Correlation of Dark Matter Fermi momentum with bulk properties of DM admixed Neutron stars

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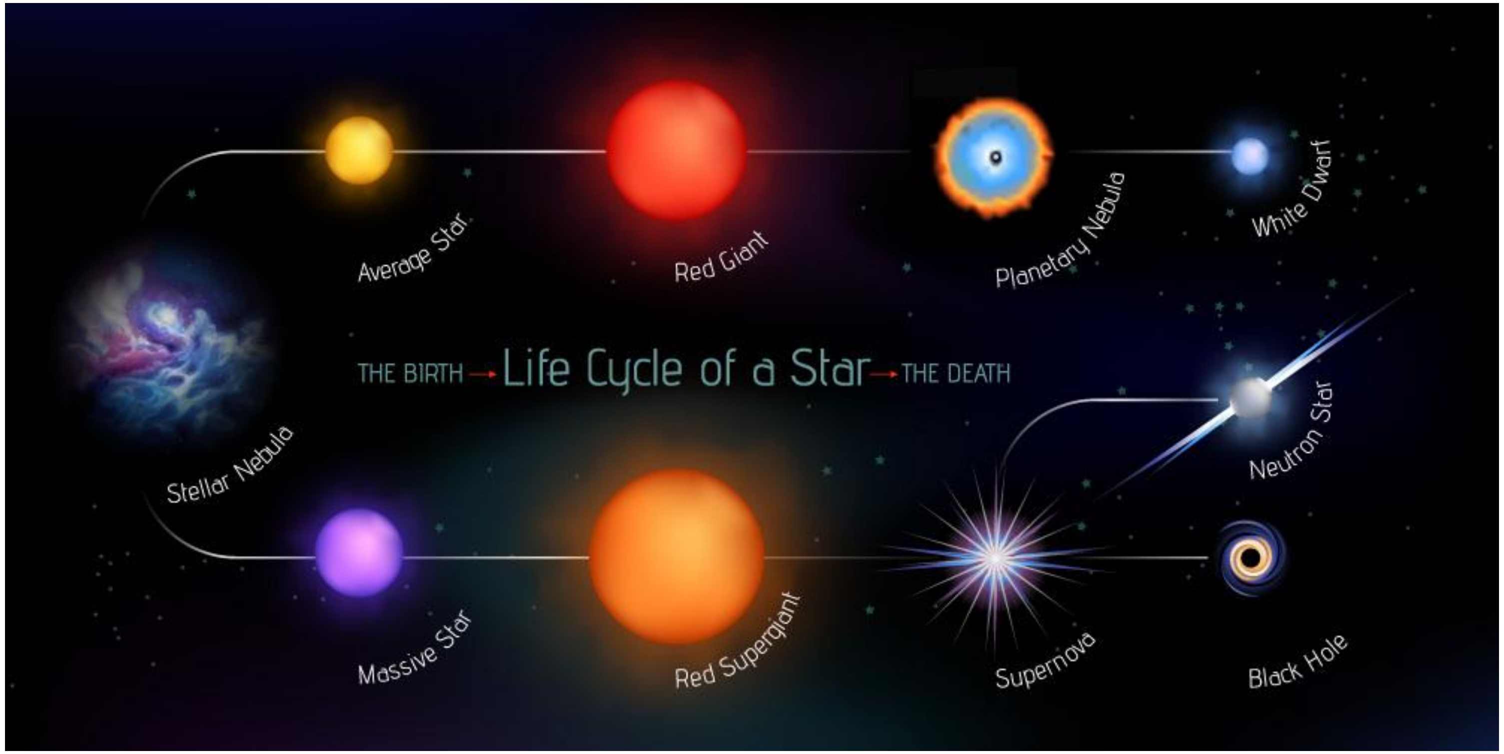
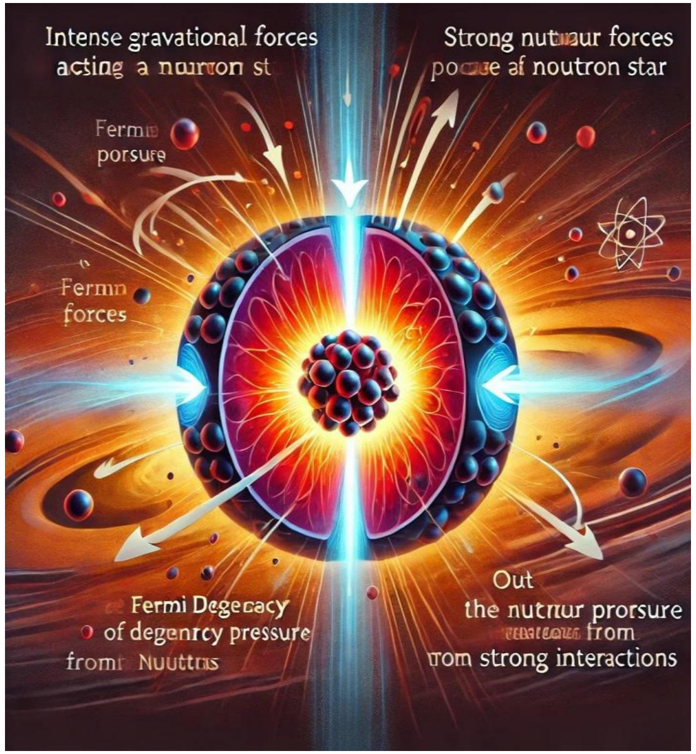
Contents

Introduction

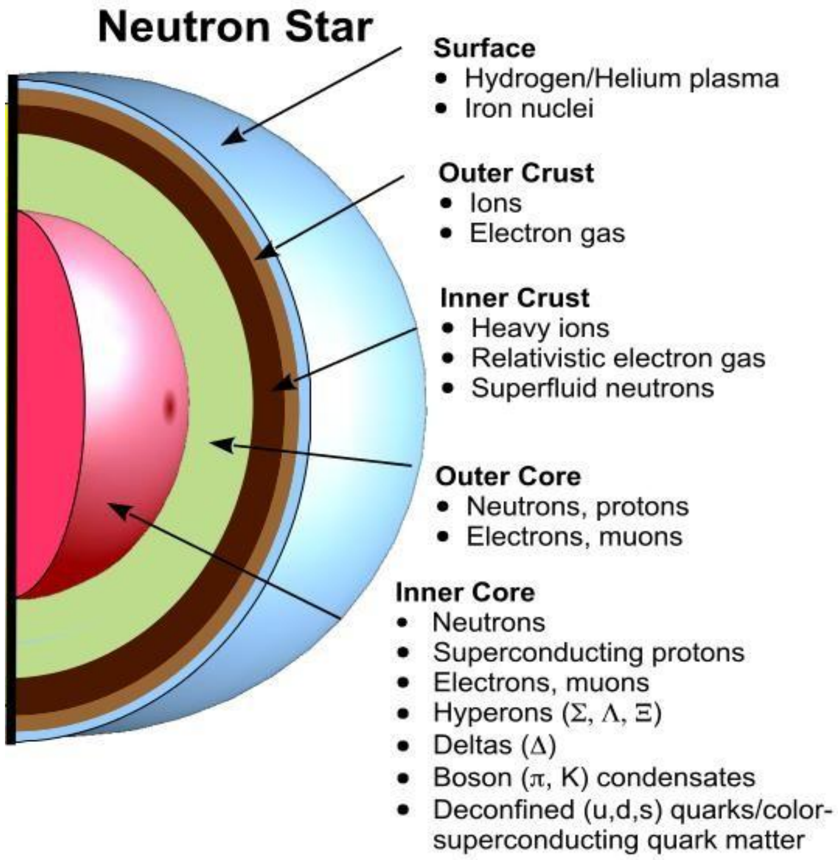
- Neutron stars: highly dense object formed by supernova explosions.
- The NS have mass around $1.2 - 2.5\ M_{\odot}$ confined within a radius $10 - 15$ km.





Introduction

- The inner core is the central region of a neutron star.
- The core contain 90% neutron and 10% of proton.
- Hyperons form in neutron star cores when nucleon chemical potentials exceed nucleons and hyperons mass difference.
- Δ–baryons can appear at 2-3 times of nuclear saturation density (ρ_0) by replacing neutron and electron pair
- At high densities; energy increase and baryons transit to quark matter.
- Kaons replace electrons as charge neutralizers if their energy matches electron chemical potential.



Role of dark matter

- Dark matter and dark energy constitute approximately 95% of the cosmos.
- Dark matter is accreted into the densely packed neutron star due to its strong gravitational field.
- Self annihilating dark matter can heat the core of the neutron star, consequently increase the surface temperature and alter the cooling rates of the NS.
- Non self-annihilating dark matter can influence the structural properties of neutron stars.
- The accretion of dark matter has been found to decrease the mass, radius, moment of inertia, binding energy and tidal deformability of NS while it increases f–mode oscillation frequency of NS.

Theoretical framework

RMF Model:

- A successful theory describing many nuclear phenomena, including nuclear masses, radii, deformation, and the properties of exotic nuclei from finite to infinite nucleus.

- Mesons (sigma (σ), omega (ω), rho (ρ) mesons, phi (ϕ) meson), delta (δ) meson.

- $\beta -$ equilibrium condition:

$$\mu_{\Delta^-} = 2\mu_n - \mu_p, \qquad \mu_p = \mu_{\Sigma^+} = \mu_n - \mu_e,$$
$$\mu_{\Delta^0} = \mu_n, \qquad \mu_n = \mu_{\Sigma^0} = \mu_{\Xi^0},$$
$$\mu_{\Delta^+} = \mu_p, \qquad \mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + \mu_e,$$
$$\mu_{\Delta^{++}} = \mu_p - \mu_n. \qquad \mu_\mu = \mu_e,$$

- Charge neutrality condition

$$n_p + n_{\Sigma^+} + 2n_{\Delta^{++}} + n_{\Delta^+} = n_e + n_{\mu^-} + n_{\Sigma^-} + n_{\Xi^-} + n_{\Delta^-}$$

Theoretical framework

$$\mathcal{L}_B = \sum_B \overline{\psi}_B \Big(i\gamma^\mu \partial_\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu \omega^\mu + g_{\delta B}\tau_B.\delta$$

$$-\frac{1}{2} g_{\rho B}\gamma_\mu \tau \rho^\mu - g_{\phi_0 B}\gamma_\mu \phi_0{}^\mu \Big)\psi_B + \frac{1}{2}\partial_\mu \sigma \partial_\mu \sigma$$

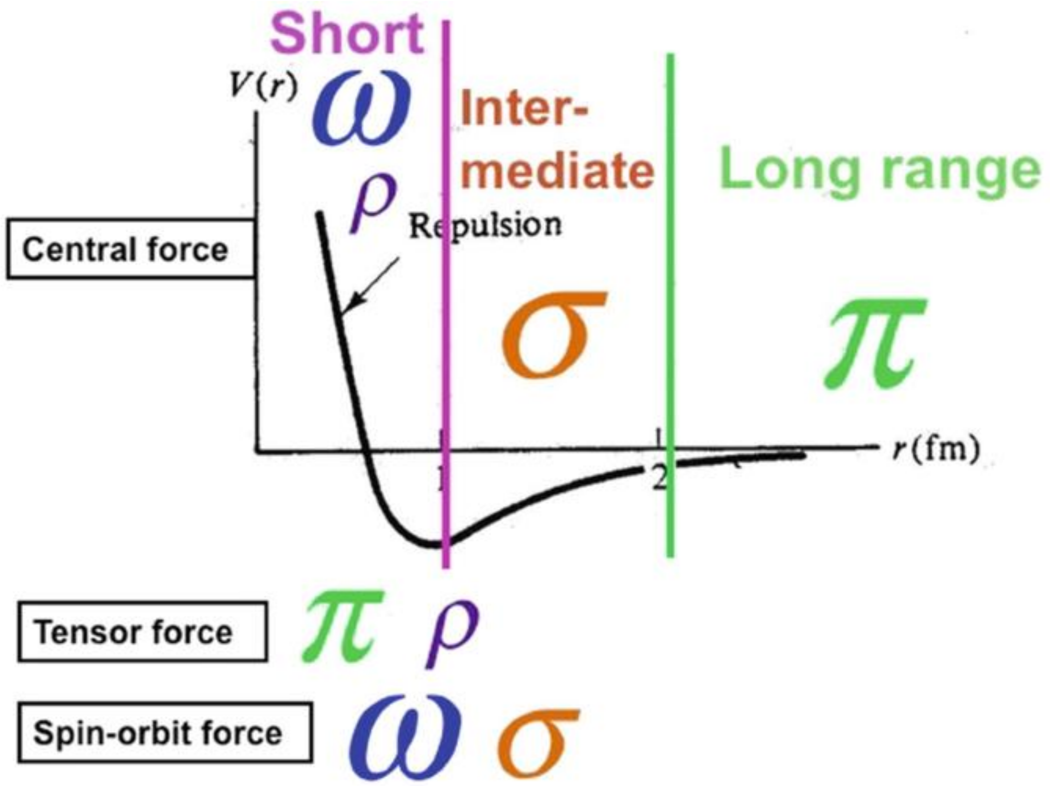
$$-m_\sigma^2 \sigma^2 \left(\frac{1}{2} + \frac{\kappa_3}{3!}\frac{g_\sigma \sigma}{m_B} + \frac{\kappa_4}{4!}\frac{g_\sigma^2 \sigma^2}{m_B^2} \right) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu}$$

$$+\frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \left(1 + \eta_1 \frac{g_\sigma \sigma}{m_B} + \frac{\eta_2}{2}\frac{g_\sigma^2 \sigma^2}{m_B^2} \right) - \frac{1}{4} R_{\mu\nu} R^{\mu\nu}$$

$$+\frac{1}{2} m_\rho^2 R_\mu R^\mu \left(1 + \eta_\rho \frac{g_\sigma \sigma}{m_B} \right) + \frac{1}{4!}\zeta_0 \left(g_\omega \omega_\mu \omega^\mu \right)^2$$

$$+\frac{1}{2} m_\phi{}^2 \phi_\mu \phi^\mu + \sum_l \overline{\psi}_l \left(i\gamma^\mu \partial_\mu - m_l \right)\psi_l$$

$$+\Lambda_v R_\mu R^\mu (\omega_\mu \omega^\mu) + \frac{1}{2}(\partial_\mu \delta \partial^\mu \delta - m_\delta^2 \delta^2),$$



Theoretical framework

EOS for nuclear matter:

- Total energy density and pressure can be calculated from energy momentum tensor $T^{\mu\nu}$ as

$$\mathcal{E}_{\mathcal{N}} = \sum_B \frac{Y_B}{(2\pi)^3}\int_0^{k_F^B} d^3k\sqrt{k^2+m_B{}^*} + \frac{1}{8}\zeta_0 g_\omega^2\omega_0^4 + m_\sigma^2\sigma_0^2\left(\frac{1}{2}+\frac{\kappa_3}{3!}\frac{g_\sigma\sigma_0}{m_B}+\frac{\kappa_4}{4!}\frac{g_\sigma^2\sigma_0^2}{m_B^2}\right)$$
$$+\frac{1}{2}m_\omega^2\omega_0^2\left(1+\eta_1\frac{g_\sigma\sigma_0}{m_B}+\frac{\eta_2}{2}\frac{g_\sigma^2\sigma_0^2}{m_B^2}\right)+\frac{1}{2}m_\rho^2\rho_{03}^2\left(1+\eta_\rho\frac{g_\sigma\sigma_0}{m_B}\right)+\frac{1}{2}m_\phi{}^2\phi^2+\frac{1}{2}m_\delta{}^2\delta_0{}^2$$
$$+\sum_l\int^{k_F^l}\sqrt{k^2+m_l{}^2}k^2dk + 3\Lambda_V\omega_0^2R_0^2+\frac{1}{2}m_{\sigma^*}^2\sigma^{*2}, \quad (5)$$

$$\mathcal{P}_{\mathcal{N}} = \sum_B \frac{Y_B}{3(2\pi)^3}\int_0^{k_F^B}\frac{k^2d^3k}{\sqrt{k^2+m_B^*{}^2}} + \frac{1}{24}\zeta_0 g_\omega^2\omega_0^4 - m_\sigma^2\sigma_0^2\left(\frac{1}{2}+\frac{\kappa_3}{3!}\frac{g_\sigma\sigma_0}{m_B}+\frac{\kappa_4}{4!}\frac{g_\sigma^2\sigma_0^2}{m_B^2}\right)$$
$$+\frac{1}{2}m_\omega^2\omega_0^2\left(1+\eta_1\frac{g_\sigma\sigma_0}{m_B}+\frac{\eta_2}{2}\frac{g_\sigma^2\sigma_0^2}{m_B^2}\right)+\frac{1}{2}m_\rho^2\rho_{03}^2\left(1+\eta_\rho\frac{g_\sigma\sigma_0}{m_B}\right)-\frac{1}{2}m_\delta{}^2\delta_0{}^2$$
$$+\Lambda_V R_0^2\omega_0^2-\frac{1}{2}m_{\sigma^*}^2\sigma^{*2}+\frac{1}{3\pi^2}\sum_l\int_0^{k_F^l}\frac{k^4dk}{\sqrt{k^2+m_l^2}}, \quad (6)$$

Theoretical framework

EOS for dark matter

- The Lagrangian for dark matter interaction with baryons is given by

$$\mathcal{L}_{\mathcal{DM}} = \overline{\mathcal{X}}[i\gamma^{\mu}\partial_{\mu} - \mathcal{M}_{\mathcal{X}} + yh]\mathcal{X} \;\; + \frac{1}{2}\partial_{\mu}h\partial^{\mu}h - \frac{1}{2}M_h^2h^2 + \sum_B f_B \frac{m_B}{v}\overline{\psi}_B h \psi_B$$

- The dark matter energy density and pressure can be calculated from energy momentum tensor as

$$\mathcal{E}_{\mathcal{DM}} = \frac{1}{\pi^2}\int_0^{k_f^{DM}} k^2\sqrt{k^2 + (M_{\mathcal{X}}^*)^2}dk + \frac{1}{2}M_h^2h_0^2,$$

$$\mathcal{P}_{\mathcal{DM}} = \frac{1}{3\pi^2}\int_0^{k_f^{DM}} \frac{k^4}{\sqrt{k^2 + (M_{\mathcal{X}}^*)^2}}dk - \frac{1}{2}M_h^2h_0^2.$$

- Total Lagrangian

$$\mathcal{L} = \mathcal{L}_{\mathcal{N}} + \mathcal{L}_{\mathcal{DM}}$$

- Total energy and pressure are

$$\mathcal{E} = \mathcal{E}_{\mathcal{N}} + \mathcal{E}_{\mathcal{DM}}, \mathcal{P} = \mathcal{P}_{\mathcal{N}} + \mathcal{P}_{\mathcal{DM}}$$

Theoretical framework

TOV (Tolman-Oppenheimer-Volkoff) equation:

- ❑ We use the output of the EOS as input of TOV for mass and radius calculation.

$$\frac{\partial P}{\partial r} = -\frac{(P+\epsilon(r))(m(r)+4\pi r^3 P)}{r(r-2m(r))},$$

$$\frac{\partial m}{\partial r} = 4\pi r^2 \epsilon(r),$$

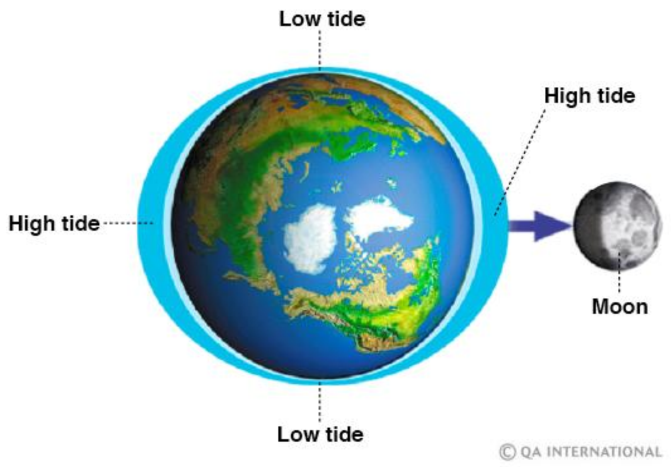
Tidal deformability:

λ depends on its radius and tidal Love number k_2

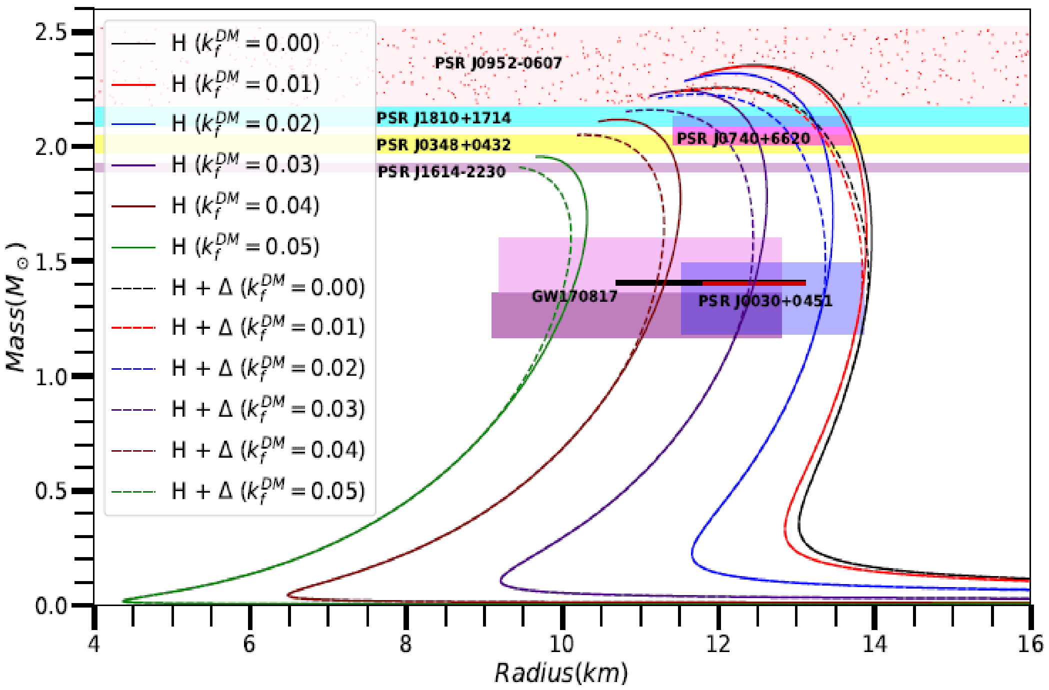
$$\lambda = \frac{2k_2 R^5}{3},$$

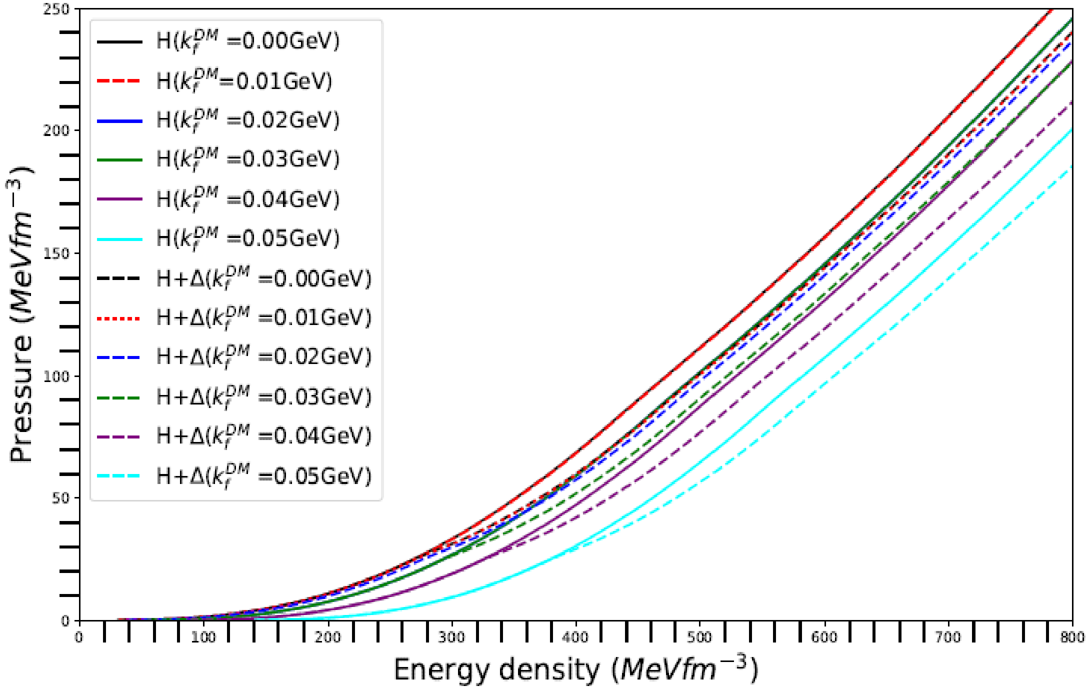
and the dimensionless tidal deformability is

$$\Lambda = \frac{2k_2}{3C^5}$$



Mass-Radius Profile and EOS



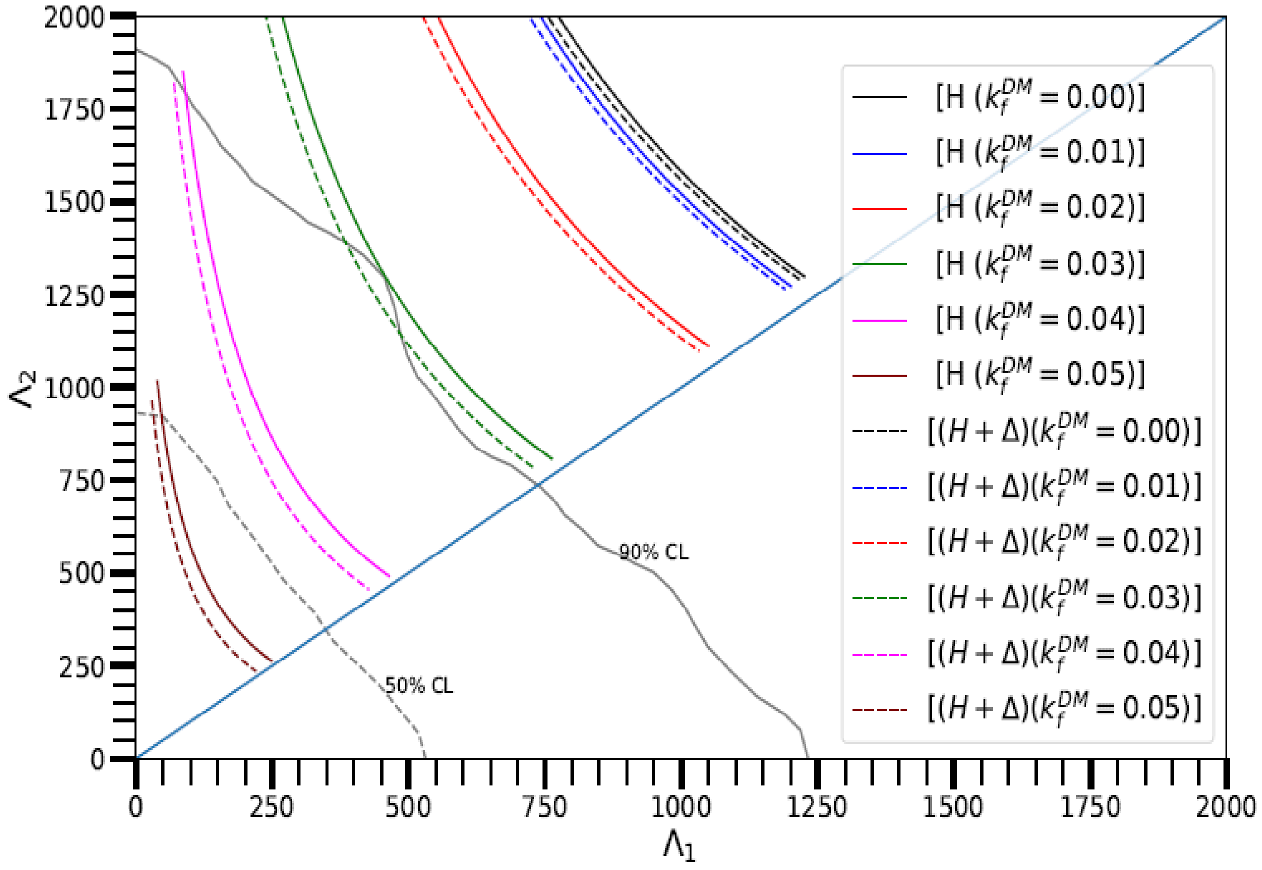


- The EOS with dark matter become softer leading to decrease in mass and radius.
- We use NLD parameter set to study the effect DM on NS bulk properties.
- Higher value of k_f^{DM} leads to softening EOS, due to which the maximum mass and radius decrease.
- The canonical radius decreases to approximately 4 km by changing k_f^{DM} from 0 GeV to 0.05 GeV.
- For $k_f^{DM} = 0.03\ GeV$, NLD parameter set satisfy most of NICER constraints.

Effect on Tidal deformability

- ❑ Tidal deformability for NLD parameter using dark matter Fermi momenta k_f^{DM} = 0.04 and 0.05 GeV lies within the 90% credible limit of the GW170817 event.

- ❑ The hyperonic star with dark matter Fermi momenta k_f^{DM} = 0.04 and 0.05 GeV satisfies the GW170817 constraints for the NLD parameter set.

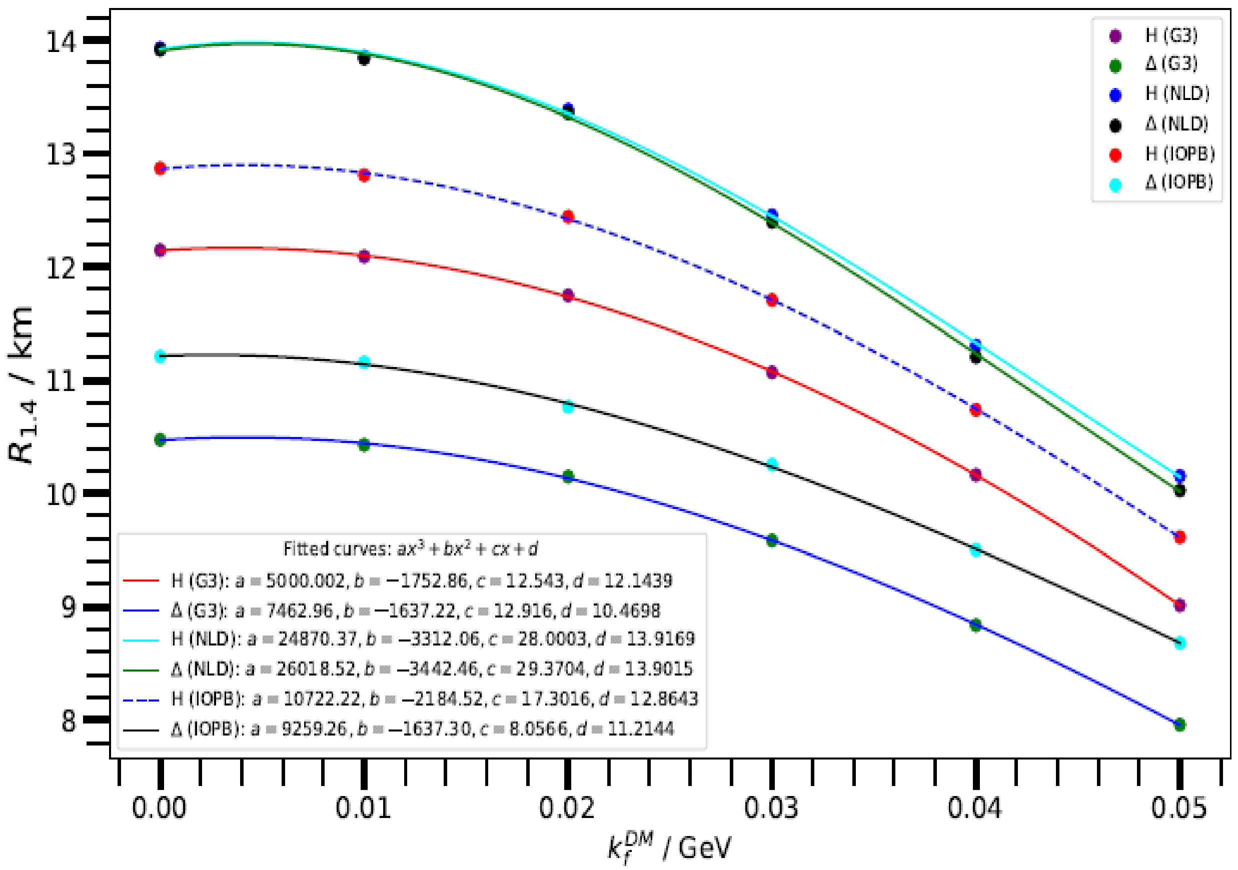


Correlation

- Parameter set : NLD, IOPB and G3.
- Each RMF parameter set exhibits a non-linear decreasing trend, which suggests that $R_{1.4}$ decreases as the value of k_f^{DM} increases.
- Correlation between $R_{1.4}$ and k_f^{DM} - represented by a third-order polynomial function

$$R_{1.4} = a\left(k_f^{DM}\right)^3 + b\left(k_f^{DM}\right)^2 + c\,k_f^{DM} + d$$

- The functional form of this correlation is same for each parameter set.
- The correlation coefficients a, b, c and d are varied for different composition and parameter set.



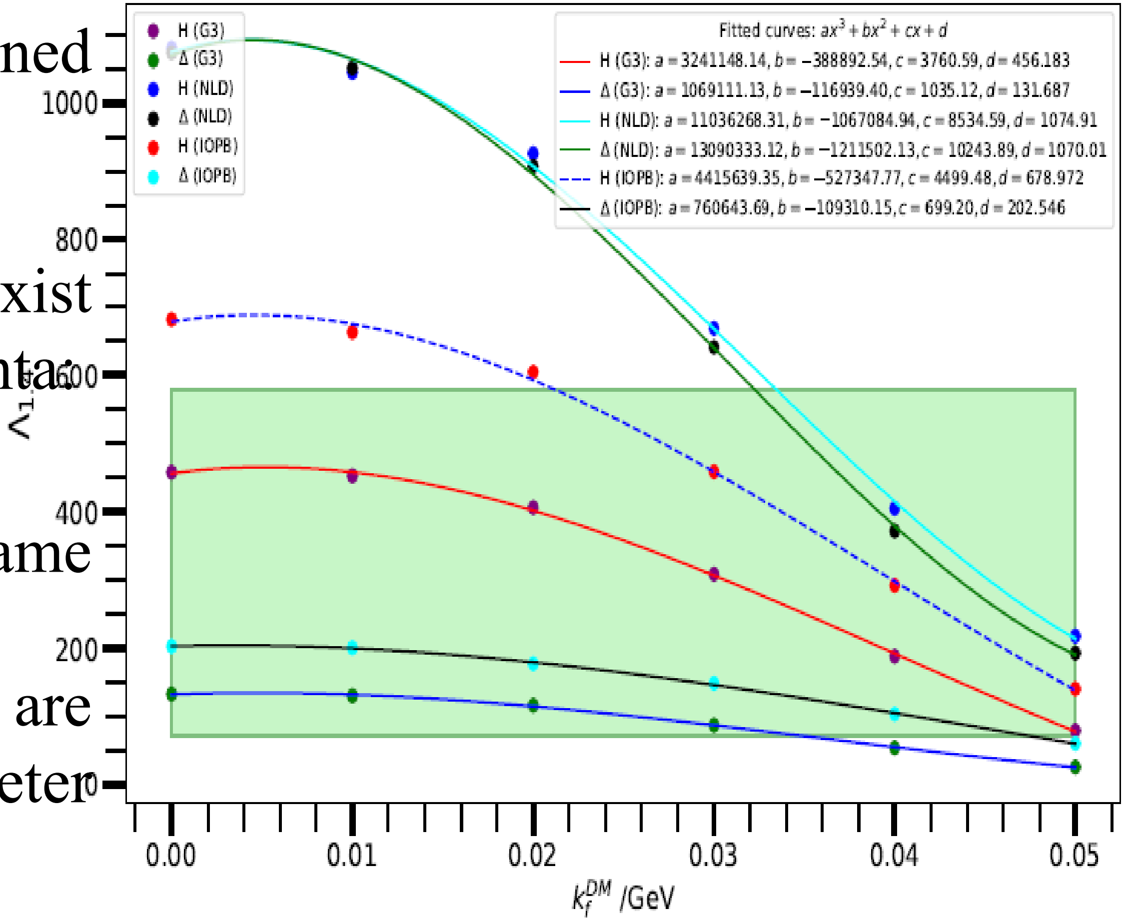
Correlation

- For k_f^{DM} =0.04 and 0.05 GeV, the mentioned parameters satisfy GW170817 constraint.

- A third-order polynomial correlation exist between tidal deformability and Fermi momenta:

$$\Lambda_{1.4} = a\left(k_f^{DM}\right)^3 + b\left(k_f^{DM}\right)^2 + c\, k_f^{DM} + d$$

- The functional form of this correlation is same for each parameter set.
- The correlation coefficients a, b, c and d are varied for different composition and parameter set.

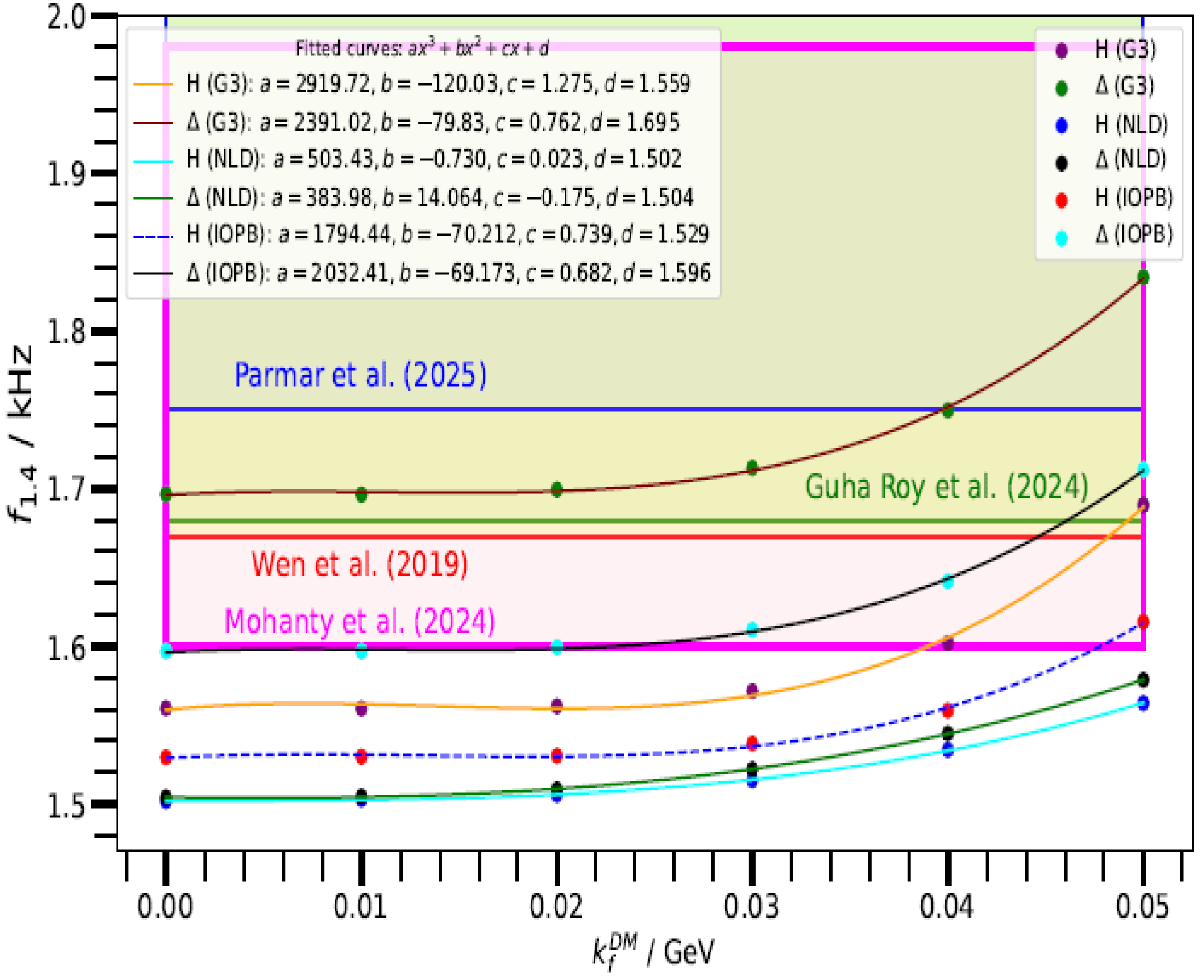


Correlation

- The canonical frequency is correlated with the dark matter Fermi momenta k_f^{DM} by a third order polynomial equation defined as:

$$f_{1.4} = a\left(k_f^{DM}\right)^3 + b\left(k_f^{DM}\right)^2 + c\,k_f^{DM} + d$$

- The functional form of this correlation is same for each parameter set.
- The correlation coefficients a, b, c and d are varied for different composition and parameter set.
- G3 parameter set with Δ−admixed hyperon composition satisfies all the constraints with DM content k_f^{DM} more than 0.04GeV while NLD parameter set does not satisfy any constraint of f1.4 with DM content.
- IOPB does not satisfy the f1.4 constraints with a lower value of k_f^{DM}, it approaches the constraints when the DM content increases.



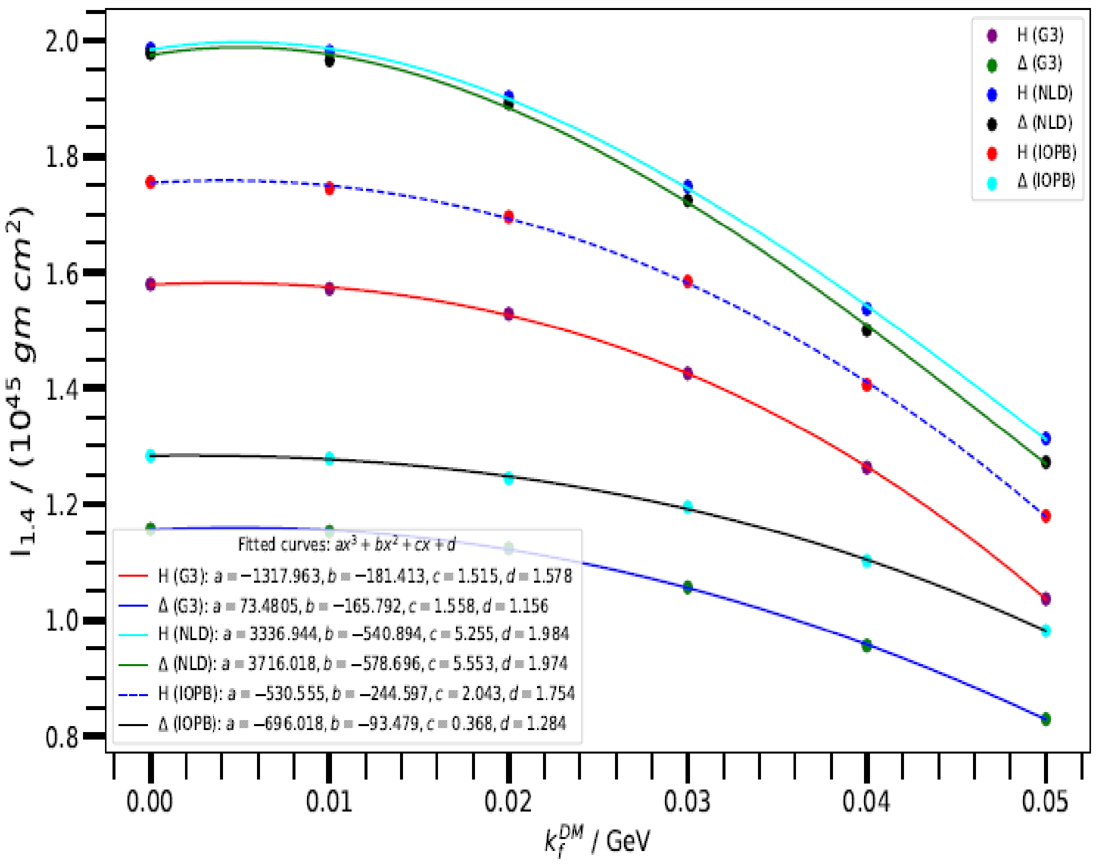
Correlation

- Cubical correlation between $I_{1.4}$ and k_f^{DM}:

$$I_{1.4} = a\left(k_f^{DM}\right)^3 + b\left(k_f^{DM}\right)^2 + c\, k_f^{DM} + d$$

- The functional form of this correlation is same for each parameter set.

- The correlation coefficients a, b, c and d are varied for different composition and parameter set.

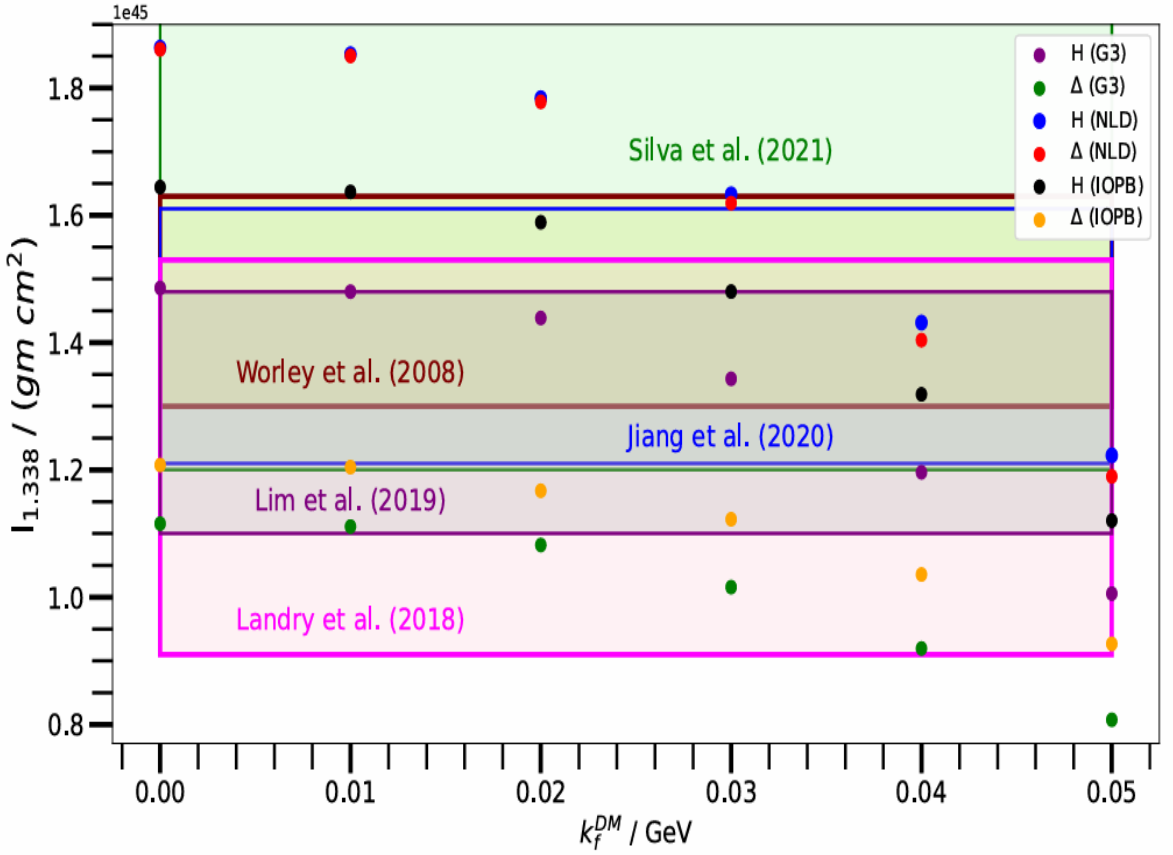


Correlation

- The NLD parameter set with k_f^{DM} =0.04 GeV satisfies all the constraints of PSR J0737-3039A pulsar with mass 1.338 $M_{\odot}$.

- IOPB parameter set with hyperon composition also satisfies all the constraints for k_f^{DM} =0.04 GeV.

- G3 parameter set with hyperon composition satisfies all the mentioned constraints for k_f^{DM} =0.00−0.03 GeV.



Conclusion

- The presence of dark matter softens the EOS; consequently, the maximum mass, corresponding radius, canonical radius, canonical tidal deformability, and moment of inertia decrease with k_f^{DM} .
- The canonical frequency increases with k_f^{DM} .
- There exist a third-order polynomial correlation between dark matter Fermi momenta and neutron star bulk properties namely canonical radius, canonical tidal deformability, canonical frequency and moment of inertia of the canonical star.
- The functional form for all these correlations are same for each parameter set; however, the correlation coefficients are varying for different parameter set.
- For $k_f^{DM} = 0.04$ and 0.05 GeV, our results align with most of the constraints.

Thank you