

Mean field approach to QCD vacuum and properties of hadrons

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CONFINEMENT

”there is little reason to doubt the validity of the Standard Model..., and there is good reason to admire a theory which can describe such a wide range of phenomena so economically... it is unfortunate that, in one very important respect, we don’t really understand how it works.

We simply do not know for sure why quarks and gluons, which are the fundamental fields of the theory, don’t show up in the actual spectrum of the theory, as asymptotic particle states...”

Jeff Greensite, An Introduction to the Confinement Problem, Springer-Verlag, Berlin, Heidelberg, 2011

Free field $\leftrightarrow$ elementary particle:

$$(\partial_0^2 - \vec{\partial}^2 - m^2)\phi(x) = 0 \leftrightarrow p_0^2 - \vec{p}^2 = m^2$$

- Effective quantum action and self-consistent mean field as the vacuum of quantum field systems.
- Mean field approach to QCD: statistical ensemble of almost everywhere homogeneous (anti-) self-dual Abelian gluon fields as QCD vacuum: confinement of dynamic and static quarks, chiral symmetry
- Collective excitations in QCD vacuum and QCD Bozonization
- Meson effective action: the mass spectrum of mesons
- Effects of strong, weak and electromagnetic interactions of mesons: decay constants and form factors.
- "Projection" to other approaches: FRG, DSE+BS, 4-dim. oscillator - harmonic confinement, AdS/QCD models
- Electromagnetic field as a trigger of deconfinement, direct photon production.

R. Feynman *et al* PRD 3 1971;
 H. Leutwyler, J. Stern, PLB 73, 1978; PLB 77, 1978;
 H. Pagels and E. Tomboulis, Nucl. Phys. B143, 485 (1978).
 A.I. Milstein, Yu.F. Pinelis, PLB 137 (1984);
 H. Leutwyler, Phys. Lett. 96B, 154 (1980); Nucl. Phys. B179, 129 (1981).
 P. Minkowski, Nucl. Phys. B177, 203 (1981)
 A.I. Milstein, Yu.F. Pinelis, PLB 137 (1984);
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 J. Greensite, Progress in Particle and Nuclear Physics 51 (2003) 1-83
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 E. M. Ilgenfritz et al., Proc. Sci., LAT2007 (2007) 311.
 L. D. Faddeev, arXiv:0911.1013.
 A. Eichhorn, H. Gies, and J. M. Pawłowski, Phys. Rev. D 83, 045014 (2011); 83, 069903 (2011).
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 S.N. , V.E. Voronin, Eur.Phys.J. A51 (2015) 4; Phys.Rev.D 93 (2016) 9, 094010; Phys.Rev.D 95 (2017) 7, 074038; Phys.Rev.D 103 (2021) 11, 114021
 S.N. , A.Nikolskii, V.E. Voronin, J.Phys.G 49 (2022) 3, 035003
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Quantum effective action of QCD

- **Confinement of both static and dynamical quarks** $\longrightarrow$
$$W(C) = \langle \text{Tr P } e^{i \int_C dz_\mu \hat{A}_\mu} \rangle$$
$$S(x, y) = \langle \psi(y) \bar{\psi}(x) \rangle$$
- **Dynamical Breaking of chiral $SU_L(N_f) \times SU_R(N_f)$ symmetry** $\longrightarrow \langle \bar{\psi}(x) \psi(x) \rangle$
- **$U_A(1)$ Problem** $\longrightarrow \eta'$ (χ , Axial Anomaly)
- **Strong CP Problem** $\longrightarrow Z(\theta)$
- **Colorless Hadron Formation:** $\longrightarrow$ Effective action for colorless collective modes:
hadron masses,
form factors, scattering
- **Light mesons, Regge spectrum of excited states of light hadrons, heavy-light hadrons, heavy quarkonia**
- **Deconfinement, chiral symmetry restoration under "extreme" conditions**

$$L = \bar{\psi}_f(x) (i \not{D} - m_f) \psi_f(x) - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

$$t^a F_{\mu\nu}^a = i [D_\mu, D_\nu], \quad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + i f^{abc} A_\mu^b A_\nu^c.$$

Color $SU(3)$ $\psi_f^U = U(x)\psi$, $A_\mu^U = U(x) [t_a A_\mu^a + i\partial_\mu] U^{-1}(x)$

Flavour $SU(N_f)$: $\psi_f \rightarrow U^f f' \psi_{f'}$, $U = \exp \{i\theta^a t^a\}$, $[t^a, t^b] = i f^{abc} t^c$

Chiral symmetry:

$$\gamma_5 \psi_\pm = \pm \psi_\pm.$$

$$P_\pm = \frac{1}{2} (I \pm \gamma_5), \quad P_+ + P_- = I, \quad P_+ P_- = 0, \quad P_\pm^2 = P_\pm,$$

$$\psi_\pm = P_\pm \psi, \quad \psi = \psi_+ + \psi_-,$$

$$L = i \bar{\psi}_+^f \not{D} \psi_+^f + i \bar{\psi}_-^f \not{D} \psi_-^f - m_f (\bar{\psi}_+^f \psi_-^f + \bar{\psi}_-^f \psi_+^f)$$

For $m_f = 0$,

$$L = i \bar{\psi}_+^f \not{D} \psi_+^f + i \bar{\psi}_-^f \not{D} \psi_-^f.$$

Lagrangian is invariant with respect to $SU_R(N_f) \times SU_L(N_f)$:

$$\psi_+ \rightarrow e^{i\theta^a t^a} \psi_+, \quad \psi_- \rightarrow e^{i\chi^b t^b} \psi_-,$$

θ^a and χ^a – independent parameters. Lagrangian is also invariant with respect to

$$\psi \rightarrow e^{i\gamma_5 \alpha} \psi \quad (\psi_+ \rightarrow e^{i\alpha} \psi_+, \quad \psi_- \rightarrow e^{-i\alpha} \psi_-)$$

that is axial $U_A(1)$ symmetry.

QCD effective action and vacuum gluon configurations

In Euclidean functional integral for YM theory one has to allow the gluon condensates to be nonzero:

$$Z = N \int_{\mathcal{F}_B} DA \int_{\Psi} D\psi D\bar{\psi} \exp\{-S[A, \psi, \bar{\psi}]\}$$

$$\mathcal{F}_B = \left\{ A : \lim_{V \rightarrow \infty} \frac{1}{V} \int_V d^4x g^2 F_{\mu\nu}^a(x) F_{\mu\nu}^a(x) = B^2 \right\}.$$

B.V. Galilo and S.N. ,
Phys. Rev. D84 (2011) 094017

L. D. Faddeev,
[arXiv:0911.1013 [math-ph]]

H. Leutwyler,
Nucl. Phys. B 179 (1981) 129

$A_\mu^a = B_\mu^a + Q_\mu^a$, background gauge fixing condition $D(B)Q = 0$:

$$1 = \int_{\mathcal{B}} DB \Phi[A, B] \int_{\mathcal{Q}} DQ \int_{\Omega} D\omega \delta[A^\omega - Q^\omega - B^\omega] \delta[D(B^\omega)Q^\omega]$$

Q_μ^a – local (perturbative) fluctuations of gluon field with zero gluon condensate: $Q \in \mathcal{Q}$;
 B_μ^a are long range field configurations with nonzero condensate: $B \in \mathcal{B}$.

$$Z = N' \int_{\mathcal{B}} DB \int_{\mathcal{Q}} DQ \int_{\Psi} D\psi D\bar{\psi} \det[D(B)D(B+Q)] \delta[D(B)Q] \exp\{-S[B+Q, \psi, \bar{\psi}]\}$$

Particular features of background fields B have yet to be identified by the dynamics of fluctuations:

$$\begin{aligned} Z &= N' \int_{\mathcal{B}} DB \int_{\Psi} D\psi D\bar{\psi} \int_{\mathcal{Q}} DQ \det[D(B)D(B+Q)] \delta[D(B)Q] \exp\{-S[B+Q, \psi, \bar{\psi}]\} \\ &= N' \int_{\mathcal{B}} DB \exp\{-S_{\text{eff}}[B]\} \end{aligned}$$

Global minima of $S_{\text{eff}}[B]$ – field configurations that are dominant in the limit $V \rightarrow \infty$.

$$Z = N'' \int_{\mathcal{B}_{\text{vac}}} D\sigma_B \int_{\Psi} D\psi D\bar{\psi} \int_{\mathcal{Q}} DQ \det[D(B)D(B+Q)] \delta[D(B)Q] \exp\{-S[B+Q, \psi, \bar{\psi}]\}$$

Background (vacuum) fields $\mathcal{B}_{\text{vac}}$ have to be treated nonperturbatively, while the rest of fields are fluctuations - perturbation expansions.

QCD vacuum as a medium characterized by certain condensates,
 quarks and gluons - elementary coloured excitations (confined),
 mesons and baryons - collective colorless excitations

L.D. Faddeev,

arXiv:1509.06186 [hep-th], Theor.Math.Phys. 181 (2014), arXiv:0911.1013[hep-th]

In general, **division of the gauge field potential into the (nonperturbative) background and fluctuation parts** is a complicated task.

Specifically, a separation of the Abelian part from the general gauge field requires application of the special gauge field parametrization suggested by [L.D. Faddeev, A. J. Niemi \(2007\)](#); [K.-I. Kondo, T. Shinohara, T. Murakami \(2008\)](#); [Y.M. Cho \(1980, 1981\)](#); [L.Prokhorov, S.V. Shabanov \(1989,1999\)](#)

The Abelian part $\hat{V}_\mu(x)$ of the gauge field $\hat{A}_\mu(x)$ is separated manifestly,

$$\begin{aligned}\hat{A}_\mu(x) &= \hat{V}_\mu(x) + \hat{X}_\mu(x), \quad \hat{V}_\mu(x) = \hat{B}_\mu(x) + \hat{C}_\mu(x), \\ \hat{B}_\mu(x) &= [n^a A_\mu^a(x)] \hat{n}(x) = B_\mu(x) \hat{n}(x), \\ \hat{C}_\mu(x) &= g^{-1} \partial_\mu \hat{n}(x) \times \hat{n}(x), \\ \hat{X}_\mu(x) &= g^{-1} \hat{n}(x) \times \left(\partial_\mu \hat{n}(x) + g \hat{A}_\mu(x) \times \hat{n}(x) \right),\end{aligned}$$

where $\hat{A}_\mu(x) = A_\mu^a(x) t^a$, $\hat{n}(x) = n_a(x) t^a$, $n^a n^a = 1$, and

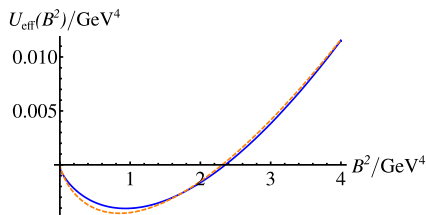
$$\partial_\mu \hat{n} \times \hat{n} = i f^{abc} \partial_\mu n^a n^b t^c, \quad [t^a, t^b] = i f^{abc} t^c.$$

$$[\hat{V}_\mu(x), \hat{V}_\nu(x)] = 0$$

Homogeneous Abelian (anti-)self-dual fields are of particular interest.

$$B_\mu = -\frac{1}{2}nB_{\mu\nu}x_\nu, \quad \tilde{B}_{\mu\nu} = \pm B_{\mu\nu}$$

$$n = T^3 \cos \xi + T^8 \sin \xi.$$

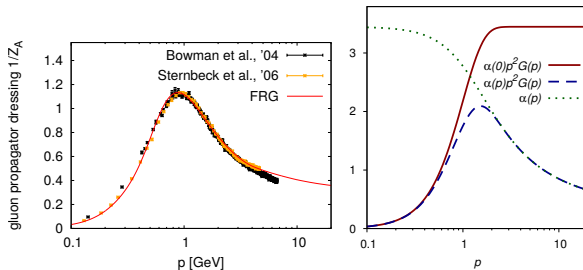


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A. Eichhorn, H. Gies and
J.M. Pawłowski, Phys. Rev.
D 83, 045014 (2011)

Discrete symmetries of the effective action – Weyl reflections & CP

(SN, Kalloniatis, B. Galilo, V. Voronin, PRD (2001, 2011, 2015); K.-I. Kondo *et al* Phys. Rep. (2015); Y.M. Cho *et al* PRD (2012))



A signature of self-consistency:

Functional RG (used for evaluation of U_{eff}) vs mean field propagators

Analytical properties of the quark and gluon propagators

$$G(z^2) \sim \frac{e^{-Bz^2}}{z^2}, \quad \tilde{G}(p^2) \sim \frac{1}{p^2} \left(1 - e^{-p^2/B}\right)$$

$\Rightarrow$ dynamical color confinement

H. Leutwyler, Phys. Lett. B 96 (1980) 154

Analytical properties of polarization diagram

$\Rightarrow$ confinement

A.I. Milstein, Yu. Pinelis, Phys. Lett. B 137 (1984)

$\Rightarrow$ Regge mass spectrum of mesons

G.V. Efimov, and S.N., Phys. Rev. D 51 (1995)

(Anti-)self-duality $\Rightarrow$ Chiral symmetry

H. Leutwyler, Phys. Lett. B 96 (1980) 154

A.I. Milshtein, Yu.F. Pinelis, Z.Phys. C27 (1985).

A.G. Grozin, Yu.F. Pinelis, Z.Phys. C33 (1987)

Gluon condensates and domain wall network

Pure gluodynamics - Ginzburg-Landau approach:

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4\Lambda^2} \left(D_\nu^{ab} F_{\rho\mu}^b D_\nu^{ac} F_{\rho\mu}^c + D_\mu^{ab} F_{\mu\nu}^b D_\rho^{ac} F_{\rho\nu}^c \right) - U_{\text{eff}}$$
$$U_{\text{eff}} = \frac{\Lambda^4}{12} \text{Tr} \left(C_1 F^2 + \frac{4}{3} C_2 F^4 - \frac{16}{9} C_3 F^6 \right),$$

B.V. Galilo, S.N. , Phys. Part. Nucl. Lett., 8 (2011) 67

D. P. George, A. Ram, J. E. Thompson and R. R. Volkas, Phys. Rev. D 87, 105009 (2013) [arXiv:1203.1048 [hep-th]]

where

$$D_\mu^{ab} = \delta^{ab} \partial_\mu - i A_\mu^{ab} = \partial_\mu - i A_\mu^c (T^c)^{ab},$$
$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - i f^{abc} A_\mu^b A_\nu^c,$$
$$F_{\mu\nu} = F_{\mu\nu}^a T^a, \quad T_{bc}^a = -i f^{abc}$$
$$C_1 > 0, \quad C_2 > 0, \quad C_3 > 0.$$

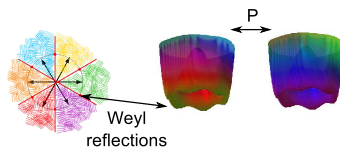
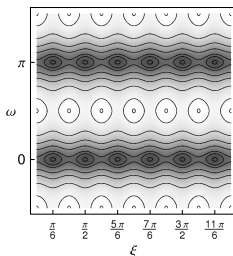
U_{eff} possesses degenerate discrete minima:

$$B_\mu = -\frac{1}{2}n_k B_{\mu\nu} x_\nu, \quad \tilde{B}_{\mu\nu} = \pm B_{\mu\nu},$$

matrix n_k belongs to the Cartan subalgebra of $su(3)$

$$n_k = T^3 \cos(\xi_k) + T^8 \sin(\xi_k), \quad \xi_k = \frac{2k+1}{6}\pi, \quad k = 0, 1, \dots, 5,$$

$$\vec{E}\vec{H} = B^2 \cos(\omega)$$



Discrete minima related by Weyl reflections and CP $\leftrightarrow$ **kinks and solitons**

L.D. Faddeev

[In "Diakonov, D. (ed.): Subtleties in quantum field theory", arXiv:1003.4854 [hep-th]]:

"Quantum equations could have soliton solutions, which are absent in the classical limit. In particular, it is not completely crazy idea that quantum Yang-Mills equations have soliton-like solutions due to the dimensional transmutation."

Search for solutions of the effective nonlinear equations, derived from the Ginzburg-Landau-like effective action.

Field strength: $\text{Tr}(F^2(x))$

Topological charge density: $\text{Tr}\tilde{F}(x)F(x)$

Color "orientation": $n(x) \leftrightarrow \xi(x)$

Domain wall network

$\xi, \langle g^2 F^2 \rangle \rightarrow$ vacuum values

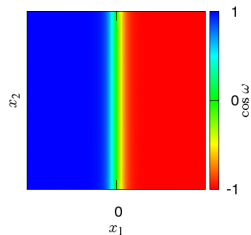
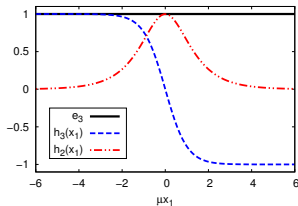
$$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\Lambda^2 b_{\text{vac}}^2 \partial_\mu \omega \partial_\mu \omega - b_{\text{vac}}^4 \Lambda^4 (C_2 + 3C_3 b_{\text{vac}}^2) \sin^2 \omega,$$

leads to sine-Gordon equation

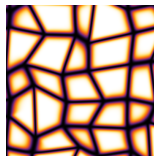
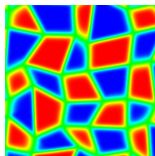
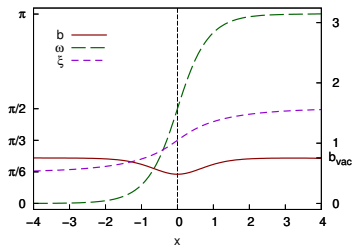
$$\partial^2 \omega = m_\omega^2 \sin 2\omega, \quad m_\omega^2 = b_{\text{vac}}^2 \Lambda^2 (C_2 + 3C_3 b_{\text{vac}}^2),$$

and the standard kink solution

$$\omega(x_\nu) = 2 \arctg(\exp(\mu x_\nu))$$



”Domain wall involving the topological charge density (in units of $\langle g^2 F^2 \rangle$), $su(3)$ angle ξ and the background action density simultaneously”



The general kink configuration can be parametrized as

$$\zeta(\mu_i, \eta_\nu^i x_\nu - q^i) = \frac{2}{\pi} \arctan \exp(\mu_i(\eta_\nu^i x_\nu - q^i)).$$

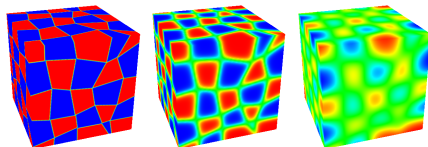
A single lump in two, three and four dimensions is given by

$$\omega(x) = \pi \prod_{i=1}^k \zeta(\mu_i, \eta_\nu^i x_\nu - q^i).$$

for $k = 4, 6, 8$, respectively. The general kink network is then given by the additive superposition of lumps

$$\omega = \pi \sum_{j=1}^{\infty} \prod_{i=1}^k \zeta(\mu_{ij}, \eta_\nu^{ij} x_\nu - q^{ij})$$

S.N., V.E. Voronin, Eur.Phys.J. A51 (2015) 4



$$\begin{aligned} \langle F^2 \rangle &= B^2 \\ \langle |F \tilde{F}| \rangle &= B^2 \end{aligned}$$

$$\begin{aligned} \langle F^2 \rangle &= B^2 \\ \langle |F \tilde{F}| \rangle &\ll B^2 \end{aligned}$$

"Phase transitions and heterophase fluctuations" V. I. Yukalov, Phys. Rep. 208, 396 (1991)

What could stabilize a finite mean size of the domains?

Lower dimensional defects?

Quark (quasi-)zero modes?

Domain bulk - harmonic confinement

Elementary color charged excitations - fluctuations, eigenmodes decay in all four directions.

Eigenvalue problem for scalar field in $\mathbb{R}^4$:

H. Leutwyler, Nucl. Phys. B 179 (1981);

$$B_\mu = B_{\mu\nu} x_\nu, \tilde{B}_{\mu\nu} = \pm B_{\mu\nu}, B_{\mu\alpha} B_{\nu\alpha} = B^2 \delta_{\mu\nu}.$$

$$-(\partial_\mu - iB_\mu)^2 G = \delta \quad \longrightarrow \quad G(x-y) \sim \frac{e^{-B(x-y)^2/4}}{(x-y)^2}$$

$$-(\partial_\mu - i\check{B}_\mu)^2 \Phi = \lambda \Phi \quad \longrightarrow \quad \left[\beta_\pm^+ \beta_\pm + \gamma_\pm^+ \gamma_\pm + 1 \right] \Phi = \frac{\lambda}{4B} \Phi,$$

$$\beta_\pm = \frac{1}{2}(\alpha_1 \mp i\alpha_2), \quad \gamma_\pm = \frac{1}{2}(\alpha_3 \mp i\alpha_4), \quad \alpha_\mu = \frac{1}{\sqrt{B}} x_\mu + \partial_\mu,$$

$$\beta_\pm^+ = \frac{1}{2}(\alpha_1^+ \pm i\alpha_2^+), \quad \gamma_\pm^+ = \frac{1}{2}(\alpha_3^+ \pm i\alpha_4^+), \quad \alpha_\mu^+ = \frac{1}{\sqrt{B}} x_\mu - \partial_\mu.$$

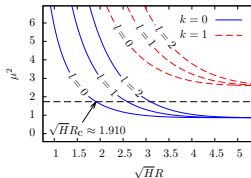
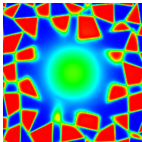
The eigenfunctions and eigenvalues - 4-dim. harmonic oscillator

$$\Phi_{nmkl}(x) = \frac{1}{\pi^2 \sqrt{n!m!k!l!}} \left(\beta_+^+ \right)^k \left(\beta_-^+ \right)^l \left(\gamma_+^+ \right)^n \left(\gamma_-^+ \right)^m \Phi_{0000}, \quad \Phi_{0000} = e^{-\frac{1}{2} B x^2}$$

$\lambda_r = 4B(r+1)$, $r = k+n$ self-dual field, $r = l+n$ anti-self-dual field

Domain wall junctions - deconfinement

S.N. , V.E. Voronin, Eur.Phys.J. A51 (2015) 4



The color charged scalar field inside junction:

$$-\left(\partial_\mu - i\check{B}_\mu\right)^2 \Phi = 0, \quad \Phi(x) = 0, \quad x \in \partial\mathcal{T}, \quad \mathcal{T} = \{x_1^2 + x_2^2 < R^2, (x_3, x_4) \in \mathbb{R}^2\}$$

The solutions are quasi-particle excitations

$$\phi^a(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[a_{akl}^+(p_3) e^{ix_0\omega_{akl} - ip_3x_3} + b_{akl}(p_3) e^{-ix_0\omega_{akl} + ip_3x_3} \right] e^{il\vartheta} \phi_{alk}(r),$$

$$\phi^{a\dagger}(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[b_{akl}^+(p_3) e^{-ix_0\omega_{akl} + ip_3x_3} + a_{akl}(p_3) e^{ix_0\omega_{akl} - ip_3x_3} \right] e^{-il\vartheta} \phi_{alk}(r),$$

$$p_0^2 = p_3^2 + \mu_{akl}^2, \quad p_0 = \pm\omega_{akl}(p_3), \quad \omega_{akl} = \sqrt{p_3^2 + \mu_{akl}^2},$$

$$k = 0, 1, \dots, \infty, \quad l \in \mathbb{Z},$$

Topology driven disorder

In general near the boundaries $\text{div}\vec{H} \neq 0$, $\text{div}\vec{E} \neq 0$

The description of the domain walls as well as separation of the Abelian part in the general network in terms of the vector potential requires application of the gauge field parametrization suggested by [L.D. Faddeev, A. J. Niemi \(2007\)](#); [K.-I. Kondo, T. Shinohara, T. Murakami \(2008\)](#); [Y.M. Cho \(1980, 1981\)](#); [L.Prokhorov, S.V. Shabanov \(1989,1999\)](#)

The Abelian part $\hat{V}_\mu(x)$ of the gauge field $\hat{A}_\mu(x)$ is separated manifestly,

$$\begin{aligned}\hat{A}_\mu(x) &= \hat{V}_\mu(x) + \hat{X}_\mu(x), \quad \hat{V}_\mu(x) = \hat{B}_\mu(x) + \hat{C}_\mu(x), \\ \hat{B}_\mu(x) &= [n^a A_\mu^a(x)]\hat{n}(x) = B_\mu(x)\hat{n}(x), \\ \hat{C}_\mu(x) &= g^{-1}\partial_\mu\hat{n}(x) \times \hat{n}(x), \\ \hat{X}_\mu(x) &= g^{-1}\hat{n}(x) \times \left(\partial_\mu\hat{n}(x) + g\hat{A}_\mu(x) \times \hat{n}(x)\right),\end{aligned}$$

where $\hat{A}_\mu(x) = A_\mu^a(x)t^a$, $\hat{n}(x) = n_a(x)t^a$, $n^a n^a = 1$, and

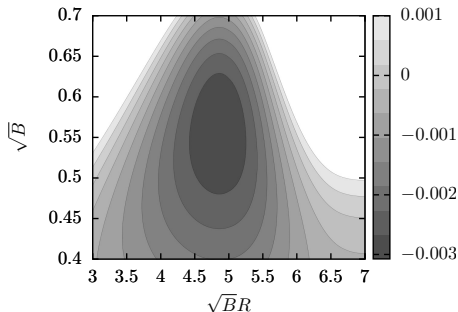
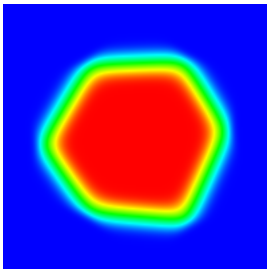
$$\partial_\mu\hat{n} \times \hat{n} = if^{abc}\partial_\mu n^a n^b t^c, \quad [t^a, t^b] = if^{abc}t^c.$$

$$[\hat{V}_\mu(x), \hat{V}_\nu(x)] = 0$$

Both the color and space orientation of the field can become frustrated at the junction location and, thus, develop the singularities in the vector potential. [Potentially singularities cover the whole range of defects – vortex-like, monopole/dyon-like and instanton-like defects.](#)

Energy driven disorder

Finite size effects in the free energy density for Abelian (anti-)self-dual gluon field in $SU(3)$ gluodynamics.

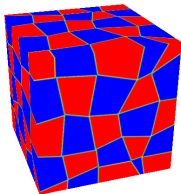


Minimum of free energy density at finite values of the **field strength and size of the domain**.

$$\begin{aligned}
 U &= V_R F(B, R) = U^{\text{cl}} + \delta U \\
 &= \frac{\pi^2 B^2 R^4}{2g^2} - \text{Tr} \ln \left[-\check{D}^2 \right] + \frac{1}{2} \text{Tr} \ln \left[-\check{D}^2 \delta_{\mu\nu} + 2i \check{B}_{\mu\nu} \right]' - \text{Tr} \ln \left[\hat{\mathcal{D}} \right]' + \delta U_0
 \end{aligned}$$

Normalization $U|_{B=0} = 0$.

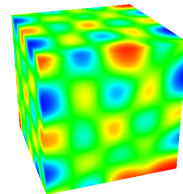
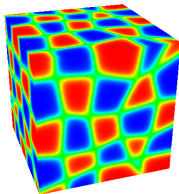
V.Voronin & SN Phys.Rev.D 95 (2017) 7, 074038; Phys.Rev.D 103 (2021)



$$\langle F^2 \rangle = B^2$$

$$\langle |F\tilde{F}| \rangle = B^2$$

Confinement - colorless
hadrons



$$\langle F^2 \rangle = B^2$$

$$\langle |F\tilde{F}| \rangle \ll B^2$$

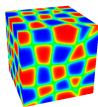
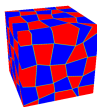
Deconfinement - color
charged quasi-particles

"Phase transitions and heterophase fluctuations",
V. I. Yukalov, Phys. Rep. 208, 396 (1991)

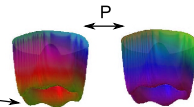
An ensemble of almost everywhere (in R^4) homogeneous Abelian (anti-)self-dual gluon fields

$$\langle : g^2 F^2 : \rangle \neq 0, \quad \chi = \int d^4x \langle Q(x) Q(0) \rangle \neq 0, \quad \langle Q(x) \rangle = 0$$

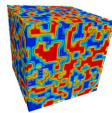
Topological charge density $Q(x) = \frac{g^2}{32\pi^2} F_{\mu\nu}^a(x) \tilde{F}_{\mu\nu}^a(x)$



Weyl
reflections



Domain wall network (S.N., V.E. Voronin, EPJA (2015); A. Kalloniatis, S.N., PRD (2001))



Lattice confining configuration (P.J. Moran, D.B. Leinweber, arXiv:0805.4246v1 [hep-lat])

Testing the model: characteristics of the domain wall network ensemble

Spherical domains

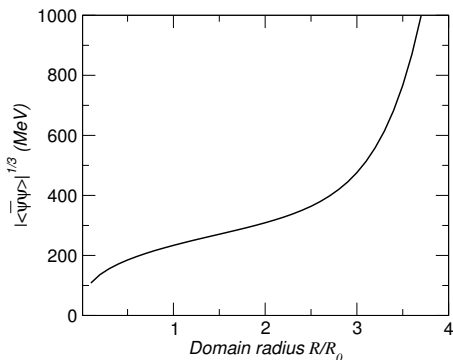
A.C. Kalloniatis and S.N. , Phys. Rev. D 64 (2001); Phys. Rev. D 69 (2004); Phys. Rev. D 71 (2005); Phys. Rev. D 73 (2006), Eur.Phys.J. A51 (2015), arXiv:1603.01447 [hep-ph] (2016)

Area law

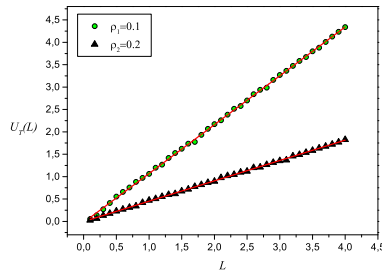
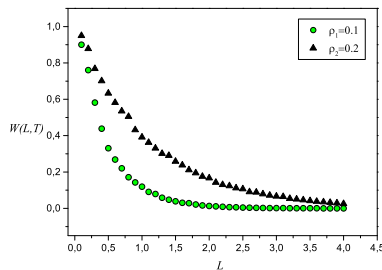
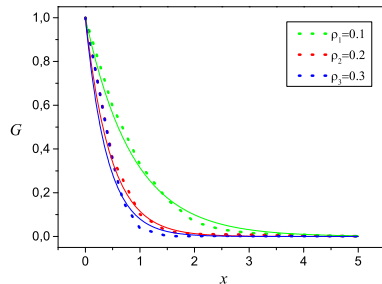
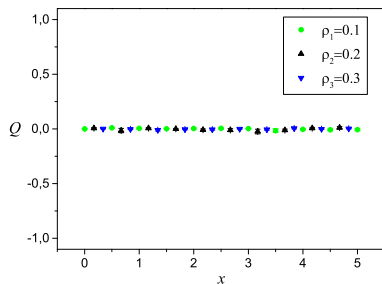
Spontaneous chiral symmetry breaking

$U_A(1)$ is broken by anomaly

There is no strong CP violation



Pure glue, domains - tubes with two finite dimensions (mean topological charge, two-point correlator of top. charge density, Wilson loop and static potential)



Collective excitations, composite fields

QCD bosonization

$$Z = \int_{\mathcal{B}} DB \int_{\Psi} \mathcal{D}\psi \mathcal{D}\bar{\psi} \int_{\mathcal{Q}} \mathcal{D}Q \delta[D(B)Q] \Delta_{\text{FP}}[B, Q] e^{-S^{\text{QCD}}[Q+B, \psi, \bar{\psi}]} =$$

$$\int_{\mathcal{B}} dB \int_{\Psi} \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp \left\{ \int dx \bar{\psi} (i \not{\partial} + g \not{B} - m) \psi \right\} W[j]$$

$$W[j] = \exp \left\{ \sum_n \frac{g^n}{n!} \int dx_1 \cdots \int dx_n j_{\mu_1}^{a_1}(x_1) \cdots j_{\mu_n}^{a_n}(x_n) G_{\mu_1 \dots \mu_n}^{a_1 \dots a_n}(x_1, \dots, x_n | B) \right\}$$

$$j_{\mu}^a = \bar{\psi} \gamma_{\mu} t^a \psi,$$

$W[j]$ is truncated up to the term including two-point gluon correlation function. Fierz transform

$$\gamma_{\mu}^{\alpha\beta} \gamma_{\nu}^{\delta\rho} = \sum_J C_{\mu\nu}^J \Gamma_{\alpha\rho}^J \Gamma_{\beta\delta}^J, \quad \Gamma^J \in \{I, \gamma_5, \gamma_{\mu}, i\gamma_5 \gamma_{\mu}, \sigma_{\mu\nu}\}$$

$$t_a^{ik} t_a^{jl} = \frac{1}{2N} \delta_{ki} \delta_{lj} + \frac{1}{2} \delta_{li} \delta_{kj} \quad \delta_{li} \delta_{kj} = \frac{1}{N} \delta_{ki} \delta_{lj} + T_{ik}^a t_{jl}^a$$

$$\bar{\psi}_f^{i\alpha}(x) \gamma_{\mu}^{\alpha\beta} t_{ik}^a \psi_f^{k\beta}(x) \times \bar{\psi}_f^{j\delta}(x) \gamma_{\mu}^{\delta\rho} t_{jl}^a \psi_f^{l\beta}(x)$$

Effective meson action for composite colorless fields:

$$Z = \mathcal{N} \lim_{V \rightarrow \infty} \int D\Phi_{\mathcal{Q}} \exp \left\{ -\frac{B}{2} \frac{h_{\mathcal{Q}}^2}{g^2 C_{\mathcal{Q}}} \int dx \Phi_{\mathcal{Q}}^2(x) - \sum_k \frac{1}{k} W_k[\Phi] \right\}, \quad \mathcal{Q} = (aJln)$$

$$1 = \frac{g^2 C_{\mathcal{Q}}}{B} \tilde{\Gamma}_{\mathcal{Q}\mathcal{Q}}^{(2)}(-M_{\mathcal{Q}}^2|B), \quad h_{\mathcal{Q}}^{-2} = \frac{d}{dp^2} \tilde{\Gamma}_{\mathcal{Q}\mathcal{Q}}^{(2)}(p^2)|_{p^2=-M_{\mathcal{Q}}^2}$$

$$W_k[\Phi] = \sum_{\mathcal{Q}_1 \dots \mathcal{Q}_k} h_{\mathcal{Q}_1} \dots h_{\mathcal{Q}_k} \int dx_1 \dots \int dx_k \Phi_{\mathcal{Q}_1}(x_1) \dots \Phi_{\mathcal{Q}_k}(x_k) \Gamma_{\mathcal{Q}_1 \dots \mathcal{Q}_k}^{(k)}(x_1, \dots, x_k|B)$$

$$\Gamma_{\mathcal{Q}_1 \mathcal{Q}_2}^{(2)} = \overline{G_{\mathcal{Q}_1 \mathcal{Q}_2}^{(2)}(x_1, x_2)} - \Xi_2(x_1 - x_2) \overline{G_{\mathcal{Q}_1}^{(1)} G_{\mathcal{Q}_2}^{(1)}},$$

$$\begin{aligned} \Gamma_{\mathcal{Q}_1 \mathcal{Q}_2 \mathcal{Q}_3}^{(3)} &= \overline{G_{\mathcal{Q}_1 \mathcal{Q}_2 \mathcal{Q}_3}^{(3)}(x_1, x_2, x_3)} - \frac{3}{2} \Xi_2(x_1 - x_3) \overline{G_{\mathcal{Q}_1 \mathcal{Q}_2}^{(2)}(x_1, x_2) G_{\mathcal{Q}_3}^{(1)}(x_3)} \\ &\quad + \frac{1}{2} \Xi_3(x_1, x_2, x_3) \overline{G_{\mathcal{Q}_1}^{(1)}(x_1) G_{\mathcal{Q}_2}^{(1)}(x_2) G_{\mathcal{Q}_3}^{(1)}(x_3)}, \end{aligned}$$

$$\begin{aligned} \Gamma_{\mathcal{Q}_1 \mathcal{Q}_2 \mathcal{Q}_3 \mathcal{Q}_4}^{(4)} &= \overline{G_{\mathcal{Q}_1 \mathcal{Q}_2 \mathcal{Q}_3 \mathcal{Q}_4}^{(4)}(x_1, x_2, x_3, x_4)} - \frac{4}{3} \Xi_2(x_1 - x_2) \overline{G_{\mathcal{Q}_1}^{(1)}(x_1) G_{\mathcal{Q}_2 \mathcal{Q}_3 \mathcal{Q}_4}^{(3)}(x_2, x_3, x_4)} \\ &\quad - \frac{1}{2} \Xi_2(x_1 - x_3) \overline{G_{\mathcal{Q}_1 \mathcal{Q}_2}^{(2)}(x_1, x_2) G_{\mathcal{Q}_3 \mathcal{Q}_4}^{(2)}(x_3, x_4)} \\ &\quad + \Xi_3(x_1, x_2, x_3) \overline{G_{\mathcal{Q}_1}^{(1)}(x_1) G_{\mathcal{Q}_2}^{(1)}(x_2) G_{\mathcal{Q}_3 \mathcal{Q}_4}^{(2)}(x_3, x_4)} \\ &\quad - \frac{1}{6} \Xi_4(x_1, x_2, x_3, x_4) \overline{G_{\mathcal{Q}_1}^{(1)}(x_1) G_{\mathcal{Q}_2}^{(1)}(x_2) G_{\mathcal{Q}_3}^{(1)}(x_3) G_{\mathcal{Q}_4}^{(1)}(x_4)}. \end{aligned}$$

$$\overline{G_{\mathcal{Q}_1 \dots \mathcal{Q}_k}^{(k)}(x_1, \dots, x_k)} = \int dB_j \text{Tr} V_{\mathcal{Q}_1} \left(x_1 | B^{(j)} \right) S \left(x_1, x_2 | B^{(j)} \right) \dots \\ \dots V_{\mathcal{Q}_k} \left(x_k | B^{(j)} \right) S \left(x_k, x_1 | B^{(j)} \right)$$

$$\overline{G_{\mathcal{Q}_1 \dots \mathcal{Q}_l}^{(l)}(x_1, \dots, x_l) G_{\mathcal{Q}_{l+1} \dots \mathcal{Q}_k}^{(k)}(x_{l+1}, \dots, x_k)} = \\ \int dB_j \text{Tr} \left\{ V_{\mathcal{Q}_1} \left(x_1 | B^{(j)} \right) S \left(x_1, x_2 | B^{(j)} \right) \dots V_{\mathcal{Q}_k} \left(x_l | B^{(j)} \right) S \left(x_l, x_1 | B^{(j)} \right) \right\} \\ \times \text{Tr} \left\{ V_{\mathcal{Q}_{l+1}} \left(x_{l+1} | B^{(j)} \right) S \left(x_{l+1}, x_{l+2} | B^{(j)} \right) \dots V_{\mathcal{Q}_k} \left(x_k | B^{(j)} \right) S \left(x_k, x_{l+1} | B^{(j)} \right) \right\},$$

Bar denotes integration over all configurations of the background field with measure dB_j .

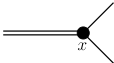
$$\langle \exp(i B_{\mu\nu} J_{\mu\nu}) \rangle = \frac{\sin W}{W}$$

$$W = \sqrt{2B^2 \left(J_{\mu\nu} J_{\mu\nu} \pm J_{\mu\nu} \tilde{J}_{\mu\nu} \right)}$$

$J_{\mu\nu}$ is a tensor, in general composed of the momenta $p_{1\mu_1} \dots p_{n\mu_n}$ - arguments of the meson interaction vertex

$$\tilde{\Gamma}^{(n)}(p_{1\mu_1} \dots p_{n\mu_n})$$

Meson-quark vertex operators $\Leftarrow J_{\mu_1 \dots \mu_l}^{aJln} = \bar{q}(x) V_{\mu_1 \dots \mu_l}^{aJln} q(x)$



$$V_{\mu_1 \dots \mu_l}^{aJln}(x) = M^a \Gamma^J \left\{ \left\{ F_{nl} \left(\frac{\overleftrightarrow{D}(x)}{B^2} \right) T_{\mu_1 \dots \mu_l}^{(l)} \left(\frac{1}{i} \frac{\overleftrightarrow{D}(x)}{B} \right) \right\} \right\},$$

$$F_{nl}(s) = s^n \int_0^1 dt t^{n+l} \exp(st) = \int_0^1 dt t^{n+l} \frac{\partial^n}{\partial t^n} \exp(st),$$

$$\overleftrightarrow{D} \! = \! \overleftarrow{D} \, \xi_{f'} \! - \! \overrightarrow{D} \, \xi_f, \xi_f = \frac{m_f}{m_f + m_{f'}}$$

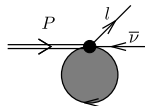
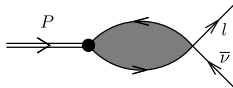
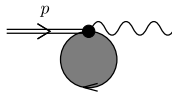
Quark propagator in homogeneous Abelian (anti-)self-dual field

$$\longrightarrow = \overrightarrow{\hspace{0.5cm}} \Big[1 + \Sigma^{\rm R}(p^2)\Big] \, ; \, \Sigma^{\rm R}(0) = 0 \qquad S(x,y) = \exp\left(-\frac{i}{2} x_\mu B_{\mu\nu} y_\nu\right) H(x-y),$$

$$\begin{aligned} \tilde{H}_f(p|B) = \frac{1}{vB^2} \int_0^1 ds e^{(-p^2/vB^2)s} \left(\frac{1-s}{1+s}\right)^{m_f^2/2vB^2} &\left[p_\alpha \gamma_\alpha \pm i s \gamma_5 \gamma_\alpha \frac{B_{\alpha\beta}}{vB^2} p_\beta + \right. \\ &\left. + m_f \left(P_\pm + P_\mp \frac{1+s^2}{1-s^2} - \frac{i}{2} \gamma_\alpha \frac{B_{\alpha\beta}}{vB^2} \gamma_\beta \frac{s}{1-s^2} \right) \right] \end{aligned}$$

$$\begin{aligned} \tilde{H}_f(p|B) = \frac{m}{2v\Lambda^2} \mathcal{H}_S(p^2) \mp \gamma_5 \frac{m}{2v\Lambda^2} \mathcal{H}_P(p^2) + \gamma_\alpha \frac{p_\alpha}{2v\Lambda^2} \mathcal{H}_V(p^2) \pm i \gamma_5 \gamma_\alpha \frac{f_{\alpha\beta} p_\beta}{2v\Lambda^2} \mathcal{H}_A(p^2) \qquad (1) \\ + \sigma_{\alpha\beta} \frac{mf_{\alpha\beta}}{4v\Lambda^2} \mathcal{H}_T(p^2). \end{aligned}$$

Weak and electromagnetic interactions



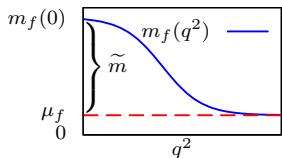
Masses of radially excited mesons

The parameters of the derived effective meson action

$$\alpha_s(0) \quad m_{u/d}(0) \quad m_s(0) \quad m_c(0) \quad m_b(0) \quad B \quad R$$

$$\langle \alpha_s F^2 \rangle = \frac{B^2}{\pi} \quad \chi_{\text{YM}} = \frac{B^4 R^4}{128\pi^2}$$

Dynamical chiral symmetry breaking:



$$\tilde{m} = 136 \text{ MeV}$$

$$\mu_{u/d} = m_{u/d} - \tilde{m}$$

$$\mu_s = m_s - \tilde{m}$$

$$\frac{\mu_s}{\mu_{u/d}} = 26.7$$

$$\Lambda^2 \Phi_{Q_1}^{(0)} = \sum_{k=1}^{\infty} \frac{g^k}{k} \sum_{Q_1 \dots Q_k} \Phi_{Q_2}^{(0)} \dots \Phi_{Q_k}^{(0)} \Gamma_{Q_1 \dots Q_k}^{(k)},$$

$$\text{---} + \text{---} \overset{\Phi_n^{(0)}}{\underset{F_{n0}}{\bullet}} \text{---} + \text{---} \bullet \text{---} \bullet \text{---} + \dots = \text{---}$$

Figure: Mass corrections to the quark propagator due to the constant scalar condensates $\Phi_n^{(0)}$ coupled to nonlocal form factor F_{n0} . Summation over the radial number n is assumed.

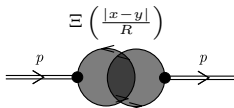
Asymptotic Regge spectrum :

$$M_n^2 \sim Bn, \quad n \gg 1$$

$$M_l^2 \sim Bl, \quad l \gg 1$$

G.V. Efimov and S.N. , Phys. Rev. D 51 (1995)

η and η' !



Polarization operator

Polarization operation for $l = 0$:

$$\Pi_J^{nn'}(-M^2; m_f, m_{f'}; B) = \frac{B}{4\pi^2} \text{Tr}_v \int_0^1 dt_1 \int_0^1 dt_2 \int_0^1 ds_1 \int_0^1 ds_2 \left(\frac{1-s_1}{1+s_1} \right)^{m_f^2/4vB} \left(\frac{1-s_2}{1+s_2} \right)^{m_{f'}^2/4vB} \times \\ \times t_1^n t_2^{n'} \frac{\partial^n}{\partial t_1^n} \frac{\partial^{n'}}{\partial t_2^{n'}} \frac{1}{\Phi_2^2} \left[\frac{M^2}{B} \frac{F_1^{(J)}}{\Phi_2^2} + \frac{m_f m_{f'}}{B} \frac{F_2^{(J)}}{(1-s_1^2)(1-s_2^2)} + \frac{F_3^{(J)}}{\Phi_2} \right] \exp \left\{ \frac{M^2}{2vB} \frac{\Phi_1}{\Phi_2} \right\}.$$

$$\Phi_1 = s_1 s_2 + 2(\xi_1^2 s_1 + \xi_2^2 s_2)(t_1 + t_2)v,$$

$$\Phi_2 = s_1 + s_2 + 2(1 + s_1 s_2)(t_1 + t_2)v + 16(\xi_1^2 s_1 + \xi_2^2 s_2)t_1 t_2 v^2,$$

$$F_1^{(P)} = (1 + s_1 s_2) [2(\xi_1 s_1 + \xi_2 s_2)(t_1 + t_2)v + 4\xi_1 \xi_2 (1 + s_1 s_2)(t_1 + t_2)^2 v^2 + s_1 s_2 (1 - 16\xi_1 \xi_2 t_1 t_2 v^2)],$$

$$F_1^{(V)} = \left(1 - \frac{1}{3} s_1 s_2\right) [s_1 s_2 + 16\xi_1 \xi_2 t_1 t_2 v^2 + 2(\xi_1 s_1 + \xi_2 s_2)(t_1 + t_2)v] + 4\xi_1 \xi_2 (1 - s_1^2 s_2^2)(t_1 - t_2)^2 v^2,$$

$$F_2^{(P)} = (1 + s_1 s_2)^2, \quad F_2^{(V)} = (1 - s_1^2 s_2^2),$$

$$F_3^{(P)} = 4v(1 + s_1 s_2)(1 - 16\xi_1 \xi_2 t_1 t_2 v^2), \quad F_3^{(V)} = 2v(1 - s_1 s_2)(1 - 16\xi_1 \xi_2 t_1 t_2 v^2).$$

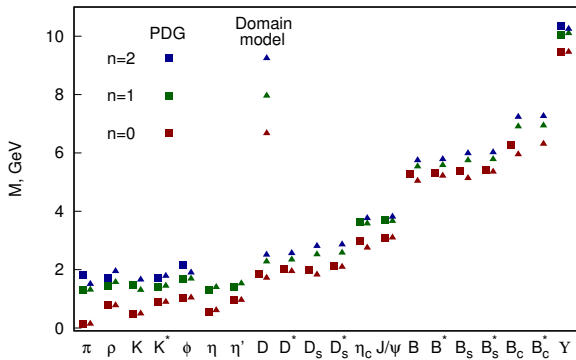


Table: Model parameters fitted to the masses of $\pi, \rho, K, K^*, \eta', J/\psi, \Upsilon$ and used in calculation of all other quantities.

$m_{u/d}, \text{ MeV}$	$m_s, \text{ MeV}$	$m_c, \text{ MeV}$	$m_b, \text{ MeV}$	$\Lambda, \text{ MeV}$	α_s	$R, \text{ fm}$
145	376	1566	4879	416	3.45	1.12

Table: Masses of light mesons. $\widetilde{M}$ denotes the value in the chiral limit.

Meson	n	M_{exp} (MeV)	M (MeV)	$\widetilde{M}$ (MeV)	Meson	n	M_{exp} (MeV)	M (MeV)	$\widetilde{M}$ (MeV)
π	0	140	140	0	ρ	0	775	775	769
$\pi(1300)$	1	1300	1310	1301	$\rho(1450)$	1	1450	1571	1576
$\pi(1800)$	1	1812	1503	1466	ρ	2	1720	1946	2098
K	0	494	494	0	K^*	0	892	892	769
$K(1460)$	1	1460	1302	1301	$K^*(1410)$	1	1410	1443	1576
K	2		1655	1466	$K^*(1717)$	1	1717	1781	2098
η	0	548	621	0	ω	0	775	775	769
η'	0	958	958	872	ϕ	0	1019	1039	769
$\eta(1295)$	1	1294	1138	1361	$\phi(1680)$	1	1680	1686	1576
$\eta(1475)$	1	1476	1297	1516	ϕ	2	2175	1897	2098

Table: Masses of heavy-light mesons and their lowest radial excitations .

Meson	n	M_{exp} (MeV)	M (MeV)	Meson	n	M_{exp} (MeV)	M (MeV)
D	0	1864	1715	D^*	0	2010	1944
D	1		2274	D^*	1		2341
D	2		2508	D^*	2		2564
D_s	0	1968	1827	D_s^*	0	2112	2092
D_s	1		2521	D_s^*	1		2578
D_s	2		2808	D_s^*	2		2859
B	0	5279	5041	B^*	0	5325	5215
B	1		5535	B^*	1		5578
B	2		5746	B^*	2		5781
B_s	0	5366	5135	B_s^*	0	5415	5355
B_s	1		5746	B_s^*	1		5783
B_s	2		5988	B_s^*	2		6021
B_c	0	6277	5952	B_c^*	0		6310
B_c	1		6904	B_c^*	1		6938
B_c	2		7233	B_c^*	2		7260

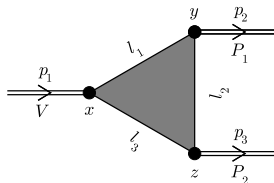
Table: Masses of heavy quarkonia.

Meson	n	M_{exp} (MeV)	M (MeV)
$\eta_c(1S)$	0	2981	2751
$\eta_c(2S)$	1	3639	3620
η_c	2		3882
$J/\psi(1S)$	0	3097	3097
$\psi(2S)$	1	3686	3665
$\psi(3770)$	2	3773	3810
$\Upsilon(1S)$	0	9460	9460
$\Upsilon(2S)$	1	10023	10102
$\Upsilon(3S)$	2	10355	10249

Table: Decay and transition constants of various mesons

Meson	n	f_P^{exp} (MeV)	f_P (MeV)	Meson	n	$g_{V\gamma}^{\text{exp}}$	$g_{V\gamma}$
π	0	130	140	ρ	0	0.2	0.2
$\pi(1300)$	1	—	29	ρ	1		0.034
K	0	156	175	ω	0	0.059	0.067
$K(1460)$	1	—	27	ω	1		0.011
D	0	205	212	ϕ	0	0.074	0.069
D	1	—	51	ϕ	1		0.025
D_s	0	258	274	J/ψ	0	0.09	0.057
D_s	1	—	57	J/ψ	1		0.024
B	0	191	187	Υ	0	0.025	0.011
B	1	—	55	Υ	1		0.0039
B_s	0	253	248				
B_s	1	—	68				
B_c	0	489	434				
B_c	1		135				

Strong decays: g_{VPP}



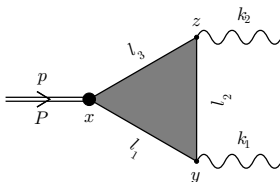
Decay	g_{VPP} [*]	g_{VPP}
$\rho^0 \rightarrow \pi^+ \pi^-$	5.95	7.58
$\omega \rightarrow \pi^+ \pi^-$	0.17	0
$K^{*\pm} \rightarrow K^\pm \pi^0$	3.23	3.54
$K^{*\pm} \rightarrow K^0 \pi^\pm$	4.57	5.01
$\varphi \rightarrow K^+ K^-$	4.47	5.02
$D^{*\pm} \rightarrow D^0 \pi^\pm$	8.41	7.9
$D^{*\pm} \rightarrow D^\pm \pi^0$	5.66	5.59

**local color
gauge
invariance**

[*] K.A. Olive et al. (Particle Data Group) Chinese Phys. C 38,090001, 2014

Pion transition form factor

$$T_a^{\mu\nu}(x, y, z) = h_P \sum_n u_n^a \int d\sigma_B \text{Tr } t_a e_f^2 V^n(x) \gamma_5 S(x, y|B) \gamma_\mu S(y, z|B) \gamma_\nu S(z, x|B),$$



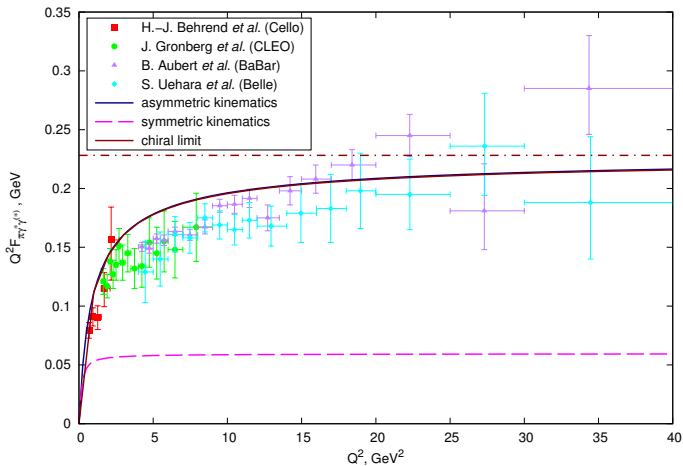
In momentum representation, the diagram has the following structure:

$$T_a^{\mu\nu}(p^2, k_1^2, k_2^2) = ie^2 \delta^{(4)}(p - k_1 - k_2) \varepsilon_{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} T_a(p^2, k_1^2, k_2^2).$$

$$F_{P\gamma}(Q^2) = T(-M_P^2, Q^2, 0).$$

$$\Gamma(P \rightarrow \gamma\gamma) = \frac{\pi}{4} \alpha^2 M_P^3 g_{P\gamma\gamma}^2$$

$$g_{P\gamma\gamma} = T(-M_P^2, 0, 0) = F_{P\gamma}(0).$$



$$g_{\pi\gamma\gamma} = 0.272 \text{ GeV}^{-1} \quad (g_{\pi\gamma\gamma}^{\text{exp}} = 0.274 \text{ GeV}^{-1}).$$

$$F_{\pi\gamma^*\gamma^*}(Q^2) = T(-M_P^2, Q^2, Q^2).$$

$$F_{\pi\gamma^*\gamma} \sim \kappa_{\gamma^*\gamma} \frac{\sqrt{2}f_\pi}{Q^2}, \quad \kappa_{\gamma^*\gamma} = 1.23, \quad (2)$$

$Q^2 F_{\pi\gamma^*\gamma}$ approaches a constant value at large Q^2 in qualitative agreement with factorization prediction, but the value of constant $\kappa_{\gamma^*\gamma}$ substantially differs from unity.

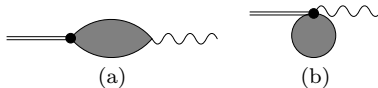
At the same time, asymptotic behavior of form factor in symmetric kinematics with two photons with equal virtuality Q^2 ,

$$F_{\pi\gamma^*\gamma^*} \sim \kappa_{\gamma^*\gamma^*} \frac{\sqrt{2}f_\pi}{3Q^2}, \quad \kappa_{\gamma^*\gamma^*} = 1. \quad (3)$$

Leptonic decays

The amplitude for leptonic decay of pseudoscalar mesons can be parametrized as

$$A(H(p) \rightarrow \ell(k) \bar{\nu}_\ell(k')) = \frac{G_F}{\sqrt{2}} V_{qq'} \bar{\ell} \gamma_\mu (1 - \gamma_5) \nu_\ell M_H^\mu = i \frac{G_F}{\sqrt{2}} V_{qq'} \bar{\ell} \gamma_\mu (1 - \gamma_5) \nu_\ell f_H p^\mu.$$



$$(2\pi)^4 \delta^{(4)}(p+q) M_\mu^{(a)} = h_H \int d^4x \int d^4y \exp(ipx + iqy) \times (-1) \int d\sigma_B \text{Tr} V^{\mathcal{H}}(x) S(x, y) V^{\text{CKM}} \gamma_\mu (1 - \gamma_5) S(y, x), \quad (\text{a})$$

$$(2\pi)^4 \delta^{(4)}(p+q) M_\mu^{(b)} = h_H \int d^4x \exp(ipx + iqx) (-1) \int d\sigma_B \text{Tr} V_\mu^{\mathcal{H}}(x; q) S(x, x). \quad (\text{b})$$

$$V_{W_\mu^+ f_1 f_2}^{\mathcal{H}}(x; q) = -\frac{g}{\sqrt{2}} \int_0^1 d\tau \frac{1}{\tau} \frac{\partial}{\partial q_\mu}$$

$$\left\{ R \begin{pmatrix} 0 & V^{\text{CKM}} \\ 0 & 0 \end{pmatrix} \right\}_{f_1 f} V_{ff'}^{\mathcal{H}} \left(\overleftrightarrow{\nabla}(x) - iq\tau\xi \right) \delta_{f' f_2} - \delta_{f_1 f} V_{ff'}^{\mathcal{H}} \left(\overleftrightarrow{\nabla}(x) + iq\tau\xi' \right) \begin{pmatrix} 0 & V^{\text{CKM}} \\ 0 & 0 \end{pmatrix}.$$

V.Voronin, Phys.Rev.D 110 (2024), 014040

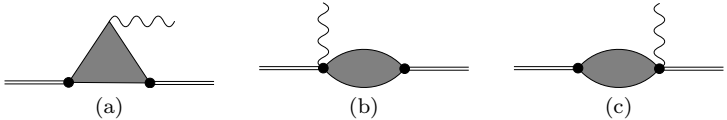
meson	decay constant f_P , MeV	f_P , MeV, this model
π	131.7 [PDG]	140.4
K	157.3 [PDG]	178.6
D	208.5 [PDG]	231.1
D_s	251.8 [PDG]	286.8
B	205.7 [PDG]	203
B_s	230.7 [PDG]	262.8
B_c	$427 \pm 6 \pm 2$ [McNeile, 2012]	450.2

[McNeile, 2012] C. McNeile, C. T. H. Davies, E. Follana, K. Hornbostel and G. P. Lepage, Phys. Rev. D **86** (2012), 074503.

Semileptonic decays

The amplitude for semileptonic decay of pseudoscalar mesons can be parametrized as

$$A(H(p) \rightarrow H'(p')\ell\bar{\nu}_\ell) = \frac{G_F}{\sqrt{2}} V_{qq'} \bar{\ell} \gamma_\mu (1 - \gamma_5) \nu_\ell M_{HH'}^\mu.$$



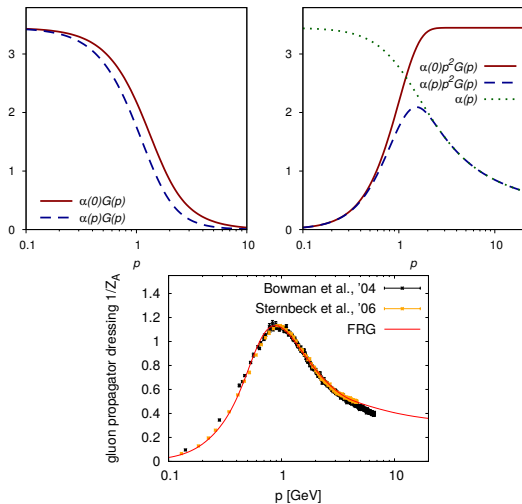
$$(2\pi)^4 \delta^{(4)}(p + p' + q) M_\mu^{(a)} = h_H h_{H'} \int d^4x \int d^4y \int d^4z \exp(ipx + ip'y + iqz) \\ \times (-1) \int d\sigma_B \text{Tr} V^\mathcal{H}(x) S(x, y) V^\mathcal{H}(y) S(y, z) V^{\text{CKM}} \gamma_\mu (1 - \gamma_5) S(z, x) \quad (\text{a})$$

$$(2\pi)^4 \delta^{(4)}(p + p' + q) M_\mu^{(b)} = h_H h_{H'} \int d^4x \int d^4y \exp(ipx + ip'y + iqx) \\ \times (-1) \int d\sigma_B \text{Tr} V_\mu^\mathcal{H}(x; q) S(x, y) V^\mathcal{H}(y) S(y, x), \quad (\text{b})$$

$$(2\pi)^4 \delta^{(4)}(p + p' + q) M_\mu^{(c)} = h_H h_{H'} \int d^4x \int d^4y \exp(ipx + ip'y + iqx) \\ \times (-1) \int d\sigma_B \text{Tr} V_\mu^\mathcal{H}(x) S(x, y) V_\mu^\mathcal{H}(y; q) S(y, x), \quad (\text{c})$$

decay	decay width Γ_{exp} , MeV [PDG]	Γ , MeV, this model
$K^- \rightarrow \pi^0 e^- \bar{\nu}_e$	$(2.70 \pm 0.021) \times 10^{-15}$	2.49×10^{-15}
$K^- \rightarrow \pi^0 \mu^- \bar{\nu}_\mu$	$(1.78 \pm 0.018) \times 10^{-15}$	1.62×10^{-15}
$K_L \rightarrow \pi^\pm e^\mp \nu_e$	$(5.22 \pm 0.014) \times 10^{-15}$	4.97×10^{-15}
$K_L \rightarrow \pi^\pm \mu^\mp \nu_\mu$	$(3.48 \pm 0.009) \times 10^{-15}$	3.24×10^{-15}
$D^0 \rightarrow K^- e^+ \nu_e$	$(5.69 \pm 0.042) \times 10^{-11}$	5.9×10^{-11}
$D^0 \rightarrow K^- \mu^+ \nu_\mu$	$(5.47 \pm 0.064) \times 10^{-11}$	5.73×10^{-11}
$D^0 \rightarrow \pi^- e^+ \nu_e$	$(4.67 \pm 0.064) \times 10^{-12}$	4.67×10^{-12}
$D^0 \rightarrow \pi^- \mu^+ \nu_\mu$	$(4.28 \pm 0.19) \times 10^{-12}$	4.57×10^{-12}
$D^+ \rightarrow K^0 e^+ \nu_e$	$(5.56 \pm 0.057) \times 10^{-11}$	5.9×10^{-11}
$D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$	$(5.58 \pm 0.12) \times 10^{-11}$	5.73×10^{-11}
$D^+ \rightarrow \pi^0 e^+ \nu_e$	$(2.37 \pm 0.11) \times 10^{-12}$	2.33×10^{-11}
$D^+ \rightarrow \pi^0 \mu^+ \nu_\mu$	$(2.23 \pm 0.096) \times 10^{-12}$	2.29×10^{-11}

"Projection" to other methods/models



Functional RG, DSE, Lattice QCD

DSE in combination with Bethe-Salpeter approach

S. Kubrak, C. S. Fischer and R. Williams, arXiv:1412.5395 [hep-ph]

C. S. Fischer, S. Kubrak and R. Williams, Eur. Phys. J. A **51**, no. 1, 10 (2015) [arXiv:1409.5076 [hep-ph]]

C. S. Fischer, S. Kubrak and R. Williams, Eur. Phys. J. A **50**, 126 (2014) [arXiv:1406.4370 [hep-ph]].

S. M. Dorkin, L. P. Kaptari and B. Kampfer, arXiv:1412.3345 [hep-ph]

S. M. Dorkin, L. P. Kaptari, T. Hilger and B. Kampfer, Phys. Rev. C **89**, no. 3, 034005 (2014)
[arXiv:1312.2721 [hep-ph]]

$$S^{-1}(p) = Z_2 S_0^{-1}(p) + 4\pi Z_2^2 C_F \int \frac{d^4 k}{(2\pi)^4} \gamma^\mu S(k+p) \gamma^\nu (\delta_{\mu\nu} - k_\mu k_\nu / k^2) \frac{\alpha_{\text{eff}}(k^2)}{k^2},$$
$$\alpha_{\text{eff}}(q^2) = \pi \eta^7 x^2 e^{-\eta^2 x} + \frac{2\pi \gamma_m (1 - e^{-y})}{\ln[e^2 - 1 + (1 + z)^2]}, \quad x = q^2 / \Lambda^2, \quad y = q^2 / \Lambda_t^2, \quad z = q^2 / \Lambda_{\text{QCD}}^2$$

Harmonic confinement - 4-dim. oscillator

R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D **3** (1971) 2706.

H. Leutwyler and J. Stern, “Harmonic Confinement: A Fully Relativistic Approximation to the Meson Spectrum,” Phys. Lett. B **73** (1978) 75;

H. Leutwyler and J. Stern, “Relativistic Dynamics on a Null Plane,” Annals Phys. **112** (1978) 94.

Laguerre polynomials

$$\begin{aligned} \mathcal{S}_2 &= -\frac{1}{2} \int d^4x \int d^4z D(z) \Phi_{Jc}^2(x, z) \\ &\quad - 2g^2 \int d^4x d^4x' d^4z d^4z' D(z) D(z') \Phi_{Jc}(x, z) \Pi_{Jc, J'c'}(x, x'; z, z') \Phi_{J'c'}(x', z'), \\ \Phi^{aJ}(x, z) &= \sum_{nl} (z^2)^{l/2} \varphi^{nl}(z) \Phi^{aJln}(x). \end{aligned}$$

Soft-wall AdS/QCD models

A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D **74**, 015005 (2006) [hep-ph/0602229]
 T. Branz, T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D **82**, 074022 (2010)
 [arXiv:1008.0268 [hep-ph]].

G. F. de Teramond and S. J. Brodsky, Phys. Rev. Lett. **102**, 081601 (2009)

$$S_\Phi = \frac{(-1)^J}{2} \int d^d x \, dz \left(\frac{R}{z} \right)^{d+1} e^{-\kappa^2 z^2} \left(\partial_N \Phi_J \partial^N \Phi_J - \mu_J^2(z) \Phi_J \Phi^J \right)$$

$$M_{nJ} = 4\kappa^2 \left(n + \frac{l+J}{2} \right) \quad \Phi_J(x, z) = \sum_n \phi_{nJ} \Phi_{nJ}(x),$$

$$\phi_{nJ} = R^{J-(d-1)/2} \kappa^{1+l} z^{l-J+2} L_n^l(\kappa^2 z^2)$$

G.V. Efimov and S.N. Nedelko, Phys. Rev. D **51**, 176 (1995)

$$J^{aJ}(x, z) = \sum_{nl} (z^2)^{l/2} f_J^{nl}(z) J_J^{aJln}(x), \quad J_J^{aJln}(x) = \bar{q}(x) V_J^{aJln}(x) q(x)$$

$$M_n^2 \propto Bn$$

$$M_l^2 \propto Bl$$

$$f_{\mu_1 \dots \mu_l}^{nl} = L_n^l(z^2) T_{\mu_1 \dots \mu_l}^{(l)}(n_z), \quad n_z = \frac{z}{\sqrt{z}},$$

$$\int_0^\infty du \rho_l(u) L_n^l(u) L_{n'}^l(u) = \delta_{nn'}, \quad \rho_l(u) = u^l e^{-u}.$$

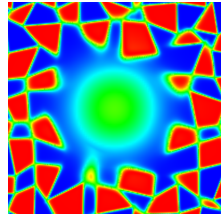
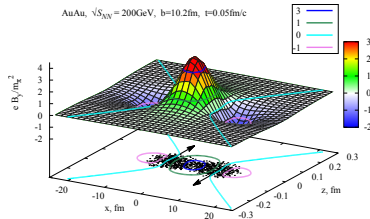
"Polarization" of QCD vacuum by the strong electromagnetic fields

- Relativistic heavy ion collisions - strong electromagnetic fields

V. Skokov, A. Y. Illarionov and V. Toneev, Int. J. Mod. Phys. A **24** (2009) 5925

V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya,

V. P. Konchakovski and S. A. Voloshin, Phys. Rev C **84** (2011)



Strong electro-magnetic field plays catalyzing role for deconfinement and anisotropies!

One-loop quark contribution to the effective potential in the background of Abelian self-dual $SU(3)$ gauge field and electromagnetic field

$$(i\mathcal{D} - m)S(x, y) = -\delta(x - y)$$

$$\mathcal{D} = \gamma_\mu D_\mu, \quad D_\mu = \partial_\mu - iG_\mu, \quad \{\gamma_\mu, \gamma_\nu\} = -2\delta_{\mu\nu}$$

$$G_\mu = \hat{B}_\mu + A_\mu,$$

$$G_\mu = -\frac{1}{2}G_{\mu\nu}x_\nu, \quad G_{\mu\nu} = F_{\mu\nu} + nB_{\mu\nu}, \quad \tilde{G}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}G_{\alpha\beta}$$

$$G_{ij} = \varepsilon_{ijk}\mathcal{H}_k, \quad G_{4k} = \mathcal{E}_k,$$

$$\vec{\mathcal{H}} = n\vec{B} + \vec{H}, \quad \vec{\mathcal{E}} = \pm n\vec{B} + \vec{E},$$

$$\vec{\mathcal{H}}\vec{\mathcal{E}} = \frac{1}{4}G_{\mu\nu}\tilde{G}_{\mu\nu} = \mathcal{Q}, \quad \frac{1}{2}(\vec{\mathcal{H}}^2 + \vec{\mathcal{E}}^2) = \frac{1}{4}G_{\mu\nu}G_{\mu\nu} = \mathcal{R}.$$

$$\begin{aligned} \mathcal{Z}(G) &= \mathcal{Z}^{-1}(0) \int D\psi D\bar{\psi} \exp \left\{ \int d^4x \bar{\psi}(x) (i\mathcal{D} - m) \psi(x) \right\} \\ &= \frac{\det(i\mathcal{D} - m)}{\det(i\mathcal{D} - m)} = \exp \{-VU_{\text{eff}}(G)\} \end{aligned}$$

$$U_{\text{eff}}(G) = -\frac{1}{V} \text{Tr} \ln \left(\frac{i\mathcal{D} - m}{i\mathcal{D} - m} \right)$$

Quark propagator in the presence of an arbitrary homogeneous Abelian gauge field takes the form:

$$\begin{aligned}
S(x, y) &= (m + i \not{D}_x) H(x, y) \\
H(x, y) &= e^{-\frac{i}{2} x_\mu G_{\mu\nu} y_\nu} \frac{\mathcal{Q}}{4\pi^2} \int_0^\infty ds \frac{e^{-m^2 s}}{\sinh(s\sqrt{\mathcal{Q}\sigma_-}) \sinh(s\sqrt{\mathcal{Q}\sigma_+})} \\
&\times \left[P_+ \cosh(s|\vec{\mathcal{E}} - \vec{\mathcal{H}}|) + P_- \cosh(s|\vec{\mathcal{E}} + \vec{\mathcal{H}}|) \right. \\
&\quad \left. - \frac{1}{2} \sigma_{\mu\nu} [G_{\mu\nu} - \tilde{G}_{\mu\nu}] \frac{\sinh(s|\vec{\mathcal{E}} - \vec{\mathcal{H}}|)}{|\vec{\mathcal{E}} - \vec{\mathcal{H}}|} - \frac{1}{2} \sigma_{\mu\nu} [G_{\mu\nu} + \tilde{G}_{\mu\nu}] \frac{\sinh(s|\vec{\mathcal{E}} + \vec{\mathcal{H}}|)}{|\vec{\mathcal{E}} + \vec{\mathcal{H}}|} \right] \\
&\times \exp \left\{ -\frac{\sqrt{\mathcal{Q}}(x-y)^2}{4(\sigma_+ - \sigma_-)} \left[\sqrt{\sigma_+} \coth(s\sqrt{\mathcal{Q}\sigma_-}) - \sqrt{\sigma_-} \coth(s\sqrt{\mathcal{Q}\sigma_+}) \right] \right. \\
&\quad \left. - \frac{G_{\mu\nu} G_{\mu\rho} (x-y)_\nu (x-y)_\rho}{4\sqrt{\mathcal{Q}}(\sigma_+ - \sigma_-)} \left[\sqrt{\sigma_+} \coth(s\sqrt{\mathcal{Q}\sigma_+}) - \sqrt{\sigma_-} \coth(s\sqrt{\mathcal{Q}\sigma_-}) \right] \right\} \\
\mathcal{Q} &= (\vec{\mathcal{E}}\vec{\mathcal{H}}), \quad \mathcal{R} = \frac{1}{2}(\vec{\mathcal{E}}^2 + \vec{\mathcal{H}}^2), \quad \sigma_\pm = \frac{\mathcal{R}}{\mathcal{Q}} \left(1 \pm \sqrt{1 - \frac{\mathcal{Q}^2}{\mathcal{R}^2}} \right) \\
\vec{\mathcal{H}} &= n\vec{B} + \vec{H}, \quad \vec{\mathcal{E}} = \pm n\vec{B} + \vec{E}.
\end{aligned}$$

One-loop quark contribution to the effective potential in the presence of arbitrary homogenous Abelian fields

$$U_{\text{eff}}(G) = -\frac{1}{V} \ln \frac{\det(i\not{D} - m)}{\det(i\not{\partial} - m)} = \frac{1}{V} \int_V d^4x \text{Tr} \int_m^\infty dm' [S(x, x|m') - S_0(x, x|m')] |$$

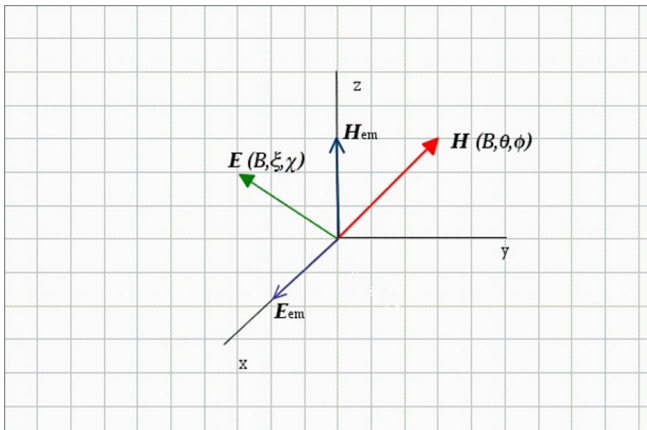
$$U_{\text{eff}}^{\text{ren}}(G) = \frac{B^2}{8\pi^2} \int_0^\infty \frac{ds}{s^3} \text{Tr}_n \left[s\kappa_+ \coth(s\kappa_+) s\kappa_- \coth(s\kappa_-) - \mathbf{1} - \frac{s^2}{3} (\kappa_+^2 + \kappa_-^2) \right] e^{-\frac{m^2}{B}s},$$

$$\kappa_{\pm} = \frac{1}{2B} \sqrt{Q\sigma_{\pm}} = \frac{1}{2B} \left(\sqrt{2(\mathcal{R} + \mathcal{Q})} \pm \sqrt{2(\mathcal{R} - \mathcal{Q})} \right),$$

$$\mathcal{R} = (H^2 - E^2)/2 + \hat{n}^2 B^2 + \hat{n} B (H \cos(\theta) + i E \cos(\chi) \sin(\xi))$$

$$\mathcal{Q} = \hat{n} B H \cos(\xi) + i \hat{n} B E \sin(\theta) \cos(\phi) + \hat{n}^2 B^2 (\sin(\theta) \sin(\xi) \cos(\phi - \chi) + \cos(\theta) \cos(\xi))$$

Y. M. Cho and D. G. Pak, Phys.Rev. Lett., 6 (2001) 1047

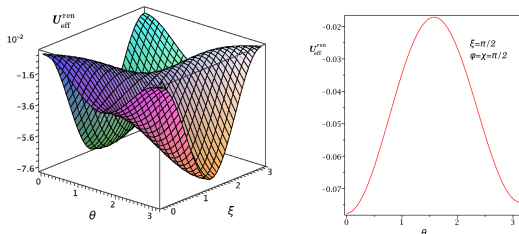


$$H_i = H\delta_{i3}, \quad E_j = E\delta_{j1}, \quad H^c = \{B, \theta, \phi\}, \quad E^c = \{B, \xi, \chi\}$$

$H \neq 0, E \neq 0$ and arbitrary gluon field

$$\Im(U_{\text{eff}}) = 0 \implies \cos(\chi) \sin(\xi) = 0, \sin(\theta) \cos(\phi) = 0$$

Effective potential (in units of $B^2/8\pi^2$) for the electric $E = .5B$ and the magnetic $H = .9B$ fields as functions of angles θ and ξ ($\phi = \chi = \pi/2$)



Minimum is at $\theta = \pi$ and $\xi = \pi/2$:

orthogonal to each other chromomagnetic and chromoelectric fields: $Q = 0$.

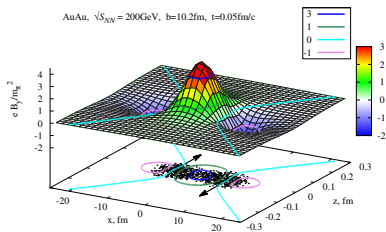
Strong electro-magnetic field plays catalyzing role for deconfinement and anisotropies?!

B.V. Galilo and S.N., Phys. Rev. D84 (2011) 094017.

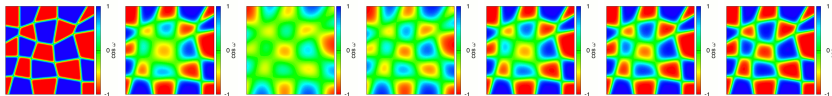
M. D'Elia, M. Mariti and F. Negro, Phys. Rev. Lett. **110**, 082002 (2013)

G. S. Bali, F. Bruckmann, G. Endrodi, F. Gruber and A. Schaefer, JHEP **1304**, 130 (2013)

V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya,
V. P. Konchakovski and S. A. Voloshin, *Phys. Rev C* 84 (2011)



Magnetic field $eB \gtrsim m_\pi^2$ in the region $5 \text{ fm} \times 5 \text{ fm} \times .2 \text{ fm} \times .2 \text{ fm}/c$



Green region ("Spaghetti vacuum") and the color charged quasi-particles

Strong electromagnetic fields, high energy and baryon density

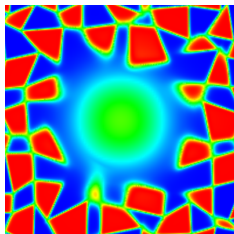
Two-stage phase transformation:

I. Colorless hadrons: $\langle |F\tilde{F}| \rangle = \langle F^2 \rangle^2 \neq 0$

→ II. Charged quasiparticles: $\langle |F\tilde{F}| \rangle = 0$, $\langle F^2 \rangle \neq 0$

→ III Weakly interacting QGP: $\langle |F\tilde{F}| \rangle = 0$, $\langle F^2 \rangle = 0$

Free energy density $F(B, R; H_{\text{em}}, T, \mu)$, EOS, ...: finite size effects, anisotropy, instabilities, ...



Photons as a signal of deconfinement: conversion of the gluons to photon $gg \rightarrow \gamma \rightarrow$ direct photon production in deconfinement phase

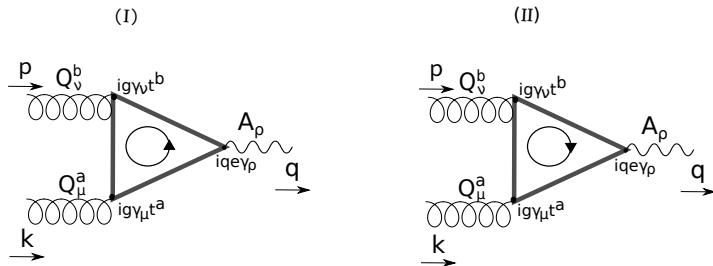


Figure: The diagrams for the process $gg \rightarrow \gamma$. Here p, k - are momenta of the gluons, q is the photon momentum. The arrows inside loop indicate the direction of loop momentum.

- S. Nedelko, A. Nikolskii. Eur.Phys. J. A **59** (2023) 4, 70; arXiv: 2208.00842 [hep-ph]

The confinement phase. $gg \rightarrow \gamma$ (via quark loop) in the presence of a random ensemble of almost everywhere homogeneous Abelian (anti-)self-dual gluon field:

$$\begin{aligned}\hat{B}_\mu &= \frac{1}{2} \hat{B}_{\mu\nu} x_\nu, \quad \hat{B}_{\mu\nu} = \hat{n} B_{\mu\nu}, \quad \hat{n} = t^8 = \frac{\lambda^8}{2}, \\ \tilde{B}_{\mu\nu} &= \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} B_{\alpha\beta} = \pm B_{\mu\nu}, \quad \hat{B}_{\rho\mu} \hat{B}_{\rho\nu} = 4v^2 B^2 \delta_{\mu\nu}, \\ \hat{f}_{\alpha\beta} &= \frac{\hat{n}}{2vB} B_{\alpha\beta}, \quad v = \text{diag} \left(\frac{1}{6}, \frac{1}{6}, \frac{1}{3} \right), \quad \hat{f}_{\mu\alpha}^{ik} \hat{f}_{\nu\alpha}^{kj} = \delta^{ij} \delta_{\mu\nu},\end{aligned}$$

where λ^8 is the Gell-Mann matrix. Field strength B sets the scale related to the value of the scalar gluon condensate.

The propagator of the quark field with mass m_f in the presence of the (anti-)self-dual field determined in Eq. (63) has the form

$$\begin{aligned}S_f(x, y) &= \exp \left(\frac{i}{2} x_\mu \hat{B}_{\mu\nu} y_\nu \right) H_f(x - y), \\ H_f(z) &= \frac{vB}{8\pi^2} \int_0^1 \frac{ds}{s^2} \exp \left(-\frac{vB}{2s} z^2 \right) \left(\frac{1-s}{1+s} \right)^{\frac{m_f^2}{4vB}} \\ &\times \left[-i \frac{vB}{s} z_\mu \left(\gamma_\mu \pm is \hat{f}_{\mu\nu} \gamma_\nu \gamma_5 \right) + m_f \left(P_\pm + \frac{1+s^2}{1-s^2} P_\mp + \frac{i}{2} \gamma_\mu \hat{f}_{\mu\nu} \gamma_\nu \frac{s}{1-s^2} \right) \right],\end{aligned}\tag{4}$$

- B. Galilo, S. Nedelko. Phys. Rev. D 84, 094017 (2011); arXiv:1107.4737 [hep-ph].

The confinement phase. The terms with **odd** powers $f_{\mu\nu}$ **violate** the conditions of the Furry theorem

$$\begin{aligned}
M^{(I)} = & i e g^2 q_f (2\pi)^4 \delta^{(4)}(p+k-q) \left(\frac{vB}{8\pi^2} \right)^3 \int_0^1 \int_0^1 \int_0^1 \frac{ds_1}{s_1^2} \frac{ds_2}{s_2^2} \frac{ds_3}{s_3^2} \frac{(-ivB)^3}{s_1 s_2 s_3} \\
& \left(\frac{1-s_1}{1+s_1} \right)^{\frac{m_f^2}{4vB}} \left(\frac{1-s_2}{1+s_2} \right)^{\frac{m_f^2}{4vB}} \left(\frac{1-s_3}{1+s_3} \right)^{\frac{m_f^2}{4vB}} \int d^4x d^4y e^{-i(px+ky)} \\
& \left\langle \text{Tr} \left[e^{i v B x^\mu \hat{f}_{\mu\nu} y^\nu - \frac{v}{2s_1} x^2 - \frac{v}{2s_2} y^2 - \frac{v}{2s_3} (y-x)^2} \hat{f}_{\alpha\omega} \hat{f}_{\beta\chi} \hat{f}_{\lambda\eta} K_{\alpha\omega\beta\chi\lambda\eta}^{\mu\nu\rho} + \right. \right. \\
& \left. \left. \hat{f}_{\alpha\eta} \hat{f}_{\beta\omega} \Pi_{\alpha\eta\beta\omega}^{\mu\nu\rho} + \hat{f}_{\alpha\omega} \Gamma_{\alpha\omega}^{\mu\nu\rho} \right] \right\rangle \epsilon_\mu^a(k) \epsilon_\nu^b(p) \epsilon_\rho(q).
\end{aligned}$$

The amplitudes differ by the **sign** of the phase factor

$$\begin{aligned}
M^{(II)} = & i e g^2 q_f (2\pi)^4 \delta^{(4)}(p+k-q) \left(\frac{vB}{8\pi^2} \right)^3 \int_0^1 \int_0^1 \int_0^1 \frac{ds_1}{s_1^2} \frac{ds_2}{s_2^2} \frac{ds_3}{s_3^2} \frac{(-ivB)^3}{s_1 s_2 s_3} \\
& \left(\frac{1-s_1}{1+s_1} \right)^{\frac{m_f^2}{4vB}} \left(\frac{1-s_2}{1+s_2} \right)^{\frac{m_f^2}{4vB}} \left(\frac{1-s_3}{1+s_3} \right)^{\frac{m_f^2}{4vB}} \int d^4x d^4y e^{-i(px+ky)} \\
& \left\langle \text{Tr} \left[e^{-i v B x^\mu \hat{f}_{\mu\nu} y^\nu - \frac{v}{2s_1} (x-y)^2 - \frac{v}{2s_2} y^2 - \frac{v}{2s_3} x^2} \hat{f}_{\alpha\omega} \hat{f}_{\beta\chi} \hat{f}_{\lambda\eta} K_{\alpha\omega\beta\chi\lambda\eta}^{\mu\nu\rho} + \right. \right. \\
& \left. \left. \hat{f}_{\alpha\eta} \hat{f}_{\beta\omega} \Pi_{\alpha\eta\beta\omega}^{\mu\nu\rho} + \hat{f}_{\alpha\omega} \Gamma_{\alpha\omega}^{\mu\nu\rho} \right] \right\rangle \epsilon_\mu^a(k) \epsilon_\nu^b(p) \epsilon_\rho(q).
\end{aligned}$$

The confinement phase. The sign of the phase factor is reflected in the result of averaging over the spacial orientation of the background field (S. Nedelko, V. Voronin. Phys. Rev. D 95, 074038 (2017); arXiv: 1612.02621 [hep-ph])

$$\left\langle \prod_{j=1}^n f_{\alpha_j \beta_j} e^{\pm i f_{\mu\nu} J_{\mu\nu}} \right\rangle = \frac{(\pm 1)^n}{(2i)^n} \prod_{j=1}^n \frac{\partial}{\partial J_{\alpha_j \beta_j}} \frac{\sin \sqrt{2 \left(J_{\mu\nu} J_{\mu\nu} \pm J_{\mu\nu} \tilde{J}_{\mu\nu} \right)}}{\sqrt{2 \left(J_{\mu\nu} J_{\mu\nu} \pm J_{\mu\nu} \tilde{J}_{\mu\nu} \right)}}, \quad (5)$$

and

$$\left\langle \prod_{j=1}^n f_{\alpha_j \beta_j} e^{-i f_{\mu\nu} J_{\mu\nu}} \right\rangle = (-1)^n \left\langle \prod_{j=1}^n f_{\alpha_j \beta_j} e^{i f_{\mu\nu} J_{\mu\nu}} \right\rangle. \quad (6)$$

Thus, the terms in $M = M^{(I)} + M^{(II)}$ with the product of an even number of the tensor $\hat{f}_{\mu\nu}$ cancel each other out identically just as in the case of the “usual” Furry theorem in QED, and the terms with the product of an odd number of the field strength tensor cancel each other upon averaging.

The amplitude $M = M^{(I)} + M^{(II)}$ vanishes in the confinement phase where averaging over random ensemble of almost everywhere homogeneous (anti-)self-dual vacuum gluon fields must be applied. **The conversion $gg \rightarrow \gamma$ does not occur in the presence of the random ensemble of confining vacuum fields.**

The deconfinement phase. This phase is characterized by the presence of the chromomagnetic field with the singled direction (B. Galilo, S. Nedelko. Phys. Rev. D 84, 094017 (2011); arXiv:1107.4737 [hep-ph].)

$$\hat{B}_{\mu\nu} = \hat{n} B_{\mu\nu} = \hat{n} B f_{\mu\nu}, \quad f_{12} = -f_{21} = 1,$$

all other components of $f_{\mu\nu}$ are equal to zero.

The coordinates and momenta (in Euclidean space-time)

$$x_{\perp} = (x_1, x_2, 0, 0), \quad x_{||} = (0, 0, x_3, x_4)$$

$$p_{\perp} = (p_1, p_2, 0, 0), \quad p_{||} = (0, 0, p_3, p_4).$$

The complete propagator of the quark field with mass m_f in the presence of an external chromomagnetic field , accounting for contribution of all Landau levels, has the form

$$S(x, y) = \exp \left\{ -\frac{i}{2} x_{\perp}^{\mu} \hat{B}_{\mu\nu} y_{\perp}^{\nu} \right\} H_f(x - y), \quad (7)$$

$$H_f(z) = \frac{\mathcal{B}}{16\pi^2} \int_0^{\infty} \frac{ds}{s} [\coth(\mathcal{B}s) - \sigma_{\rho\lambda} f_{\rho\lambda}] \exp \left\{ -m_f^2 s - \frac{1}{4s} z_{||}^2 - \frac{1}{8s} [\mathcal{B}s \coth(\mathcal{B}s) + 1] z_{\perp}^2 \right\}$$

$$\left\{ m_f - \frac{i}{2s} \gamma_{\mu} z_{||}^{\mu} - \frac{1}{2} \gamma_{\mu} \hat{B}_{\mu\nu} z_{\perp}^{\nu} - \frac{i}{4s} [\mathcal{B}s \coth(\mathcal{B}s) + 1] \gamma_{\mu} z_{\perp}^{\mu} \right\},$$

$$\mathcal{B} = B|\hat{n}|, \sigma_{\rho\lambda} = \frac{i}{2} [\gamma_{\rho}, \gamma_{\lambda}].$$

The deconfinement phase. The probability of photon production is given by the squared amplitude averaged over the initial gluon polarization states and summed over the final polarizations of photon

$$\overline{T}(p, k, q) = \Delta v \Delta \tau (2\pi)^4 \delta^4(p + k - q) T(p, k) ,$$

here $\Delta v \Delta \tau$ - is a space-time volume,

$$\begin{aligned} T(p, k) &= \frac{2\alpha\alpha_s^2}{\pi} \int_0^\infty ds_1 ds_2 ds_3 dr_1 dr_2 dr_3 F(s_1, s_2, s_3, r_1, r_2, r_3 | p, k) \\ &\times \exp \left\{ p_\perp^2 \Phi_1(s_1, s_2, s_3, r_1, r_2, r_3) + p_\perp k_\perp \Phi_2(s_1, s_2, s_3, r_1, r_2, r_3) + \right. \\ &\quad \left. k_\perp^2 \Phi_3(s_1, s_2, s_3, r_1, r_2, r_3) - m_f^2(s_1 + s_2 + s_3 + r_1 + r_2 + r_3) \right\} , \end{aligned} \quad (8)$$

where α and α_s – electromagnetic and strong coupling constants, and Φ_1, Φ_2, Φ_3 , - some functions of quark proper time

SN, A. Nikolskii. Eur.Phys. J. A **59** (2023) 4, 70; arXiv: 2208.00842 [hep-ph].

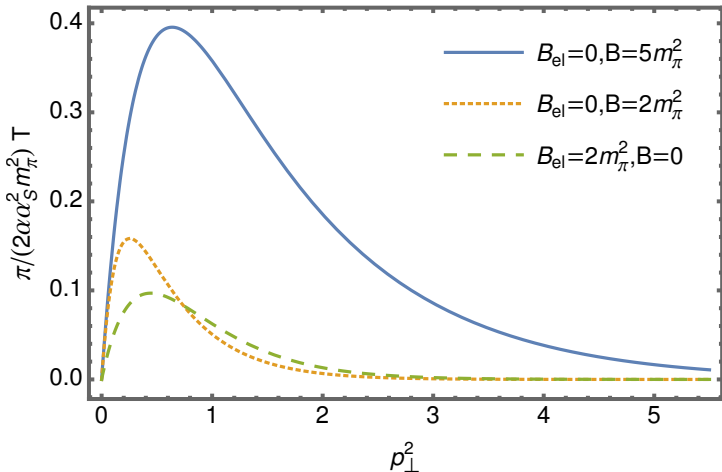


Figure: Dependence of $T(p, k)$ in regime $k_{\perp}^2 = p_{\perp}^2$. The dashed line corresponds to the purely magnetic field B_{el} , dotted and solid lines represent the case of pure chromomagnetic field B with different values of strength.

- the gluon conversion in pure magnetic field B_{el} : A. Ayala et al. Eur. Phys. J. A 56, 53 (2020); arXiv: 1904.02938 [hep-ph].

THANK YOU!