

Implementation of final-state lepton polarization for quasielastic and 2p2h processes in GENIE

I. D. Kakorin & V. A. Naumov

Joint Institute for Nuclear Research, Bogoliubov Laboratory of Theoretical Physics

The 2nd India—JINR Workshop on Particle, Nuclear, Neutrino Physics and Astrophysics
National Institute of Science Education and Research (NISER), Bhubaneswar, India

November 10–12, 2025



Why is lepton polarization interesting?

- The polarization of the final lepton (especially τ) produced in the νN and νA CC interactions is an important characteristic, the knowledge of which is necessary in the current and future experiments with accelerator and atmospheric neutrinos, like
 - IceCube-Gen2/DeepCore/PINGU,
 - Baikal GVD,
 - KM3NeT/ORCA/P2O,
 - TRIDENT,
 - P-ONE,
 - JUNO,
 - DUNE,
 - Super-Kamiokande, Hyper-Kamiokande,...
- The τ leptons at relatively low energies ($\lesssim 10$ GeV) typically cannot be detected directly because their tracks are too short. Instead, they are identified through the kinematics of their decay products, whose angular and energy distributions depend on the τ -lepton polarization.
 \Rightarrow The latter must be predicted within some theoretical framework (model).
- The lepton polarization vector $\mathcal{P} = (\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3)$ is a strongly model-dependent quantity, so much so that small “inaccuracies” in a particular model for νA interactions, unnoticeable in the predicted exclusive or inclusive cross sections, can lead to unphysical values of perpendicular ($\mathcal{P}_P = \mathcal{P}_1$) or longitudinal ($\mathcal{P}_L = \mathcal{P}_3$) polarization.
- Non-zero transverse polarization, $\mathcal{P}_T = \mathcal{P}_2$, is excluded in the Standard Model due to the absence of scalar and tensor interactions in it (see below).

What is GENIE?

GENIE is the world's leading neutrino event generator, bridging theory and experiment in modern neutrino physics. It underpins data interpretation and exploitation across major experiments.

The international GENIE collaboration plays the leading role in

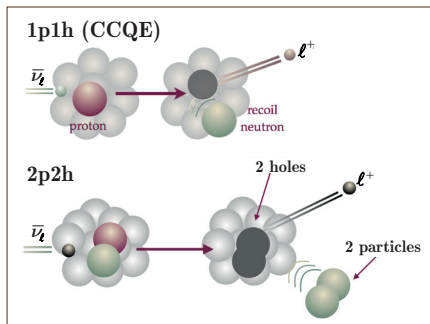
- development of a modern event generation framework, including experimental interfaces and analysis-related tools in support of neutrino experiments,
- validated and efficient implementation of a constellation of alternative physics models within a common platform,
- development and characterisation of novel and predictive comprehensive physics models including all processes relevant from MeV to PeV energy scales,
- development of an advanced global analysis of neutrino scattering data for model tuning and data-driven model uncertainty evaluation.



The collaboration maintains a suite of well-known software products.

- Numerous representatives of the experimental collaborations mentioned above have requested that GENIE include an option to calculate the lepton polarization vector for all interaction models used in the generator.
- In this report, we outline the work carried out to implement the lepton polarization calculations for Charged Current Quasi-Elastic (**CCQE** or **1p1h**) neutrino–nucleus interactions, as well as for two-nucleon emission (**2p2h**) processes induced by meson-exchange currents (MEC) and related mechanisms.

The processes under discussion



Artist's (naive) illustration¹ of the standard charged-current process ("1p1h" or "CCQE"), where an antineutrino $\bar{\nu}_\ell$ interacts with a single proton and produces a recoil neutron and lepton ℓ^+ (e^+ , μ^+ , or τ^+) [top], and the "2p2h" process, where $\bar{\nu}_\ell$ interacts with an np pair in the nucleus, producing a final state of two nucleons and lepton ℓ^+ [bottom]. Similar processes occur in neutrino-induced CC interactions.

¹Figure adapted from M. Sajjad Athar et al., "Status and perspectives of neutrino physics," Prog. Part. Nucl. Phys. **124** (2022) 103947, e-Print: 2111.07586 [hep-ph]. Credit: Cheryl Patrick (Northwestern University).

Polarization density matrix (PDM) formalism

The production of *polarized* leptons in CC (anti)neutrino scattering on an *unpolarized* nucleon or nuclear target can be described using a *polarization density matrix* (PDM). The generic form of PDM, ρ , for arbitrary charged-current reaction on nucleon/nuclear target

$$\nu_\ell^{(-)}(k) + N(p) \rightarrow \ell^\mp(k') + X(p')$$

(where $k' \equiv p_\ell = (E_\ell, \mathbf{p}_\ell)$, $p' = \{p_i\}$, and $p_i = (E_i, \mathbf{p}_i)$) is defined by

$$d\Sigma = \left\| d\sigma_{\lambda\lambda'} \right\| \equiv \rho d\sigma, \quad d\sigma = d\sigma_{++} + d\sigma_{--},$$

where

$$d\sigma_{\lambda\lambda'} = \frac{1}{8(kp)} (2\pi)^4 \delta \left(k + p - k' - \sum_{i=1}^n p_i \right) \overline{\mathcal{M}_\lambda \mathcal{M}_{\lambda'}^*} \frac{d\mathbf{p}_\ell}{(2\pi)^3 2E_\ell} \prod_{i=1}^n \frac{d\mathbf{p}_i}{(2\pi)^3 2E_i}.$$

The matrix elements for the final-lepton helicity λ are

$$\mathcal{M}_\lambda \propto \begin{cases} j_\lambda^\alpha(k, k') J_\alpha(p, p') & \text{for neutrino,} \\ \bar{j}_\lambda^\alpha(k, k') \bar{J}_\alpha(p, p') & \text{for antineutrino,} \end{cases}$$

where the SM weak leptonic currents with the fixed lepton helicities, j_λ and \bar{j}_λ , are well known:

$$j_{\lambda}^{\alpha}(k, k') = \langle k', \lambda | \hat{j}_{\lambda}^{\dagger} | k \rangle = \bar{u}_{\lambda}(k') \gamma^{\alpha} \left(\frac{1 - \gamma_5}{2} \right) u(k)$$

$$\bar{j}_\lambda^\alpha(k, k') = \langle k', \lambda | \hat{j}_\alpha | k \rangle = \bar{v}(k) \gamma^\alpha \left(\frac{1 - \gamma^5}{2} \right) v_\lambda(k'),$$

while the hadronic currents, J and \bar{J} , are the subject of study of a specific model.

Lepton polarization vector

The lepton polarization vector $\mathcal{P} = (\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3)$ is defined through the *polarization density matrix*²

$$\frac{d^2\Sigma}{dE_\ell d\cos\theta} \equiv \left\| \frac{d^2\sigma_{\lambda\lambda'}}{dE_\ell d\cos\theta} \right\| = \frac{1}{2} (1 + \sigma\mathcal{P}) \frac{d^2\sigma}{dE_\ell d\cos\theta}. \quad (1)$$

Here $d^2\sigma/dE_\ell d\cos\theta$ is the differential cross section for an *unpolarized* lepton production in νA (νN) collisions.

According to Eq. (1), the *perpendicular* (\mathcal{P}_1), *transverse* (\mathcal{P}_2), and *longitudinal* (\mathcal{P}_3) components of the polarization vector are given by

$$\begin{aligned} \mathcal{P}_1 \equiv \mathcal{P}_P &= \rho_{+-} + \rho_{-+} = \frac{d^2\sigma_{+-} + d^2\sigma_{-+}}{d^2\sigma}, \\ \mathcal{P}_2 \equiv \mathcal{P}_T &= i(\rho_{+-} - \rho_{-+}) = i \frac{d^2\sigma_{+-} - d^2\sigma_{-+}}{d^2\sigma}, \\ \mathcal{P}_3 \equiv \mathcal{P}_L &= \rho_{++} - \rho_{--} = \frac{d^2\sigma_{++} - d^2\sigma_{--}}{d^2\sigma}, \\ \rho &= \frac{1}{2} (1 + \sigma\mathcal{P}) = \begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 + \mathcal{P}_3 & \mathcal{P}_1 - i\mathcal{P}_2 \\ \mathcal{P}_1 + i\mathcal{P}_2 & 1 - \mathcal{P}_3 \end{pmatrix} \end{aligned}$$

Clearly $d^2\sigma_{++}$ ($d^2\sigma_{--}$) is the cross-sections for the production of right (left) handed lepton and $d^2\sigma = d^2\sigma_{++} + d^2\sigma_{--}$. The following inequalities are valid:

$$0 \leq d^2\sigma_{++}d^2\sigma_{--} - |d^2\sigma_{+-}|^2 \leq \frac{1}{4} (d^2\sigma_{++} + d^2\sigma_{--})^2.$$

²We use notation: $\sigma = (\sigma_1, \sigma_2, \sigma_3)$, where σ_i are the usual Pauli matrices.

The elements of the polarization density matrix for an **unpolarized nuclear or nucleon target**:³

$$\frac{d^2\sigma_{++}}{dE_\ell d\cos\theta} = K \left(\frac{E_\ell \mp P_\ell}{2M} \right) \left\{ (1 \pm \cos\theta) \left(W_1 \pm \frac{E_\nu \mp P_\ell}{2M} W_3 \right) + \frac{1 \mp \cos\theta}{2} \left[W_2 + \frac{E_\ell \pm P_\ell}{M} \left(\frac{E_\ell \pm P_\ell}{M} W_4 - W_5 \right) \right] \right\},$$

$$\frac{d^2\sigma_{--}}{dE_\ell d\cos\theta} = K \left(\frac{E_\ell \pm P_\ell}{2M} \right) \left\{ (1 \mp \cos\theta) \left(W_1 \pm \frac{E_\nu \pm P_\ell}{2M} W_3 \right) + \frac{1 \pm \cos\theta}{2} \left[W_2 + \frac{E_\ell \mp P_\ell}{M} \left(\frac{E_\ell \mp P_\ell}{M} W_4 - W_5 \right) \right] \right\},$$

$$\frac{d^2\sigma_{+-}}{dE_\ell d\cos\theta} = K \left(\frac{m_\ell \sin\theta}{4M} \right) \left[\mp \left(2W_1 - W_2 - \frac{m_\ell^2}{M^2} W_4 + \frac{E_\ell}{M} W_5 \right) - \frac{E_\nu}{M} W_3 + i \frac{P_\ell}{M} W_6 \right],$$

$$\frac{d^2\sigma_{-+}}{dE_\ell d\cos\theta} = K \left(\frac{m_\ell \sin\theta}{4M} \right) \left[\mp \left(2W_1 - W_2 - \frac{m_\ell^2}{M^2} W_4 + \frac{E_\ell}{M} W_5 \right) - \frac{E_\nu}{M} W_3 - i \frac{P_\ell}{M} W_6 \right].$$

The upper (lower) signs in the above formulas are, as above, for neutrino (antineutrino) and

$$K = \frac{G_F^2 P_\ell}{\pi} \left(1 + \frac{Q^2}{M_W^2} \right)^{-2};$$

M is the target mass; m_ℓ , E_ℓ , P_ℓ and θ are, respectively, the lepton mass, energy, momentum, and scattering angle; W_i ($i = 1, \dots, 6$) are the **model-dependent** nuclear (nucleon) structure functions.

³K. S. Kuzmin, V. V. Lyubushkin, V. A. Naumov, *Mod. Phys. Lett. A* **19** (2004) 2815–2829, hep-ph/0312107.

Lepton polarization vector (continuation)

- The components of the polarization vector can be written in terms of structure functions W_i :

$$\mathcal{P}_P = \mp \frac{m_\ell \sin \theta}{2M\mathcal{R}_\mp} \left(2W_1 - W_2 \pm \frac{E_\nu}{M} W_3 - \frac{m_\ell^2}{M^2} W_4 + \frac{E_\ell}{M} W_5 \right),$$

$$\mathcal{P}_T = -\frac{m_\ell P_\ell \sin \theta}{2M^2\mathcal{R}_\mp} W_6, \quad [W_6 \equiv 0 \text{ in SM}]$$

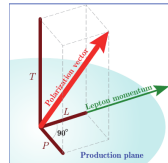
$$\mathcal{P}_L = \mp 1 \pm \frac{m_\ell^2}{M^2\mathcal{R}_\mp} \left\{ \left[\left(\frac{2M}{E_\ell + P_\ell} \right) W_1 \pm \left(\frac{E_\nu - P_\ell}{E_\ell + P_\ell} \right) W_3 \right] \cos^2 \frac{\theta}{2} + \left[\left(\frac{M}{E_\ell + P_\ell} \right) W_2 + \left(\frac{E_\ell + P_\ell}{M} \right) W_4 - W_5 \right] \sin^2 \frac{\theta}{2} \right\},$$

where

$$\mathcal{R}_\pm = \left(\frac{E_\ell - P_\ell \cos \theta}{M} \right) \left(W_1 + \frac{m_\ell^2}{2M^2} W_4 \right) + \left(\frac{E_\ell + P_\ell \cos \theta}{2M} \right) W_2 \pm \left[\left(\frac{E_\nu + E_\ell}{M} \right) \left(\frac{E_\ell - P_\ell \cos \theta}{2M} \right) - \frac{m_\ell^2}{2M^2} \right] W_3 - \frac{m_\ell^2}{2M^2} W_5.$$

are the Lorentz-invariant dimensionless functions.

- Degree of polarization $|\mathcal{P}| = \sqrt{\mathcal{P}_1^2 + \mathcal{P}_2^2 + \mathcal{P}_3^2}$ satisfies the condition $0 \leq |\mathcal{P}| \leq 1$.
- The density matrix ρ and degree of polarization $|\mathcal{P}|$ are relativistic invariants.
- $\mathcal{P}_P = \mathcal{P}_T = 0$ and $\mathcal{P}_L = \mp 1$ for a massless lepton (ℓ^\mp), in practice, for electron or positron, and to some extent for muons. This is not valid for the τ lepton, the main focus of this study.



Summary of models under consideration:

- **SM RFG**: Relativistic Fermi gas model by Smith-Monitz (degenerate Fermi gas with relativistic kinematics)¹.
- **SuSAv2**² and **Valencia**³ both describe 1p1h and 2p2h reactions. **SuSAv2** combines the superscaling function approach with a relativistic microscopic 2p2h contribution. **Valencia** uses a (local) Fermi-gas description of the nucleus and implements a fully microscopic many-body calculation.
- **HF-PW**: Hartree-Fock ground state; scattered nucleon treated as a plane wave (no final-state potential)⁴.
- **HF**: Hartree-Fock ground state; scattered nucleon treated as a distorted wave with HF potential⁴.
- **HF+CRPA**: Hartree-Fock ground state; scattered nucleon treated as a distorted wave with HF potential plus continuum RPA corrections for long-range nucleon correlations⁴.

The last three models are based on the Hartree-Fock formalism and differ in the treatment of the scattered nucleon – plane wave (**HF-PW**) or distorted wave (**HF**); the last (most advanced) version (**HF+CRPA**) additionally takes into account the Random Phase Approximation (RPA) effects.

¹R. A. Smith, E. J. Moniz, *Nucl. Phys. B* **43** (1972) 605–622 [Erratum: *Nucl. Phys. B* **101** (1975) 547]; here we use a modification of the SM RFG model by K. S. Kuzmin, V. V. Lyubushkin, V. A. Naumov, *Eur. Phys. J. C* **54** (2008) 517–538.

²S. Dolan, G. D. Megias, S. Bolognesi, *Phys. Rev. D* **101** (2020) 033003.

³J. Schwehr, D. Cherdack, R. Gran, arXiv:1601.02038 [hep-ph]; J. Nieves, I. Ruiz Simo, M. J. Vicente Vacas, *Phys. Rev. C* **83** (2011) 045501; R. Gran et al., *Phys. Rev. D* **88** (2013) 113007.

⁴S. Dolan, A. Nikolakopoulos, O. Page, et al. *Phys. Rev. D* **106** (2022) 073001.

We have carefully analyzed all the above-mentioned models for all experimentally relevant nuclear targets and classified them according to the qualitative criterion of “goodness”, defined as the fraction of unphysical polarization appearing in the MC event simulation.

Goodness of models in terms of polarization:

(a summary of the adequacy of the models included in the GENIE generator with respect to the calculation of the final-state lepton polarization)

Model(s)	Characteristic
SM RFG, HF-PW, HF, SuSAv2-2p2h	Physical polarization over the full phase space for all nuclei
Valencia-1p1h and SuSAv2-1p1h	Unphysical polarization in certain regions of phase space
Valencia-2p2h	Physical polarization over the full phase space, but only for carbon
HF+CRPA	Physical polarization for all nuclei over the full phase space, with anomalies near phase-space boundaries

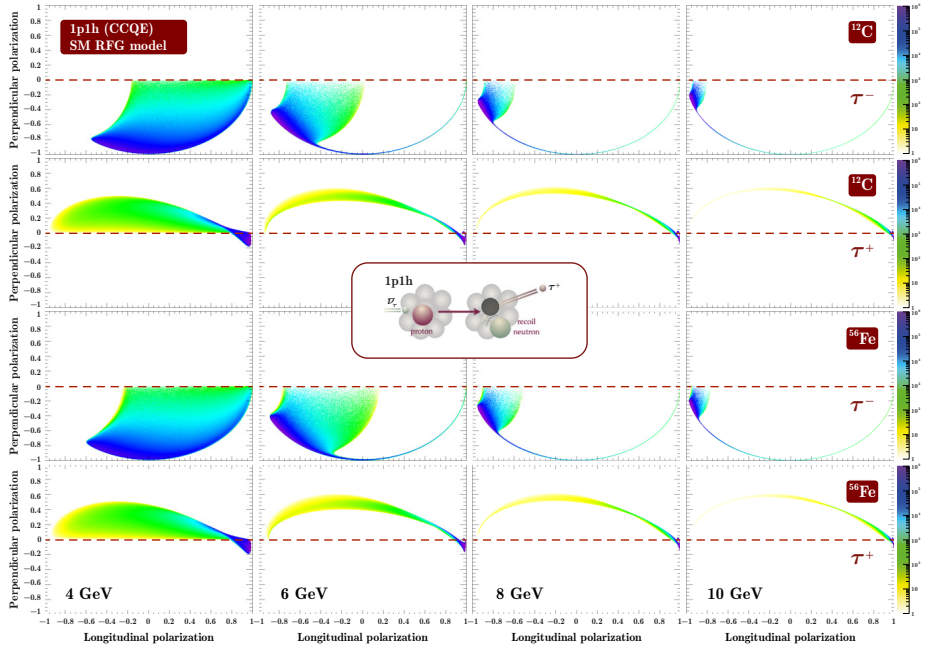
Fraction of events with unphysical polarization

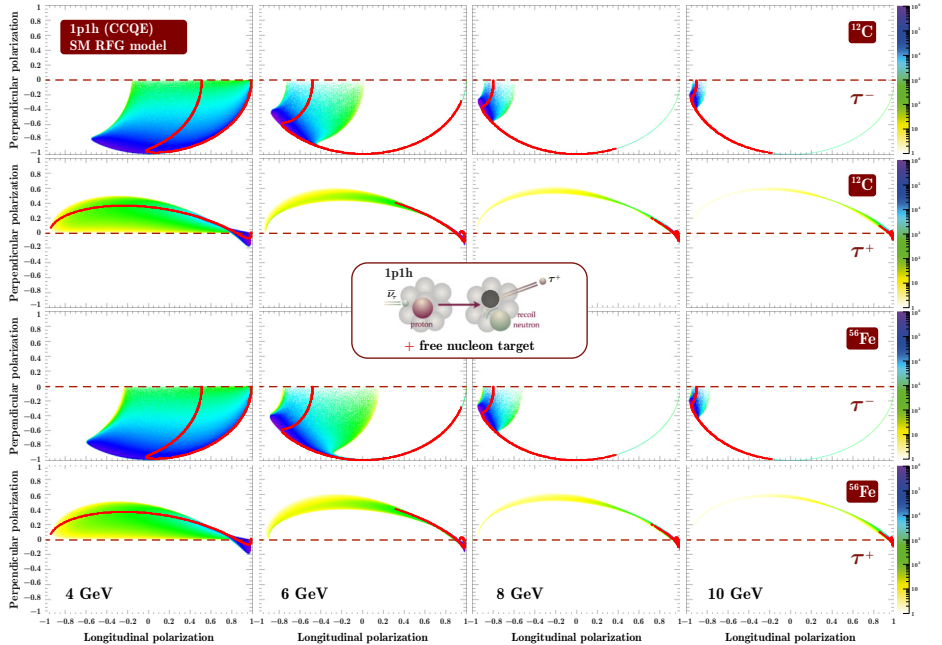
(defined as $|\mathcal{P}| > 1$ with tolerance 10^{-5})

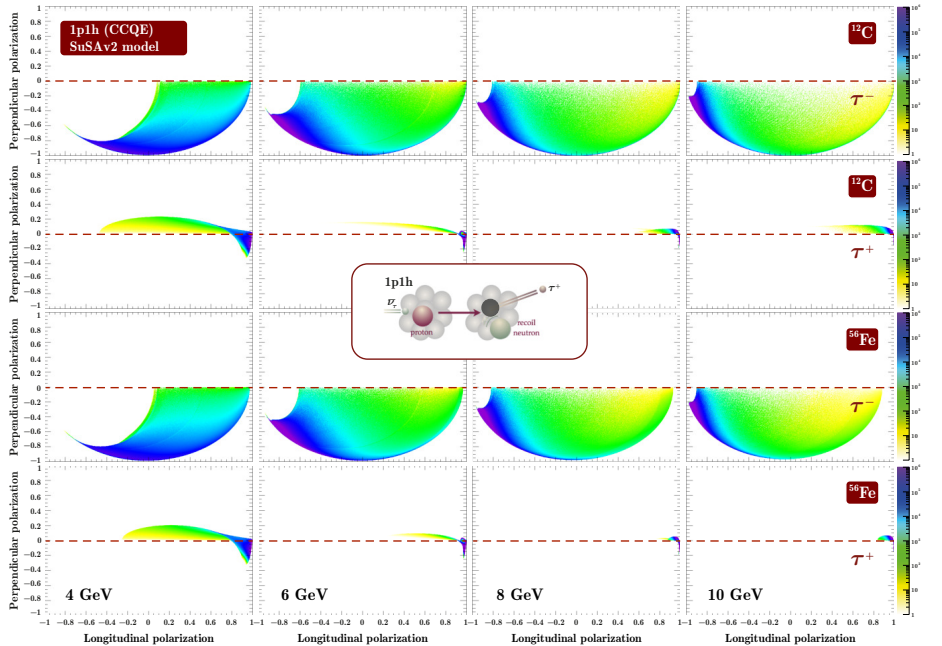
Model & Channel	4 GeV	6 GeV	8 GeV	10 GeV
SuSAv2-1p1h, ν_τ on ^{12}C	5.5×10^{-7}	4.7×10^{-7}	6.0×10^{-4}	1.1×10^{-3}
SuSAv2-1p1h, $\bar{\nu}_\tau$ on ^{12}C	1.5×10^{-5}	4.2×10^{-4}	1.1×10^{-3}	1.5×10^{-3}
SuSAv2-1p1h, ν_τ on ^{16}O	1.4×10^{-4}	1.8×10^{-2}	1.9×10^{-2}	2.0×10^{-2}
SuSAv2-1p1h, $\bar{\nu}_\tau$ on ^{16}O	1.7×10^{-6}	4.3×10^{-4}	1.1×10^{-3}	1.5×10^{-3}
SuSAv2-1p1h, $\bar{\nu}_\tau$ on ^{40}Ar	4.0×10^{-4}	5.0×10^{-4}	1.3×10^{-3}	1.7×10^{-3}
SuSAv2-1p1h, $\bar{\nu}_\tau$ on ^{56}Fe	7.4×10^{-5}	4.3×10^{-4}	1.2×10^{-3}	1.6×10^{-3}
Valencia-2p2h, ν_τ on ^{56}Fe	1.1×10^{-14}	3.7×10^{-15}	1.9×10^{-15}	2.4×10^{-15}
Valencia-2p2h, $\bar{\nu}_\tau$ on ^{12}C	7.1×10^{-4}	4.6×10^{-10}	1.4×10^{-10}	8.4×10^{-11}
Valencia-2p2h, $\bar{\nu}_\tau$ on ^{16}O	3.1×10^{-4}	5.6×10^{-9}	1.0×10^{-9}	1.6×10^{-10}
Valencia-2p2h, $\bar{\nu}_\tau$ on ^{40}Ar	2.1×10^{-4}	3.3×10^{-9}	2.6×10^{-10}	8.9×10^{-11}
Valencia-2p2h, $\bar{\nu}_\tau$ on ^{56}Fe	2.0×10^{-4}	2.3×10^{-9}	7.5×10^{-11}	3.1×10^{-11}

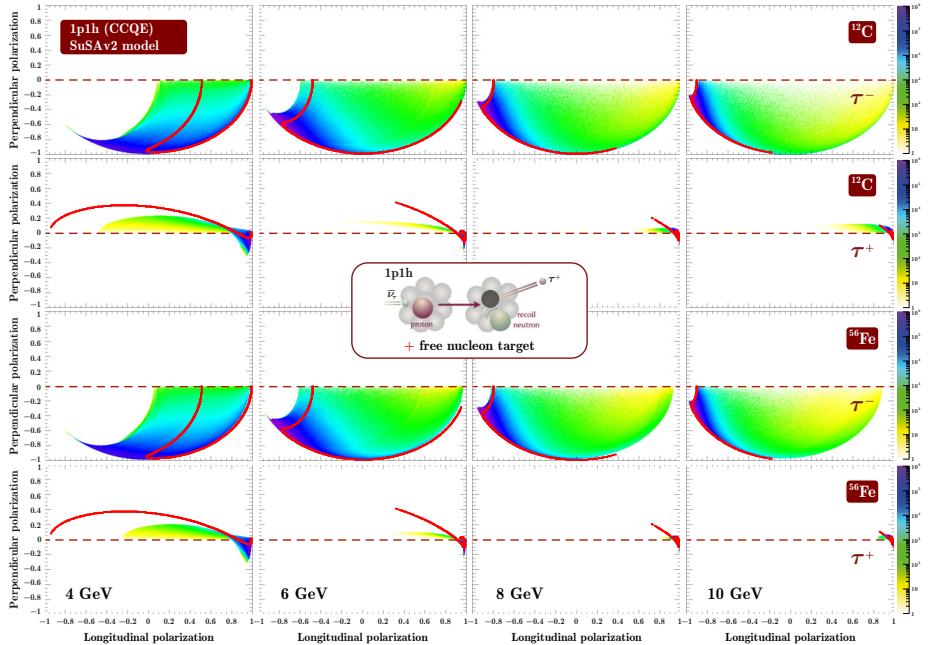
Thus, even the indicated “non-ideal” models (with the exception of SuSAv2-1p1h for ^{16}O) are suitable for use in simulations, since the events with unphysical polarization do not contribute to the total event rate.

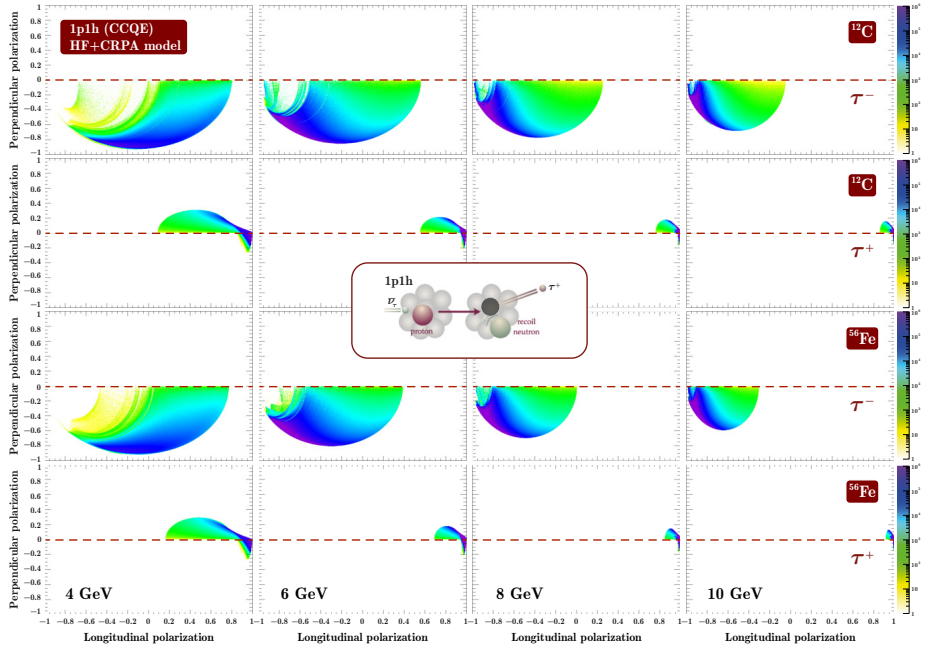
Below are some typical illustrative examples of points discussed above.

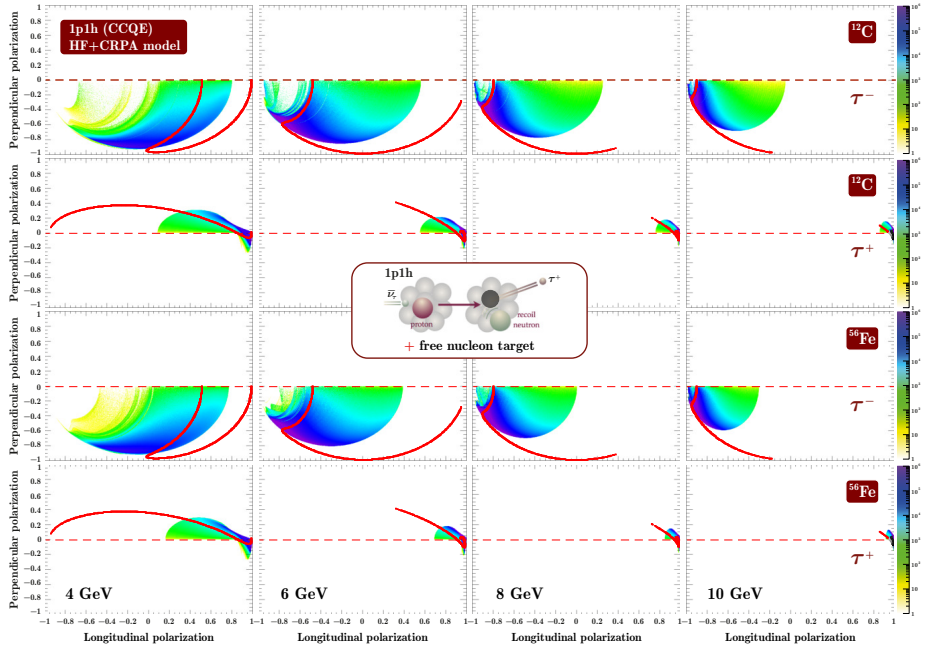


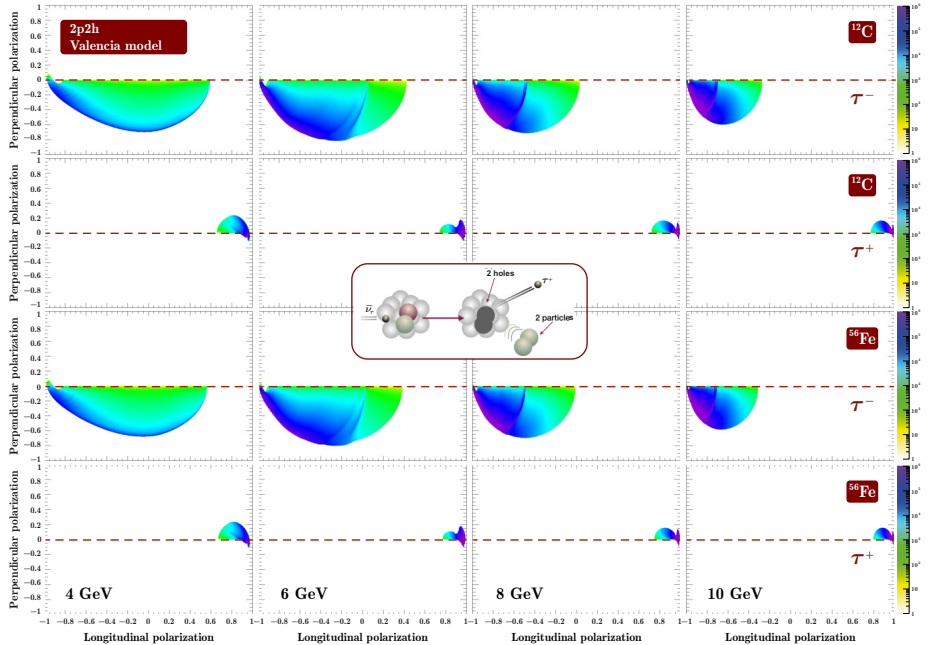


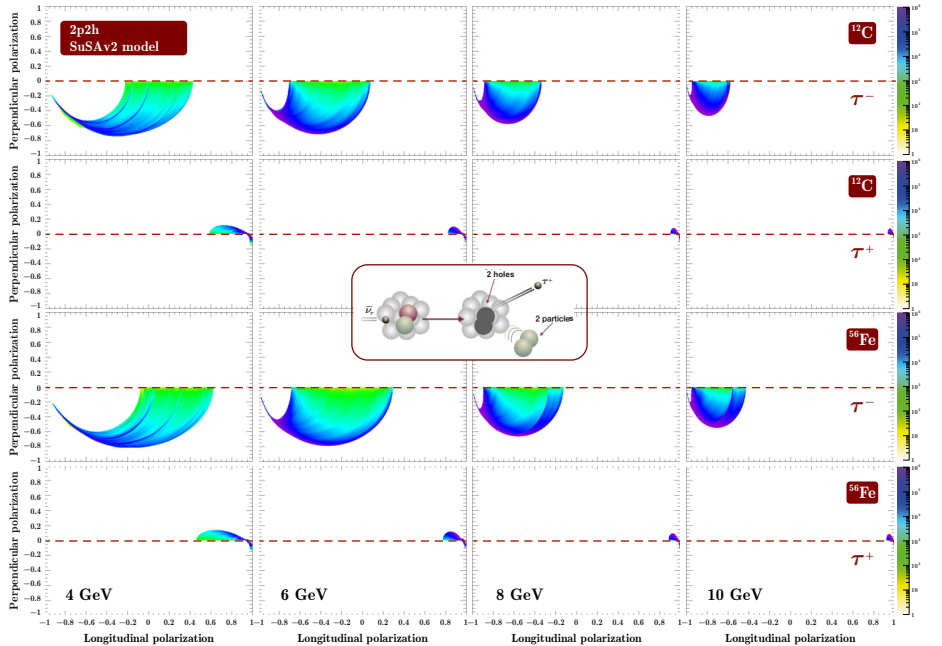












Summary

- Final-state lepton polarizations are implemented in GENIE for the 1p1h (CCQE) and 2p2h channels based on the polarization density matrix approach. All models have been tested.
- Model status (w.r.t. polarization):
 - **SM RFG, HF-PW, HF 1p1h and SuSAv2 2p2h** \Rightarrow physical in the entire phase space.
 - **Valencia 2p2h** \Rightarrow physical (currently limited in target coverage);
 - **HF+CRPA 1p1h** \Rightarrow physical in the entire phase space, but small boundary effects may appear near kinematic edges;
 - **SuSAv2 1p1h and Valencia 1p1h**⁴ \Rightarrow localized anomalies in limited kinematic regions.
- For realistic event samples, the fraction of configurations with non-physical polarization vector \mathcal{P} ($|\mathcal{P}| > 1$) is numerically negligible (except isolated edge cases), i.e. no impact on total and differential yields.
- The implemented polarization output is **directly usable** for τ -decay chains: The angular and momentum distributions of τ -decay secondaries depend on \mathcal{P}_L and \mathcal{P}_P ; integration with decay analysis packages (e.g. **TAUOLA**) is a natural next step.
- The implementation provides the ability to include non-standard neutrino interactions that may lead to the emergence of non-zero transverse polarization, \mathcal{P}_T .

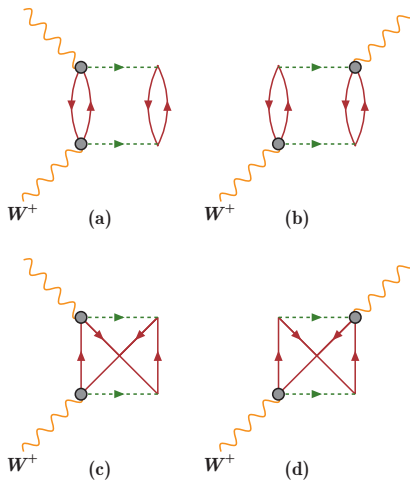
⁴The Valencia 1p1h model, one of the most advanced and comprehensive microscopic models on the market, becomes suitable (**w.r.t. polarization!**) in the whole phase space, if (presumably small) RPA effects are neglected, which is equivalent to the Local Fermi-Gas (LFG) model.

Backup

Key Differences: Valencia vs. SuSAv2 (2p2h)⁵

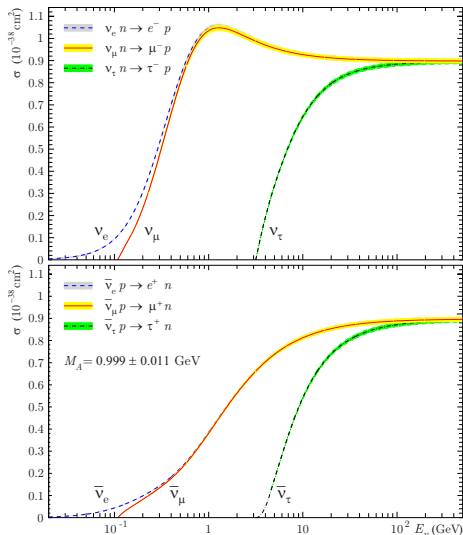
Feature	Valencia	SuSAv2
Δ -resonance propagator in pion-exchange diagrams	Partially real and imaginary parts, including higher energy ρ -resonance exchange	Real part only to avoid double counting in inelastic regime (empirical approach)
Direct-exchange interference terms	Neglected	Fully included
Momentum transfer range	Up to 1.2 GeV	Up to 2 GeV

⁵S. Dolan, G. D. Megias, S. Bolognesi, *Phys. Rev. D* **101** (2020) 033003.



An example of 2p-2h diagrams. The circle stands for the elementary $W^+ N \rightarrow \pi N$ amplitude. Diagrams (a,b) represent the direct contribution. Diagrams (c,d) are the exchange contributions⁶.

⁶I. Ruiz Simo et al., *Phys. Lett. B* **762** (2016) 124–130.



Total cross sections for the e , μ , and τ neutrinos and antineutrinos CCQE interactions with free nucleons calculated using the dipole model of the axial-vector form factor with $M_A = 0.999 \pm 0.011 \text{ GeV}$ and so-called BBBA(07) model for the vector form factors of the nucleon with $d/u = 0$. Shaded bands represent the uncertainty due to the 1σ error in extraction of the parameter M_A .

Recently updated value of M_A extracted from the Deuterium data is

$$M_A^{(D)} = 1.003^{+0.085(0.112)}_{-0.084(0.111)} \text{ GeV}.$$

Inclusion of the more recent data from heavier nuclear (mainly carbonaceous) targets yields (within the framework of the so-called “RFG M_A^{run} ” model⁷)

$$M_A = 1.008 \pm 0.025(0.029) \text{ GeV}$$

with $\chi^2/\text{NDF} \approx 0.7$.

⁷I. D. Kakorin, K. S. Kuzmin, V. A. Naumov, Eur. Phys. J. C **81** (2021) 1142; e-Print:2112.13745 [hep-ph].

GENIE COLLABORATION

Luis Alvarez-Rusp [4], Costas Andreopoulos [8], Adi Ashkenazi [11], Joshua Barrow [9], Steve Dytman [10], Hugh Gallagher [12], Alfonso Andres García Soto [4], Steven Gardiner [3], Matan Goldenberg [11], Robert Hatcher [3], Igor Kakorin [6], Konstantin Kuzmin [5,6], Liang Liu [3], Xianguo Lu [13], Anselmo Mereaglia [1], Vadim Naumov [6], Afroditi Papadopoulou [7], Gabriel Perdue [3], Komninos-John Plows [8], Marco Roda [8], Alon Sportes [11], Júlia Tena Vidal [11], Qi Yu Yan [3,13]

(1) **CENBG, Université de Bordeaux, CNRS/IN2P3**
33175 Gradignan, France

(2) **University of the Chinese Academy of Sciences**
Beijing 100864, China

(3) **Fermi National Accelerator Laboratory**
Batavia, Illinois 60510, USA

(4) **Instituto de Física Corpuscular (IFIC)**
Consejo Superior de Investigaciones Científicas (CSIC) y de la Universitat de València (UV)
46980, Paterna, València, Spain

(5) **Alikhanov Institute for Theoretical and Experimental Physics (ITEP) of NRC "Kurchatov Institute"**
Moscow, 117218, Russia

(6) **Joint Institute for Nuclear Research (JINR)**
Dubna, Moscow region, 141980, Russia

(7) **Los Alamos National Laboratory (LANL)**
Los Alamos, NM 87545, USA

(8) **University of Liverpool**, Dept. of Physics
Liverpool L69 7ZE, UK

(9) **University of Minnesota**, School of Physics and Astronomy
Minneapolis, MN 55455, USA

(10) **University of Pittsburgh**, Dept. of Physics and Astronomy
Pittsburgh, PA 15260, USA

(11) **Tel Aviv University**, School of Physics and Astronomy
Tel Aviv 6997801, Israel

(12) **Tufts University**, Dept. of Physics and Astronomy
Medford MA 02155, USA

(13) **Warwick University**, Dept. of Physics
Coventry CV4 7AL, UK