

Large relative phase induced by charmonium in baryon-antibaryon production in electron-positron annihilation

Yu. M. Bystritskiy

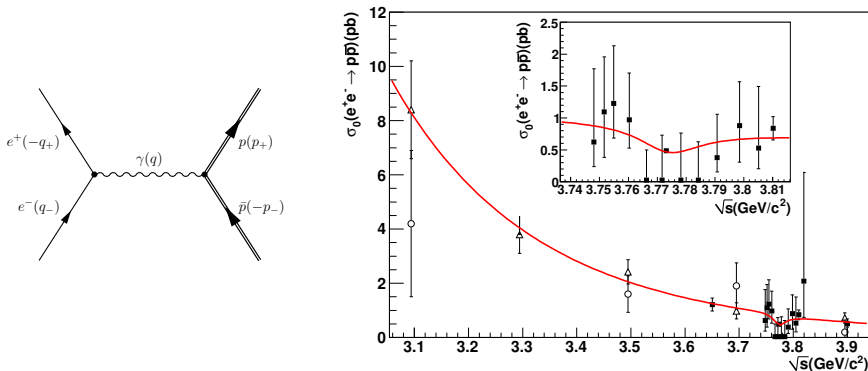
JINR BLTP

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November 11 2025

Charmonium $\psi(3770)$ production at BES

Large statistics of J/ψ , $\psi(2S)$ and $\psi(3770)$ samples have been obtained in recent years by BEPCII/BESIII facility [BESIII Collaboration, M. Ablikim *et al.*, Phys.Lett. **B710**, 594 (2012)]. It provides the possibility to study many decay channels of J/ψ , $\psi(2S)$ and $\psi(3770)$ resonances.

In a profound work BESIII has measured the phase angle ϕ between the continuum and the resonant amplitude [M. Ablikim *et al.* [BESIII Collaboration], Phys. Lett. B **735**, 101 (2014)].



Charmonium $\psi(3770)$ production at BES

Fit parameterization was like

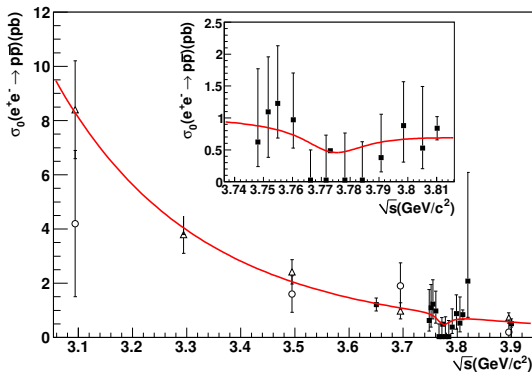
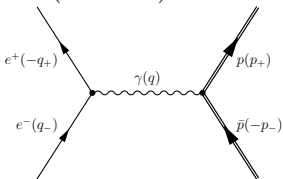
$$\sigma(s) = \left| A_{con} + A_{\psi} e^{i\phi} \right|^2 = \left| \sqrt{\sigma_{con}(s)} + \sqrt{\sigma_{\psi}} \frac{M_{\psi} \Gamma_{\psi}}{s - M_{\psi}^2 + i M_{\psi} \Gamma_{\psi}} e^{i\phi} \right|^2, \quad (1)$$

$$\sigma_{con}(s) = \frac{4\pi\alpha^2\beta}{3s} \left(1 + \frac{2M_p^2}{s} \right) |G(s)|^2, \quad (2)$$

where effective electromagnetic proton form factor:

$$|G(s)| = \frac{C}{s^2 \ln^2(s/\Lambda^2)},$$

with $\Lambda = 0.3$ GeV is the QCD scale parameter and $C = (62.6 \pm 4.1) \text{ GeV}^4$.

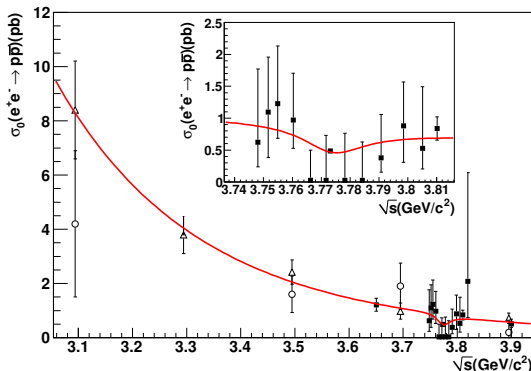
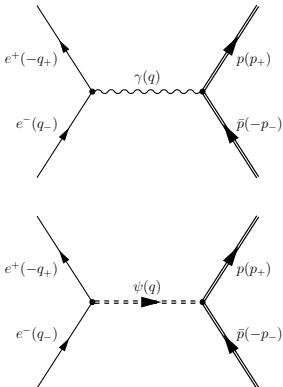


Charmonium $\psi(3770)$ production at BES

$$\sigma(s) = \left| A_{con} + A_{\psi} e^{i\phi} \right|^2 = \left| \sqrt{\sigma_{con}(s)} + \sqrt{\sigma_{\psi}} \frac{M_{\psi} \Gamma_{\psi}}{s - M_{\psi}^2 + i M_{\psi} \Gamma_{\psi}} e^{i\phi} \right|^2, \quad (3)$$

Fitting procedure found two possible solutions:

Solution	σ_{ψ} (pb)	ϕ (°)
(1)	$0.059 \pm 0.032 \pm 0.012$	$255.8 \pm 37.9 \pm 4.8$
(2)	$2.57 \pm 0.12 \pm 0.12$	$266.9 \pm 6.1 \pm 0.9$



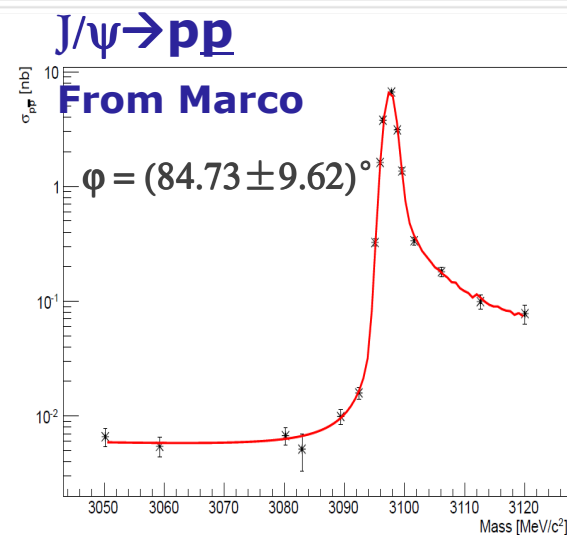
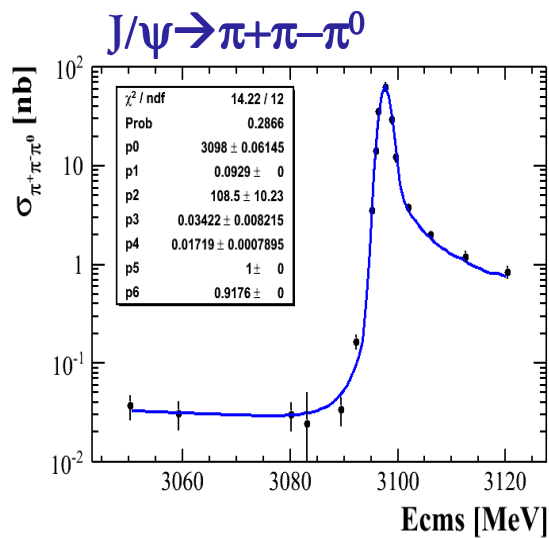
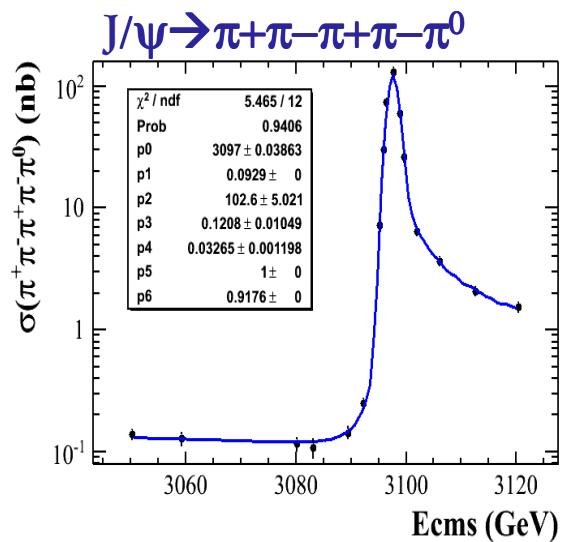
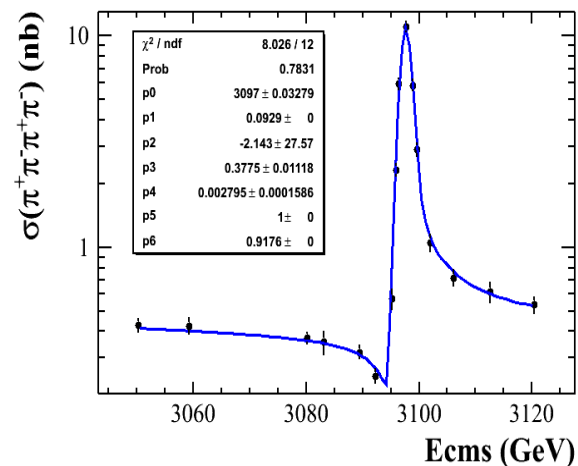
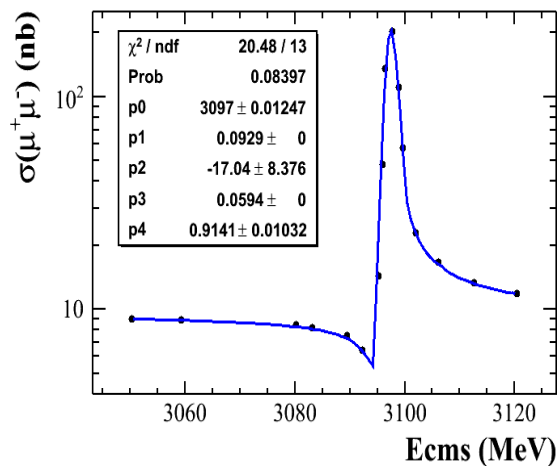
Experimental evidences for $\Psi(3770)$ imaginary strong decay widths

$\Psi''(3770)$:

- ❖ non DDbar (small) \rightarrow through the interference with continuum
- ❖ For a wide resonance Φ from interference at the peak
 $- 2|A_{3g}|/\Gamma_{\text{tot}} \sin \Phi \times \text{continuum}$
- ❖ CLEOc and BESIII: $\Phi \sim -90^\circ$, since continuum sign

decay	continuum	$\Psi''(3770)$	sign	
$\rho \pi$	13.1 ± 2.8	7.4 ± 1.3	-	CLEOc, PRD 73(2006)012002
$\phi \eta$	2.1 ± 1.6	4.5 ± 0.7	+	CLEOc, PRD 73(2006)012002
$\rho \bar{\rho}$	0.74 ± 0.08	0.4 ± 0.02	-	BESIII Y.Liang, Nov (2012)

BESIII J/ Ψ scan



10/09/2014

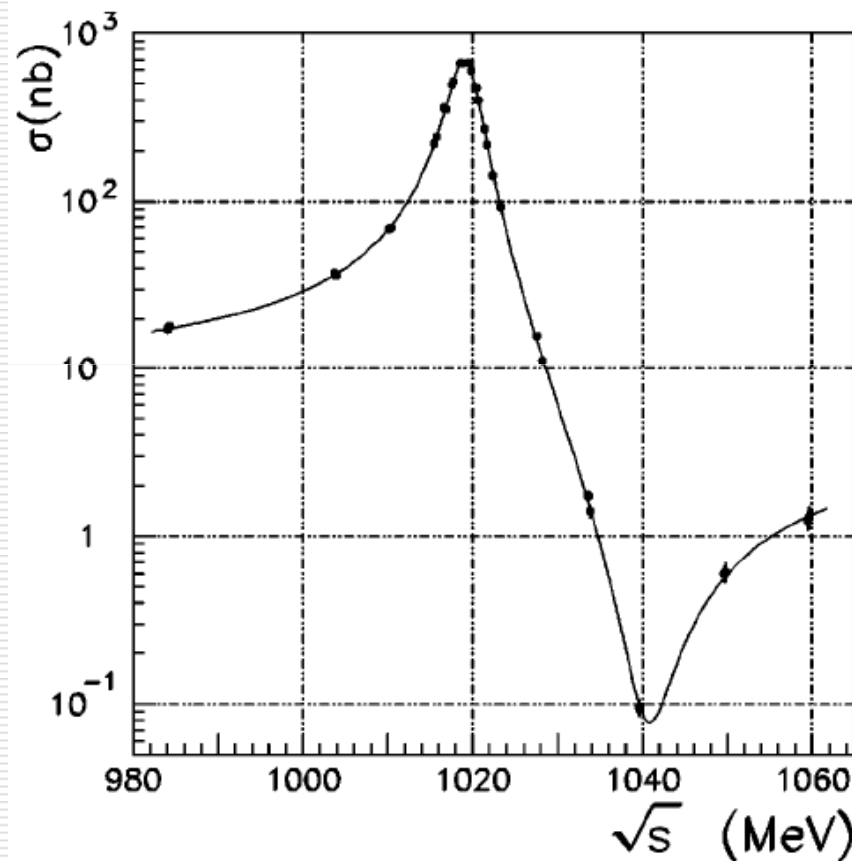
September 10th, 2014, Frascati

R. Baldini

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The interference pattern
is not always the same.

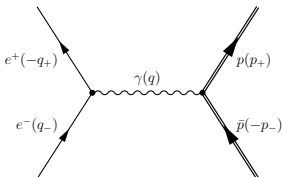
$e^+e^- \rightarrow \pi^+\pi^-\pi^0$ @ *VEPP2M*



$\phi \sim 180^\circ$

Phys. Rev.D 63, 072002

Born approximation



We consider the mechanisms of creation of a $p\bar{p}$ pair in electron–positron collisions which proceeds through virtual photon intermediate state

$$\mathcal{M}_B = \frac{1}{s} J_\mu^{e\bar{e} \rightarrow \gamma}(q) J_{\gamma \rightarrow p\bar{p}}^\mu(q), \quad (4)$$

where lepton $J_\mu^{e\bar{e} \rightarrow \gamma}$ and proton $J_{\gamma \rightarrow p\bar{p}}^\mu$ currents have a form:

$$J_\mu^{e\bar{e} \rightarrow \gamma}(q) = -e [\bar{v}(q_+) \gamma_\mu u(q_-)], \quad (5)$$

$$J_\mu^{\gamma \rightarrow p\bar{p}}(q) = e [\bar{u}(p_+) \Gamma_\mu(q) v(p_-)], \quad (6)$$

where proton electromagnetic vertex $\Gamma_\mu(q)$ is parameterized in a standard way:

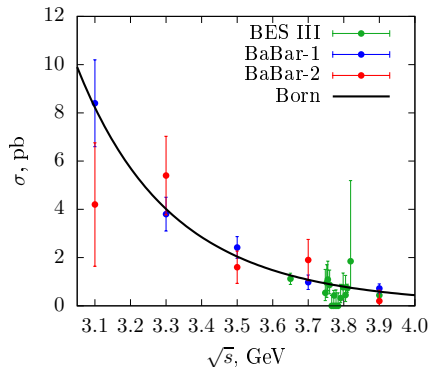
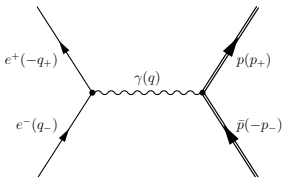
$$\Gamma_\mu(q) = F_1(q^2) \gamma_\mu - \frac{F_2(q^2)}{4M_p} (\gamma_\mu \hat{q} - \hat{q} \gamma_\mu), \quad (7)$$

where form factors $F_{1,2}(q^2)$ are related to electric $G_E(q^2)$ and magnetic $G_M(q^2)$ form factors of the proton. We use effective form factor approach and assume that $|G_E| = |G_M|$ and thus:

$$F_1(s) = |G(s)| = \frac{C}{s^2 \log^2(s/\Lambda^2)}, \quad F_2(s) = 0, \quad (8)$$

where $\Lambda = 0.3 \text{ GeV}$ is the QCD scale parameter and $C = (62.6 \pm 4.1) \text{ GeV}^4$ was fitted in [M. Ablikim *et al.* [BESIII Collaboration], *Phys. Lett. B* **735**, 101 (2014)].

Born approximation

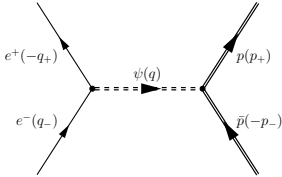


The corresponding contribution to the differential cross section then reads as

$$\frac{d\sigma}{d\cos\theta_p} = \frac{\pi\alpha^2\beta_p}{2s} (2 - \beta_p^2 \sin^2\theta_p) |F_1(s)|^2, \quad (9)$$

where $s = q^2 = (q_+ + q_-)^2 = 4E^2$ is the total invariant energy of the process, E is the electron beam energy in the center-of-mass reference frame, β_p is the proton velocity ($\beta_p^2 = 1 - M_p^2/E^2$). The total cross section then is equal to:

$$\sigma_B(s) = \frac{2\pi\alpha^2}{3s} \beta_p (3 - \beta_p^2) |F_1(s)|^2. \quad (10)$$



The second mechanism describes the conversion of electron–positron pair to $\psi(3770)$ with the subsequent conversion to the proton–antiproton pair. For this aim we put the whole matrix element as

$$\mathcal{M} = \mathcal{M}_B + \mathcal{M}_\psi, \quad (11)$$

where the contribution with $\psi(3770)$ intermediate state is

$$\mathcal{M}_\psi = \frac{g^{\mu\nu} - q^\mu q^\nu / M_\psi^2}{s - M_\psi^2 + i M_\psi \Gamma_\psi} J_\mu^{e\bar{e} \rightarrow \psi}(q) J_\nu^{\psi \rightarrow p\bar{p}}(q). \quad (12)$$

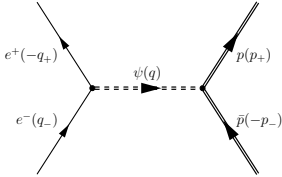
Here we assumed that vertex $\psi \rightarrow e^+e^-$ has the same structure as $\gamma \rightarrow e^+e^-$, i.e.:

$$J_\mu^{e\bar{e} \rightarrow \psi}(q) = g_e [\bar{v}(q_+) \gamma_\mu u(q_-)], \quad (13)$$

and the constant g_e is defined via $\psi \rightarrow e^+e^-$ decay

$$|g_e| = \sqrt{\frac{12\pi\Gamma_{\psi \rightarrow e^+e^-}}{M_\psi}} = 1.6 \cdot 10^{-3}. \quad (14)$$

We should also note that in paper [E.A. Kuraev, Yu.M. Bystritskiy and E. Tomasi-Gustafsson, Nucl. Phys. A **920**, 45 (2013).] it was shown that the vertexes of this type do not generate large imaginary part and thus we can neglect their contribution to the relative phase ϕ .



The second mechanism describes the conversion of electron–positron pair to $\psi(3770)$ with the subsequent conversion to the proton–antiproton pair. For this aim we put the whole matrix element as

$$\mathcal{M} = \mathcal{M}_B + \mathcal{M}_\psi, \quad (15)$$

where the contribution with $\psi(3770)$ intermediate state is

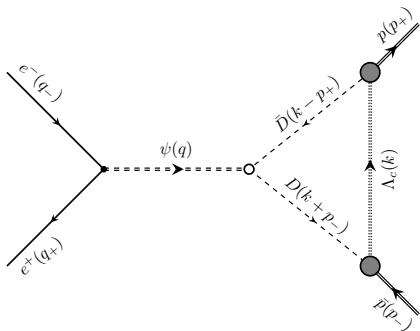
$$\mathcal{M}_\psi = \frac{g^{\mu\nu} - q^\mu q^\nu / M_\psi^2}{s - M_\psi^2 + iM_\psi \Gamma_\psi} J_\mu^{e\bar{e} \rightarrow \psi}(q) J_\nu^{\psi \rightarrow p\bar{p}}(q). \quad (16)$$

Thus we can present the interference contribution to the total cross section in the form:

$$\sigma_{int}(s) = \text{Re} \left(\frac{S_i(s)}{s - M_\psi^2 + iM_\psi \Gamma_\psi} \right), \quad (17)$$

where $S_i(s)$ contains all the dynamics of the transformation of charmonium into proton–antiproton pair and has the following explicit form:

$$S_i(s) = \frac{eg_e\beta_p}{48\pi s} \int_{-1}^1 d\cos\theta_p \sum_{s'} (J_{\gamma \rightarrow p\bar{p}}^\alpha)^* J_\alpha^{\psi \rightarrow p\bar{p}}. \quad (18)$$



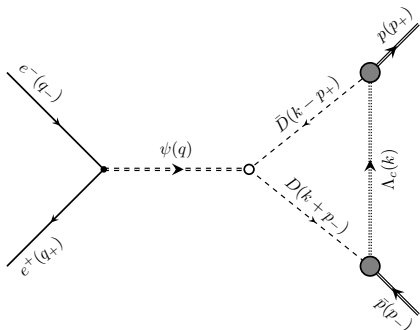
The main contribution seems to be generated by *D*-meson loop mechanism:

$$S_D(s) = \frac{\alpha g_e}{2^4 3\pi^2} \beta_P F_1(s) Z_D(s),$$

$$Z_D(s) = \frac{1}{s} \int \frac{dk}{i\pi^2} G_{\psi D \bar{D}}(s, (k+p_-)^2, (k-p_+)^2) \times \\ \times \frac{SpD(s, k^2)}{(k^2 - M_\Lambda^2) ((k-p_+)^2 - M_D^2) ((k+p_-)^2 - M_D^2)} \times \\ \times G_{\Lambda DP}(k^2, (k-p_+)^2) G_{\Lambda DP}(k^2, (k+p_-)^2),$$

and $SpD(s, k^2)$ is the trace of γ -matrices over the baryon line:

$$SpD(s, k^2) = \text{Sp} \left[(\hat{p}_+ + M_P) \gamma_5 (\hat{k} + M_\Lambda) \gamma_5 (\hat{p}_- - M_P) (\hat{k} - M_P) \right]. \quad (19)$$



The calculation is done using dispersion relation with one subtraction and the imaginary part of Z_D is found by using Cutkosky rule:

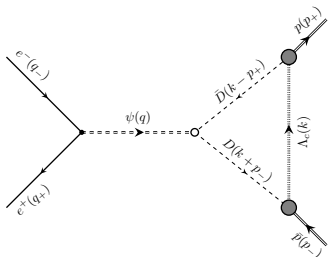
$$\frac{1}{(k+p_-)^2 - M_D^2} \rightarrow -2\pi i \delta((k+p_-)^2 - M_D^2),$$

$$\frac{1}{(k-p_+)^2 - M_D^2} \rightarrow -2\pi i \delta((k-p_+)^2 - M_D^2),$$

which gives the following result for the $\text{Im } Z_D(s)$ which is non-zero above the threshold ($s > 4M_D^2$):

$$\text{Im } Z_D(s) = -\frac{2\pi}{s^{3/2}} G_{\psi D \bar{D}}(s, M_D^2, M_D^2) \times$$

$$\times \int_{C_k^{(1)}}^1 \frac{dC_k}{\sqrt{D_1}} \sum_{i=1,2} \frac{k_{(i)}^2}{k_{(i)}^2 + M_\Lambda^2} \text{Sp} D(s, -k_{(i)}^2) G_{\Lambda D P}^2(-k_{(i)}^2, M_D^2).$$



The dynamical dependence of function $G_{\psi D\bar{D}}(s, M_D^2, M_D^2)$ we propose following to [G.P. Lepage, S.J. Brodsky, Phys. Lett. B 87, 359 \(1979\)](#) in the form:

$$G_{\psi D\bar{D}}(s, M_D^2, M_D^2) = g_{\psi D\bar{D}} \frac{M_\psi^2}{s} \frac{\log(M_\psi^2/\Lambda_D^2)}{\log(s/\Lambda_D^2)},$$

where constant $g_{\psi D\bar{D}}$ is fixed by the $\psi \rightarrow D\bar{D}$ decay width:

$$g_{\psi D\bar{D}} \equiv G_{\psi D\bar{D}}(M_\psi^2, M_D^2, M_D^2) = 4 \sqrt{\frac{3\pi \Gamma_{\psi \rightarrow D\bar{D}}}{M_\psi \beta_D^3}} = 18.4,$$

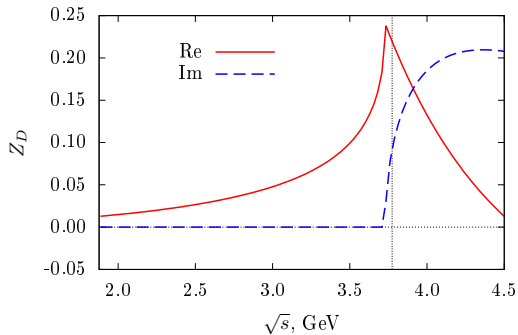
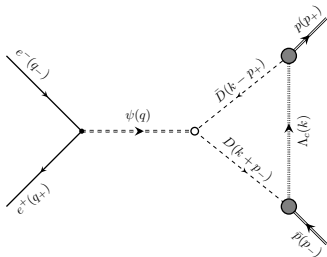
where $\beta_D = \sqrt{1 - 4M_D^2/M_\psi^2}$ is the D -meson velocity in this decay. For ΛDP -vertex with the off-mass-shell D -meson we follow the results of [L. Reinders, H. Rubinstein, S. Yazaki, Phys. Rept. 127, 1 \(1985\)](#):

$$g_D(p^2) = \frac{2M_D^2 f_D}{m_u + m_c} \frac{g_{D N \Lambda}}{p^2 - M_D^2}, \quad (20)$$

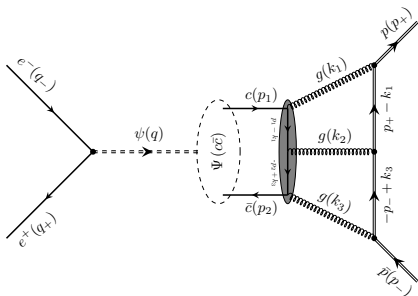
where $f_D \approx 180$ MeV. The constant $g_{D N \Lambda} \approx 6.74$ was estimated in [F. Navarra, M. Nielsen, Phys. Lett. B 443, 285 \(1998\)](#) in scattering regime, i.e. for $p^2 < 0$. Thus in our calculation in we use the following expression:

$$G_{\Lambda DP}(-k_{(i)}^2, M_D^2) = \frac{f_D g_{D N \Lambda}}{m_u + m_c}. \quad (21)$$

D-meson loop mechanism



Three gluon mechanism



and

$$Z_{3g}(s) = \frac{4}{\pi^5 s} \int \frac{dk_1}{k_1^2} \frac{dk_2}{k_2^2} \frac{dk_3}{k_3^2} \frac{Sp3g \delta(q - k_1 - k_2 - k_3)}{((p_+ - k_1)^2 - M_p^2)((p_- - k_3)^2 - M_p^2)}, \quad (22)$$

where the quantity $Sp3g$ is the product of traces over proton and c -quark lines:

$$Sp3g = \text{Sp} \left[\hat{Q}_{\alpha\beta\gamma} (\hat{p}_1 + m_c) \gamma^\mu (\hat{p}_2 - m_c) \right] \times \\ \times \text{Sp} \left[(\hat{p}_+ + M_p) \gamma^\alpha (\hat{p}_+ - \hat{k}_1 + M_p) \gamma^\beta (-\hat{p}_- + \hat{k}_3 + M_p) \gamma^\gamma (\hat{p}_- - M_p) \gamma_\mu \right],$$

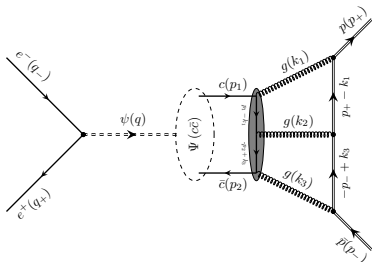
In paper A.I. Ahmadv, Yu.M. Bystritskiy, E.A. Kuraev and P. Wang, Nucl. Phys. B **888**, 271-283 (2014) it was shown that important possible contribution to relative phase comes from OZI-violated three gluon mechanism:

$$S_{3g}(s) = \frac{\alpha \alpha_s^3}{2^3 3} g_e g_{col} \phi \beta_p F_1(s) G_\psi(s) Z_{3g}(s),$$

where quantity ϕ is related with the charmonium wave function:

$$\phi = \frac{|\Psi(\mathbf{r} = \mathbf{0})|}{M_\psi^{3/2}} = \frac{\alpha_s^{3/2}}{3\sqrt{3}\pi},$$

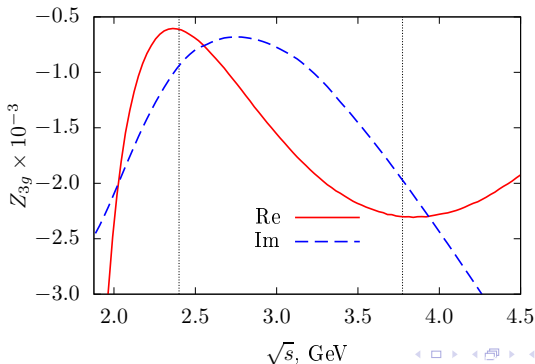
Three gluon mechanism



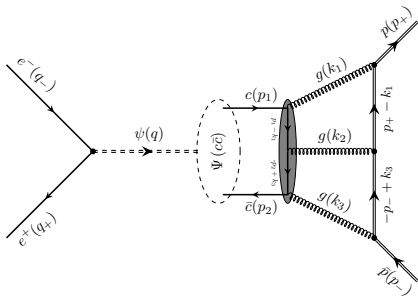
Again we use dispersion relation with one subtraction and the imaginary part of Z_{3g} is found by using Cutkosky rule:

$$\frac{1}{k_1^2 + i0} \frac{1}{k_2^2 + i0} \frac{1}{k_3^2 + i0} \rightarrow (-2\pi i)^3 \delta(k_1^2) \delta(k_2^2) \delta(k_3^2),$$

which gives the following result for the $\text{Im } Z_{3g}(s)$:



Three gluon mechanism



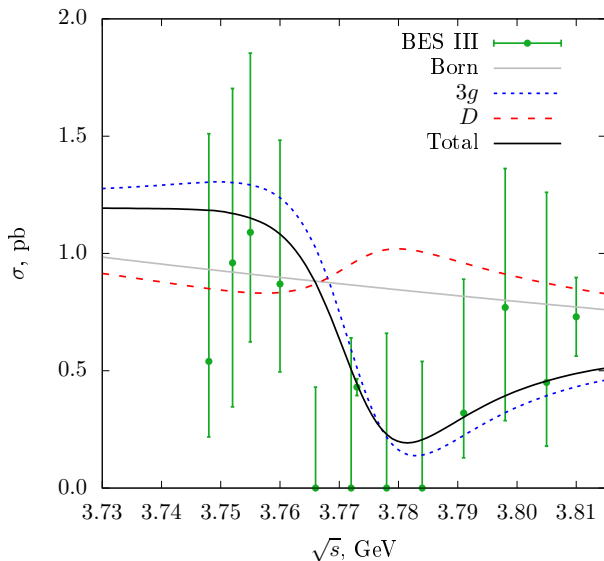
Let us remind that interference contains some number of form factors:

$$S_{3g}(s) = \frac{\alpha_s^3}{2^3 3} g_e g_{col} \phi \beta_p F_1(s) G_\psi(s) Z_{3g}(s),$$

where $F_1(s)$ is the electromagnetic form factor of the proton in time-like region while $G_\psi(s)$ is some similar form factor which describes the transition of three gluons into proton-antiproton final state:

$$|G_\psi(s)| = \frac{C_\psi}{s^2 \log^2(s/\Lambda^2)}.$$

Compare to data of $e^+e^- \rightarrow p\bar{p}$ process



Fitting the total curve with BES data gives us:

$$|G_\psi(s)| = \frac{C_\psi}{s^2 \log^2(s/\Lambda^2)},$$

with

$$C_\psi = (45 \pm 9) \text{ GeV}^4.$$

Compare to data of $e^+e^- \rightarrow p\bar{p}$ process

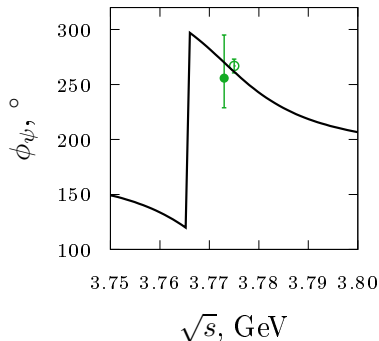
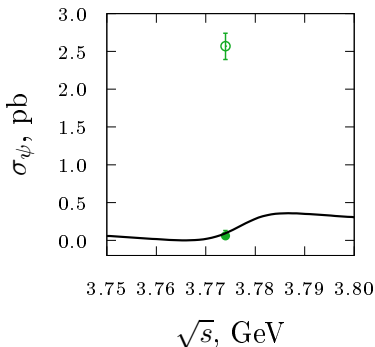
Now we recall the fittings of BES III collaboration from **M. Ablikim *et al.* [BESIII Collaboration], Phys. Lett. B **735**, 101 (2014)** where they used empirical formula:

$$\sigma(s) = \left| A_{con} + A_\psi e^{i\phi} \right|^2 = \left| \sqrt{\sigma_{con}(s)} + \sqrt{\sigma_\psi} \frac{M_\psi \Gamma_\psi}{s - M_\psi^2 + iM_\psi \Gamma_\psi} e^{i\phi_\psi} \right|^2, \quad (23)$$

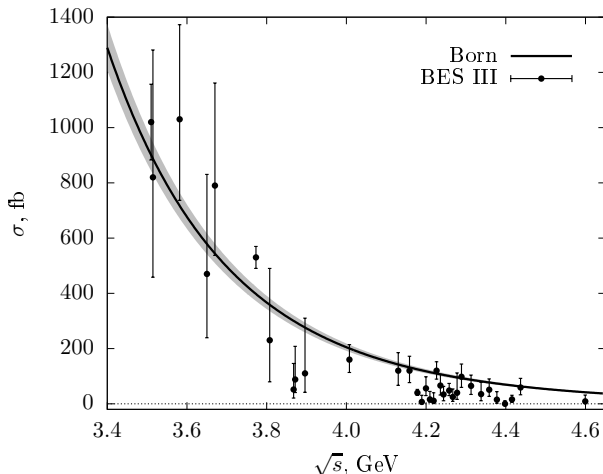
and obtained two possible solutions:

Solution	σ_ψ (pb)	ϕ_ψ (°)
(1)	$0.059 \pm 0.032 \pm 0.012$	$255.8 \pm 37.9 \pm 4.8$
(2)	$2.57 \pm 0.12 \pm 0.12$	$266.9 \pm 6.1 \pm 0.9$

So one can see that our fit ($\sigma_\psi = 0.075$ pb, $\phi_\psi = 270^\circ$) prefers one of these solutions:



The process $e^+e^- \rightarrow \Lambda\bar{\Lambda}$ at BES III



The data for the cross section is taken from [BESIII, M. Ablikim et al., Phys. Rev. D **104**, L091104 \(2021\)](#).

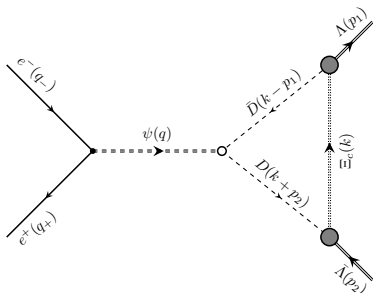
We fit this data with the standard Born cross section with the electromagnetic formfactor of Λ -baryon:

$$|G_{\Lambda}(s)| = \frac{C_{\Lambda}}{s^2 \log^2(s/\Lambda^2)},$$

and get

$$C_{\Lambda} = (43.1 \pm 1.4) \text{ GeV}^4.$$

The process $e^+e^- \rightarrow \Lambda\bar{\Lambda}$: Two mechanisms

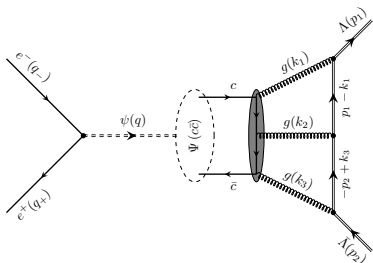


Here we use the same form factor:

$$G_{\Lambda D\Xi}(k^2, M_D^2) = \frac{f_D g_{\Lambda D\Xi}}{m_u + m_c}, \quad k^2 < 0,$$

where

$$g_{\Lambda D\Xi} \approx g_{D\Lambda\Xi} = 6.7 \pm 2.1.$$



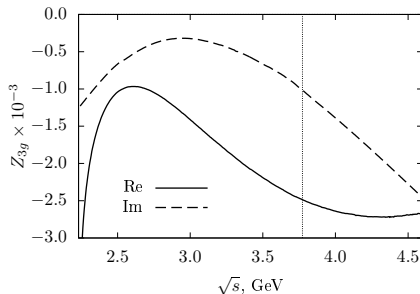
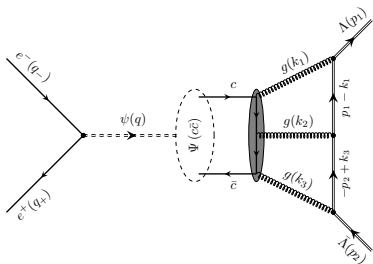
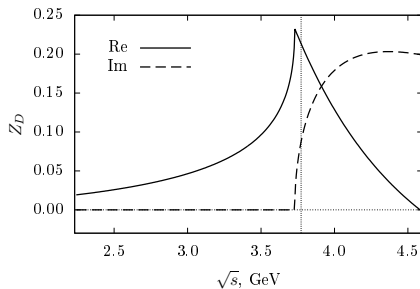
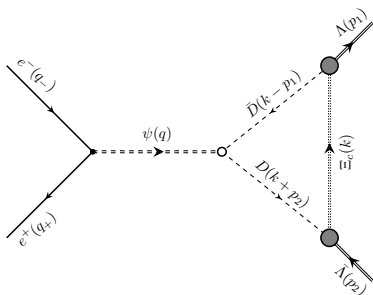
The factor of three gluon transition into $\Lambda\bar{\Lambda}$ has the form:

$$|G_\psi(s)| = \frac{C_\psi}{s^2 \log^2(s/\Lambda^2)},$$

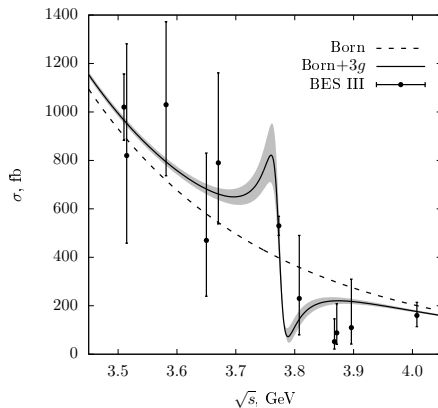
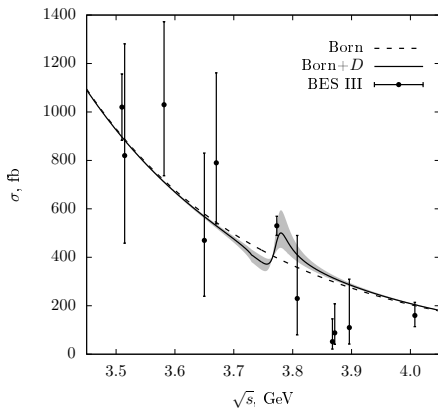
and the constant C_ψ here is the same as in the case of proton-antiproton production since gluons do not feel the flavour of the quarks in the final baryons:

$$C_\psi = (45 \pm 9) \text{ GeV}^4.$$

The process $e^+e^- \rightarrow \Lambda\bar{\Lambda}$: Two mechanisms

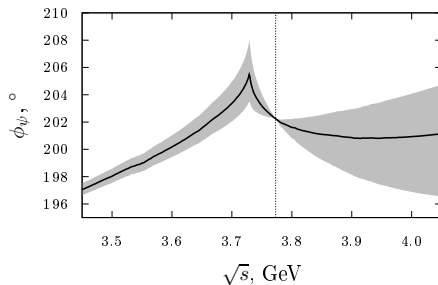
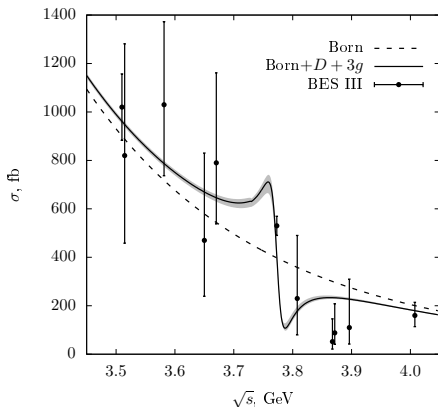


The process $e^+e^- \rightarrow \Lambda\bar{\Lambda}$: Comparison to data



The data are from BESIII, M. Ablikim *et al.*, Phys. Rev. D **104**, L091104 (2021).

The process $e^+e^- \rightarrow \Lambda\bar{\Lambda}$: Comparison to data



The data are from BESIII, M. Ablikim *et al.*, Phys. Rev. D **104**, L091104 (2021).

Large phase as systematic result

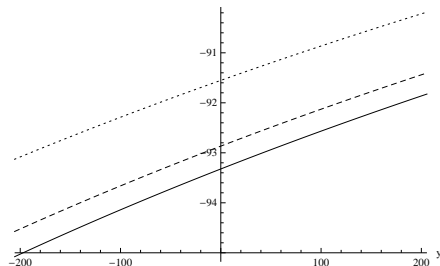
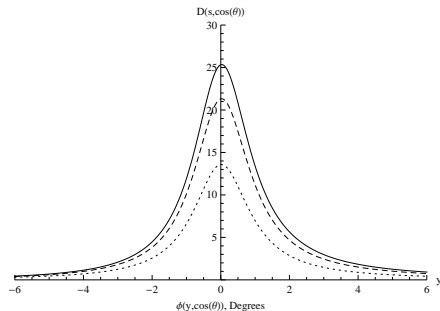
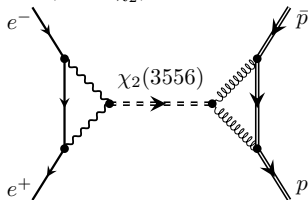
The large phase was obtained earlier in
 E.A. Kuraev, Yu.M. Bystritskiy and E. Tomasi-
 Gustafsson, Nucl. Phys. A **920**, 45 (2013):

$$\mathcal{A}_\chi = \frac{d\sigma(c) - d\sigma(-c)}{d\sigma(c) + d\sigma(-c)} = \mathcal{A}_0 D,$$

$$\mathcal{A}_0 = \frac{5\alpha_s}{864\pi} (M_p^2 g_\gamma g_g) \left(\ln \frac{M_{\chi_2}^2}{m_e^2} - i\pi \right),$$

$$D = \frac{y H_{\text{odd}} + G_{\text{odd}}}{y^2 + 1} \frac{1}{2 - \beta^2(1 - c^2)},$$

where $y = (s - M_{\chi_2}^2) / M_{\chi_2} \Gamma_{\chi_2}$.

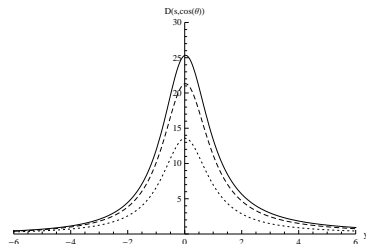


Charmonium $\chi_2(3556)$ contribution:

$$\mathcal{A}_\chi = \frac{d\sigma(c) - d\sigma(-c)}{d\sigma(c) + d\sigma(-c)} = \mathcal{A}_0 D = (4.88 \pm 0.18) \cdot 10^{-4},$$

$$\mathcal{A}_0 = \frac{5\alpha_s}{864\pi} (M^2 g_\gamma g_g) \ln \frac{M_{\chi_2}^2}{m_e^2} = (1.95 \pm 0.07) \cdot 10^{-5},$$

$$D = \frac{y H_{\text{odd}} + G_{\text{odd}}}{y^2 + 1} \frac{1}{2 - \beta^2(1 - c^2)}, \quad y = \frac{s - M_{\chi_2}^2}{M_{\chi_2} \Gamma_{\chi_2}}.$$

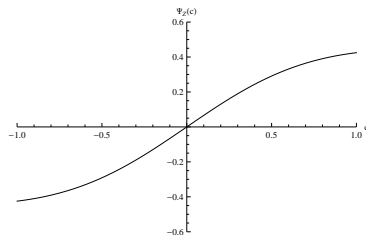


Contribution of Z -boson:

$$\mathcal{A}_Z = \mathcal{A}_0^Z \Psi_Z(c) = (8.776 \pm 0.008) \times 10^{-4},$$

$$\mathcal{A}_0^Z = \frac{s}{M_Z^2 \sin^2(2\theta_W)} \approx (2.194 \pm 0.002) \times 10^{-3},$$

where θ_W is the Weinberg angle.



Forward-backward asymmetry

Contributions to forward-backward asymmetries:

$$\mathcal{A}_\chi = (4.88 \pm 0.18) \cdot 10^{-4},$$

$$\mathcal{A}_Z = (8.776 \pm 0.008) \times 10^{-4}.$$

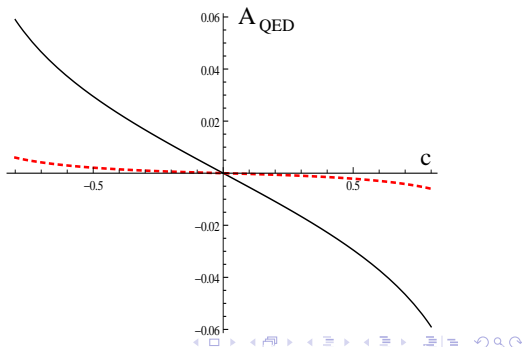
Contributions from pure electrodynamics:

E.A. Kuraev, G.V. Meledin, Nucl.Phys.B 122 (1977) 485-492,

A.I. Ahmadv, V.V. Bytev, E.A. Kuraev, E. Tomasi-Gustafsson, Phys.Rev.D 82 (2010) 094016

Red-dashed line is Born-box interference.

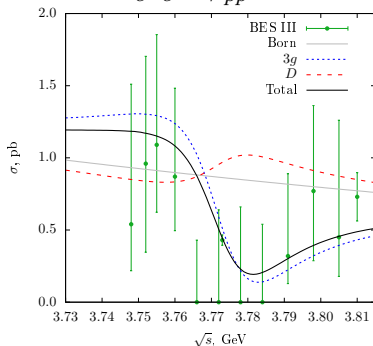
Black solid line is soft and hard real photon emission.



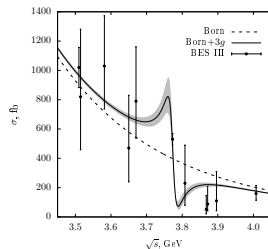
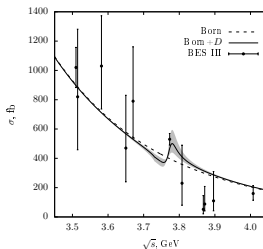
Propositions for Experiment

First Proposition for Experiment: Shoulders are important

$$e^+e^- \rightarrow p\bar{p}$$

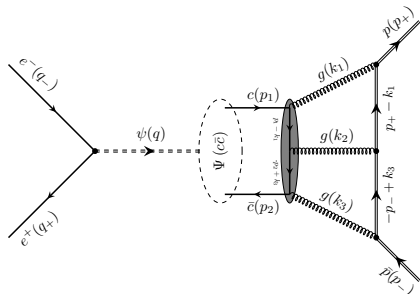


$$e^+e^- \rightarrow \Lambda\bar{\Lambda}$$



The data are from BESIII: M. Ablikim *et al.*, Phys. Lett. B **735**, 101 (2014),
M. Ablikim *et al.*, Phys. Rev. D **104**, L091104 (2021).

Second Proposition for Experiment: $\alpha_s(M_c)$ issue



We note that our calculation of **three gluon mechanism**

$$S_{3g}(s) = \frac{\alpha \alpha_s^3}{2^3 3} g_e g_{col} \phi \beta_D F_D(s) G_\psi(s) Z_{3g}(s),$$

contains the QCD coupling constant α_s at charmonium scale (i.e. at $s \sim M_c^2$) in rather high degree and thus it **is very sensitive to its values**.

We use the value $\alpha_s(M_c) = 0.28$ which is expected by the QCD evolution of α_s from the b -quark scale to the c -quark scale.

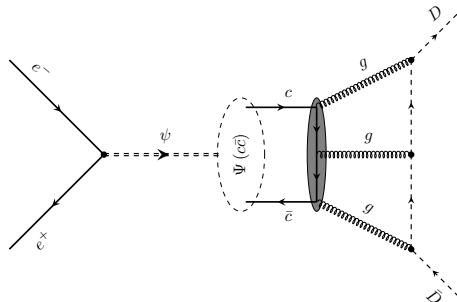
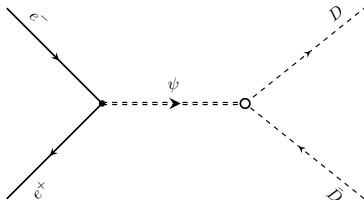
We should note that this value differs from the one for J/ψ charmonium for which one must use much smaller value of parameter $\alpha_s(M_c) = 0.19$ (see **H. Chiang, J. Hufner, H. Pirner, Phys. Lett. B 324, 482 (1994)**).

Thus assuming that C_ψ weakly depends on energy and measuring effective parameter:

$$C_\psi \alpha_s^3, \quad (24)$$

around different charmonia J/ψ , $\chi_{c2}(3556)$, $\psi(2S)$, $\psi(3770)$ we can extract $\alpha_s(M_c)$.

Third Proposition for Experiment: D -meson pair production



Conclusions

- Three gluon mechanism of charmonium $\psi(3770)$ transition into proton–antiproton pair gives dominant contribution and may explain cross section behavior in the vicinity of the resonance.
- For $\psi(3770)$ transition into $\Lambda\bar{\Lambda}$ pair both mechanisms contribute noticeably, but still three gluon one seems to be responsible for the "shoulders" around the charmonium peak.
- The large phase between Born and charmonium contributions is mostly generated by three gluon mechanism.

Publications

- E.A. Kuraev, Yu.M. Bystritskiy and E. Tomasi-Gustafsson, Nucl. Phys. A **920**, 45 (2013).
- A.I. Ahmadov, Yu.M. Bystritskiy, E.A. Kuraev and P. Wang, Nucl. Phys. B **888**, 271-283 (2014).
- Yu.M. Bystritskiy, Phys. Rev. D **103**, no.11, 116029 (2021).
- Yu.M. Bystritskiy and A.I. Ahmadov, Phys. Rev. D **105**, no.11, 116012 (2022).

The QCD radiative corrections to three gluon-mechanism of charmonium decay were studied long ago in paper **P.B. Mackenzie, G.P. Lepage, Phys.Rev.Lett. 47 (1981) 1244**. The calculation was done for Υ -meson decay, giving the following form of corrected decay width:

$$\Gamma_g = \Gamma_g^{(0)} \left(1 + \frac{\alpha_s(M)}{\pi} \left(-19.4 + \frac{3}{2} \beta_0 \left(1.161 + \ln \left(\frac{2M}{M_\Upsilon} \right) \right) \right) + O \left(\frac{v^2}{c^2}, \frac{\alpha_s^2}{\pi^2} s \right) \right)_{\overline{MS}}, \quad (25)$$

where $\beta_0 = 11 - (2/3)n_f$ and $M \sim M_\Upsilon/2$, thus giving $\alpha_s(M) \approx 0.158 \pm 0.01$.

This entire calculation is applicable for charmonium decay as well. The correction for the branching is:

$$\frac{\Gamma_g}{\Gamma_{\mu\mu}} = \frac{10(\pi^2 - 9)}{81\pi e_q^2} \frac{\alpha_s^3(M)}{\alpha^2} \left(1 + C \left\langle \frac{v^2}{c^2} \right\rangle \right), \quad (26)$$

where $\langle v^2/c^2 \rangle \approx 0.23$ within potential model and $C = -2.6$. Thus QCD corrections are rather big: $C \langle v^2/c^2 \rangle \approx -0.598$.

But in any case our expression for the amplitude is proportional to:

$$C_\psi \alpha_s^3, \quad (27)$$

and C_ψ is fitted to the $p\bar{p}$ -data. Inclusion of QCD correction will just rescale the constant C_ψ leaving the conclusions about the phase untouched.

Backup slide: The dispersion relation

The dispersion relations we use can be obtained from the standard dispersion relation written over the variable s with subtraction at the point $s = 0$:

$$\operatorname{Re} Z_i(s) = \operatorname{Re} Z_i(0) + \frac{s}{\pi} \mathcal{P} \int_{s_{\min}}^{\infty} \frac{ds_1}{s_1(s_1 - s)} \operatorname{Im} Z_i(s_1), \quad (28)$$

where s_{\min} is the minimal threshold at which $\operatorname{Im} Z_i(s_1)$ becomes non-zero. The subtraction constant $\operatorname{Re} Z_i(0)$ vanishes (i.e. $\operatorname{Re} Z_i(0) = 0$) since there are no open charm in the proton and thus in the Compton limit the vertex $\psi \rightarrow p\bar{p}$ must vanish.

Next we move to the β_p :

$$\beta_1 = \sqrt{1 - \frac{4M_p^2}{s_1}}, \quad \rightarrow \quad ds_1 = \frac{8M_p^2\beta_1}{(1 - \beta_1^2)^2} d\beta_1,$$

and get the final form of dispersion relation:

$$\operatorname{Re} Z_i(\beta) = \frac{1}{\pi} \left\{ \operatorname{Im} Z_i(\beta) \log \left| \frac{1 - \beta^2}{\beta_{\min}^2 - \beta^2} \right| + \int_{\beta_{\min}}^1 \frac{2\beta_1 d\beta_1}{\beta_1^2 - \beta^2} [\operatorname{Im} Z_i(\beta_1) - \operatorname{Im} Z_i(\beta)] \right\}. \quad (29)$$

We note that for the D -meson loop contribution the imaginary part $\operatorname{Im} Z_D(\beta)$ is non-zero above threshold ($s > 4M_D^2$), thus the lower limit of integration is $\beta_{\min} = \sqrt{1 - M_p^2/M_D^2}$. For the three gluon contribution the threshold for the imaginary part $\operatorname{Im} Z_{3g}(\beta)$ coincides with the threshold of the reaction, i.e. $s_{\min} = 4M_p^2$ and thus lower limit of integration is $\beta_{\min} = 0$.