E', E) = (2J+1) \frac{\pi\hbar^2}{2\mu_{\alpha ^8\mathrm{Be}}} \frac{\Gamma_\alpha(^{12}\mathrm{C}^J, E')\Gamma_{\gamma(E\lambda)}(^{12}\mathrm{C}^J, E' + E)}{(E' - E_r^J + E - E_r^8\mathrm{Be})^2 + \frac{1}{4}\Gamma(^{12}\mathrm{C}^J, E', E)^2}$$

$$\Gamma_\alpha(^{12}\mathrm{C}^J, E') = \Gamma_\alpha(^{12}\mathrm{C}^J) \frac{P_J(E')}{P_J(E_r^J)} \qquad \Gamma_{\gamma(E\lambda)}(^{12}\mathrm{C}^J, E' + E) = \Gamma_{\gamma(E\lambda)}(^{12}\mathrm{C}^J) \frac{\left(E_T + E' + E - E_r^{^8\mathrm{Be}} \right)^{2\lambda+1}}{(E_T + E_r^J)^{2\lambda+1}}$$

α-penetrability from WKB theory

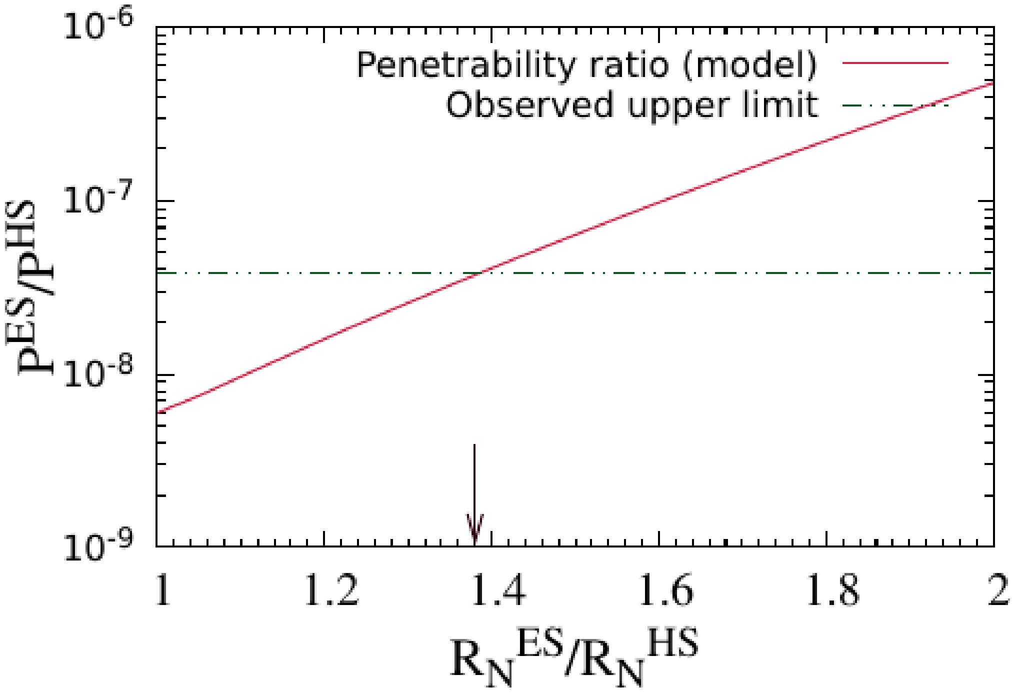
$\Gamma_i = 2P_i\gamma_i^2$,

P_i is the penetrability factor &

γ_i^2 is related to the structure

$\gamma^{2}{}_{3\alpha}{}^{HS} \approx \gamma^{2}{}_{3\alpha}{}^{ES}$ (assumption)

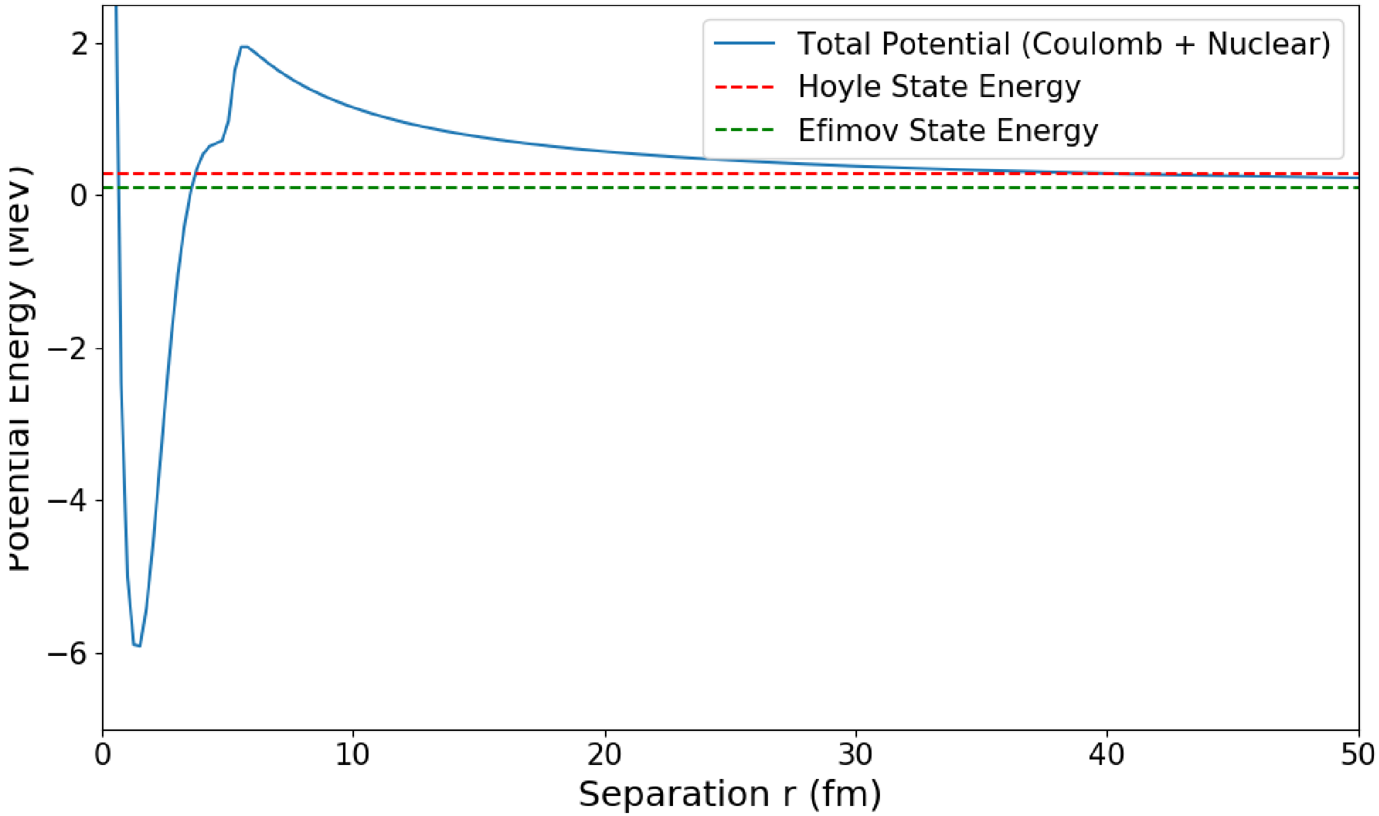
$\Gamma_{3\alpha}^{ES} = \Gamma_{3\alpha}^{HS} * P_{3\alpha}^{ES}/P_{3\alpha}^{HS}$



From the WKB theory

$$P_{wkb} = \frac{1}{1 + exp(2S)}$$

$$S = \frac{1}{\hbar}\int_{r_1}^{r_2} \sqrt{2m\left(V_{eff}(r) - E\right)}\, dr$$

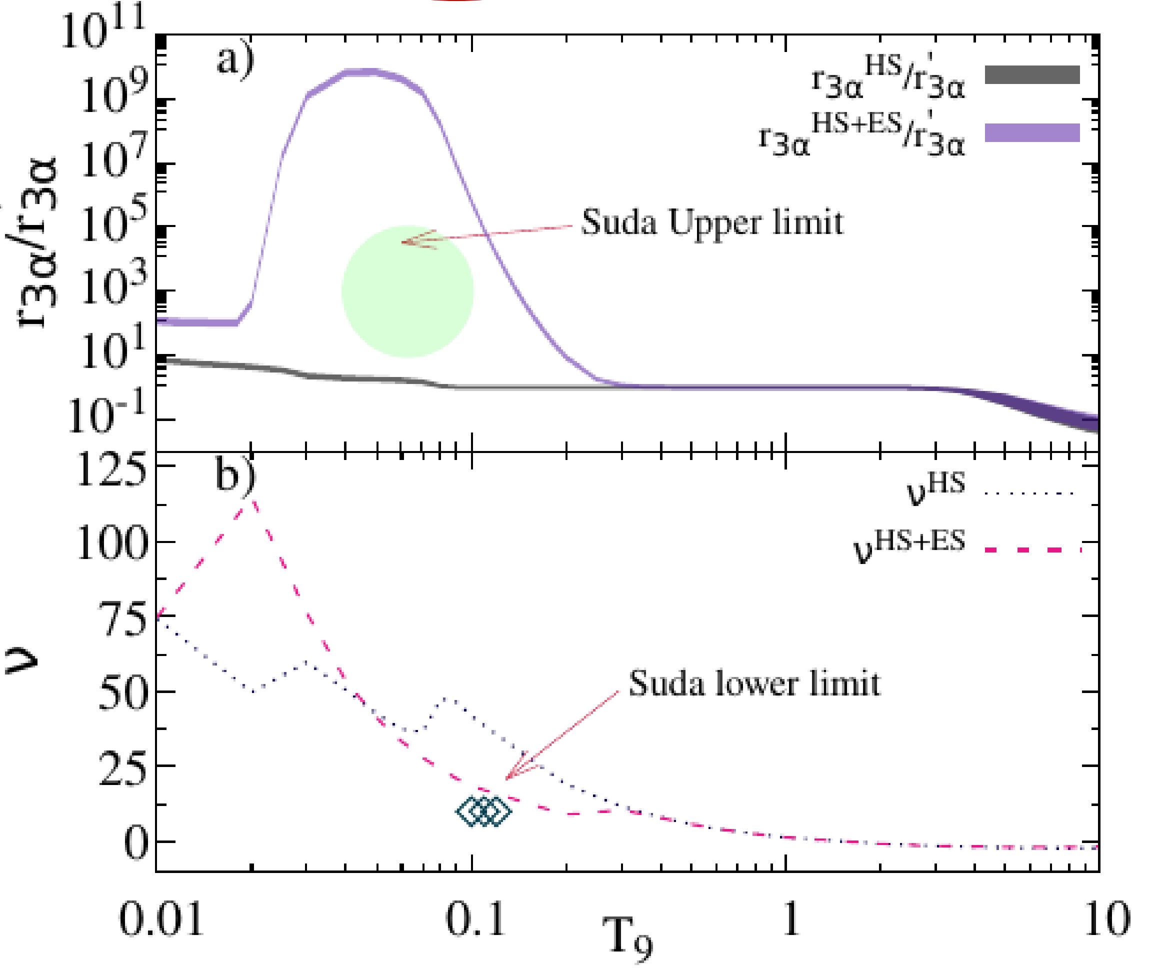


Triple-α Reaction rate

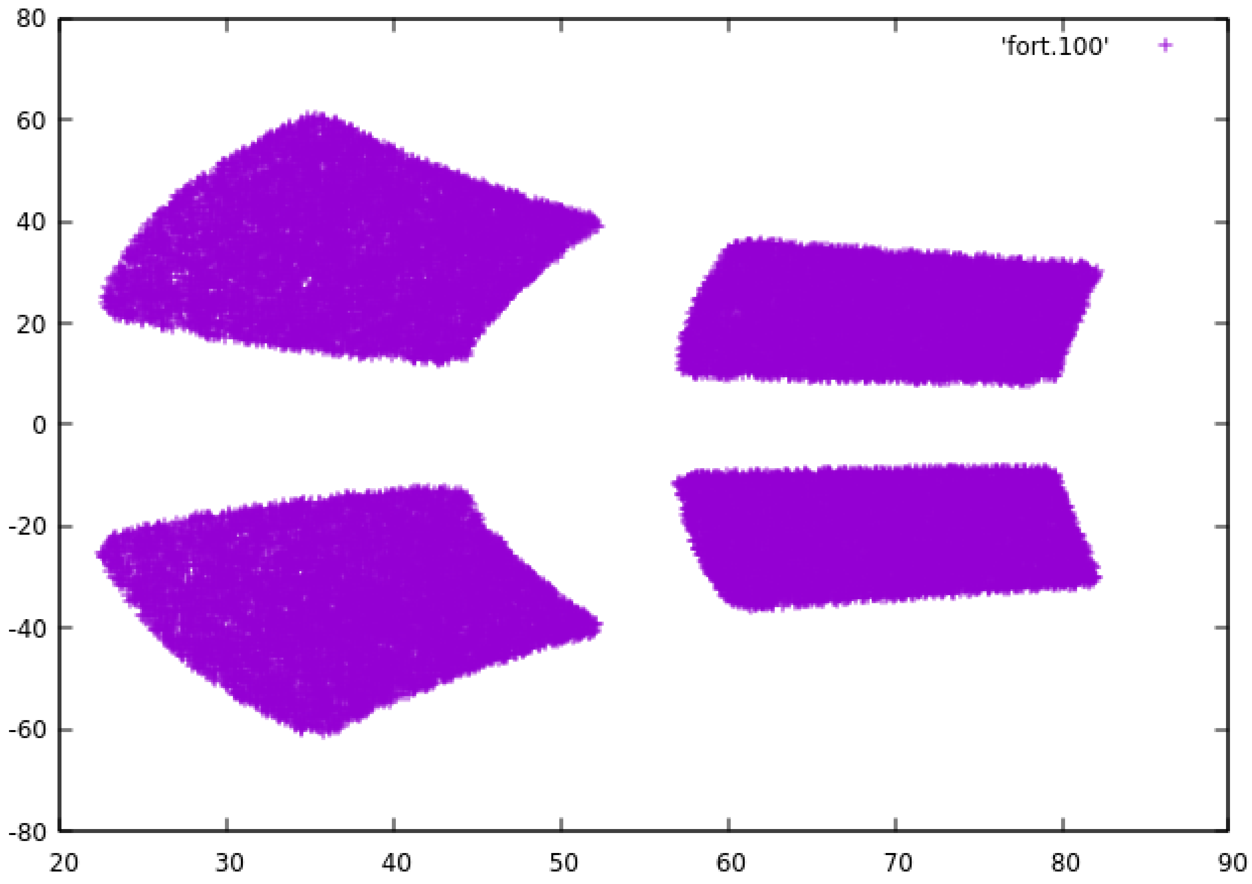
$$r_{3\alpha} = N_A^2 \langle \sigma v \rangle^{\alpha\alpha\alpha} = 3N_A \left(\frac{8\pi\hbar}{\mu_{\alpha\alpha}^2}\right)\left(\frac{\mu_{\alpha\alpha}}{2\pi k_B T}\right)^{3/2} \int_0^\infty \frac{\sigma_{\alpha\alpha}(E)}{\Gamma_\alpha(^8\mathrm{Be}, E)} \exp\left(-\frac{E}{k_B T}\right) N_A \langle \sigma v \rangle^{\alpha^8\mathrm{Be}} E \, dE$$

Developed in Fortran with the help of external subroutines COUL90 and QUADPACK.

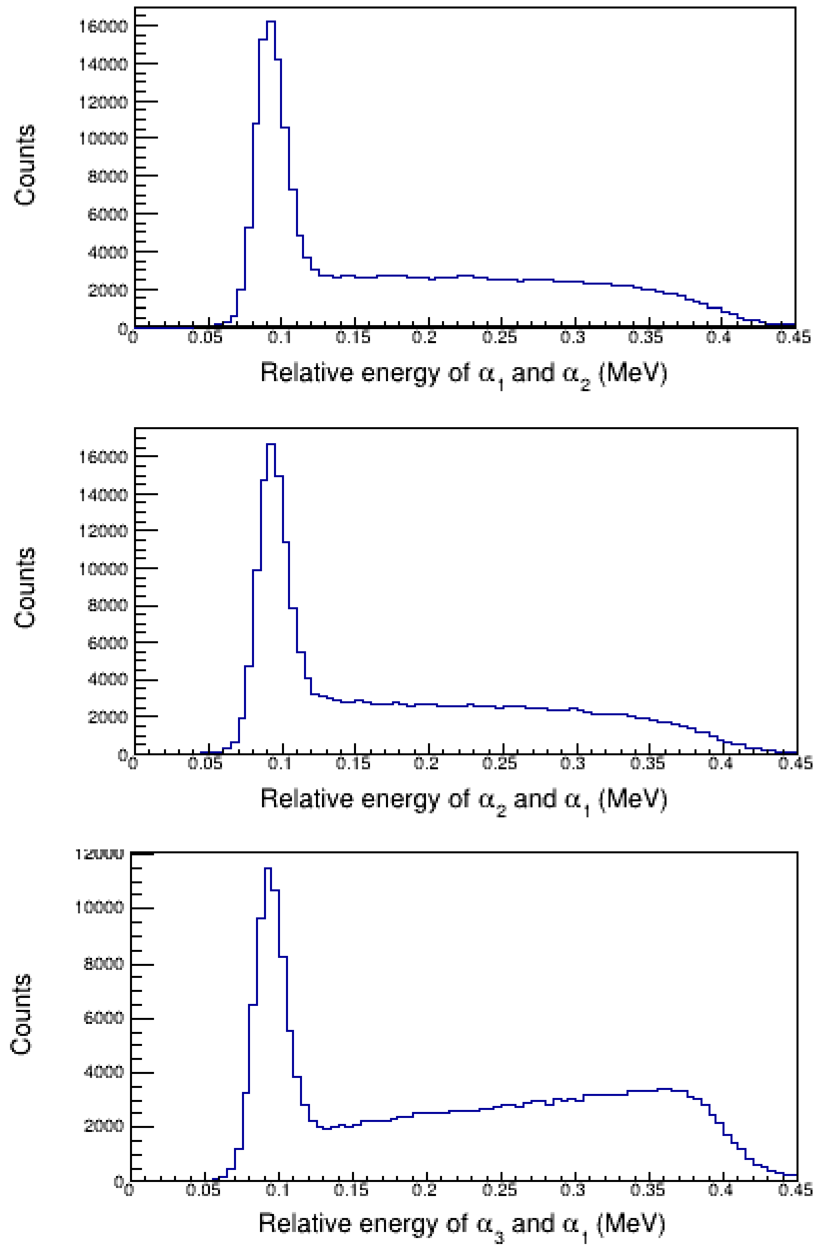
Nucleus	J_n^π	E_r (keV)	Γ_α (eV)	$\Gamma_{\gamma(E\lambda)}$ (meV)
^{8}Be	0_1^+	91.84 ± 0.04	5.57 ± 0.25	–
^{12}C	0_2^+	287.7 ± 0.2	9.3 ± 0.9	3.81 ± 0.39
	0^+ (Efimov)	91.84 ± 0.04	2.64×10^{-7} $\pm 8.16 \times 10^{-8}$	2.78 ± 0.21



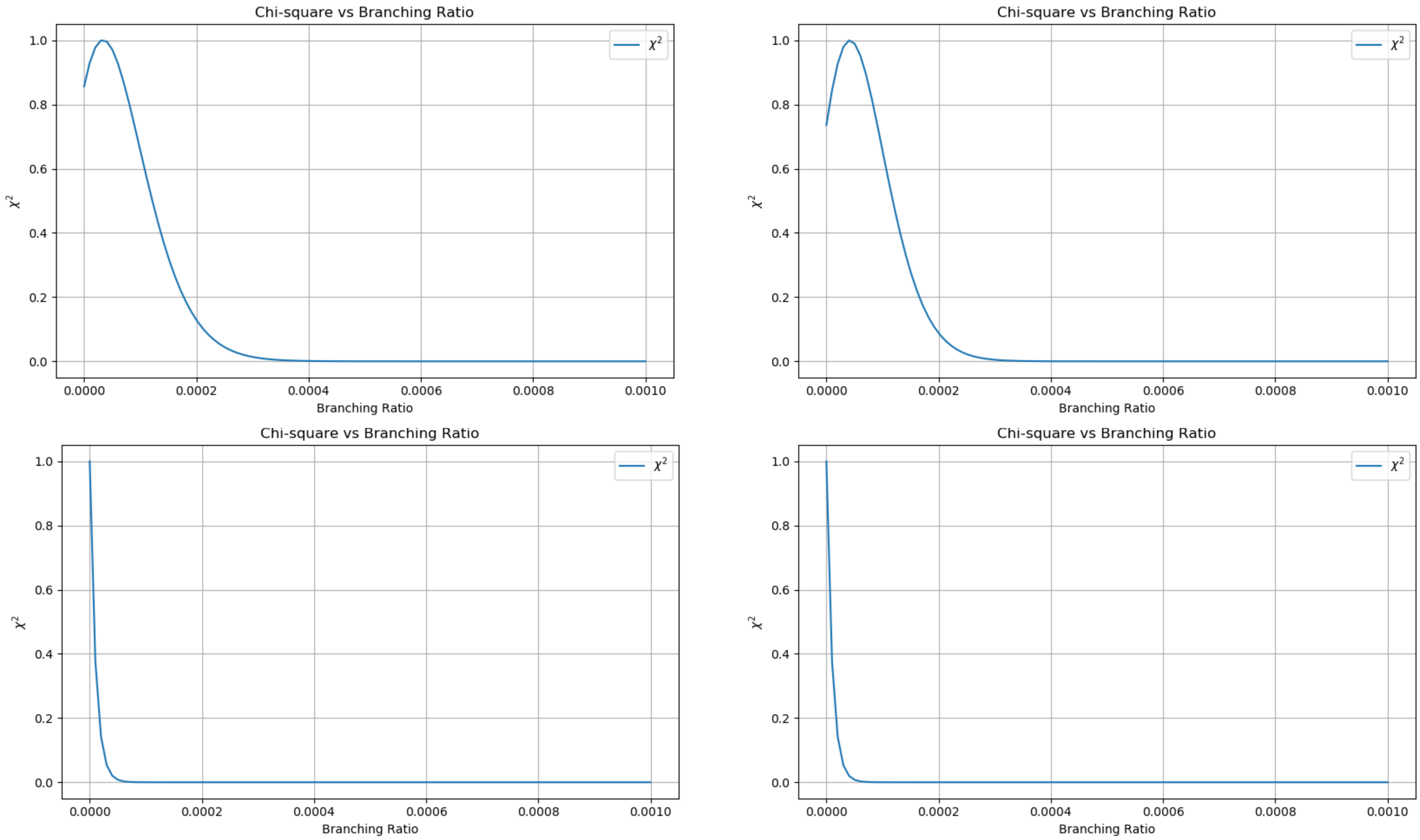
$r'_{3\alpha}$ = triple-α rate for the HS with updated parameters form Tsumura *et al.* (PLB 817 (2021) 136283)



'fort.100'



Counts
Relative energy of α_1 and α_2 (MeV)
Counts
Relative energy of α_2 and α_1 (MeV)
Counts
Relative energy of α_3 and α_1 (MeV)


Chi-square vs Branching Ratio
χ^2
Branching Ratio
1.0
0.8
0.6
0.4
0.2
0.0
0.0000
0.0002
0.0004
0.0006
0.0008
0.0010

Events after the Big Bang in chronological order