Probing nuclear matter through intermediate energy heavy-ion reactions

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Heavy-ion Reactions (at different energies):-

- **Low Energy reaction:-**
  - Projectile beam energy <20 MeV/nucleon
  - **Dominated** by **mean field**
- **High Energy reaction:-**
  - Projectile beam energy > 1 GeV/nucleon
  - **Dominated** by **n-n collision**
- **Intermediate energy reaction:-**
  - Projectile beam energy ~20 MeV/nucleon to 1 GeV/nucleon
  - **Competition** between **mean field** and **n-n collision**
    - Temperature~3-10 MeV (1 MeV=1.2×10^{10} Kelvin)
    - Reaction time~ 100 fm/c (1 fm/c=3.3×10^{-24} sec)

✓ **Nuclear multifragmentation**

✓ **Searching for the "nuclear equation of state"**

✓ **Nuclear liquid gas phase transition**

Intermediate Energy Heavy-ion Reactions:-

Discovery of multiple fragments (multifragmentation) → 90 years ago from cosmic rays

No suitable accelerator facility

↓

Observation of multiple intermediate mass fragments at Bevalac experiment in 1982 (at Lawrence Berkley Laboratory, USA)

↓

Experimental facilities

- NSCL (Michigan, USA)
- Texas A&M (Texas, USA)
- GANIL (Caen, France)
- GSI (Darmstadt, Germany)
- RIKEN (Wako, Japan)
- LNS (Catania, Italy)
- **JINR (Dubna, Russia)**
- **VECC (Kolkata, India)**
- HIAF (Huinzhou, China)
- RAON (Daejeon, South Korea)

Theoretical models of Intermediate Energy Heavy-ion Reactions :-

- ***Statistical Models:-***

  Basic Assumption: Equilibrium @ freeze-out

  - **Canonical Thermodynamical Model (CTM)**
  - Statistical Multifragmentation Model (SMM)
  - **Grand Canonical Model (GCM)**

  etc….

- ***Dynamical Models:-***

  Time evolution of projectile and target nucleons

  - Quantum Molecular Dynamics (QMD) Model
  - **Boltzmann Uehling Uhlenbeck (BUU) Model**

  etc….

- ***Hybrid Models:-***

  - **Combination of dynamical and statistical models**

- ***Other Models:-***

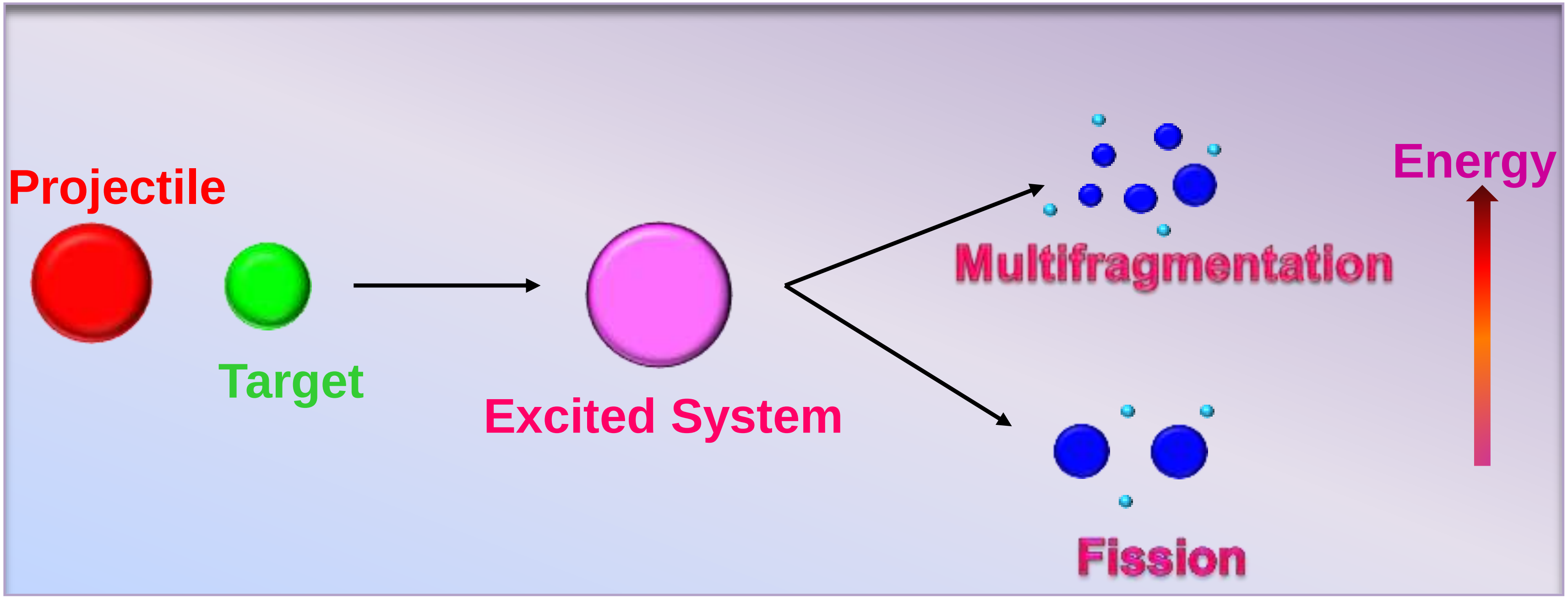
  - Abrasion-Ablation Model
  - Lattice Gas Model
  - Percolation Model
  - EPAX

  etc….

1. Nuclear Multifragmentation

Nuclear Multifragmentation:-

Due to the collision of the projectile & target nuclei, an excited system is formed. If it's excitation energy is greater than a few MeV/nucleon, then it breaks into many nuclear fragments of different masses.

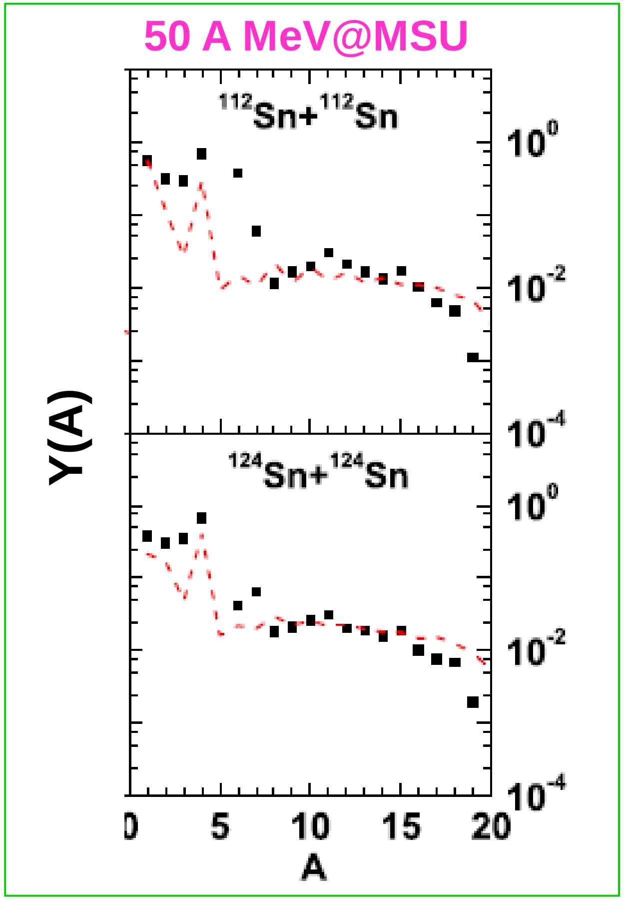


- "Multi" = More than 2
- Multifragmentation ⟶ Higher energy version of fission/evaporation

Ref: "Heavy Ion Collisions at Intermediate Energy: Theoretical Models" S. Das Gupta, S. Mallik and G. Chaudhuri World Scientific Publishers (2019)

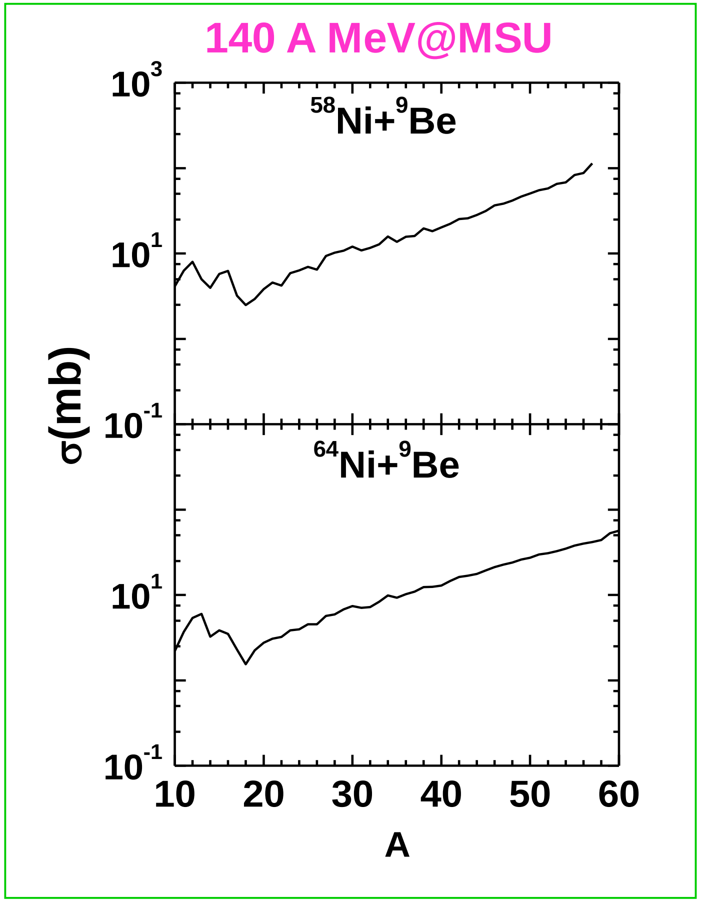
Experimental approaches for multifragmentation from heavy-ion reaction studies:-

- Central heavy-ion collisions at incident energies of $20 \leq E/A \leq 100$ MeV



T. X. Liu et. al. ; Phys. Rev. C 69, 014603 (2004)

- Larger impact parameter heavy ion collisions at E/A > 100 MeV (Projectile fragmentation)

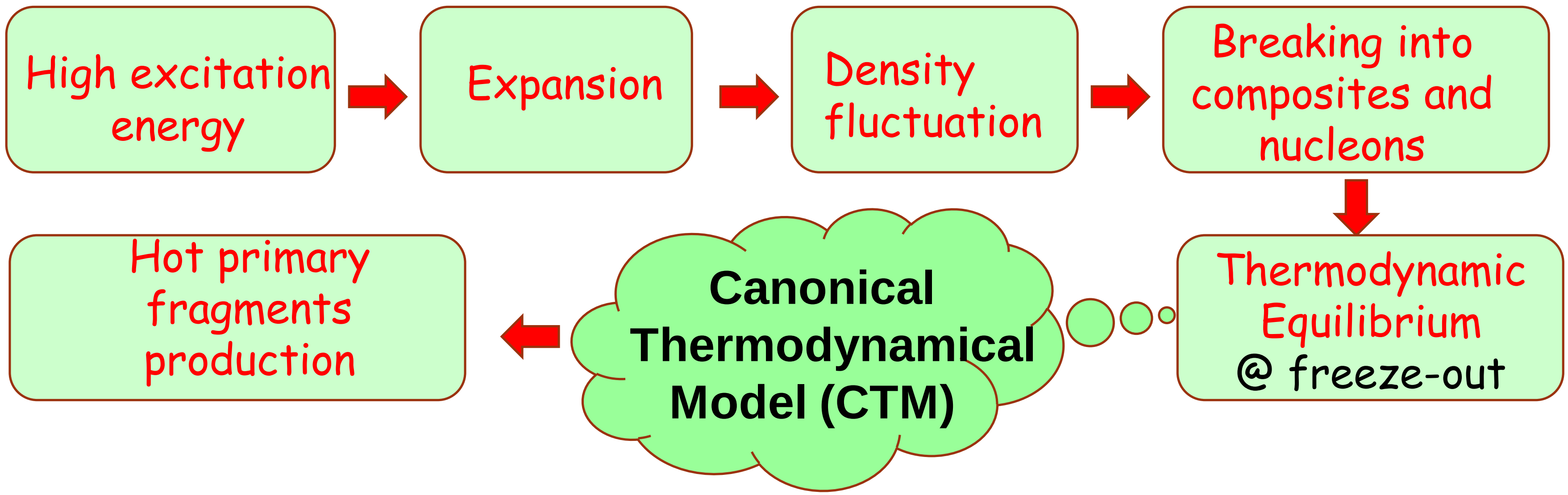


M. Mocko et. al. ; Phys. Rev. C 78, 024612 (2008)

Hybrid Model of Multifragmentation:-

- *Initial Excitation calculation from dynamical BUU Model.*
- *Fragmentation form Canonical Thermodynamical Model (CTM)*
- *Decay of excited fragments by statistical evaporation model.*

Canonical Thermodynamical Model (CTM):-



Ref: C. B. Das , S. Das Gupta et al. , Phys . Rep. 406,1 (2005)

Canonical Partition function of fragmenting system A_0 (Z_0 ,N_0)

$$Q_{Z_0,N_0} = \sum \prod_{Z,N} \frac{\omega_{Z,N}^{n_{Z,N}}}{n_{Z,N}!} \delta\Big(\sum_Z Z \times n_{Z,N} - Z_0\Big) \delta\Big(\sum_N N \times n_{Z,N} - N_0\Big)$$

$n_{Z,N}$=No of fragments with Z protons & N neutrons

Charge & baryon conservation

$\omega_{Z,N}$=Partition function of the fragment $n_{Z,N}$

Computationally difficult !

CTM *contd…*

Recursion relation

$$Q_{Z_0,N_0} = \frac{1}{Z_0}\sum_{Z,N} Z\omega_{Z,N}\, Q_{Z_0-Z,N_0-N}$$

An exact computational method which avoids Monte Carlo by exploiting some properties of the partition function

Most important feature of our model → Possible to calculate partition function of very large nuclei within seconds

Average no. of composites {Z,N}
or
Multiplicity

$$\langle n_{Z,N}\rangle = \omega_{Z,N}\frac{Q_{Z_s-Z,N_s-N}}{Q_{Z_s,N_s}}$$

Ref: C. B. Das, S. Das Gupta et al. , Phys. Rep. 406 (2005) 1

Evaporation Model :-

The excited fragments obtained from Canonical Thermodynamical Model decay to stable isotopes in their ground state.

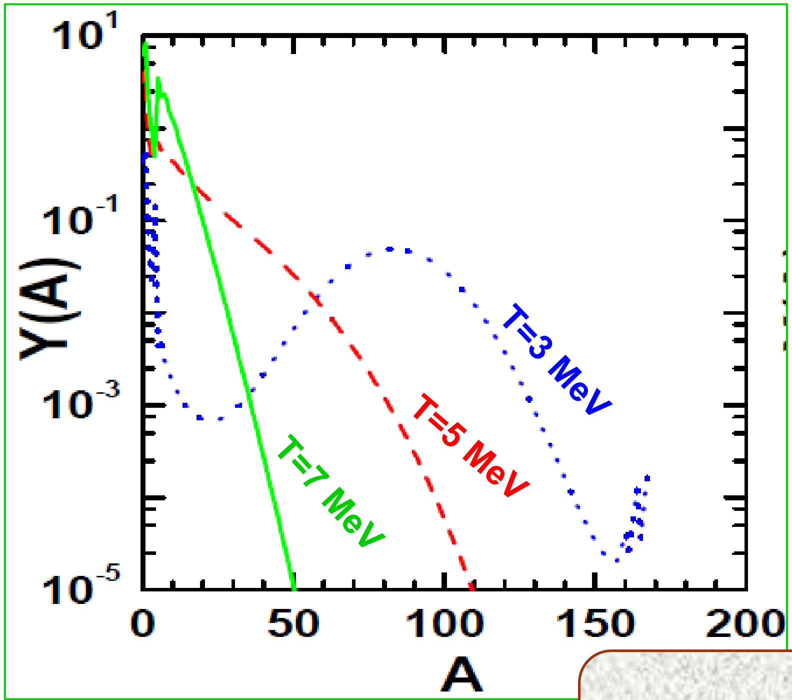
Decay Channels:- p, n, α, d, t, He^3, γ, fission

Ref: G.Chaudhuri & S.Mallik; Nucl. Phys. A 849, 190 (2011)

Multifragmentation observables from theoretical calculation:-

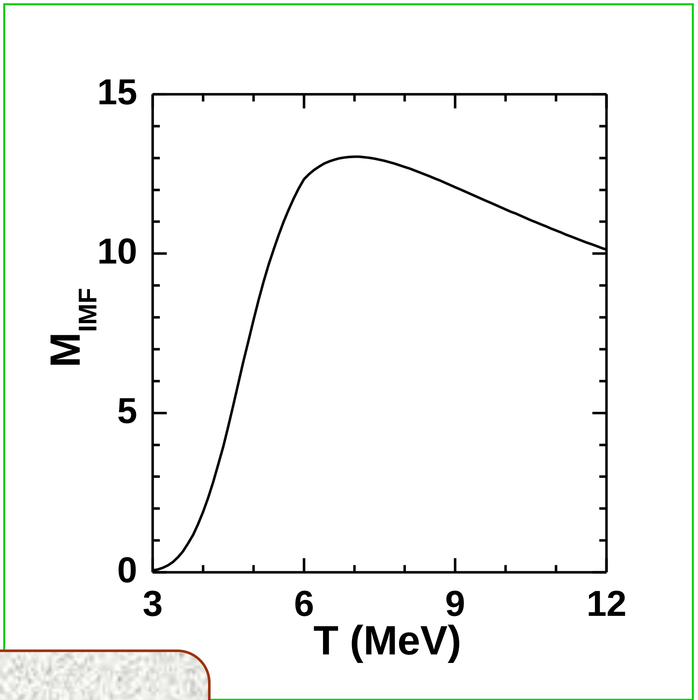
Studied Reaction: $^{112}Sn+^{112}Sn$

Mass Distribution



Intermediate Mass Fragment

(2<Z ≤20)



Unambiguous signals of multifragmentation

Ref: S. Mallik; Phys. Rev. C 107, 054605 (2023)

2. Searching for the Nuclear Equation of State

Nuclear Equation of State:-

- Thermodynamic relationships between variables like temperature, pressure, density, energy…
- Most common are ideal gas law and Van der Walls equation

✓Air density @NTP~1.2 x10^{-3} gm/cm^3
✓ Normal nuclear density~2.3 x10^{14} gm/cm^3

- The main goal is to express the properties of compact nuclear objects (from finite nuclei to neutron stars), using our thermodynamics knowledge.

First Attempt: Binding energy of nuclei

❖ Liquid-drop worked properly only for stable nuclei

$$B(Z,N) = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_{sym} \frac{(N-Z)^2}{A} - \ldots\ldots$$

❖What happens as we move away from saturation density and away from beta stability???

How to proceed further??

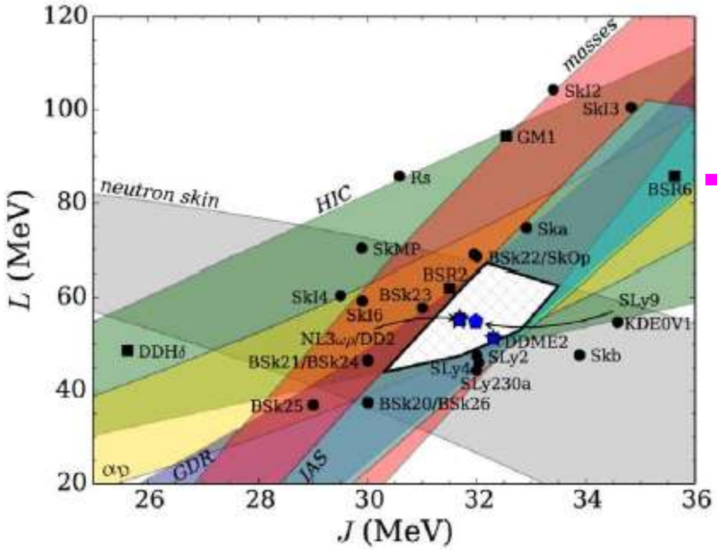
- Since nuclei are not incompressible, addition of density dependence [$\rho = \rho_n + \rho_p$]
- Addition of asymmetry [$\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$]
- Energy per particle,

$$E(\rho,\delta) = E(\rho,\delta=0) + S(\rho)\delta^2$$

Nuclear Equation of State (Cont..)

- Symmetry energy is expressed as a Taylor's expansion about saturation density

$$S(\rho) = J + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{1}{2!}K_{sym}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + + \frac{1}{3!}Z_{sym}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3 + \ldots$$



Ref: M. Fortin et. al., Phys . Rev. C 94, 035804 (2016)

Present Challenge:- Precise measurement of nuclear EoS.

Intermediate energy heavy ion reaction can be an useful probe!!

Symmetry energy measurement from Intemediate Energy Heavy ion collision :-

Important Observables:-

- **Isoscaling**
- Isobaric yield ratio
- π^-/π^+
- **Isospin tranport** ……

Ref: "Heavy Ion Collisions at Intermediate Energy: Theoretical Models" S. Das Gupta, S. Mallik and G. Chaudhuri World Scientific Publishers (2019)

Isoscaling:-

The ratio of yields from two different reactions(having different isospin asymmetry), exhibit an exponential relationship as a function of the neutron(N) & proton(Z) number.

Two reactions:- 1 and 2

$Y_1(N,Z)$ yield of (N,Z) from reaction 1 (N_{01}, Z_0, $A_{01}= N_{01}+Z_0$)

$Y_2(N,Z)$ yield of (N,Z) from reaction 2 (N_{02}, Z_0, $A_{02}= N_{02}+Z_0$)

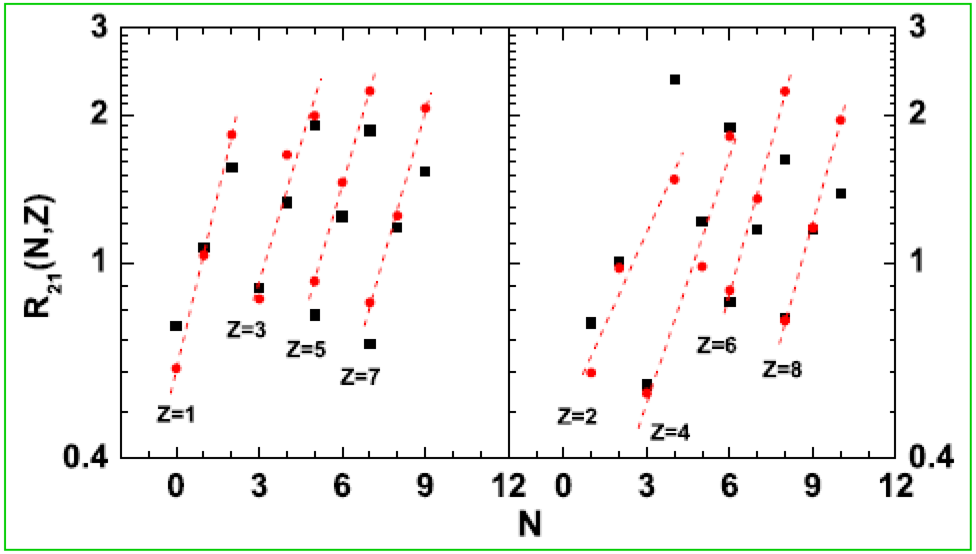
$$R_{21}(N,Z) = \frac{Y_2(N,Z)}{Y_1(N,Z)} = c\exp(\alpha N + \beta Z)$$

α and β are the isoscaling parameters.

Ref: M.B.Tsang , et al. , Phys. Rev. Lett. 85, 716 (2000)

Studied Reaction: ^{124}Sn+^{124}Sn and ^{112}Sn+^{112}Sn@50 MeV/nucleon

(MSU Experiment)



Red circles → Theoretical
Black squares → Experimental

Ref: S. Mallik and G. Chaudhuri; Phys. Lett. B 727, 282 (2013); Nucl. Phys. A 1002, 121948 (2020)

Isoscaling study@VECC Superconducting Cyclotron:-

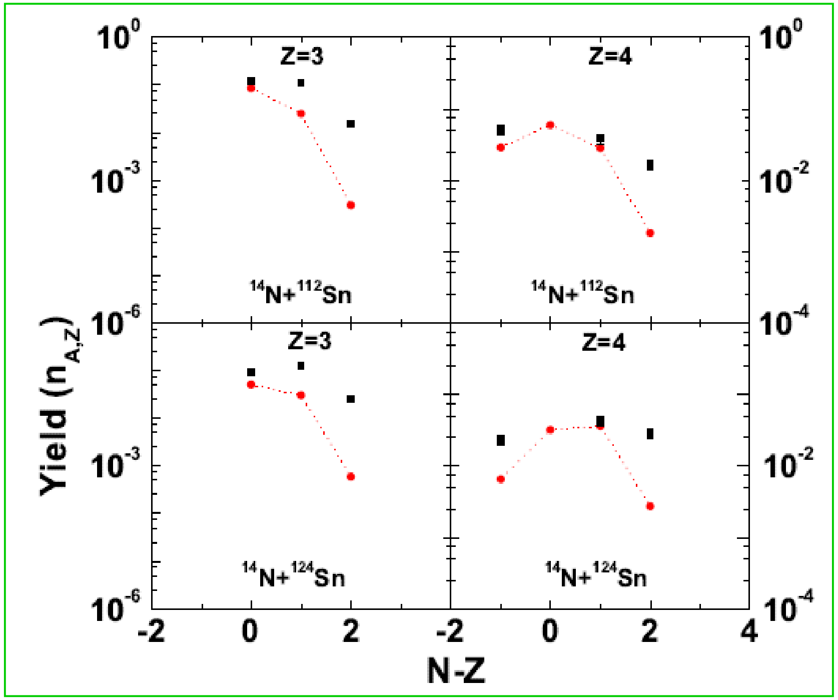
Reactions: $^{14}N+^{124,116,112}Sn$ E_p=30 and 19.2 MeV/nucleon

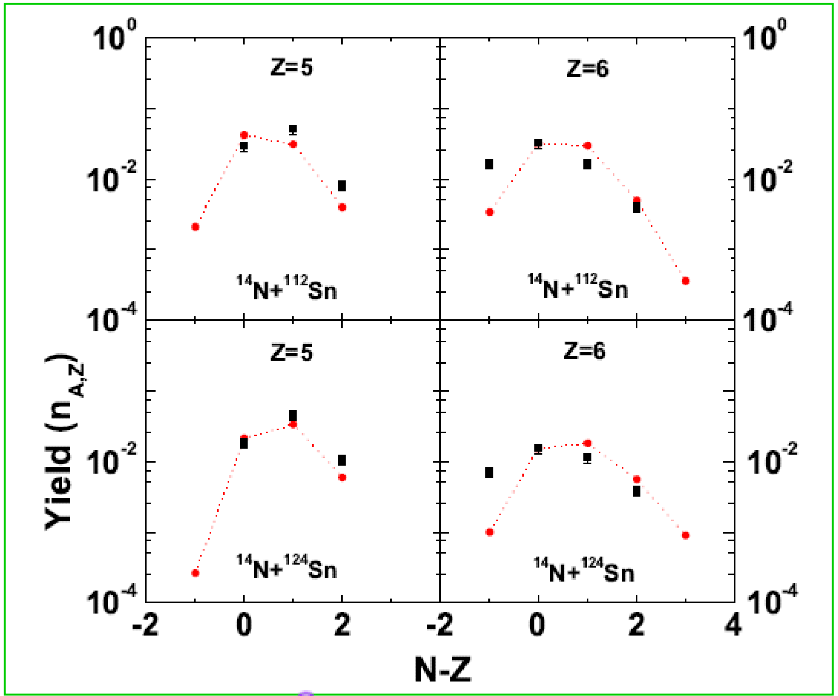
$^{20}Ne+^{124,116,112}Sn$ E_p=22 and 18.2 MeV/nucleon

Theoretical calculation@Isospin dependent hybrid model of nuclear multifragmentation

1. Isotopic Distribution:-

$^{14}N+^{124}Sn$ and $^{14}N+^{112}Sn$ E_p=30 MeV/nucleon

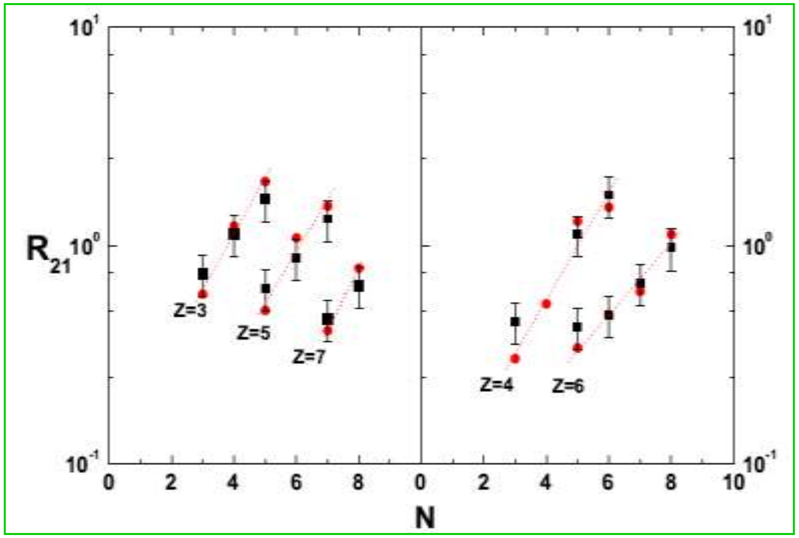


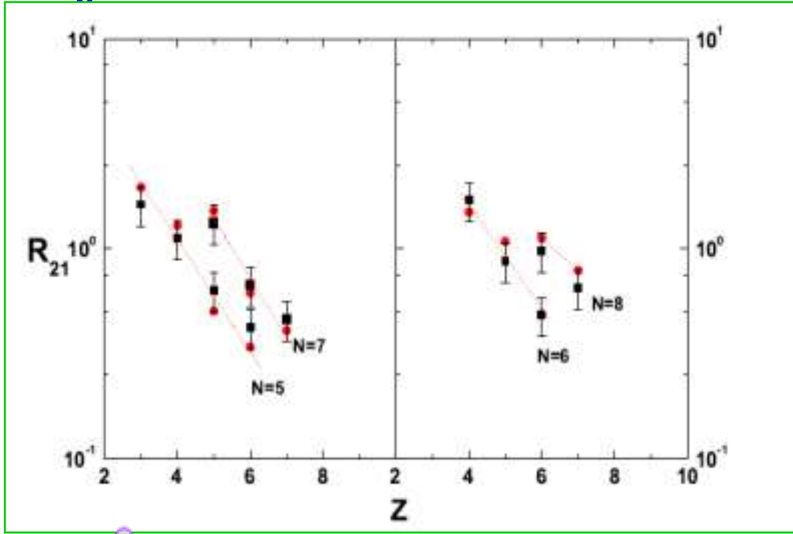


Red circles → Theoretical
Black squares → Experimental

Isoscaling study@VECC Superconducting Cyclotron:-

2. Isoscaling:- $^{14}N+^{124}Sn$ and $^{14}N+^{112}Sn$ E_p=30 MeV/nucleon





Red circles → Theoretical
Black squares → Experimental

Reactions	E_p (MeV/n)	$<E^*_{theo}>$ (MeV/n)	α_{th}	α_{ex}	β_{th}	β_{ex}	$C_{sym,ex}$
$^{14}N+^{124,112}Sn$	30	2.70	0.55	0.51	-0.55	-0.53	17.0
$^{20}Ne+^{124,112}Sn$	22	2.61	0.69	0.64	-0.72	-0.68	22.4
$^{20}Ne+^{124,112}Sn$	18.2	2.48	0.77	0.80	-0.86	-0.70	23.9
$^{14}N+^{124,112}Sn$	19.2	1.89	0.88	0.63	-1.08	-0.67	20.8

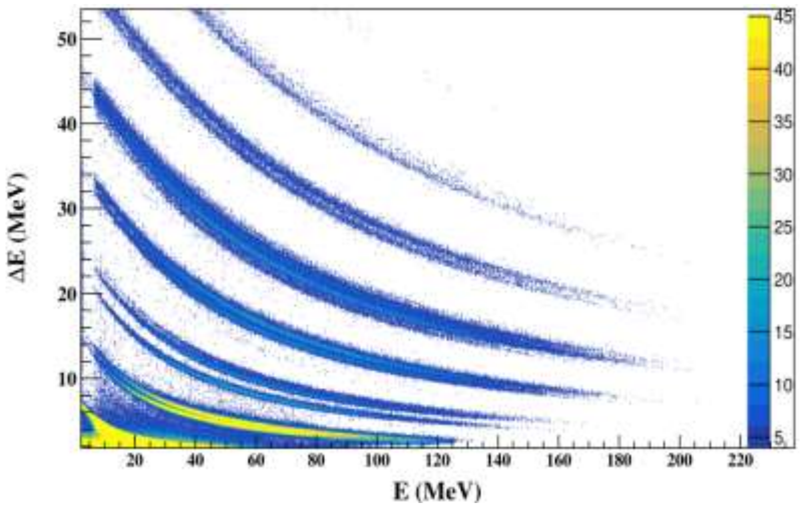
- Eestimations of T (both from theory & experiment) verifies the assumption of isoscaling equation i.e., the temperature of both the reactions are very close.

- Change of reaction mechanism at lower excitation.

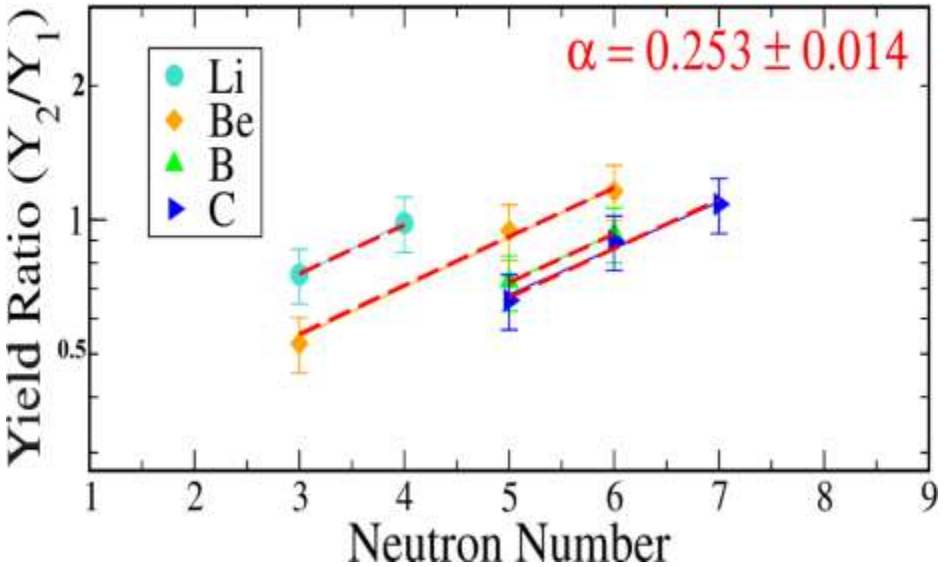
Ref: P. Karmakar, S. Kundu, T. K. Rana, S. Mallik et.al., Phys. Rev. C 112, 024614 (2025)

2^{nd} Set of Experiments of Isoscaling study@VECC Superconducting Cyclotron:-

^{20}Ne (18.2 MeV/nucleon) + ^{56}Fe $\rightarrow$ ^{76}Kr $\rightarrow$ Fragments
^{16}O (21.4 MeV/nucleon) + ^{58}Ni $\rightarrow$ ^{74}Kr $\rightarrow$ Fragments



Particle identification 2D histogram



PRELIMINARY

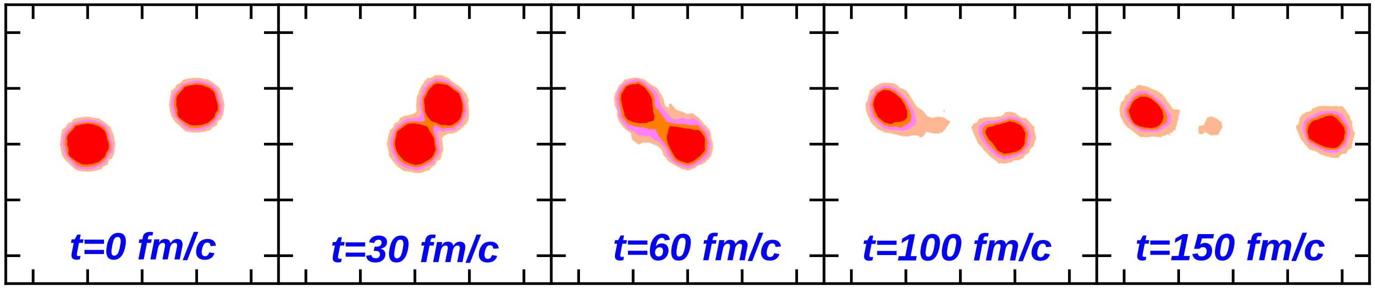
Courtesy: T. K. Rana, S. Ghosh; VECC

Isospin Transport :-

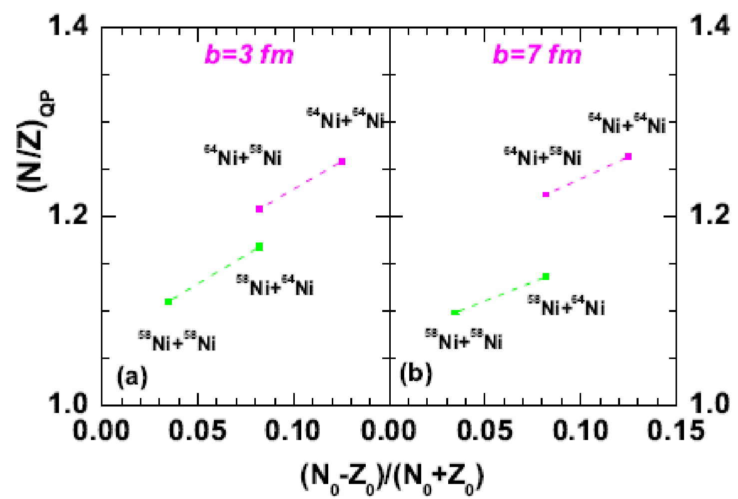
- *Transfer of isospin from more isospin asymmetric system to less isospin asymmetric system*

Time evolution :-

^{64}Ni+^{58}Ni **E_p=32 MeV/nucleon** **b=7 fm** **(BUU@VECC-McGill model Result)**



N/Z of Quasiprojectile :-



Ref: S. Mallik , F. Gulminelli, D. Gruyer; Jour. Phys. G. 49, 015102 (2021)

Isospin transport ratio:-

$$R_i = \frac{2x_i - x_{^{124}Sn+^{124}Sn} - x_{^{112}Sn+^{112}Sn}}{x_{^{124}Sn+^{124}Sn} - x_{^{112}Sn+^{112}Sn}}$$

- *Less sensitive to secondary decay*

Ref: M. B. Tsang et. al, Phys. Rev. Lett. 92, 062701 (2004)
Phys. Rev. Lett. 102, 122701 (2009)

Boltzmann-Uehling-Uhlenbeck model (BUU@VECC-McGill) for heavy ion collision :-

❖Based on the BUU equation,

$$\frac{\partial f_i}{\partial t}+\vec{v}_i.\vec{\nabla}_r f_i-\vec{\nabla}_r U.\vec{\nabla}_p f_i=\frac{1}{(2\pi)^6}\int d^3\vec{p}_j d^3\vec{p}_{j'} d\Omega \frac{d\sigma}{d\Omega} v_{ij}$$
$$\times\left\{f_{i'}f_{j'}(1-f_i)(1-f_j)-f_i f_j(1-f_{i'})(1-f_{j'})\right\}$$
$$\times(2\pi)^3\delta^3(\vec{p}_i+\vec{p}_j-\vec{p}_{i'}-\vec{p}_{j'})$$

where, $f_i \equiv f(\vec{r}_i,\vec{p}_i,t)$

❖ Can not be solved exactly, test particle method (N_{test}=100) is used for numerical calculation.

G. F. Bertsch et al. , Phys. Rep. 160, 189 (1988)

❖Initial positions and momenta of the test particles are selected by Monte-Carlo simulations of initial phase space density (obtained from variational method over Myer's density profile).

❖Mean field propagation is studied by Lattice Hamiltonian method.

❖ Bulk part of the mean field is determined from meta-modelling of EoS.

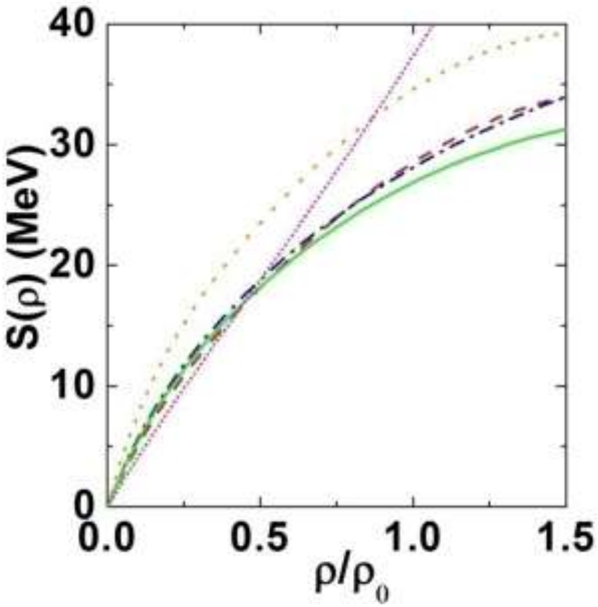
J. Margueron et. al, Phys. Rev. C. 97, 025805 (2018)

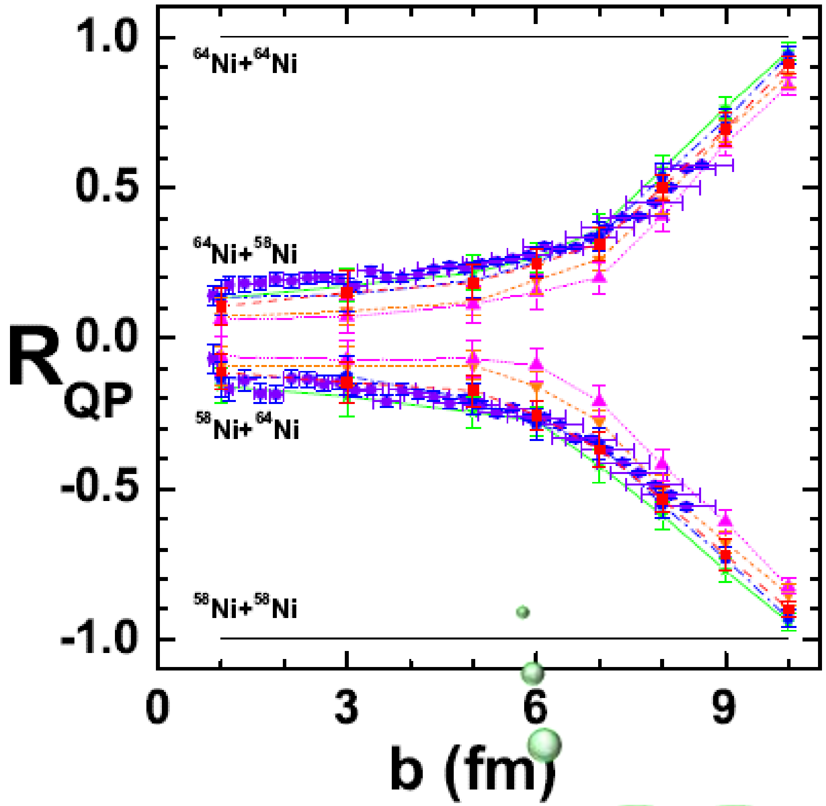
Isospin Transport Ratio from different realistic EoS:-

In collaboration with INDRA-FAZIA

GANIL E789 Experiment: ^{58,64}Ni+^{58,64}Ni@32 MeV/nucleon

Theoretical Calculation: BUU@VECC-McGill model





Violet Circles → Experimental data
Orange dotted line → ab-initio-1 EoS
Red dashed line → ab-initio-7 EoS
Blue dash dotted line → SAMI EoS
Green solid line → SGII EoS
Magenta short dotted line → NL3 EoS

Isospin transport ratio is sensitive to nuclear EoS

Ref: S. Mallik et. al., Nuovo Simento C 48, 65 (2025)
C. Cimapi et. al., Phys. Rev. C 111, 044601 (2025)

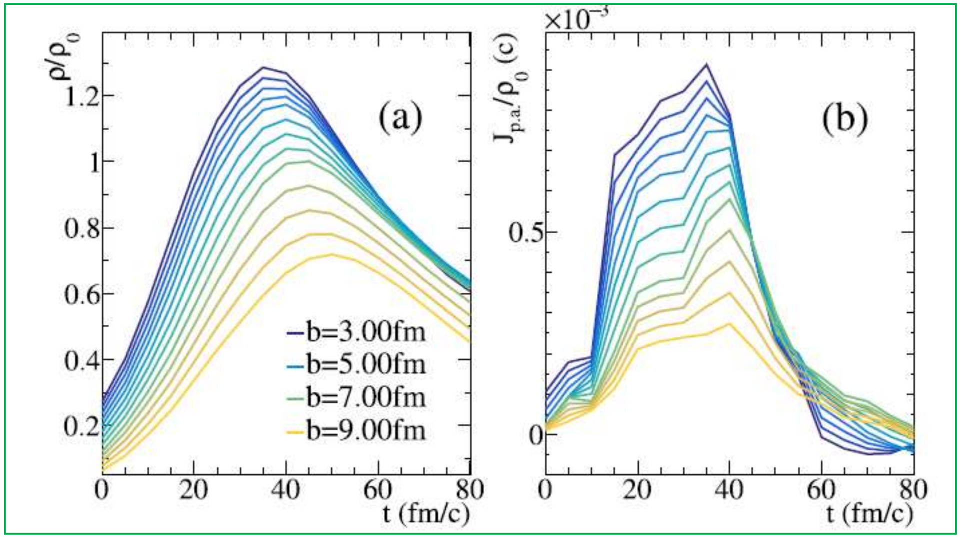
Isospin current density:-

Studied Reaction: $^{64}Ni+^{58}Ni@E_p=32$ MeV/nucleon

BUU@VECC-McGill calculation with ab-initio-7 EoS

$$J_{I,w} = J_w^n - J_w^p = \left(j_{w,PT}^n - j_{w,TP}^n\right) - \left(j_{w,PT}^p - j_{w,TP}^p\right)$$

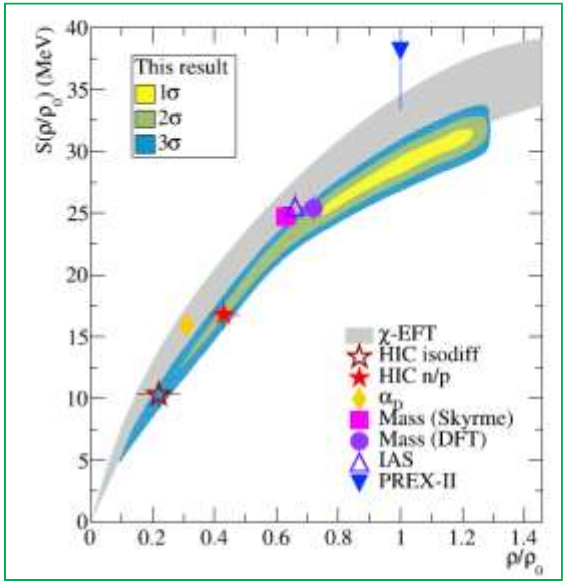
w=x,y and z direction



Extracted symmetry energy constraint:-

- ❖ *Isospin current helps to identify the probed baryonic density (ρ) region.*
- ❖ *Isospin diffusion measures symmetry energy (E_{sym}).*

Precise determination of nuclear equation of state $E_{sym}(\rho)$



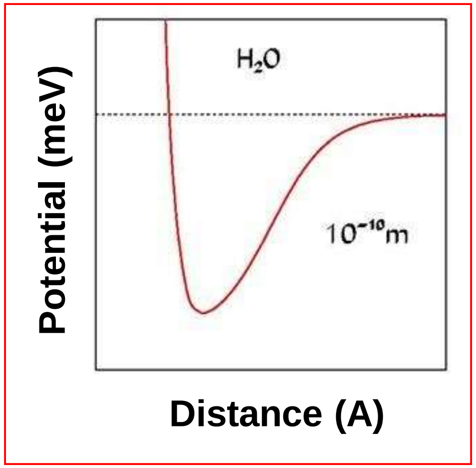
Ref: C. Cimapi , S. Mallik, F. Gulminelli, D. Gruyer et. al; Phys. Lett. B 868, 139815 (2025)

3. Nuclear Liquid Gas Phase Transition

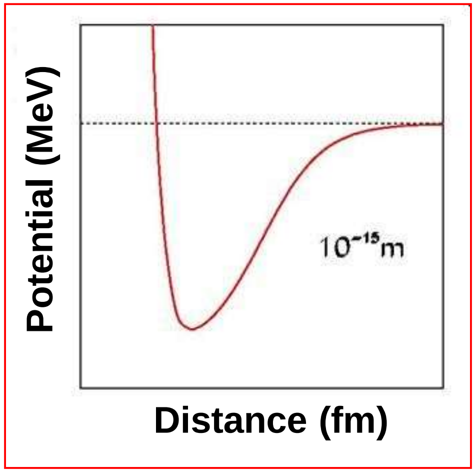
Nuclear Liquid Gas Phase Transition:-

Ref: P.J. Siemens, Nature, 305, 410 (1983)

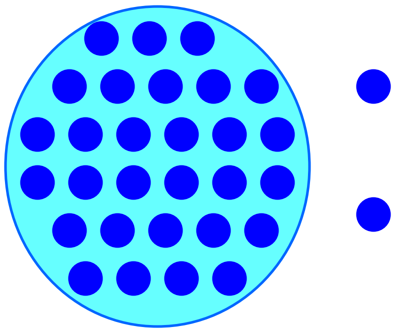
Molecular Interaction



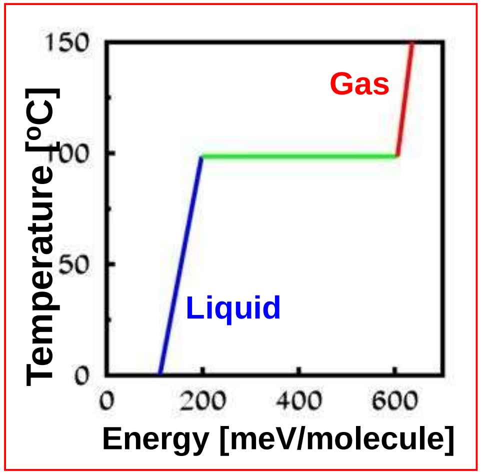
Nucleon-nucleon Interaction



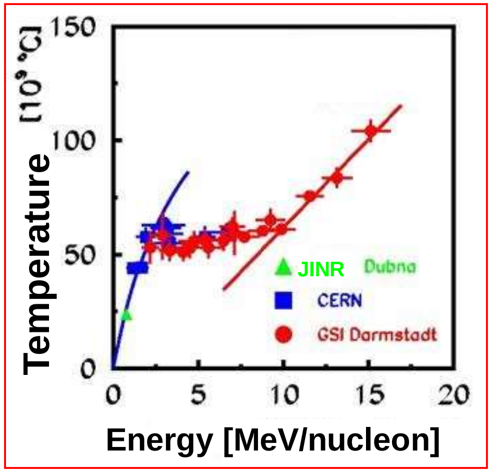
Nuclear Liquid



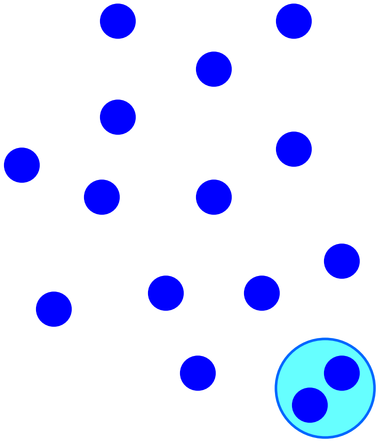
Liquid→Gas



Liquid→Gas (Nuclear)



Nuclear Gas



Difficulties for studying Nuclear Liquid Gas Phase Transition:-

- Very short time scale (10^{-21} Sec)
- System does not stay at fixed density and temperature.
- No direct way of measuring state variables
- Finite size effect

Signatures of Nuclear Liquid Gas Phase Transition:-

1st Order

- Caloric Curve
- Specific Heat
- Bimodality
- Spinodal Decomposition
- **Multiplicity Derivative**

2nd Order

- Critical Exponent
- Zip's Law
- Δ-scaling
- Maximal Fluctuation

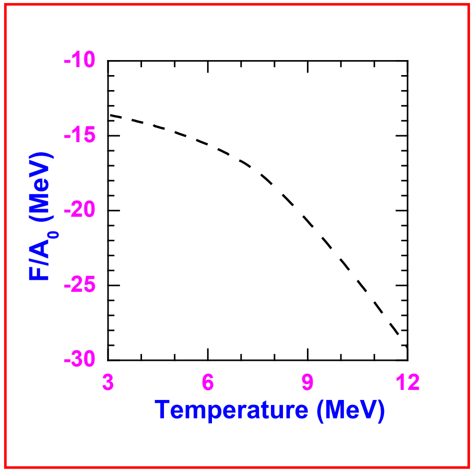
Ref: B. Borderie and J. D. Frankland, Prog. Part. Nucl. Phys. 105, 82 (2019)

Important Signatures of Nuclear Phase Transition:-

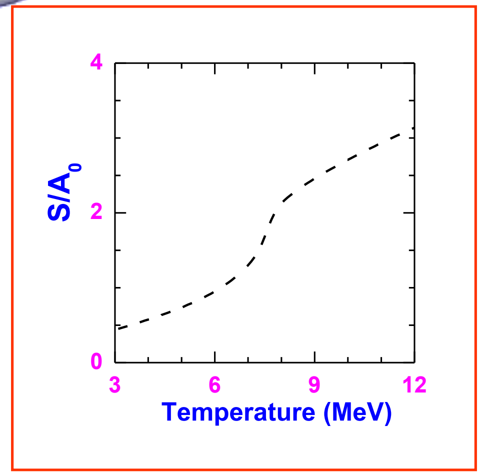
From Canonical Thermodynamical Model (CTM) Calculation-

A_0=192

Helmholtz Free Energy

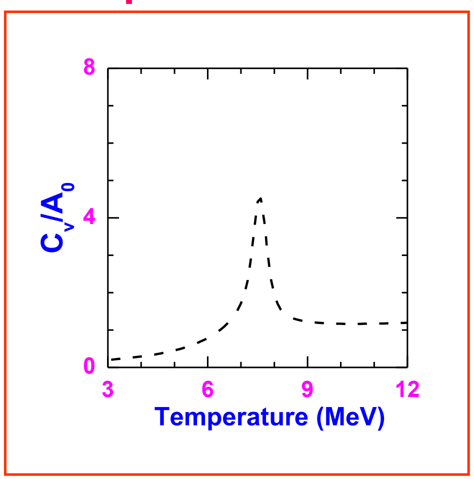


Entropy

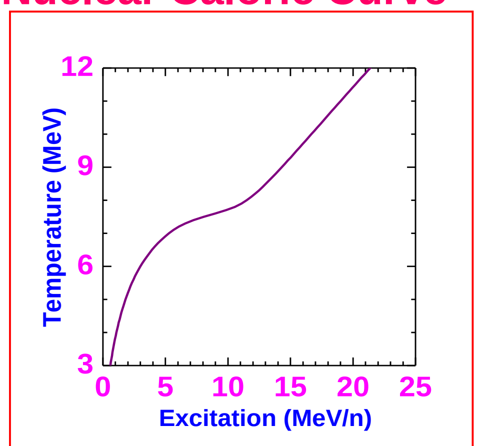


Difficult to study in experiments

Specific Heat



Nuclear Caloric Curve



Looking for a new experimentally accessible signature

Total Multiplicity and it's derivative:-

M and dM/dT much more accessible both theoretically as well as experimentally.

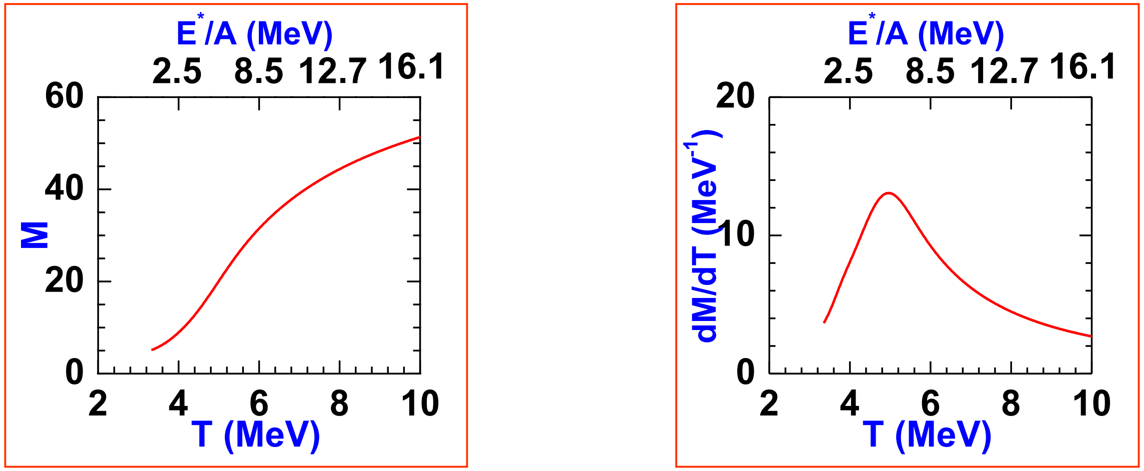
Average no. of composites {i,j}

$$\langle n_{i,j} \rangle = \omega_{i,j} \frac{Q_{Z_0-i,N_0-j}}{Q_{Z_0,N_0}}$$

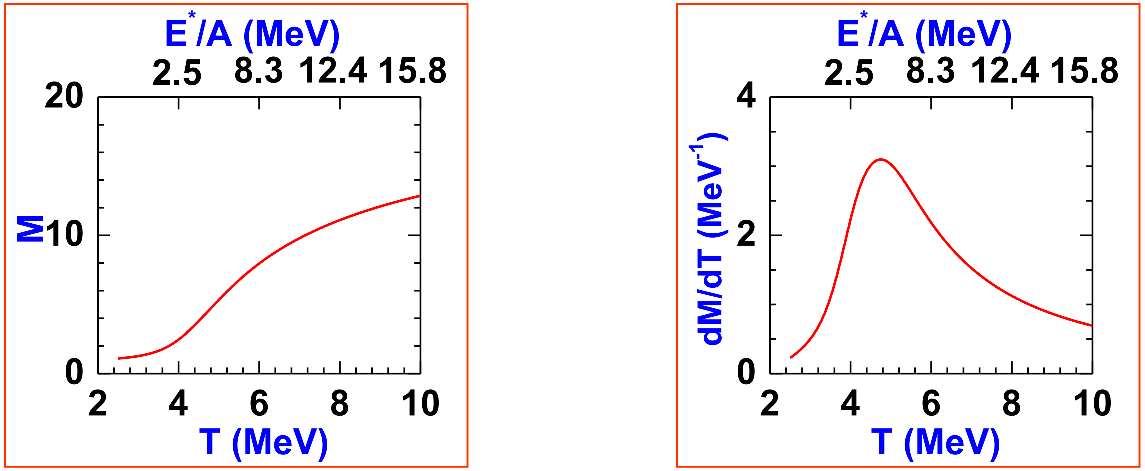
Total multiplicity

$$M = \sum_{i,j=1}^{N_0,Z_0} < n_{i,j} >$$

Fragmenting system Z_0=82 A_0=208



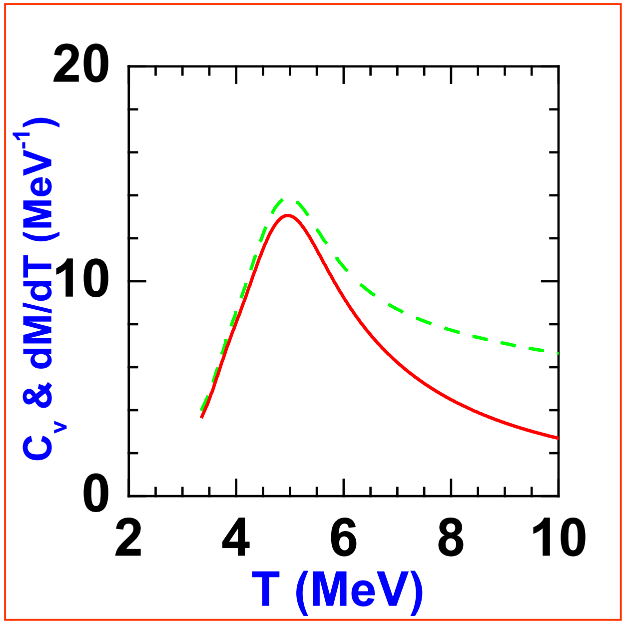
Fragmenting system Z_0=28 A_0=58



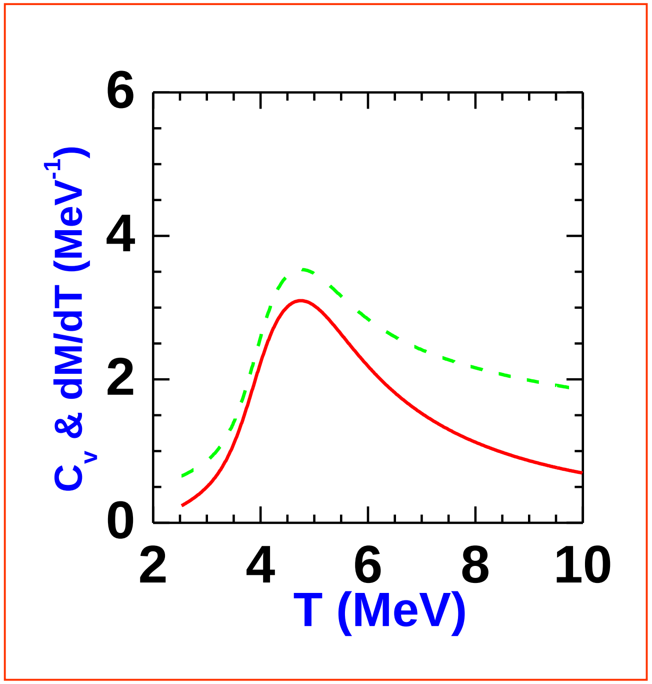
Peak in multiplicity derivative!!!

Multiplicity derivative and specific heat:-

Z_0=82 A_0=208



Z_0=28 A_0=58



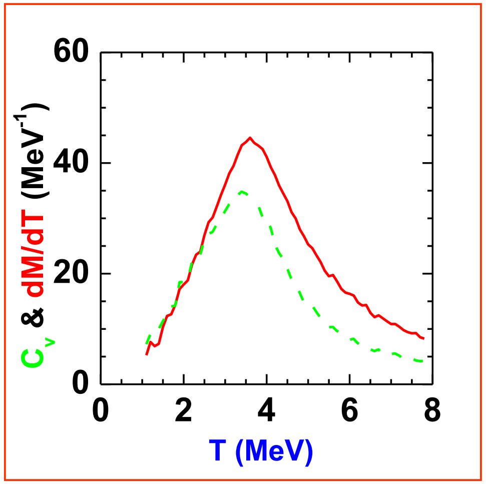
Multiplicity derivative peak coincides with specific heat

Red solid line → dM/dT
Green dashed line → C_v

Ref: S. Mallik, G. Chaudhuri, P. Das and S. Das Gupta; Phys. Rev. C 95, 061601 (2017)
. Editor's Suggestion (Rapid Communication)
P. Das, S. Mallik and G. Chaudhuri; Phys. Lett. B 783, 364 (2018)

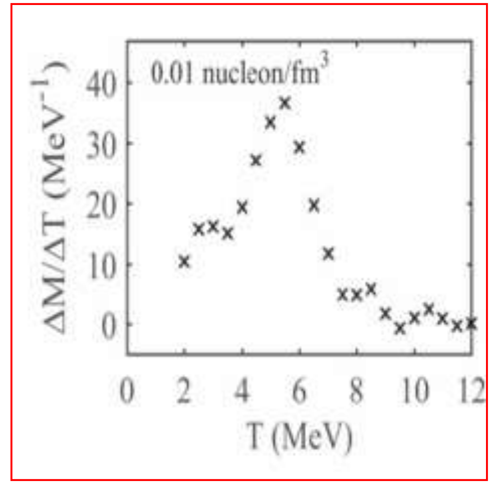
Multiplicity derivative from other theoretical models:-

Lattice Gas model



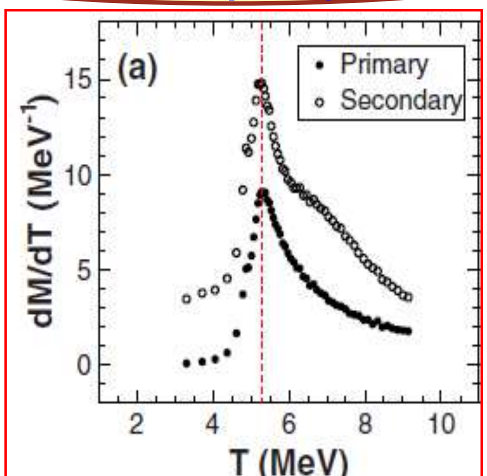
S. Das Gupta, S. Mallik & G.Chaudhuri; Phys. Rev. C 97, 044605 (2018)

Nuclear Equilibrium model (NEM)



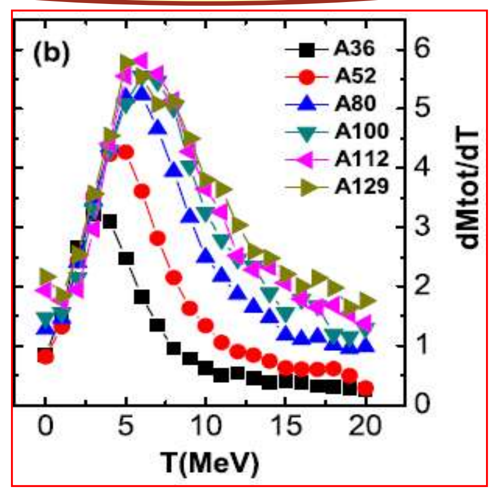
R. Bakeer et. al.; Jour. Phys. G 46, 0254105 (2019)

Statistical Multifragmentation model (SMM)



W. Lin et. al. ; Phys. Rev. C 97, 054615 (2018)

Quantum Molecular Dynamics model (IQMD)



H. Liu et. al. ; Phys. Rev. C 99, 054614 (2019)

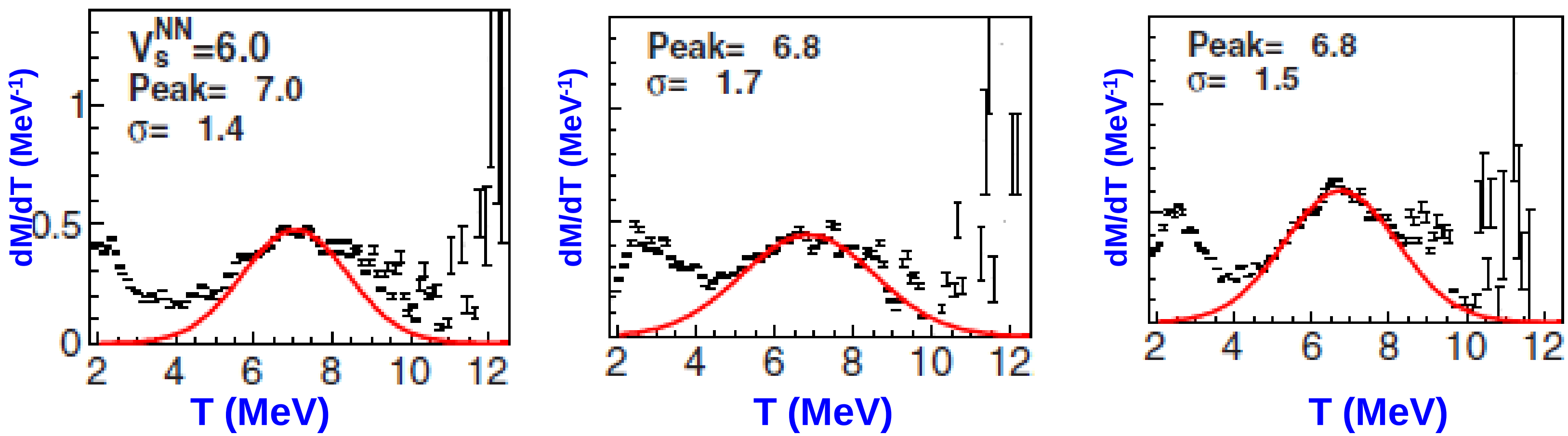
Experimental verification of multiplicity derivative:-

K=500 Superconducting Cyclotron@Texas A&M University

Studied Reaction:- $^{40}Ar+ ^{58}Ni$, $^{40}Ar+ ^{27}Al$ and $^{40}Ar+ ^{48}Ti$ @47 MeV/nucleon

- ❖ Measured the quasiprojectiles by NIRMOD 4π detector

$^{40}Ar+ ^{48}Ti$ $^{40}Ar+ ^{58}Ni$ $^{40}Ar+ ^{27}Al$

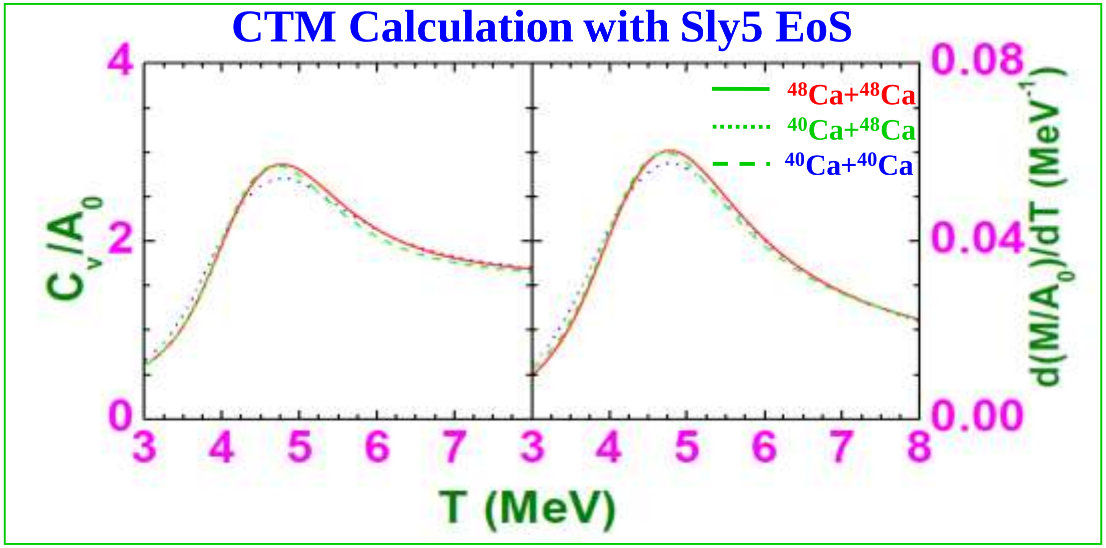


Experimentally Verified !!!

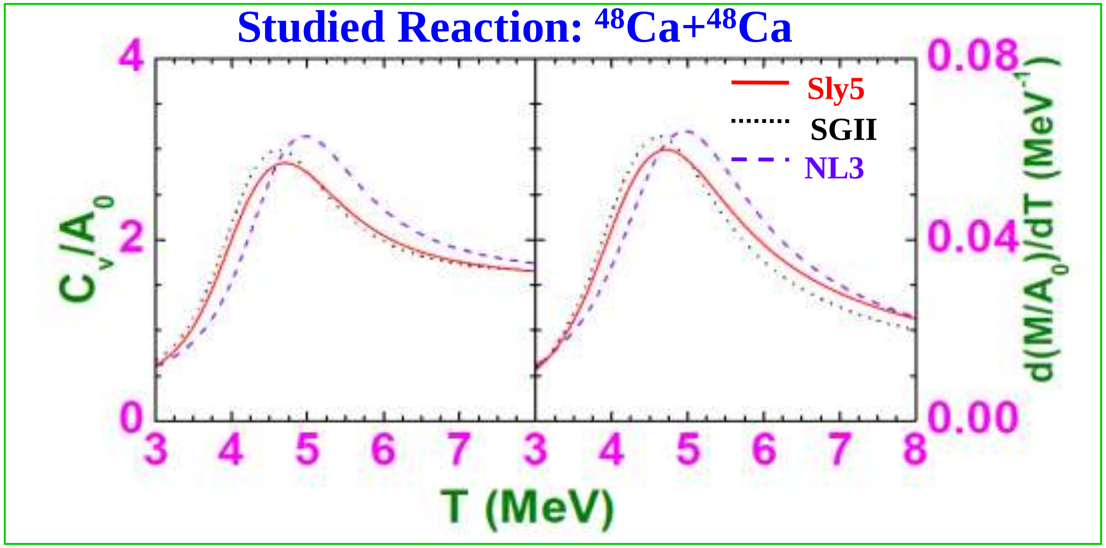
Ref: R. Wada et. al.; Phys. Rev. C 99, 024616 (2019)

Isospin effect on nuclear phase transition:-

1. Effect of isospin asymmetry of reaction:-



2. Effect of nuclear EoS:-



- The behavior of specific heat and multiplicity derivative is **largely independent** of **isospin asymmetry in the dissociating system**.

- Excitation of fragments from the SGII (NL3) EoS, is higher (lower) compared to that from the Sly5 EoS. Hence, the phase transition temperature, derived from specific heat and multiplicity derivative, is **sensitive** to different **EoS**.

Ref: S. Mallik, Nucl. Phys. A 1055, 123009 (2025)

Conclusions:-

❖Nuclear multifragmentation around the Fermi energy domain provides a unique opportunity to study intermediate mass fragments, the binding of neutron-rich isotopes, isoscaling, and more. The superconducting cyclotron facility at VECC will undoubtedly provide significant impetus to multifragmentation studies.

❖A combined theoretical and experimental approach to isoscaling and isospin transport is being utilized to reduce the current uncertainty in the isovector EoS.

❖The multiplicity derivative is proposed as a new signature of the nuclear liquid-gas phase transition, and it is easily accessible in experiments.

❖The field is wide open for different theoretical and experimental challenges.

Collaborators:-

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- C. Cimapi (Ganil, France)
- J. D. Frankland (Ganil, France)
- S. Kundu (VECC, Kolkata)
- P. Karmakar (VECC, Kolkata)

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The work is in progress…….

Thank you…