Lattice study of QCD properties under Extreme Conditions

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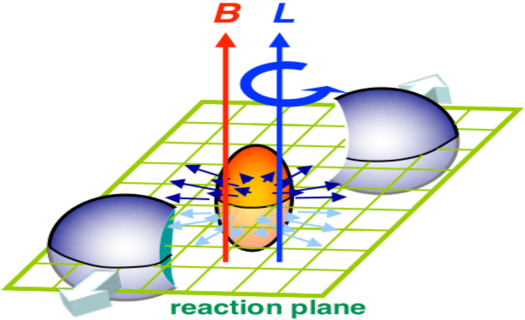
QCD under extreme conditions



- High temperature
- Large density
- Intense magnetic field
- Relativistic rotation
- Strong acceleration

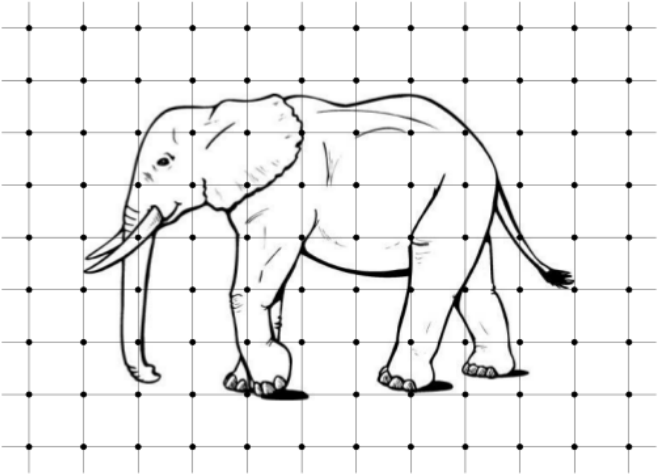
Theory of strong interactions (QCD)

- Degrees of freedom
  - Quarks q
  - Gluons A
- The QCD Lagrangian is well known

$$L = -\frac{1}{4}\sum_{a=1}^{8} F_a^{\mu\nu} F_{\mu\nu}^a + \sum_{f=u,d,s,\dots} \bar{q}_f (i\gamma^\mu \partial_\mu - m) q_f + g \sum_{f=1}^{N_f} \bar{q}_f \gamma^\mu \hat{A}_\mu q_f$$

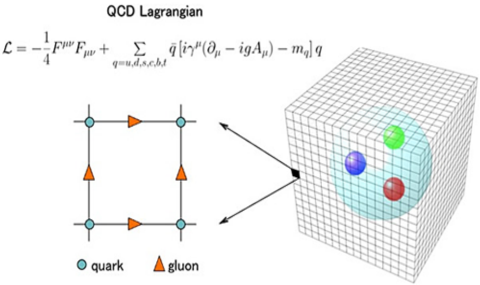
- Non-linear equations of motion with $g \sim 1$
- The main problem: calculation of observables based on the QCD Lagrangian (Millennium problem)
- Theoretical approaches contain assumptions with systematic errors which are difficult to estimate

Lattice QCD



- Allows to study strongly interacting nonlinear systems
- Based on the first principles of quantum field theory
- The most perspective approach due to supercomputers and new algorithms

Building lattice QCD



- Introduce regular cubic four dimensional lattice $N_s \times N_s \times N_s \times N_t = N_s^3 \times N_t$
- Lattice spacing–a
- Degrees of freedom
  - **Gluon fields:** 3x3 matrices $U \in SU(3)$, live on links
  - **Quarks fields:** column $q, \bar{q}$, live on sites

Building lattice QCD

- We study QCD in thermodynamic equilibrium
- QCD partition function

  $Z = \int DU \exp\left(-S_G(U)\right) \times \prod_{i=u,d,s\ldots} \det\left(\hat{D}_i(U) + m_i\right)$

- In continuum lattice partition function exactly reproduces QCD partition function
  - Gluon contribution: $S_G(U)\Big|_{a\to 0} = -\frac{1}{4}\sum_{a=1}^{8} F_a^{\mu\nu} F_{\mu\nu}^a$
  - Quark contribution:

    $\bar{q}(\hat{D}(U) + m)q\Big|_{a\to 0} = \bar{q}(\gamma^\mu \partial_\mu + ig\gamma^\mu A_\mu + m)q$

Lattice simulation of QCD

Properties

- We calculate partition function at finite a

$$Z \sim \int DU e^{-S_G(U)} \prod_{i=u,d,s...} \det\left(\hat{D}_i(U) + m_i\right)$$

- Carry out continuum extrapolation $a \to 0$
- Uncertainties (discretization and finite volume effects) can be systematically reduced
- The first principles based approach. No assumptions!
- Parameters: $\alpha_s(a)$ and $m_q(a)$

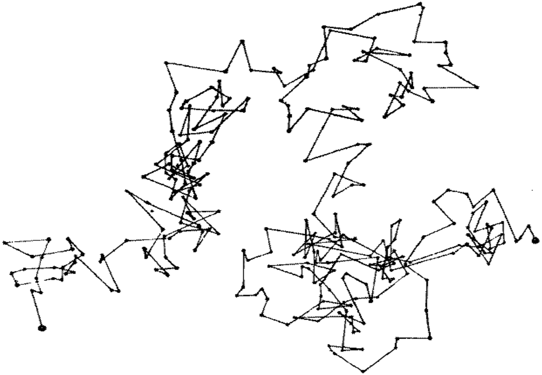
Lattice simulation of QCD

- Partition function

$$Z \sim \int DU e^{-S_G(U)} \prod_{i=u,d,s...} \det\left(\hat{D}_i(U) + m_i\right) = \int DU e^{-S_{eff}(U)}$$

  - 96×48^3 $(N_x = N_y = N_z = 48,\ N_t = 96)$
  - Variables: $96 \cdot 48^3 \cdot 4 \cdot 8 \sim 300 \cdot 10^6$
  - Matrices: $100 \cdot 10^6 \times 100 \cdot 10^6$
- To calculate Z one uses Monte Carlo approach
- HMC: stochastic process which gives $p(U) \sim e^{-S_{eff}(U)}$

Hybrid Monte Carlo algorithm



- HMC can be considered as Brownian motion of the system
- Accept/reject step at the end of the trajectory
  - if $S_{eff}(U_{n+1}) < S_{eff}(U_n)$ the U_{n+1} is accepted
  - otherwise U_{n+1} is accepted with $p \sim e^{-[S_{eff}(U_{n+1}) - S_{eff}(U_n)]}$
- Simulation of quantum system!
- For large number of the trajectories $p(U) \sim e^{-S_{eff}(U)}$

Applications

- Spectroscopy
- Matrix elements and correlations functions
- Thermodynamic properties of QCD
- Transport properties of QCD
- Phase transitions
- Nuclear physics
- Properties of QCD under extreme conditions (magnetic field, baryon density, relativistic rotation,...)
- Topological properties of QCD
- ...

talk of Prof. Saumen Datta, 11.11 at 9.00

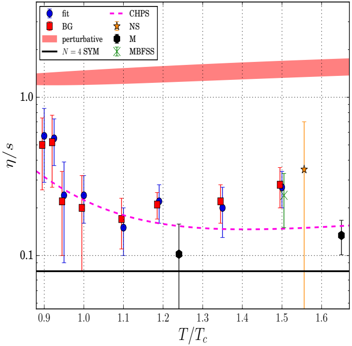
Lattice simulation of QCD in JINR

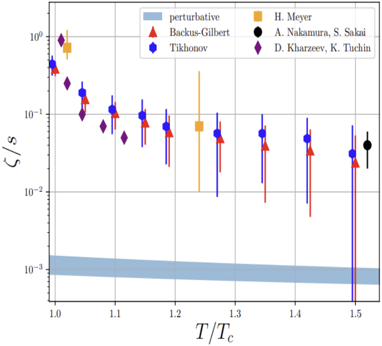
- Supercomputer "Govorun"
- SU(2) and SU(3) codes: CPU, CPU+GPU, multi-GPU
- Simulation with 2 or 2+1 dynamical quarks
- Wilson and Staggered stout fermions

QCD properties under extreme conditions

- Transport and anomalous transport phenomena
- Non-zero baryon density
- QCD with non-zero chiral density
  V.V. Braguta, Phys.Rev.D93(2016) 034509; JHEP06(2015) 094;
  N. Astrakhantsev, Eur.Phys.J.A57(2021), 15
- Strong Magnetic field
- Relativistic rotation
  Artem Roenko, 12.11 at 14.30
- Strong acceleration

Transport properties: shear and bulk viscosities



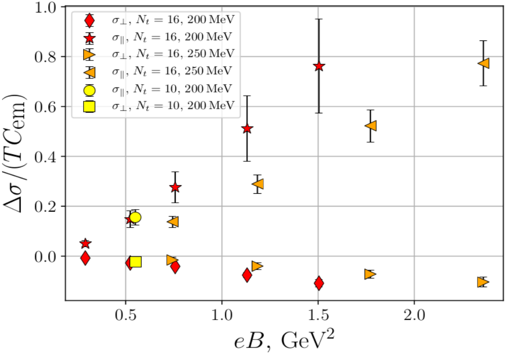


- Results for η/s are close to N=4 SYM $\frac{\eta}{s} = \frac{1}{4\pi}$

  N. Astrakhantsev et.al., JHEP 04 (2017) 101
- Peak of the ζ/s at $T = T_c$

  N. Astrakhantsev et.al., Phys.Rev.D 98 (2018) 5, 054515
- Lattice results deviate from perturbation theory
- Nonpertubative QGP!

Conductivity of QGP in strong magnetic field



- Electrical conductivity of QGP with 2+1 quarks at physical masses
  N. Astrakhantsev et.al. Phys.Rev.D102(2020), 054516;
  G. Almirante, Phys.Rev.D 111 (2025) 3, 034505
- Conductivity along magnetic field $\sigma_\parallel$ increases $\Rightarrow$ **CME**
- Conductivity perpendicular to magnetic field $\sigma_\perp$ decreases $\Rightarrow$ **magnetoresistance**
- QGP in magnetic field becomes anisotropic

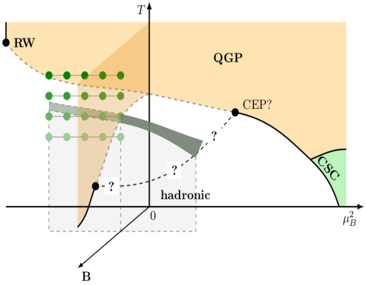
QCD at finite (T, μ_B, eB)

- First study of QCD phase transition in (T, μ_B, eB) space

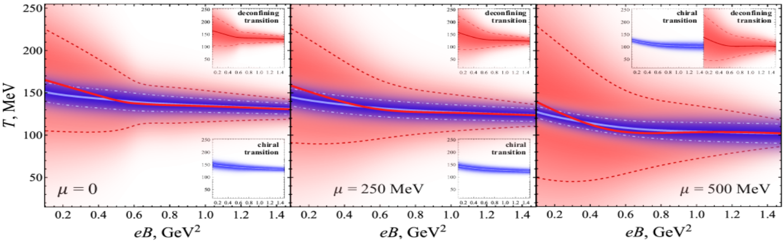
V. Braguta et.al., Phys.Rev.D100(2019), 114503

- Simulation at imaginary μ_B and physical quark masses
- Inverse magnetic at finite density
- Study of EoS $p = p(T, \mu_B, eB)$

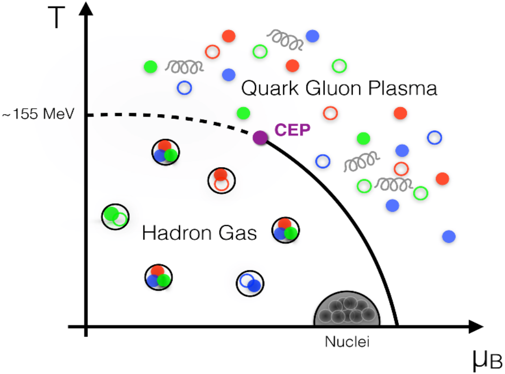
N. Astrakhantsev, et.al., Phys.Rev.D109(2024), 094511



S.Borsanyi, LATTICE2023(2024) 164



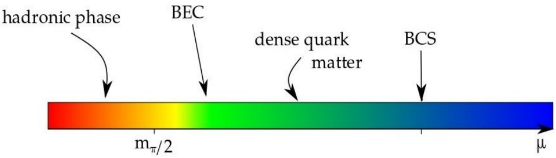
Hunting for the critical end point



- The width of the chiral transition decreases with μ
  Zero transition width $\Rightarrow$ first order phase transition
- Our estimation is $(T_c^{CEP}, \mu_B^{CEP}) = (100(25), 800(140))\,\text{MeV}$

V. Braguta et.al., Phys.Rev.D100(2019), 114503

QCD properties at high baryon density



Observation of the quarkyonic phase

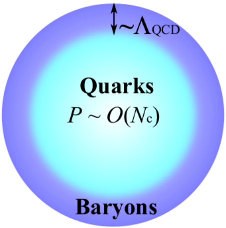
L.McLerran, R.D. Pisarski, Nucl.Phys.A796 (2007)

- Lattice simulation SU(2) at finite μ_B

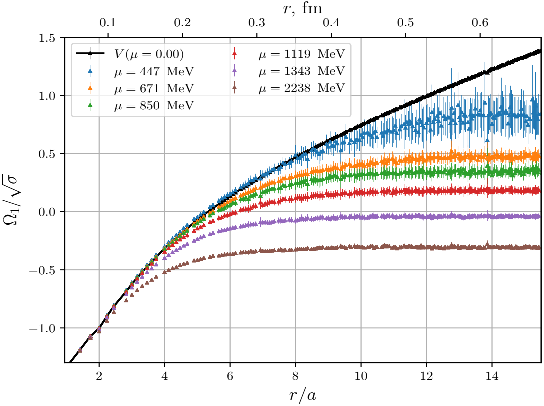
V. Braguta et al., Phys.Rev.D94(2016) 114510

N. Astrakhantsev et al., Phys.Rev.D102(2020) 074507

- Formation of the Fermi sphere $n_q \simeq n_{SB}$
- Baryons on the surface $\Sigma \sim \mu^2$
- Confinement
- The chiral symmetry is restored



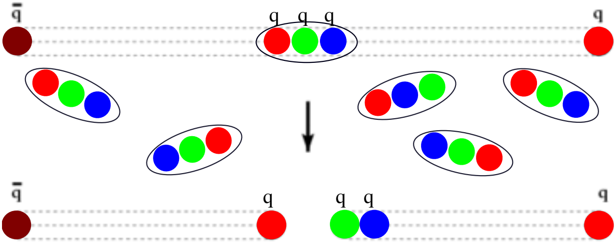
String breaking in dense matter



- The larger the baryon density the smaller string breaking distance

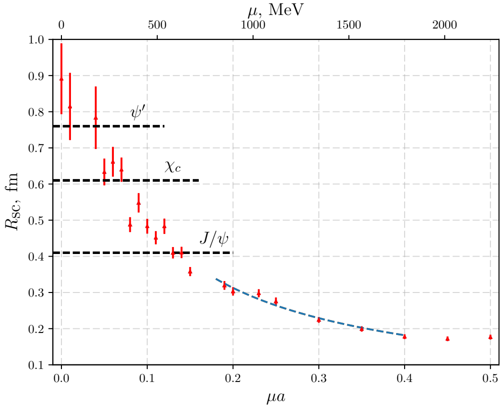
N. Astrakhantsev, JHEP 05(2019) 171

String breaking in dense QCD



- String breaking in dense matter creates $\bar{Q}q$ and Qqq
- Contrary to string breaking in vacuum: $\bar{Q}q$ and Qq

Dissociation of charminia in dense matter



- String breaking at distance R_{SC}
- Dissociation of charminia might take place before the deconfinement
- **Dissociation of charminia can be seen at NICA**

Typical accelerations

Earth	$\sim 2 \cdot 10^{-29}$ MeV
Sun	$\sim 6 \cdot 10^{-28}$ MeV
Neutron star	$\sim 2 \cdot 10^{-16}$ MeV
HIC experiments, vicinity of black hole horizon	~ 1000 MeV

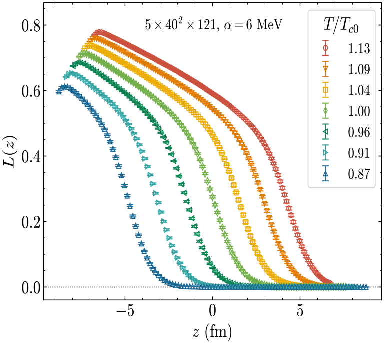
HIC experiments allow to study physics close to black hole horizon

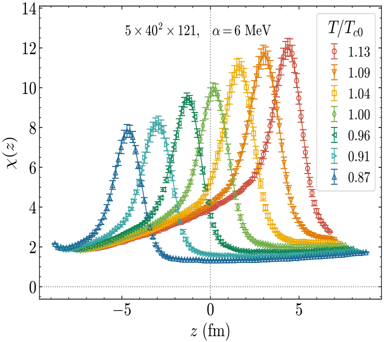
Accelerated observer and QCD

- HIC is complicated process
- Toy model:
  uniformly accelerated observer ($\vec{a} \parallel \hat{z}$)
  homogeneous magnetic field

  $\frac{d^2 z}{dt^2} = \alpha = const$
- The trajectory:
  $t(\tau) = \frac{1}{\alpha} \sinh \alpha\tau, \ z(\tau) = \frac{1}{\alpha} \cosh \alpha\tau$
- Aim: investigation of gluodynamics/QCD in the accelerated frame
- Influence of non-inertia frame can be reduced to external gravitational field
- Gluodynamics/QCD in external gravitational field

Accelerated gluodynamics





- Spatially separated confinement and deconfinement phases
- We observe: spatial confinement/deconfinement phase transition in accelerated gluodynamics

  V. Braguta, report at Lattice 2025, 07.11

Thank you!