Entanglement in Neutrino System

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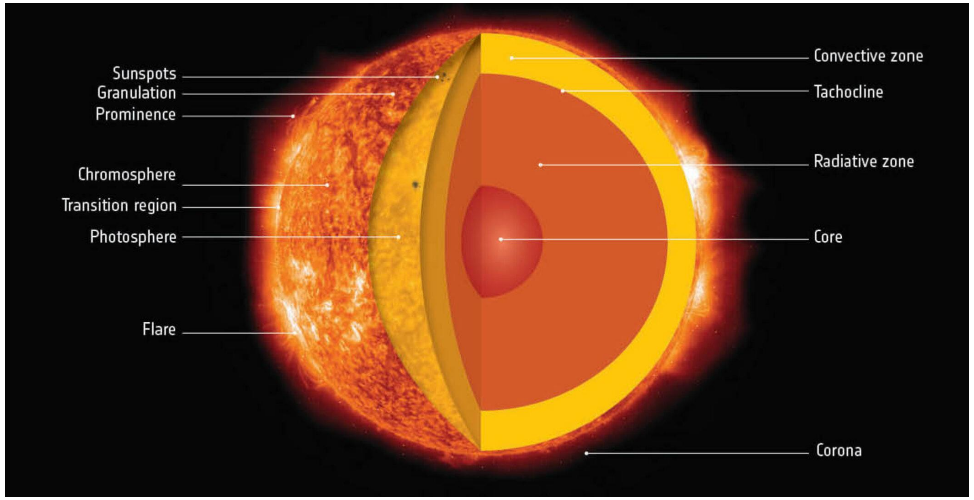
Solar neutrino Anomaly and Resolution:

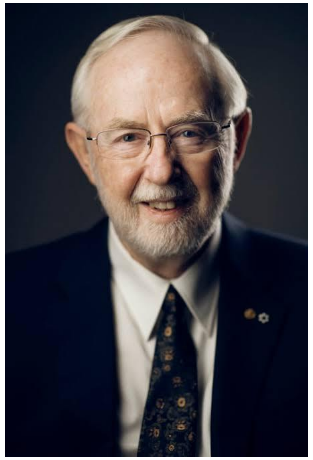


- The total number of neutrino produced at different energies within the Sun has been calculated theoretically .

- Experimentally the number of secondary particle has also been calculated in the detection process of neutrinos.

- But in practical scenario only 1/3 of the estimated numbers of neutrinos of a particular flavour is detected.

Nobel prize in 2002 for cosmic neutrino and solar neutrino problem



Nobel prize in 2015 for solving solar neutrino problem by neutrino oscillation

Why do neutrinos oscillate?

- Neutrino Oscillation is the direct evidence of quantum mechanical superposition principle
- Here each flavour eigenstates are the superposition of three mass eigenstates.
- As the propagation phases are different the flavour states evolve with time and can change to other flavour states

Appearance Probability →

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2(2\theta)\sin^2(\frac{\Delta m^2_{21}}{4E}L)$$

Disappearance Probability →

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - \sin^2(2\theta)\sin^2(\frac{\Delta m^2_{21}}{4E}L)$$

Production

$$|\nu_\mu\rangle = \sum_i U_{\mu i}|\nu_i\rangle$$

Propagation

$$\nu_1 : \quad e^{-iE_1 t}$$
$$\nu_2 : \quad e^{-iE_2 t}$$
$$\nu_3 : \quad e^{-iE_3 t}$$

Detection: projection over

$$\langle \nu_e |$$

Interacting ES

Propagating ES

Implication of existance oscillation →

1. Survival Probability
2. Transition probability

Superposition versus Entanglement: Conceptual Distinction

A superposition involves a single system existing coherently in multiple states of the same Hilbert space. Interference phenomena, such as neutrino flavor oscillation or photon polarization rotation, arise from phase differences among these components.

Mass-basis superposition (single Hilbert space)

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}\,|\nu_i\rangle,$$

where {|νi⟩} are basis states in the single-particle Hilbert space. This describes quantum superposition—one particle in a coherent mixture of its mass components.

Entanglement involves two or more subsystems whose total Hilbert space is a tensor product. The composite state cannot be expressed as a product of individual subsystem states. The subsystems exhibit nonlocal correlations that persist even when they are spatially separated.

Mode (occupation-number) entanglement (tensor-product picture):

$$|\Psi(t)\rangle = A_{e\alpha}(t)\,|1_e, 0_\mu\rangle + A_{\mu\alpha}(t)\,|0_e, 1_\mu\rangle,$$

This state cannot, in general, be factorized into a product of independent mode states—hence it represents entanglement between flavor modes. In this field-theoretic view, neutrino oscillation corresponds to the dynamical redistribution of coherence among these coupled modes.

Superposition — Single Particle Mode Entanglement

Superposition

1. Coherent mixture of mass eigenstates
2. Single Hilbert space
3. Interference of amplitudes in mass or flavor modes
4. Explains Neutrino Oscillation and Oscillation Probabilities

Single Particle Mode Entanglement

1. Quantum correlation between flavor modes
2. Tensor product of different mass or flavor mode Hilbert sub-spaces
3. Distributed coherence among mass or flavor modes
4. Connects to entanglement, decoherence, and information-theoretic measures

Single Particle mode-entanglement framework **does not replace** the standard superposition picture; rather it **extends it** by providing a **quantitative language** for coherence, decoherence, and quantum correlations — much like entanglement is used in quantum optics to describe single-photon multi-mode correlations.

Future precision experiments (DUNE, Hyper-K, IceCube-Gen2) probing **wave-packet separation, matter effects, and CP-violation** could indirectly test how such entanglement-like correlations evolve — providing a bridge between **neutrino oscillation phenomenology** and **quantum information theory.**

Separable state and entangled state

Separable state: Any state of a composite system called separable or classically correlated state iff we can represent the state as -

$$\rho_{ABCD...} = \sum_i w_i \rho_A^i \otimes \rho_B^i \otimes \rho_C^i \otimes \rho_D^i \otimes \cdots$$

Entangled state: The states that can not be represented as the product state of its constituent system are called the entangled state.state

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B)$$

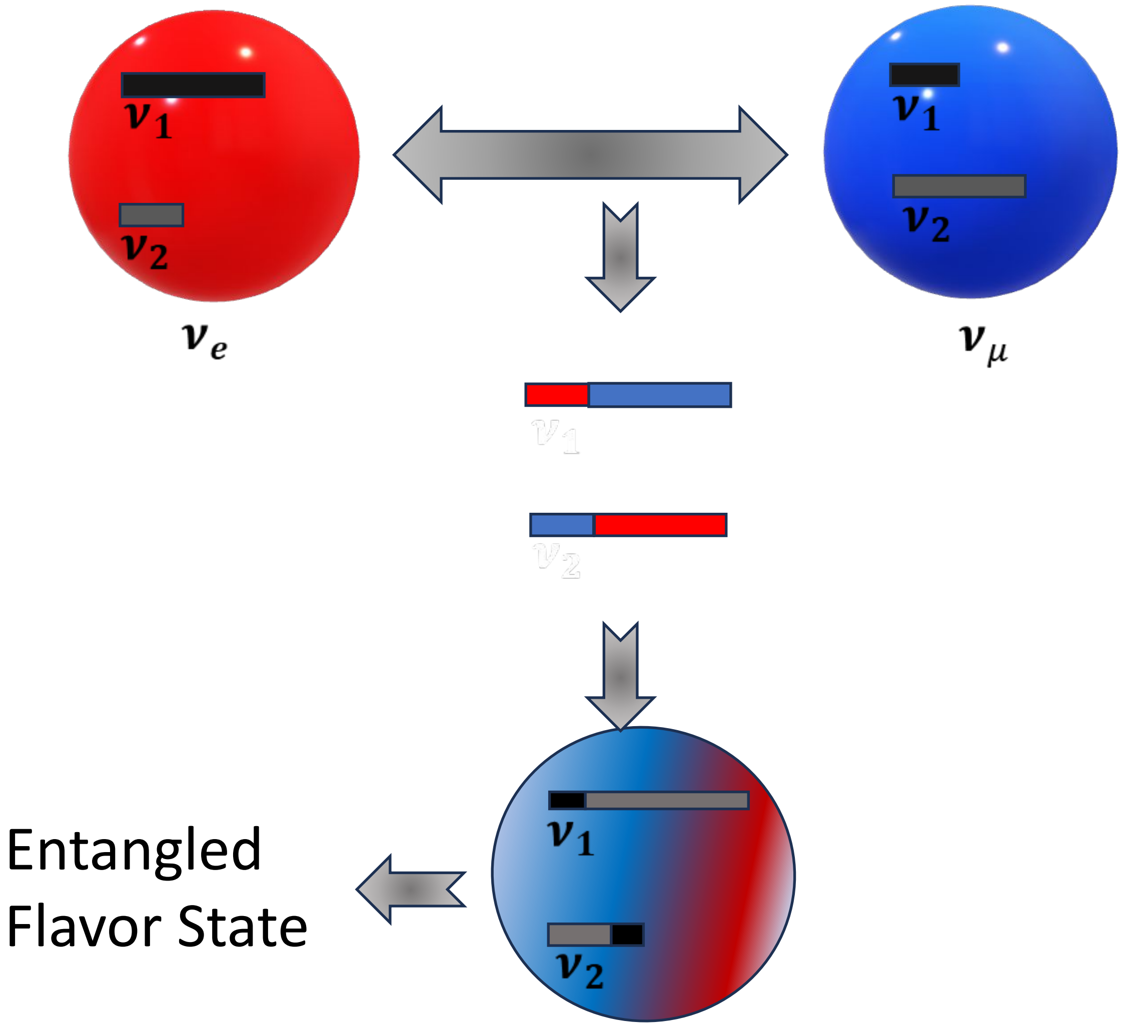
The next question is if the state is pure or mixed?

Any density operator

For pure state $\rho^2 = \rho$

For mixed state $\rho^2 < \rho$

Entangled two Flavor State



1. Consider a pure state of electron(red) and muon(blue) neutrino with superimposed mass eigenstate as ν_1(black) and ν_2(grey).

$$|\nu_\beta\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

$$|\nu_\alpha\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

2. While propagating the mass eigenstate acquires some flavor state by the rotation of basis.

$$|\nu_e\rangle = |0\rangle_e \otimes |1\rangle_\mu = |01\rangle_e$$
$$|\nu_\mu\rangle = |1\rangle_e \otimes |\nu\rangle_\mu = |10\rangle_\mu$$

3. Therefore, the superimposed mass eigenstate associated with each flavor basis now gained some flavor state with it, which implies the state is no longer pure.

$$|\nu_e(\theta,\phi)\rangle = (\cos^2\theta + \sin^2\theta e^{-i\phi})\,|\nu_e\rangle - \sin\theta\cos\theta(1-e^{-i\phi})\,|\nu_\mu\rangle$$

Tensor Product Algebra and Density Matrix :

The system of neutrino is considered a bipartite system with two Flavors, state ν_e and ν_μ

$$a \otimes b = \begin{pmatrix} a_1 b_1 \\ a_1 b_2 \\ a_2 b_1 \\ a_2 b_2 \end{pmatrix}.$$

$$\mathcal{H}_1 \simeq \mathbb{C}^2, \qquad \mathcal{H}_2 \simeq \mathbb{C}^2,$$

$$\mathcal{H}_1 \otimes \mathcal{H}_2 \cong \mathbb{C}^2 \otimes \mathbb{C}^2.$$

$$\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$$

$$|1_1, 0_2\rangle \equiv |1_1\rangle \otimes |0_2\rangle \equiv |10\rangle$$

Each qubit lives
in a two-dimensional Hilbert space

$$\mathcal{H}_i = \text{span}\{|0_i\rangle, |1_i\rangle\}, \qquad i \in \{e, \mu$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|10\rangle = |1_1\rangle \otimes |0_2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Density Matrix for 2 Qubit System $\rho_{\nu_e} = |\nu_e\rangle \langle \nu_e|.$

$$\rho_{\nu_e} = \Big[\cos\theta \big(|0_1\rangle \otimes |1_2\rangle \big) + \sin\theta \, e^{-i\phi} \big(|1_1\rangle \otimes |0_2\rangle \big) \Big]$$
$$\times \Big[\cos\theta \big(\langle 0_1| \otimes \langle 1_2| \big) + \sin\theta \, e^{+i\phi} \big(\langle 1_1| \otimes \langle 0_2| \big) \Big]$$

$$\rho^{ee}_{4\times4} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos^2\theta & \cos\theta \, \sin\theta \, e^{+i\phi} & 0 \\ 0 & \cos\theta \, \sin\theta \, e^{-i\phi} & \sin^2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Identification of Entanglement

The system of neutrino is considered a bipartite system with two Flavors, state ν_e and ν_μ

To quantify mode entanglement one often considers the reduced state of one mode.

Partial transpose is a useful tool to identify if the system is separable or not.

Trace out mode 2 to obtain the reduced density matrix for mode 1

$$\rho^{e\mu}(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |\tilde{U}_{ee}(t)|^2 & \tilde{U}_{ee}(t)\tilde{U}^*_{e\mu}(t) & 0 \\ 0 & \tilde{U}_{e\mu}(t)\tilde{U}^*_{ee}(t) & |\tilde{U}_{e\mu}(t)|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

All positive eigenvalue

$$\rho^T_\mu(t) = \begin{pmatrix} 0 & 0 & 0 & \tilde{U}_{ee}(t)\tilde{U}^*_{e\mu}(t) \\ 0 & |\tilde{U}_{ee}(t)|^2 & 0 & 0 \\ 0 & 0 & |\tilde{U}_{\mu\mu}(t)| & 0 \\ \tilde{U}_{e\mu}(t)\tilde{U}^*_{ee}(t) & 0 & 0 & 0 \end{pmatrix}$$

Not all eigenvalues are positive

After positive transpose some density matrices have non negative eigenvalues those are the PPT state and others are called the negative partial transpose, NPT nonnegative. Immediate consequence of the above classification is as follows

1. All separable states are PPT states
2. All NPT states are entangled.
   This is demonstrating mode entanglement tracing out one mode leaves the other in mixed density matrix.

$$\lambda_1 = P_d,\ \lambda_2 = P_a,\ \lambda_3 = \sqrt{P_d P_a},\ \lambda_4 = -\sqrt{P_d P_a}.$$

The electron and muon flavor states of neutrino are entangled!!!

Measurement of Entanglement

Linear Entropy → For a pure bipartite state, the entanglement of the state is defined by the Von-Neumann entropy of its reduced density matrices. Linear entropy is a lower approximation to it.

$$: S_L = 1 - \mathrm{Tr}(\rho^2).$$

Negativity → Measures by how much the partial transpose fails to be positive definite.

$$N_{e\mu} = N(\rho^{e\mu}(t)) = \frac{||\rho^{T_\mu}(t)|| - 1}{2}$$

Concurrence → Measures the quantum correlation

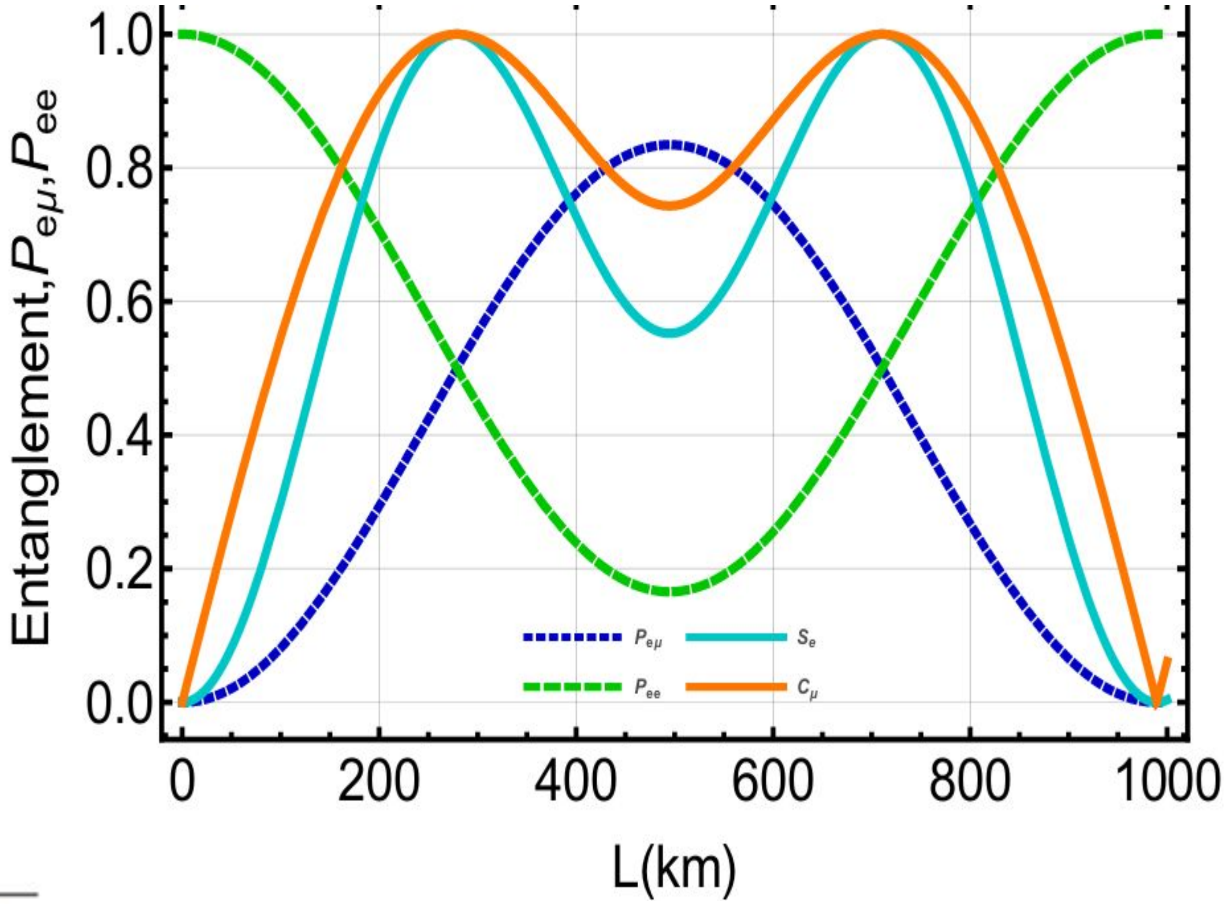
$$C_{AB} = 2\sqrt{det(\rho_A)}$$

$$C(L) = 2\sqrt{P_{e\to e}(L)\, P_{e\to\mu}(L)}.$$

Von Neumann entanglement entropy

$$S_e(t) = -\,\mathrm{Tr}\left[\rho_e(t) \ln \rho_e(t)\right]$$

$$S_L = 2\, P_{ee}(L)\, P_{e\mu}(L),$$



Q1

Neutrinos are entangled?

a) Fully separable state

b) Two-partite entangled state

c) Genuine Tripartite Entanglement ?

Q2

The nature of entanglement?

Does there exist monogamy inequality?

How does it affect the oscillation parameters of the neutrino?

Does there exist monogamy inequality?

Q3

How it affects three fundamental unknowns to the neutrino sector?

CP Sensitivity

Mass Hierarchy

Octant degeneracy

Analysis of Neutrino Oscillation parameters in the light of Quantum Entanglement

Rajrupa Banerjee, Papia Panda, Rukmani Mohanta, Sudhanwa Patra;
NPB, 2025

Entanglement in three flavor neutrino oscillation

- Construction of three qubit state of neutrino flavor mode

$$|\nu_\mu(t)\rangle = \tilde{U}_{\mu e}|\nu_e\rangle + \tilde{U}_{\mu\mu}|\nu_\mu\rangle + \tilde{U}_{\mu\tau}|\nu_\tau\rangle$$

$$|\nu_e\rangle \equiv |1\rangle_e \otimes |0\rangle_\mu \otimes |0\rangle_\tau$$

$$|\nu_\mu\rangle \equiv |0\rangle_e \otimes |1\rangle_\mu \otimes |0\rangle_\tau$$

$$|\nu_\tau\rangle \equiv |0\rangle_e \otimes |0\rangle_\mu \otimes |1\rangle_\tau$$

- Formulation of density matrix

$$\rho_\mu(t) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & |\tilde{U}_{\mu\tau}|^2 & \tilde{U}_{\mu\tau}\tilde{U}^*_{\mu\mu} & 0 & \tilde{U}_{\mu\tau}\tilde{U}^*_{\mu e} & 0 & 0 & 0 \\ 0 & \tilde{U}_{\mu\mu}\tilde{U}^*_{\mu\tau} & |\tilde{U}_{\mu\mu}|^2 & 0 & \tilde{U}_{\mu\mu}\tilde{U}^*_{\mu e} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{U}_{\mu e}\tilde{U}^*_{\mu\tau} & \tilde{U}_{\mu e}\tilde{U}^*_{\mu\mu} & 0 & |\tilde{U}_{\mu e}|^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- Introduction of Entanglement Measures:

- **Entanglement of Formation (EOF)**

The entanglement of formation is the most basic measurement of entanglement. Given a density matrix ρ of a pair of quantum systems A and B, consider all possible pure state decompositions of ρ.

- **Concurrence**

Concurrence is a quantitative measure of entanglement for a bipartite system.

Single Particle time evolved flavor neutrino state

- For three qubits, the basis vectors are defined as

$$|q_1q_2q_3\rangle = |q_1\rangle \otimes |q_2\rangle \otimes |q_3\rangle, \qquad q_i \in \{0,1\}.$$

$$= |1\rangle_e \otimes |0\rangle_\mu \otimes |0\rangle_\tau = \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}$$

$$|\nu_\mu(t)\rangle = \tilde{U}_{\mu e}|\nu_e\rangle + \tilde{U}_{\mu\mu}|\nu_\mu\rangle + \tilde{U}_{\mu\tau}|\nu_\tau\rangle$$

$$|\nu_e\rangle \equiv |1\rangle_e \otimes |0\rangle_\mu \otimes |0\rangle_\tau$$

$$|\nu_\mu\rangle \equiv |0\rangle_e \otimes |1\rangle_\mu \otimes |0\rangle_\tau$$

$$|\nu_\tau\rangle \equiv |0\rangle_e \otimes |0\rangle_\mu \otimes |1\rangle_\tau$$

$$\{|000\rangle, |100\rangle, |010\rangle, |001\rangle, |110\rangle, |101\rangle, |011\rangle, |111\rangle\}$$

- The full Hilbert space for three flavour modes are 8 dimensional and span over above 8 basis states.
  A flavour state may be embedded in the full 8-dimensional space

$$|\nu_\alpha(t)\rangle = \sum_{\beta=e,\mu,\tau} \tilde{U}_{\alpha\beta}(t)\ |1_\beta, 0_\gamma, 0_\delta\rangle$$

- For a single neutrino, the physically relevant sector of the full Hilbert space is the single excitation subspace:

$$\mathcal{H}_{\text{phys}} = \text{span}\{|100\rangle, |010\rangle, |001\rangle\}$$

$$\dim(\mathcal{H}_{\text{phys}}) = 3.$$

Embedding of $\widetilde{U}(t)$ into the 8×8 Structure

$$|\nu_\alpha(t)\rangle = \sum_{\beta=e,\mu,\tau} \widetilde{U}_{\alpha\beta}(t)\ |1_\beta, 0_\gamma, 0_\delta\rangle$$

8×8 operator $U^{(8)}_{\text{flav}}(t)$ that acts as $\widetilde{U}(t)$

$$\widetilde{U}(t) = U\,\text{diag}(e^{-iE_1 t}, e^{-iE_2 t}, e^{-iE_3 t})\,U^\dagger$$

$$U^{(8)}_{\text{flav}}(t) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \widetilde{U}_{ee} & \widetilde{U}_{e\mu} & \widetilde{U}_{e\tau} & 0 & 0 & 0 & 0 \\ 0 & \widetilde{U}_{\mu e} & \widetilde{U}_{\mu\mu} & \widetilde{U}_{\mu\tau} & 0 & 0 & 0 & 0 \\ 0 & \widetilde{U}_{\tau e} & \widetilde{U}_{\tau\mu} & \widetilde{U}_{\tau\tau} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

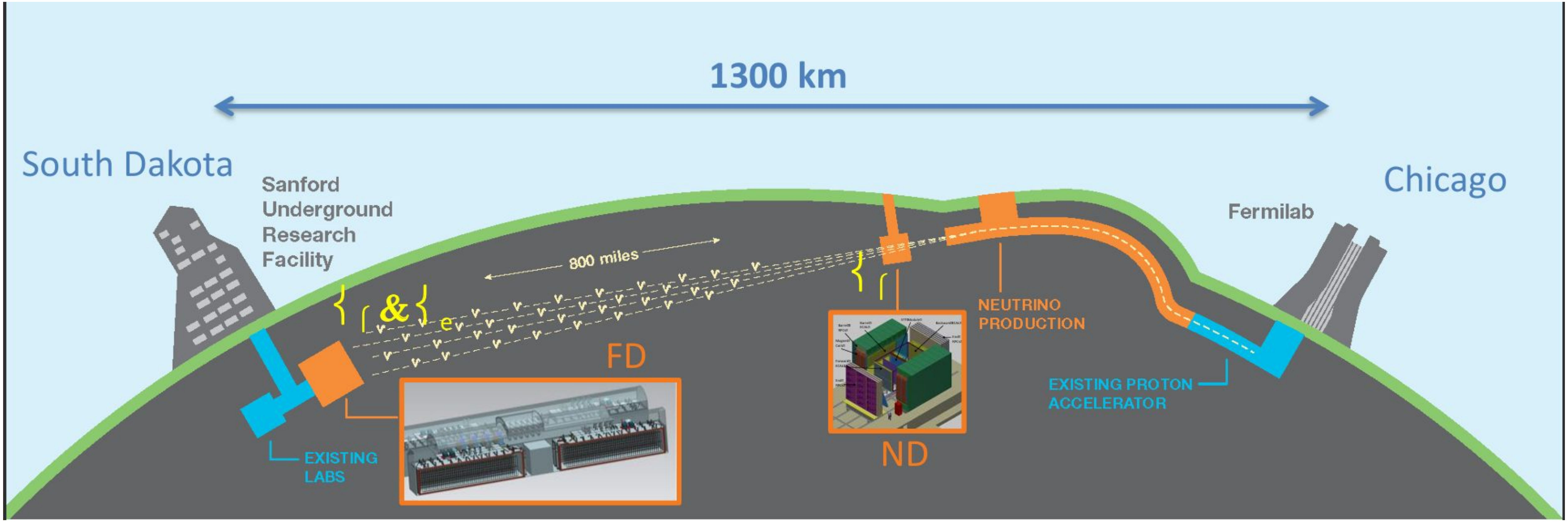
Relevant single-particle three mode reduced density matrix is derived with projecting into physical modes (1 denoted here as muon mode):

$$\mathcal{P}_1 = |100\rangle\langle 100| + |010\rangle\langle 010| + |001\rangle\langle 001|.$$

$$\rho_{1\text{-ex}}(t) = \mathcal{P}_1\,\rho_{\text{full}}(t)\,\mathcal{P}_1 = \begin{pmatrix} |\widetilde{U}_{\mu e}(t)|^2 & \widetilde{U}_{\mu e}(t)\widetilde{U}^*_{\mu\mu}(t) & \widetilde{U}_{\mu e}(t)\widetilde{U}^*_{\mu\tau}(t) \\ \widetilde{U}_{\mu\mu}(t)\widetilde{U}^*_{\mu e}(t) & |\widetilde{U}_{\mu\mu}(t)|^2 & \widetilde{U}_{\mu\mu}(t)\widetilde{U}^*_{\mu\tau}(t) \\ \widetilde{U}_{\mu\tau}(t)\widetilde{U}^*_{\mu e}(t) & \widetilde{U}_{\mu\tau}(t)\widetilde{U}^*_{\mu\mu}(t) & |\widetilde{U}_{\mu\tau}(t)|^2 \end{pmatrix}$$

Overview of DUNE

1.Muon neutrino and anti-neutrinos from high-power proton beam- 1.2MW upgradable to 4MW
2. Massive underground liquid Ar time projection chamber- 4*17 kton fidusial mass.
3. Near detector to characterize the beam



▪ Graphical Implimentation for DUNE

$$\mathrm{EOF} = S_{\mu e} + S_{\mu\tau} + S_{\mu(e\tau)}.$$

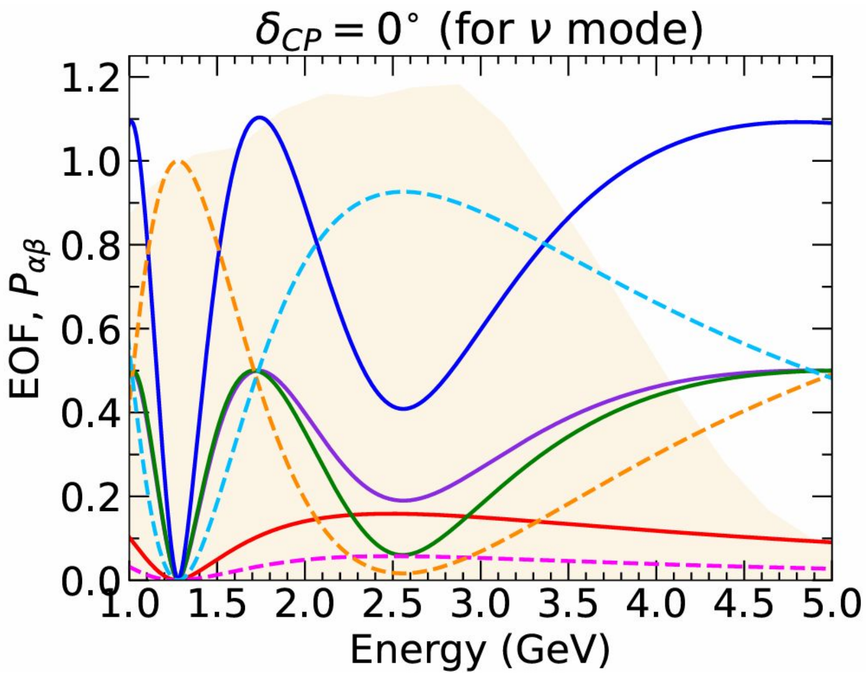
→

$$S_{\mu e} = -\frac{1}{2}\Big[P_{\mu e}\log_2 P_{\mu e} + (P_{\mu\mu} + P_{\mu\tau})\log_2(P_{\mu\mu} + P_{\mu\tau})\Big],$$
$$S_{\mu\tau} = -\frac{1}{2}\Big[P_{\mu\tau}\log_2 P_{\mu\tau} + (P_{\mu\mu} + P_{\mu e})\log_2(P_{\mu\mu} + P_{\mu e})\Big],$$
$$S_{\mu(e\tau)} = -\frac{1}{2}\Big[P_{\mu\mu}\log_2 P_{\mu\mu} + (P_{\mu e} + P_{\mu\tau})\log_2(P_{\mu e} + P_{\mu\tau})\Big].$$

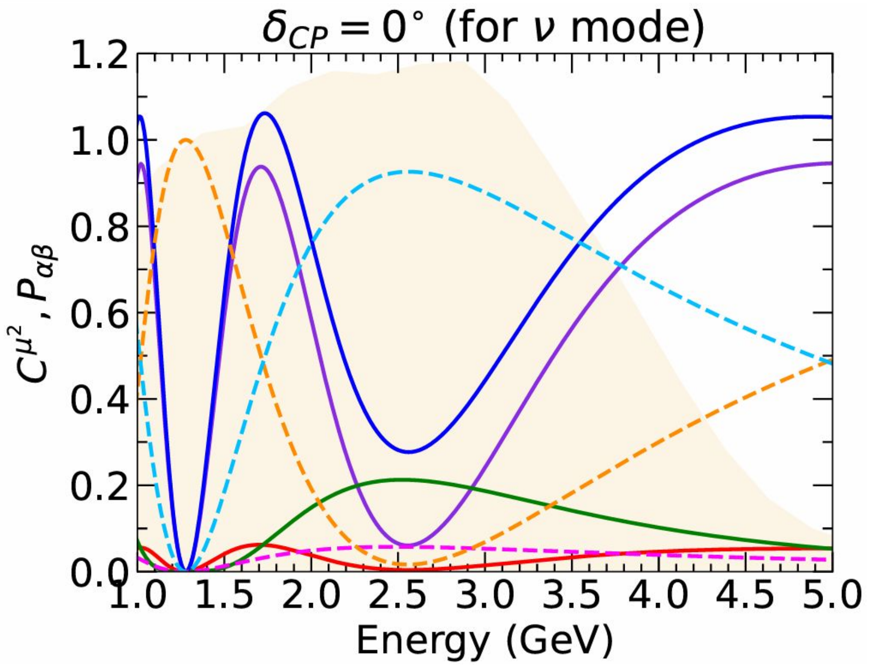
$$C^{\alpha} = \left[3 - \mathrm{Tr}(\rho_\alpha)^2 + \mathrm{Tr}(\rho_\beta)^2 + \mathrm{Tr}(\rho_\gamma)^2\right]^{1/2}.$$

→

$$C^{\mu^2} = C^2_{\mu e;\mu} + C^2_{\mu\tau;\mu} + 4\mathrm{P}_{\mu e}\mathrm{P}_{\mu\tau},$$



Dynamical representation of entanglement measurements along with probabilities



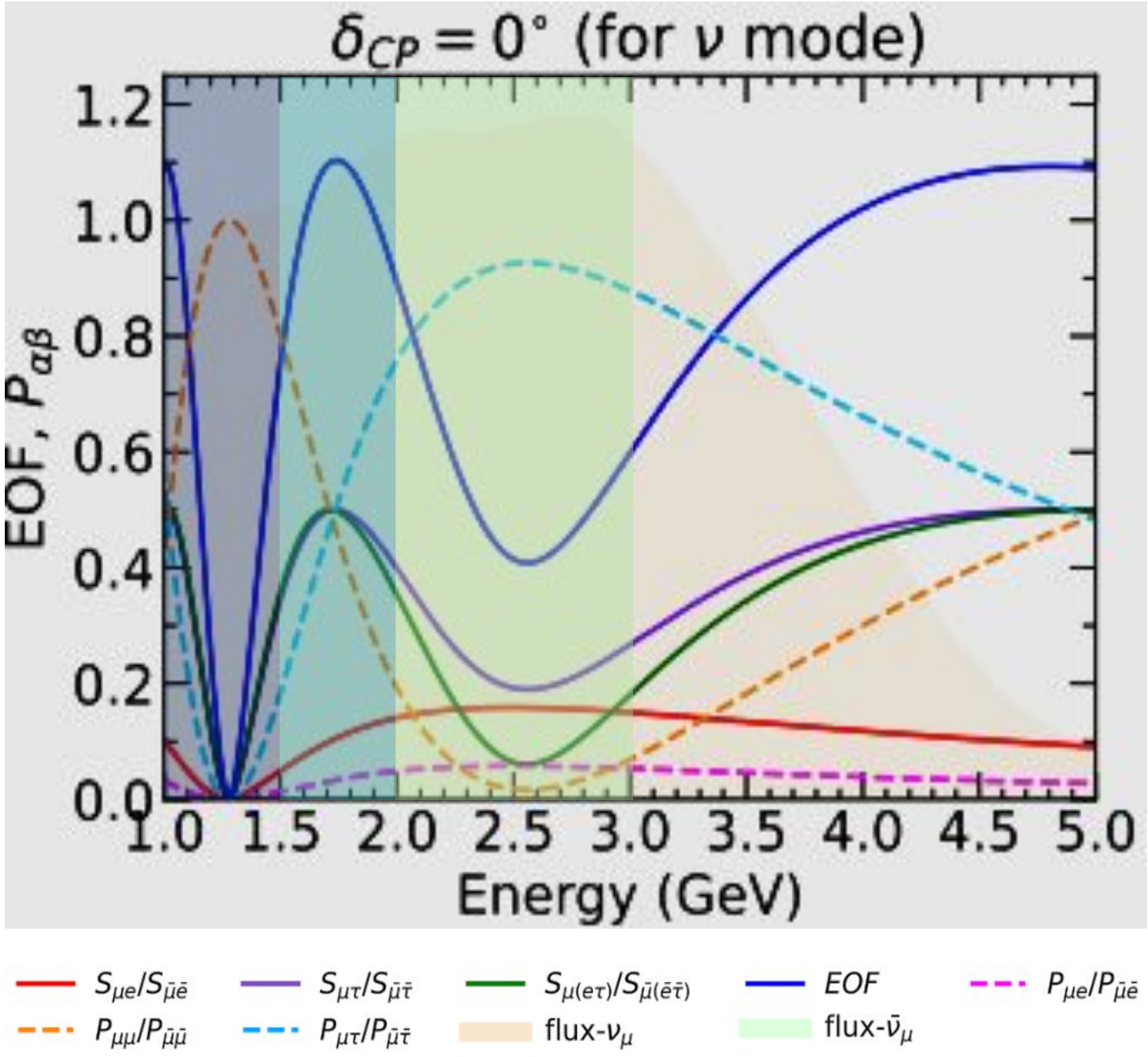
Explanation of the plots:

- Here, we have divided the total range of energy with substantial flux into three energy bins

1. Energy window I: 1.0-1.5 GeV
2. Energy Window II: 1.5-2.0 GeV
3. Energy Window III: 2.0-3.0 GeV

- The solid line depicts the entanglement measures, i.e., EOF and concurrence, while the dashed lines represent the probability.

- The beige background shows the total flux for neutrinos.



EW I

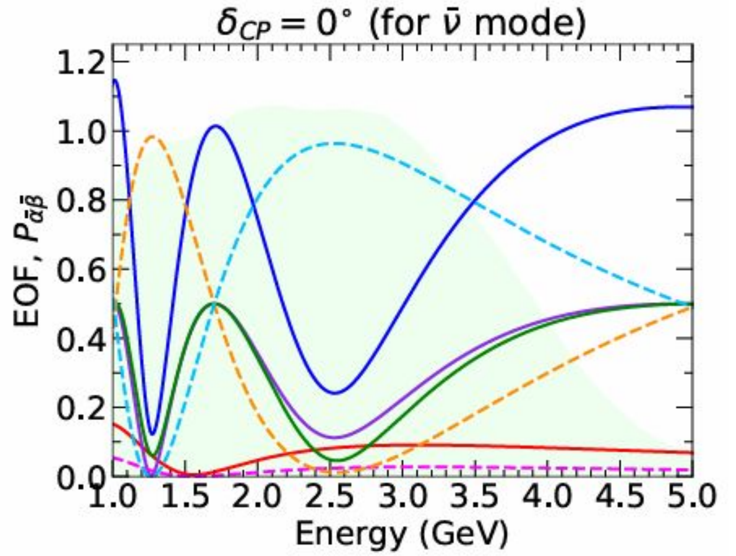
EW II

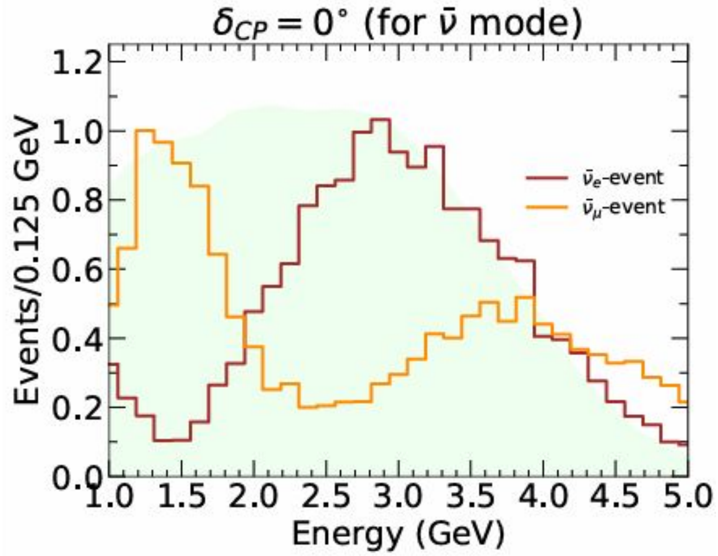
EW III

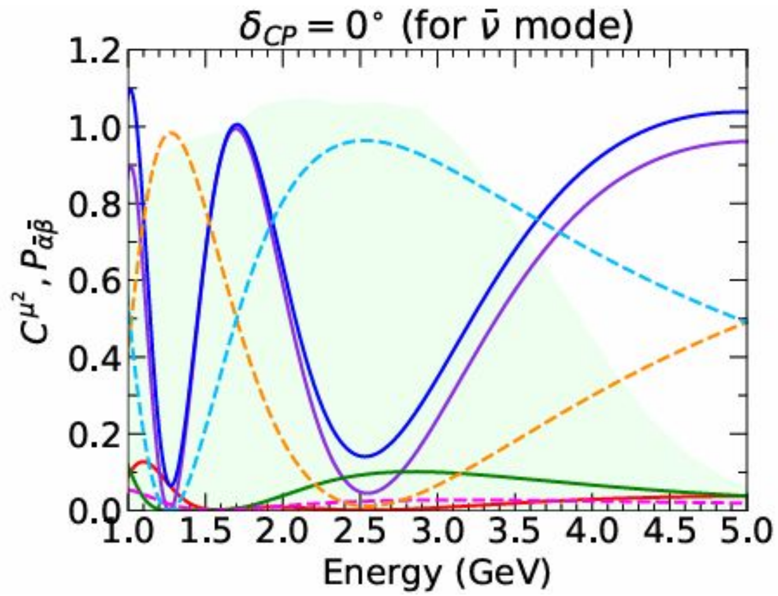
- **Analysis of the plots:**

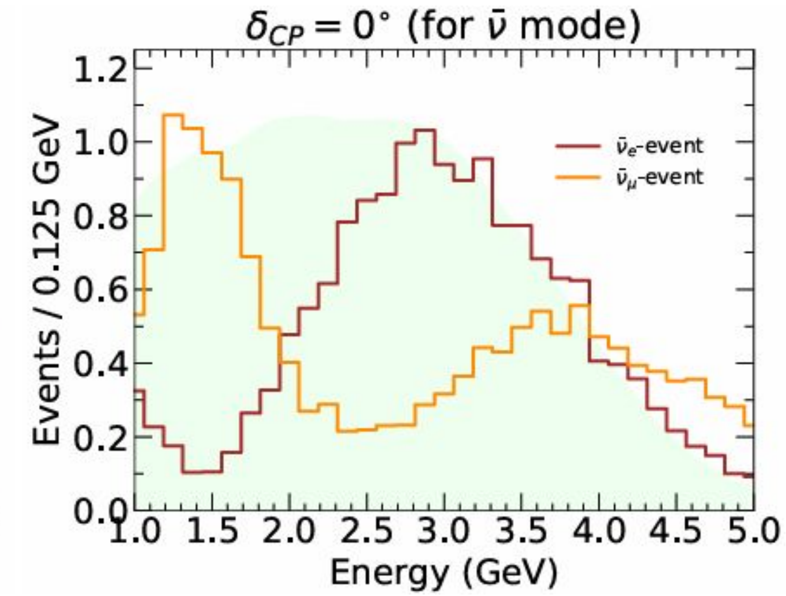
Range of Energy Windows	Analysis
Energy Window I: 1.0-1.5 GeV	1. Both entanglement measures i.e. EOF and concurrence achieve minima representing all the three flavor state behaves as separable state. 2. Also this minima attains almost 0 depicting an global minimum.
Energy Window II: 1.5-2 GeV	1. Entanglement measures attains a maxima that portaits the existence of maximally entangled state. 2. It achieves the maxima when appearance probability of tau neutrino and disappearance probability of muon neutrino is equal 0.5.
Energy Window III: 2-3 Gev	1. Entanglement measures dips into a region of local minima. 2. Both the appearance probabilities attains a maxima while disappearance probability attains a minima.

Entanglement of Formation (EOF) and Concurrence for Antineutrino



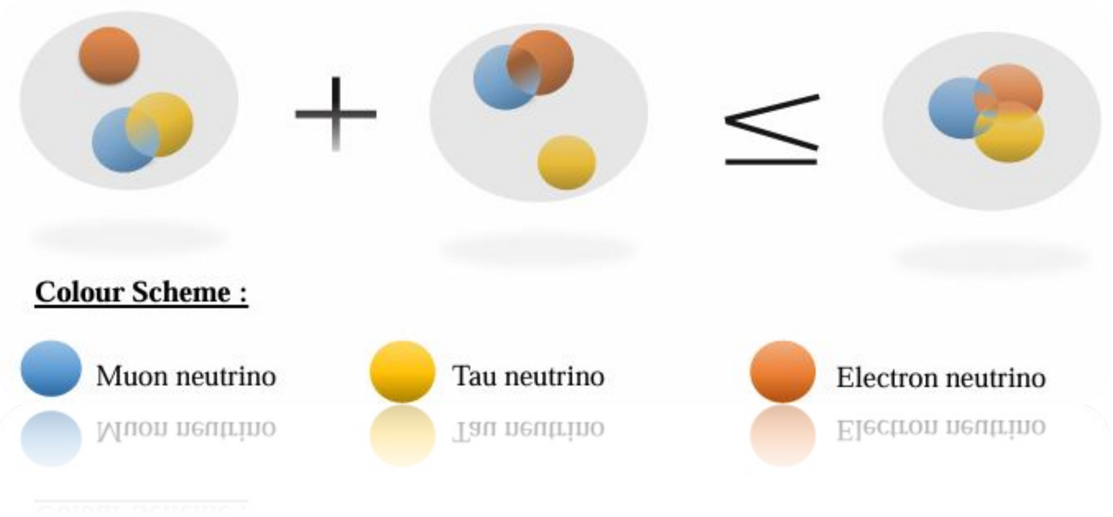






▪ Experimental Implementation

Using squared concurrence as an entanglement measure, Coffman, Kundu, and Wooters established the first monogamy inequality for three-qubit states, CKW inequality.



$$C^2_{\mu e;\mu} + C^2_{\mu\tau;\mu} \leq C^{\mu^2}$$

$$|\psi(t)\rangle = \tilde{U}_{\mu e}\big(|1_e\rangle \otimes |0_\mu 0_\tau\rangle\big) + |1_e\rangle \otimes \big(\tilde{U}_{\mu\mu}\,|1_\mu 0_\tau\rangle + \tilde{U}_{\mu\tau}\,|0_\mu 1_\tau\rangle\big)$$

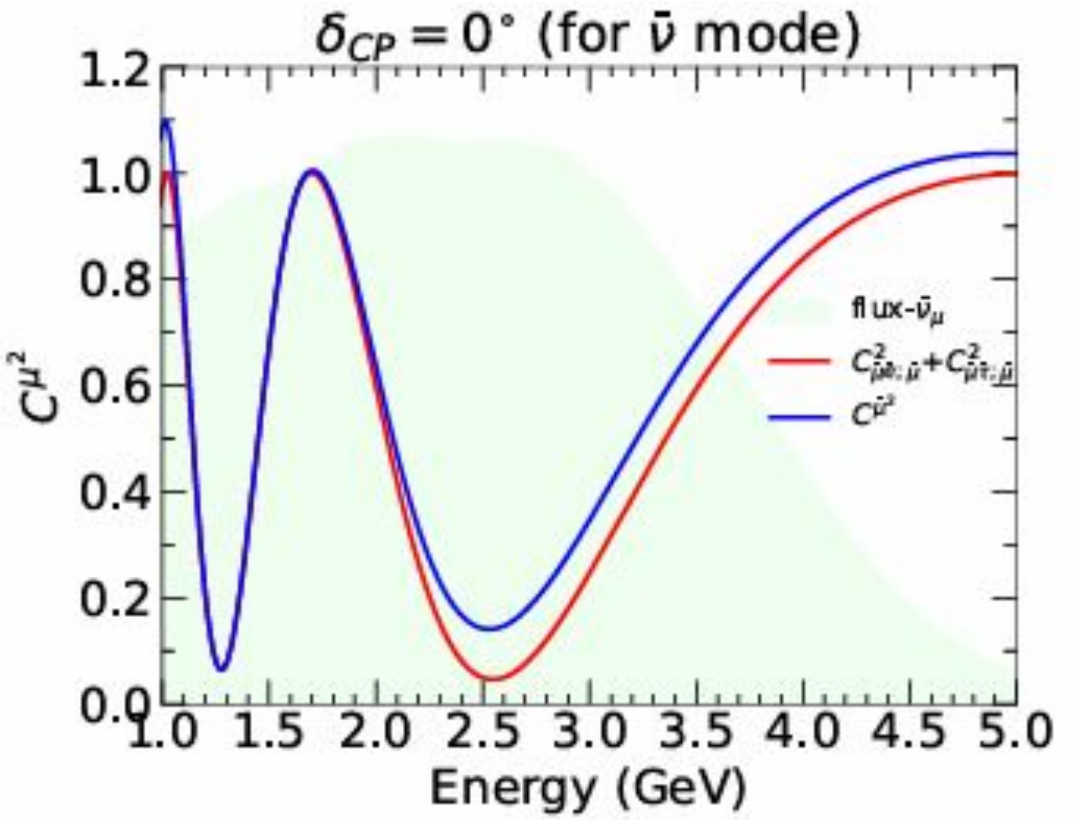
1. Qubit A shared a certain amount of entanglement with the combined system BC.

2. This total entanglement places an upper limit on how much entanglement A can share with qubit B and C individually.

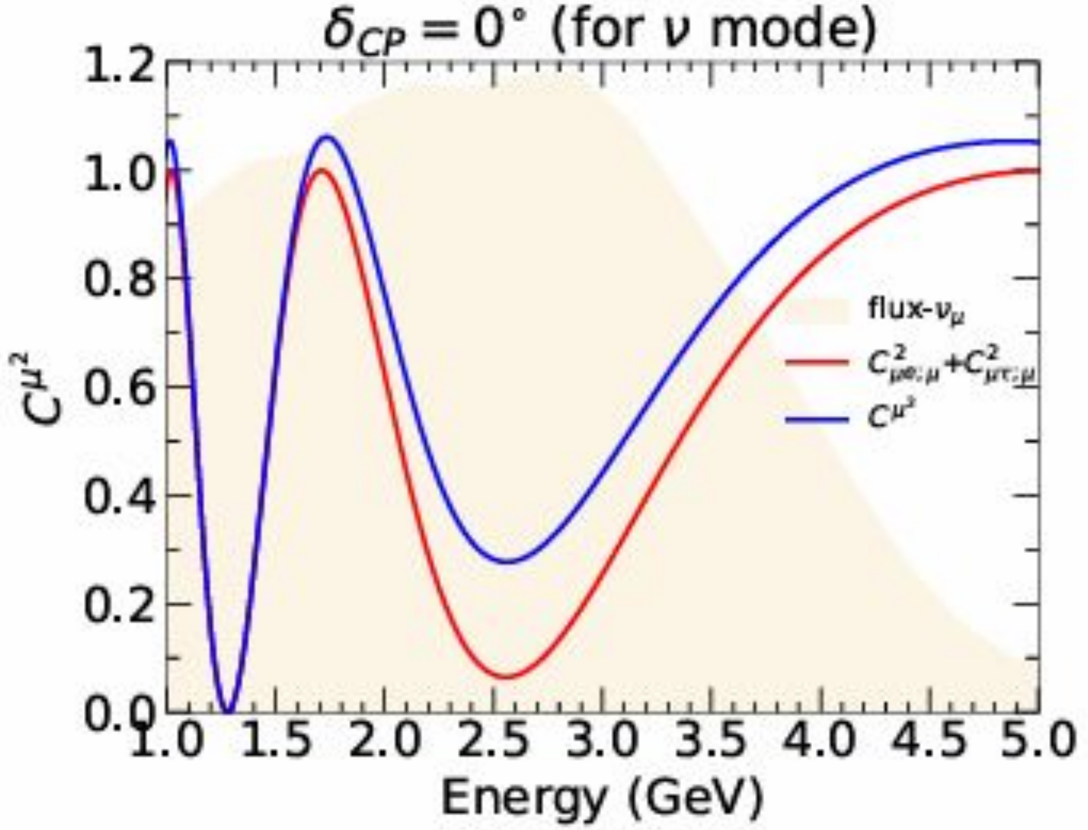
3. The portion of entanglement that A shares with qubit B is not available for sharing with qubit C, highlighting the restricted shareability of entanglement in quantum system.

- <u>Plot of CKW inequality</u>

For neutrino mode



For anti-neutrino mode



▪ Application of the entanglement in measurement :

From an experimental perspective, precise measurements of neutrino oscillation parameters are most effectively conducted in energy windows where one of the three flavor states is either in a pure state or nearly disentangled. Specifically, EW1 provides a pure state for νμ, while EW3 offers a nearly separable state for νe.

EW I 1.27 GeV

EW III 2.50 GeV

Nufit v5.2

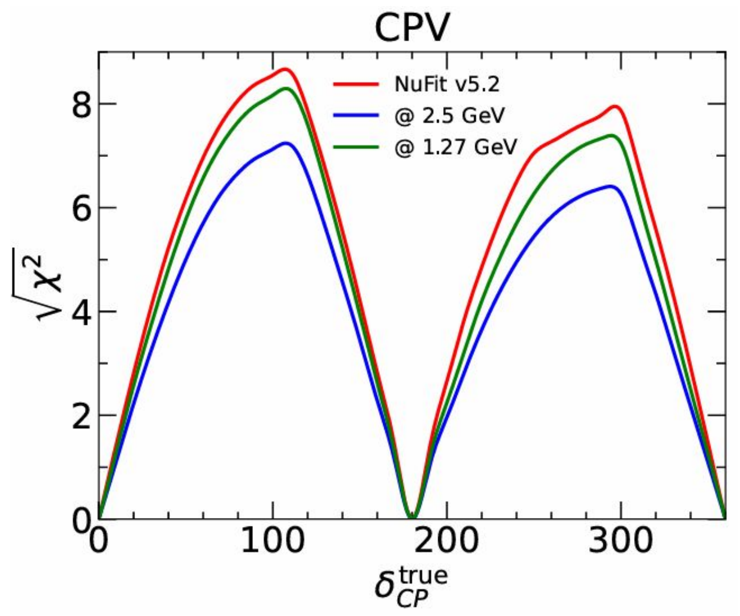
Separable State
- **EW I**

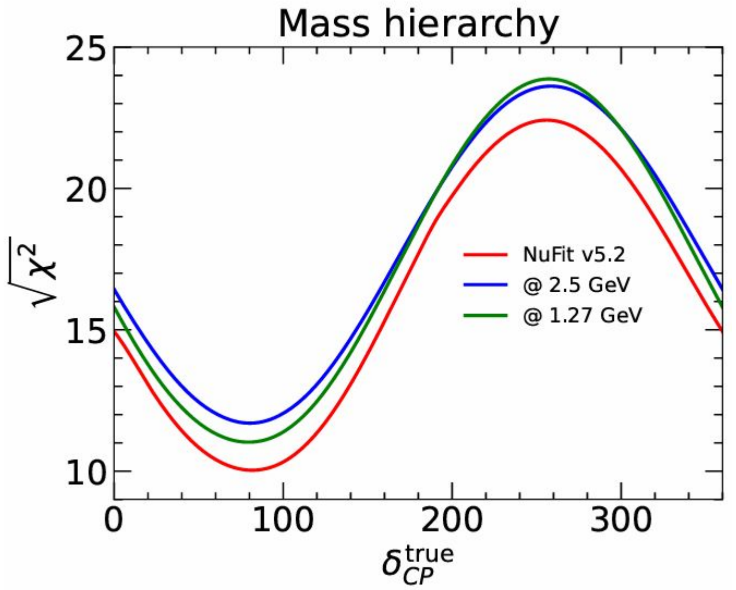
Maximally Entangled State
- **EW II**

Partially Separable State
- **EW III**

Condition						
Nufit v5.2	33.41^{o}	8.58^{o}	42.2^{o}	7.41×10^{-5}	2.507×10^{-3}	232^{o}
@ 2.50 GeV (0.03225)	31.31^{o}	8.43^{o}	45.2^{o}	7.01×10^{-5}	2.487×10^{-5}	270^{o}
@1.27 GeV (0.00314)	35.71^{o}	8.23^{o}	45.2^{o}	7.61×10^{-5}	2.587×10^{-5}	270^{o}

▪ Effect of entanglement in sensitivity analysis





CP sensitivity

- Sensitivity decreases for both global and local minima concurrence points, dropping from 8.5to 8.2 and 7.0.

Mass Hierarchy

- Mass hierarchy sensitivity in absence of quantum entanglement, with “local minima” and “global minima” of the concurrence.
- The results shows that the sensitivity is higher for those energy points where the concurrence is minimized.

Octant Degeneracy

- Given that th23 is 45.2 at the minima of the concurrence the octant sensitivity from these points is negligible.
- The results shows maximal mixing

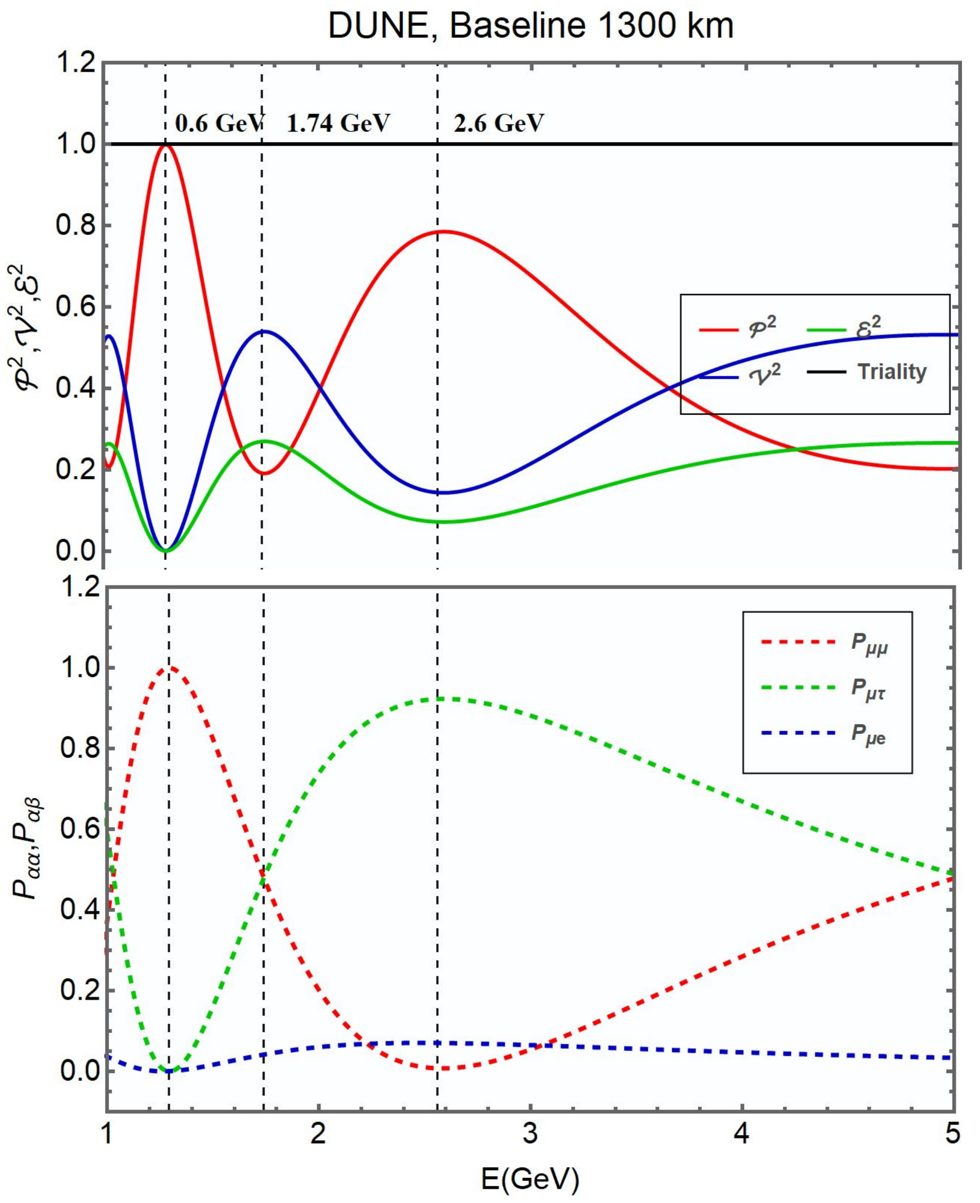
“Quantum Triality in Neutrino Oscillation”

Rajrupa Banerjee, Pratidhwani Swain, Prasanta K. Panigrahi, Sudhanwa Patra

We analyze neutrino oscillations from the perspective of quantum complementarity by extending the familiar wave–particle duality into a exact quantum triality relation among visibility, predictability, and entanglement.

The long-baseline neutrino oscillations realize the triality relation in an energy-dependent way, with T2K probing vacuum-like coherence–entanglement balance and DUNE revealing the decisive role of matter effects in redistributing the quantum information budget.

Quantum Triality in Neutrino Oscillations



- Neutrino oscillations are analyzed using *quantum complementarity*, extending the familiar **wave–particle duality** into a **triality** among *visibility*, *predictability*, and *entanglement*.

- A **density-matrix approach** defines explicit measures for these three quantities in both **two- and three-flavor** systems, revealing an **exact conservation-like relation** during oscillation.

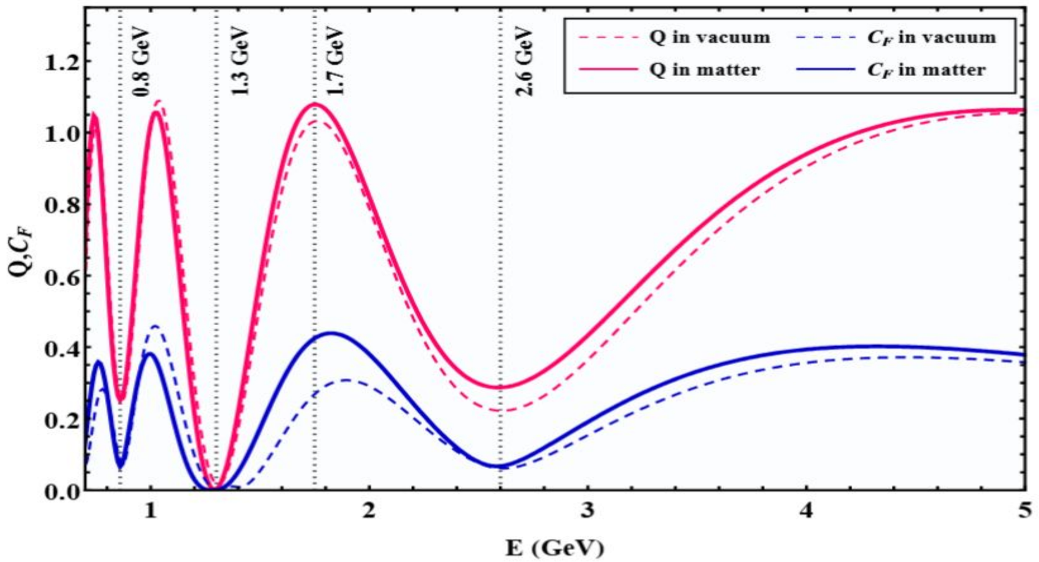
- **Physical Interpretation:**
  - *Visibility* → Strength of interference
  - *Predictability* → Flavor imbalance
  - *Entanglement* → Purity reduction from flavor correlations.

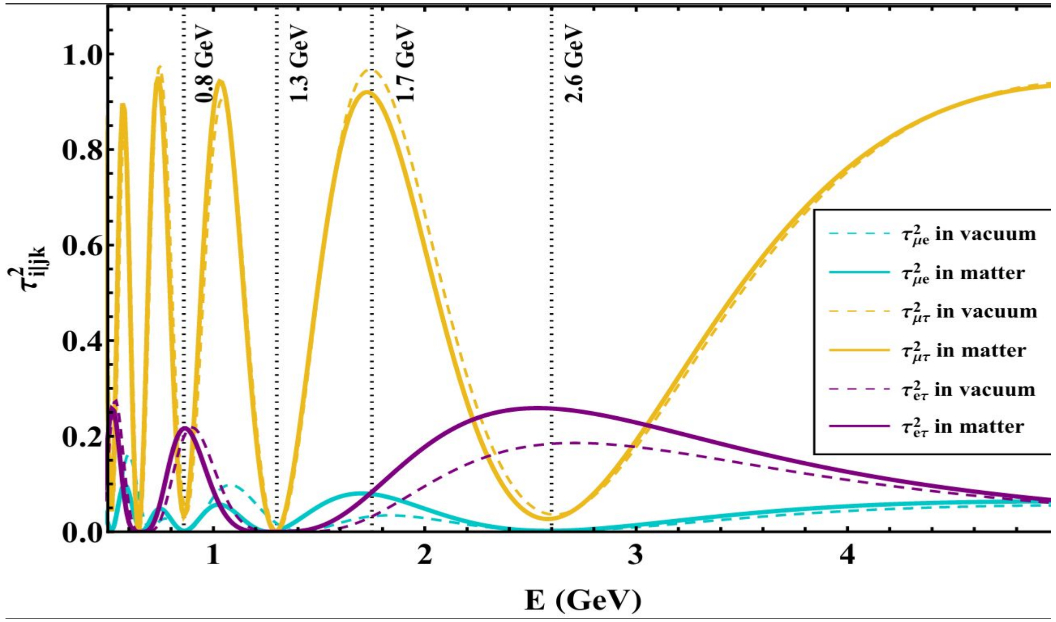
- **Energy dependence:**

  T2K: Vacuum-like coherence–entanglement balance
  DUNE: Matter effects reshape the quantum information flow

"Concurrence Fill and mode distribution of Entanglement in Neutrino Oscillation

Rajrupa Banerjee, Prasanta K. Panigrahi, Hiranmaya Mishra, Sudhanwa Patra





We investigate the detailed measures of multipartite entanglement i.e., three-tangle (or tangle), partial tangle and concurrence fill in the three-flavor neutrino system by expressing these quantum correlations directly in terms of measurable oscillation probabilities.

The tangle quantifies the extent of genuine tripartite entanglement, the partial tangle characterizes entanglement distribution within each partition, and the concurrence fill captures the total sharing of entanglement beyond pairwise correlations.

Entanglement Measures in 3-Flavor Neutrino Oscillation

- **Neutrino oscillations as W-type states:**
  Neutrino oscillations naturally realize a **quantum superposition** among three flavor modes, forming a **W-type single-particle entangled state**.

- We analyze **three key measures**
  - *Tangle (three-tangle):* Quantifies genuine tripartite entanglement.
  - *Partial tangle:* Describes how entanglement is distributed across partitions.
  - *Concurrence:* Represents the overall sharing of entanglement beyond pairwise correlations.

- The oscillation dynamics induce **mode entanglement**, where the general state remains within the **W-class**, marked by a **vanishing tangle**, distinguishing it from a GHZ-type configuration.

- These entanglement measures exhibit **distinct energy-dependent behavior**, offering **experimentally accessible signatures** in **long-baseline experiments** like **DUNE**, providing a **quantum-information perspective** on flavor evolution.

Previous and ongoing work

Concurrence fill and mode distribution of entanglement in neutrino oscillation

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Wave-particle duality and entanglement in neutrino oscillation

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We analyze neutrino oscillations from the perspective of quantum complementarity by extending the familiar wave–particle duality into a full triality relation among visibility, predictability, and en-

Analysis of neutrino oscillation parameters in the light on quantum entanglement

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Numerous neutrino experiments have confirmed the phenomenon of neutrino oscillation, providing direct evidence of the quantum mechanical nature of neutrinos. In this work, we investigate the entanglement properties of neutrino flavor states within the framework of three-flavor neutrino oscillation using two major entanglement measures: entanglement of formation (EOF) and concurrence, utilizing the DUNE experimental setup. Our findings indicate that the maximally entangled state appears between ν_μ and ν_τ whereas, ν_e behaves as a nearly separable state. To further explore the nature of bipartite entanglement, we introduce the concept of the monogamy of entanglement, which allows us to investigate the distinction between genuine three-flavor entanglement and bipartite entanglement. Our analysis confirms that the three-flavor neutrino system forms a bipartite entanglement

Summary:

- ✔ We investigated entanglement using concurrence and EOF, aka "entanglement measures."
- ✔ The maximally entangled state also occurs between muon and tau flavor modes.

- ✔ To investigate the nature of entanglement, we introduced the monogamy of entanglement in terms of CKW inequality
- ✔ CP sensitivity decreases at the local and global minima of concurrence than Nufit value whereas the sensitivity at the Mass Hiarachy sector increases.

- ✔ Neutrino oscillation can be viewed as a dynamic process of entanglement redistribution.
- ✔ Quantum information tools like concurrence, EOF, tangle and concurrence fill provide new insight into neutrino phenomenology.
- ✔ DUNE and future experiments may indirectly probe these quantum features

Verification of entanglement

Nature of entanglement

Application of entanglement

Thank You