Neutron star matter equation of state from the Bayesian analysis

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arXiv: 2505.02618 (under review)

(Collaborators: Pradip K. Sahu)

- The concept of neutron stars was first introduced by Walter Baade and Fritz Zwicky.
- In 1965, Antony Hewish and Samuel Okoye detected an unusual radio signal emanating from space, sparking curiosity.
- By 1967, Franco Pacini proposed that the search for neutron stars should target the electromagnetic waves generated by their rapid rotation. This insight led Antony Hewish and Jocelyn Bell to identify the radio pulses emitted by a pulsar, a type of neutron star.
- The year 1974 marked a breakthrough when Joseph Taylor and Russell Hulse discovered the first pair of neutron stars orbiting each other.

- **Neutron star** : Good laboratory to study the state of matter at low temperature and high densities average density $(2 \sim 10)\ \rho_0$ (ρ_0 : nuclear matter saturation density).
- Generally referred as Compact Objects with mass, $M = (1-2)M_\odot$ and radius, $R = (10-12)$ km.
- **Interior of neutron star** : Hyperons, Meson condensation, strange matter, quark matter,
- **Descriptions** : NN interaction, Mean field approach…
- **Equation of State** : A key ingredient to understand the bulk properties of neutron star -- Mass, Radius, Thermal evolution, …
- **Observations** : as pulsars

Plan:

- Introduction:
- Formalism:
  - Relativistic mean field model
  - Bayesian inference method
- Neutron star (cold)=>Equation of state
- Summary

QCD Phase diagram

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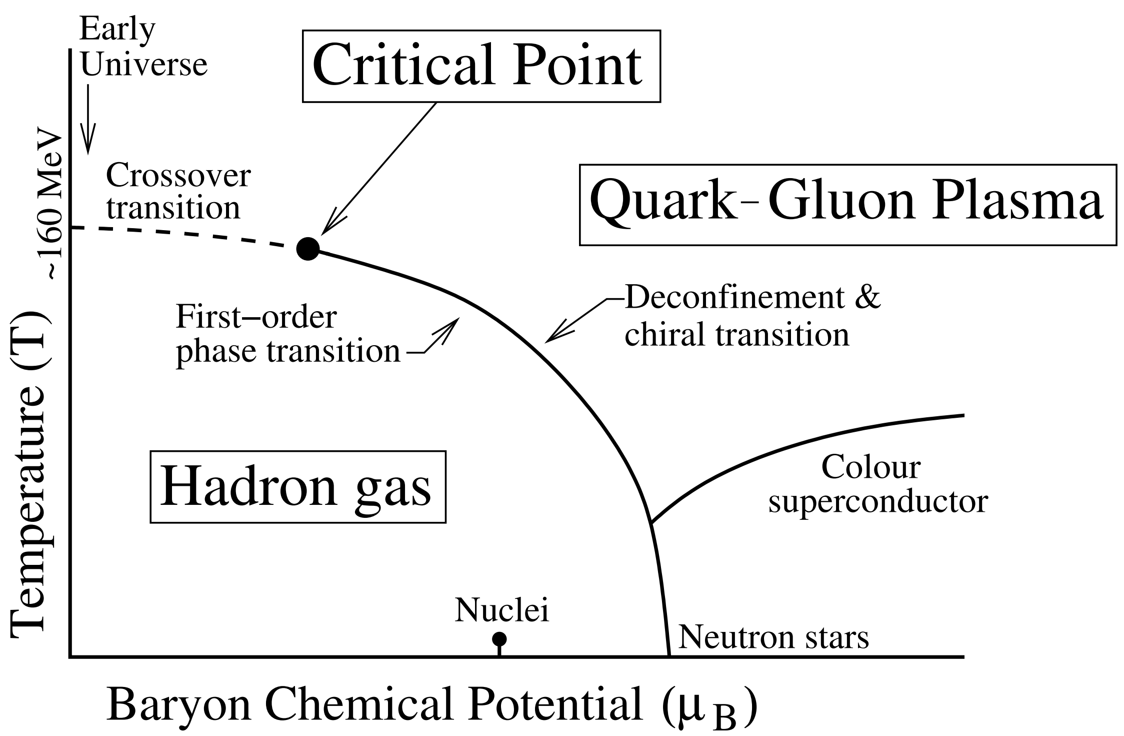
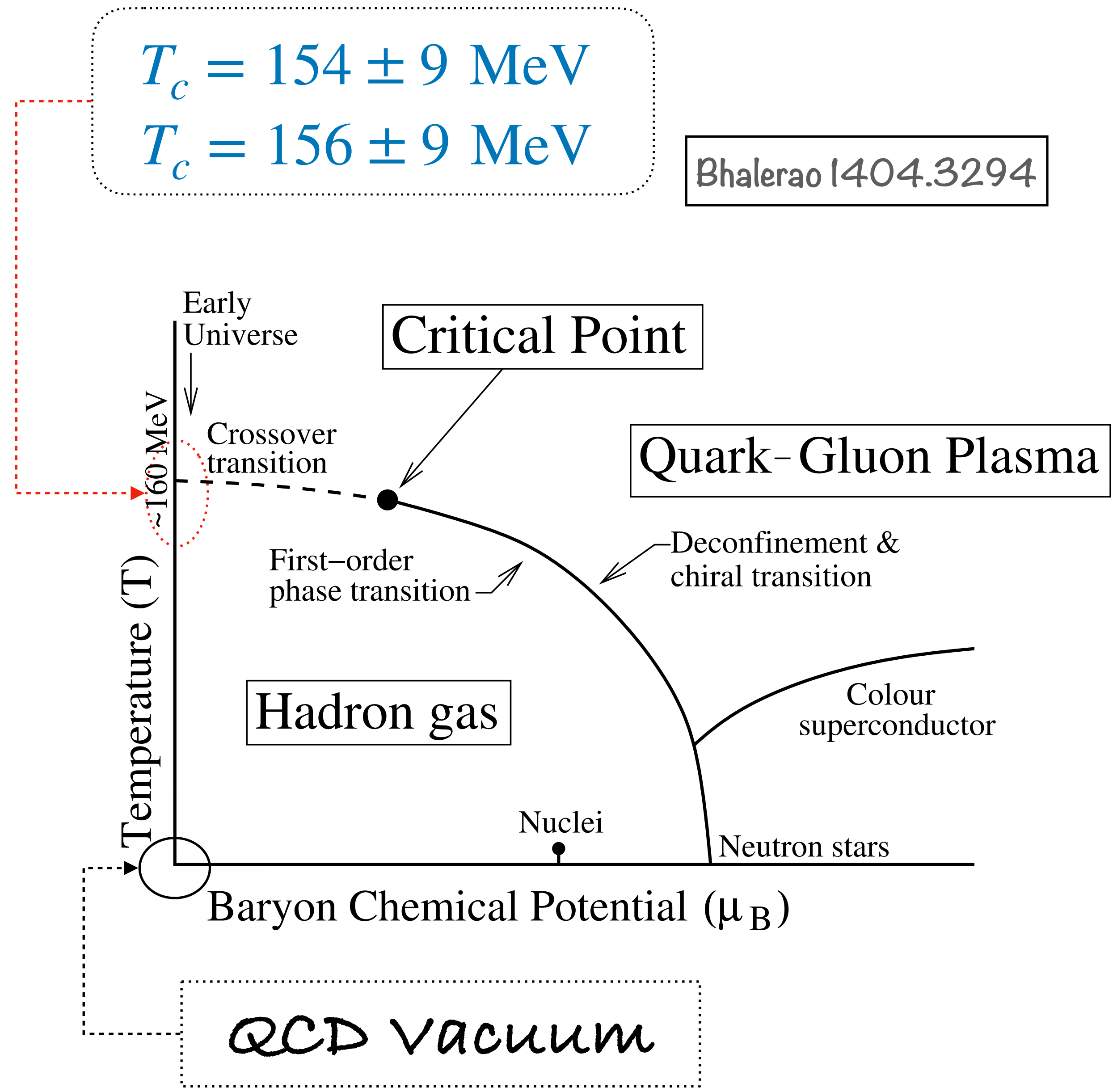


QCD Phase diagram

- Heavy-ion collisions



Introduction: Neutron Star

QCD Phase diagram

- Heavy-ion collisions
- Compact stars, (NS/HS)

PRD 90, 094503

JHEP 09, (2003) 073

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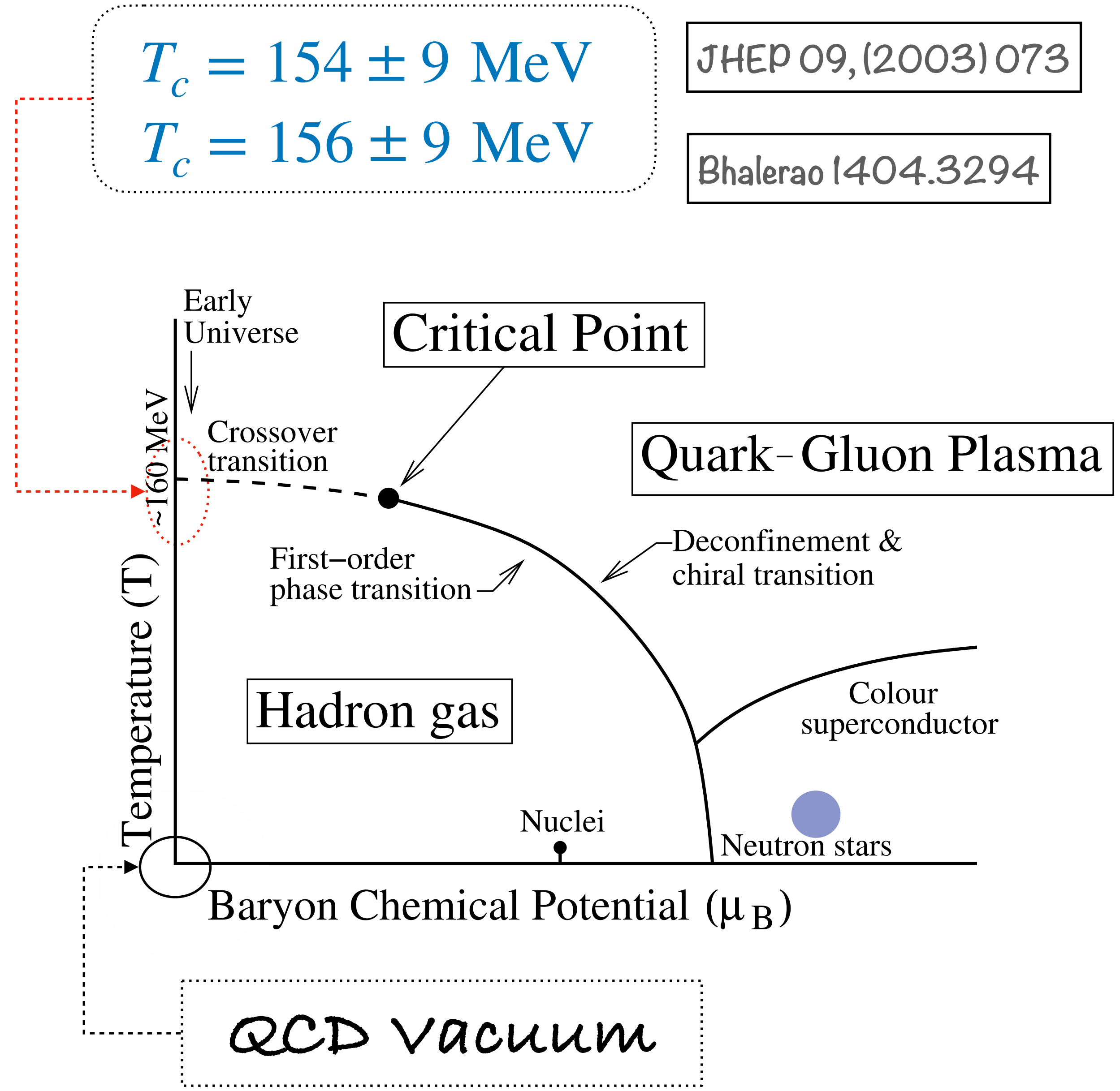

JCAP 02 (2023) 015

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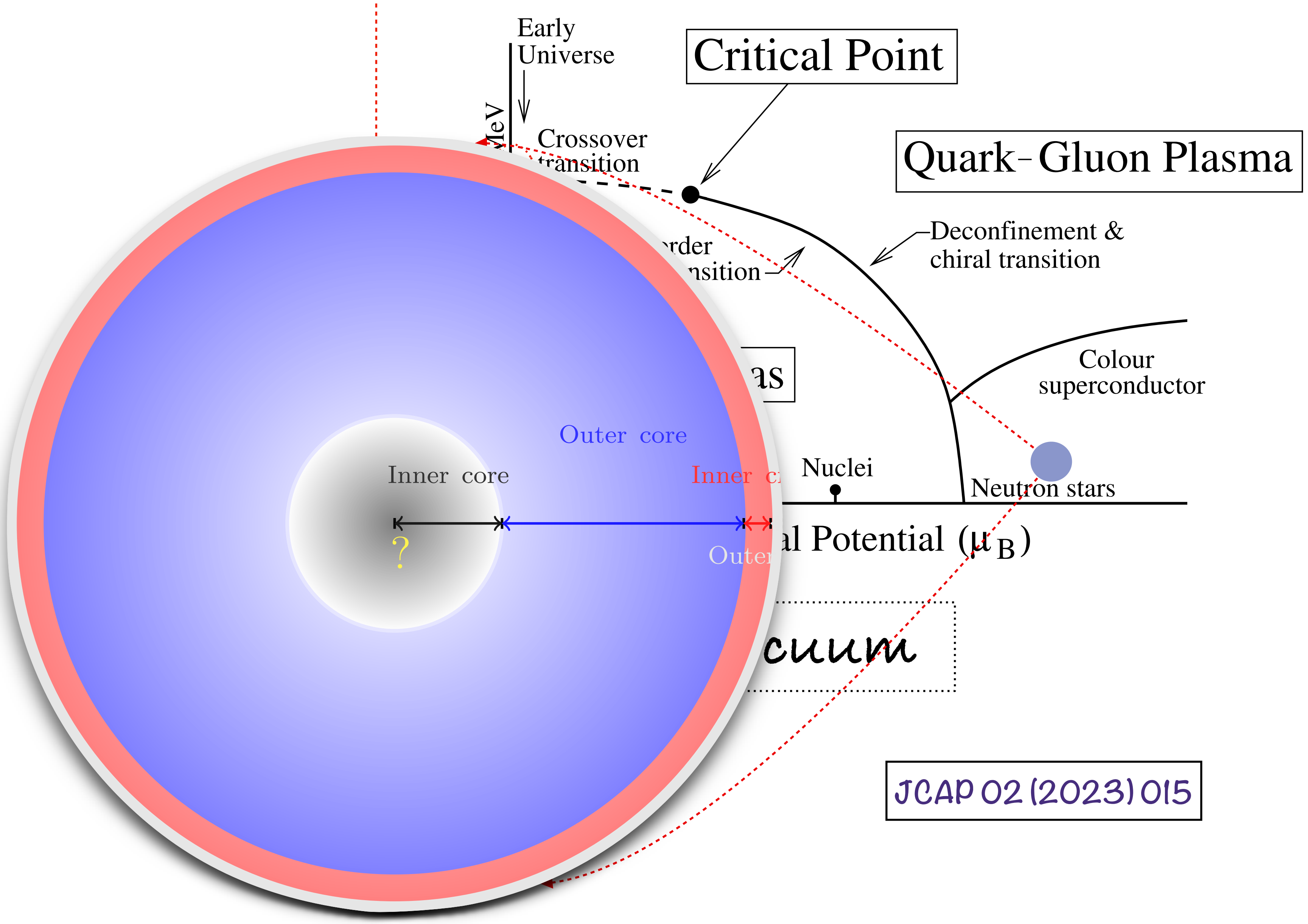

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QCD Phase diagram

- Heavy-ion collisions
- Compact stars, (NS/HS)

✤ Macroscopic properties of such NS like mass, radius, moment of inertia, tidal deformability depend on the equation of state of matter of NS.

$$\frac{dp}{dr} = -(\epsilon + p)\frac{m + 4\pi r^3 p}{r(r-2m)}, \qquad \frac{dm}{dr} = 4\pi r^2 \epsilon$$

The boundary conditions:

$$m(0) = p(R) = 0, \qquad p(r=0) = p_0,$$

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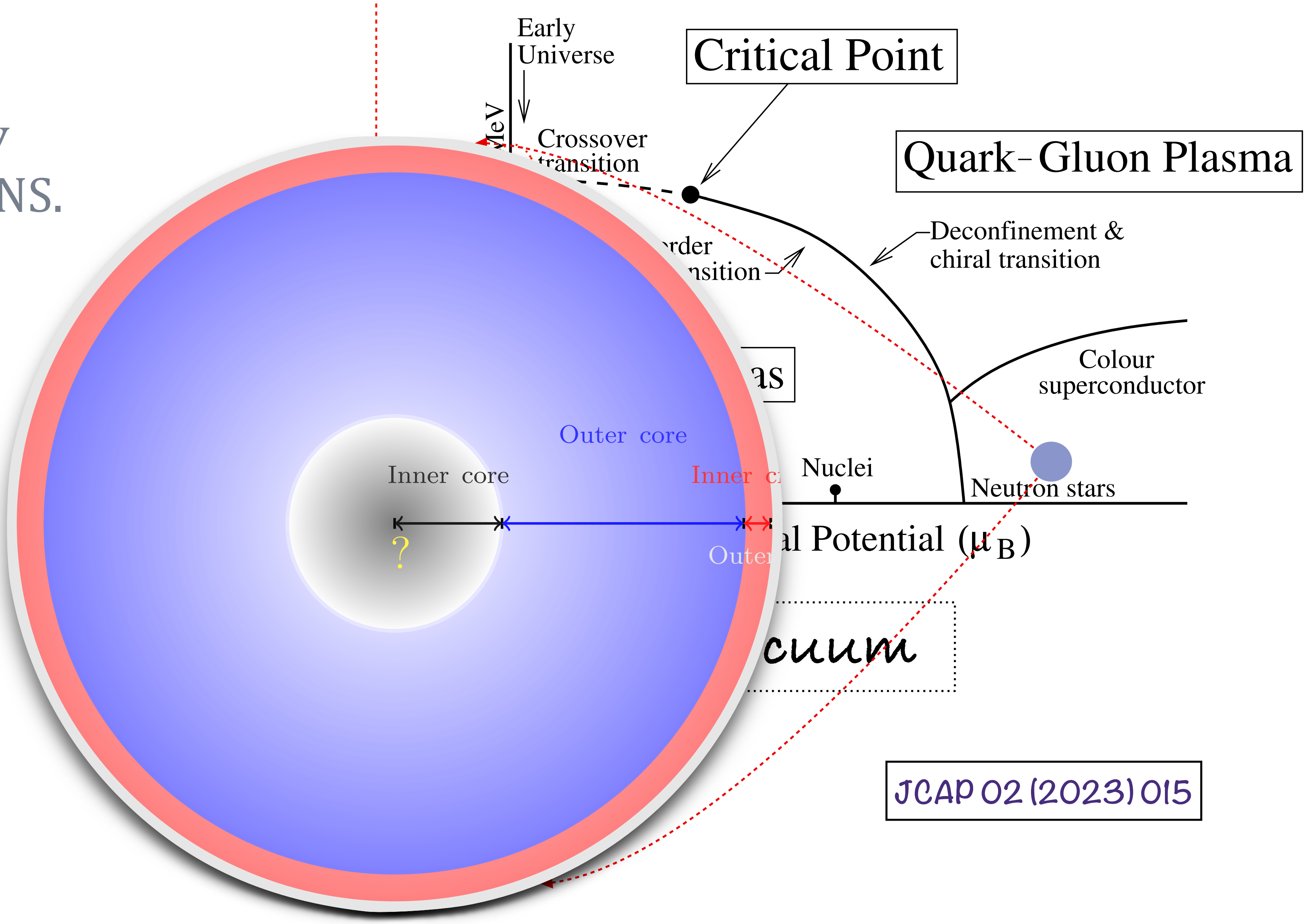

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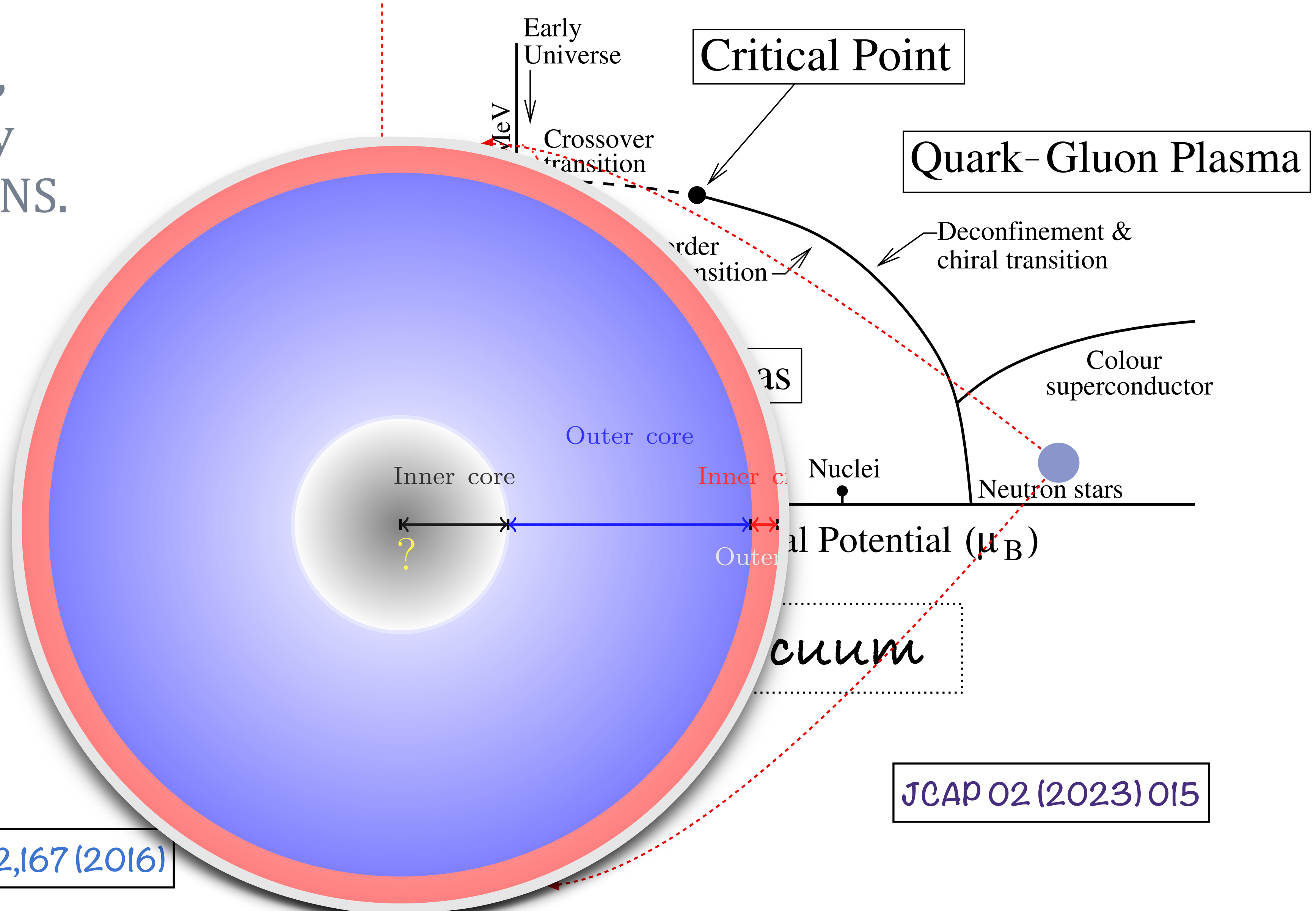

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✤ Typical observations:

$M \sim 2\ M_\odot$ and $R \sim 10$ km

- PSR J1614-2230 $M = 1.97 \pm 0.04 M_\odot$ ApJ 832,167 (2016)
- PSR J0348+0432 $M = 2.01 \pm 0.04 M_\odot$ Science 340,1233232
- PSR J1810+1744 $M = 2.13 \pm 0.07 M_\odot$ ApJL 908 L46 (2021)

Recent observation (Binary merger): GW190814

$M_1 \sim (22 - 25)\ \mathrm{M}_\odot$

$M_2 \sim (2.5 - 2.6)\ \mathrm{M}_\odot$

Black Hole

Still debatable

Abbott et al., Astrophys. J. Lett., 892(1):L3, 2020

Abbott et al., Astrophys. J. Lett., 896(2):L44, 2020

- It falls in the so-called mass-gap between heaviest neutron star and lightest known black hole.
- Astrophysical Implications: (i) If it's a neutron star, it places stringent constraints on the equation of state (EOS) of dense matter, (ii) If it's a black hole, it challenges current formation models of black holes.

Gives challenges to revisit the theoretical models for high-mass NS's equations of states i.e. $p = p(\epsilon)$.

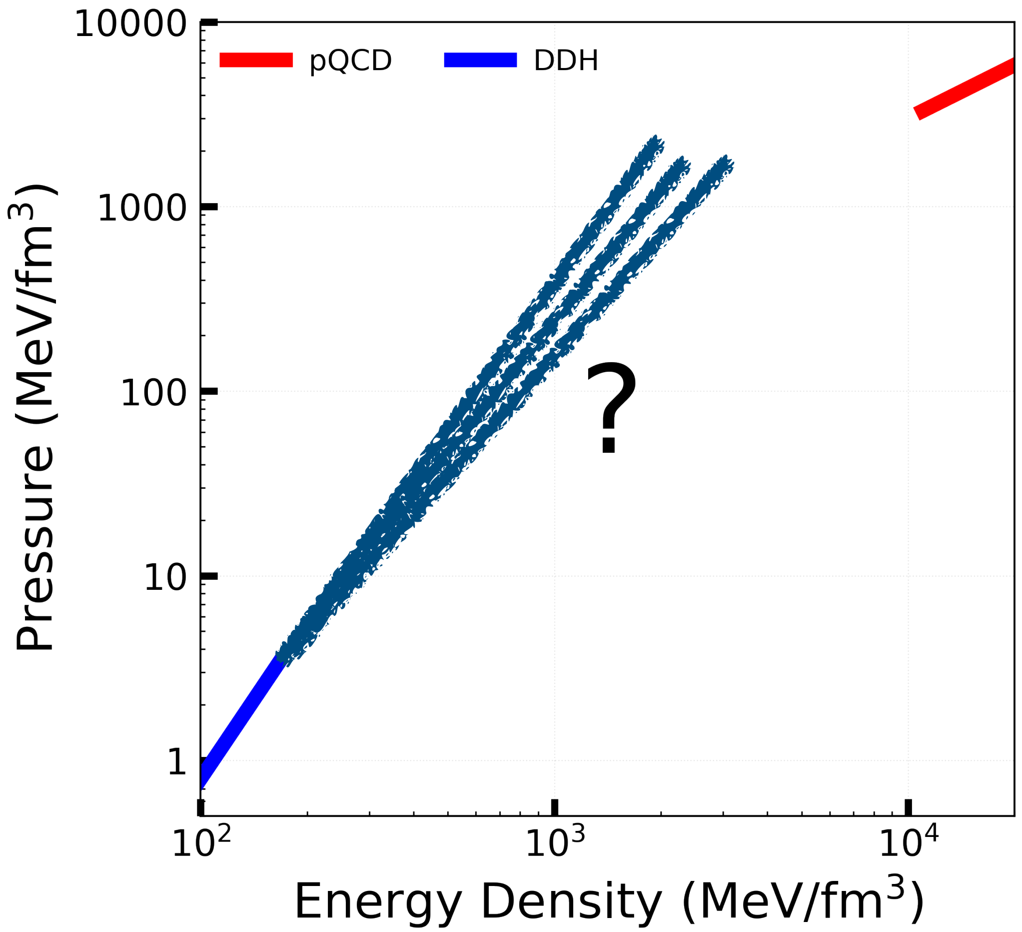
Introduction: Equation of state

- The equation of state must satisfy all the saturation properties of nuclear matter at saturation density (ρ_0).

- where energy per baryon (E/A) of symmetric nuclear matter reached its minimum value:

$$\left.\frac{\partial(E/A)}{\partial\rho}\right|_{(\rho=\rho_0)} = 0$$

- At this point, pressure of symmetric nuclear matter vanishes.



- **Saturation density**: represents the equilibrium density inside heavy nuclei.
- **Binding energy**: defines the nuclear matter stability and the average nuclear binding.
- **Incompressibility**: controls how the EOS responds to compression.
- **Symmetry energy and its slope**: determine properties of neutron-rich systems like neutron stars.

Introduction: Saturation properties

- Nuclear saturation properties (Numerical values):

  Saturation density, $\rho_0 = 0.16 \pm 0.02\ \mathrm{fm}^{-3}$

  Binding energy per nucleon, $\mathrm{BE} = -16.5 \pm 0.05\ \mathrm{MeV}$

  Incompressibility, $K_0 = 240 \pm 20\ \mathrm{MeV}$

  Symmetry energy, $J_0 = 32.5 \pm 1.5\ \mathrm{MeV}$

  Slope of Symmetry energy, $L_0 = 40 \pm 20\ \mathrm{MeV}$

  Curvature of Symmetry energy, $K_{\mathrm{sym},0} = -100 \pm 50\ \mathrm{MeV}$

Miller et al., Astrophys. J. Lett., 918(2):L28, 2021

Li et al., Phys. Rev. C, 102(4):045807, 2020

Dittmann et al., Astrophys. J., 974(2):295, 2024

Vinas et al., Symmetry, 16(2), 2024.

Garg et al., Prog. Part. Nucl. Phys., 101:55–95, 2018

Shlomo et al., Eur. Phys. J. A, 30(1):23– 30, 2006

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The equation of state (EOS) could be written as follows:

$$\epsilon(\rho, \delta) = \epsilon(\rho,0) + S_{sym}(\rho)\,\delta^2$$

$$\delta = \frac{\rho_n - \rho_p}{\rho}$$

Symmetric matter — $\epsilon(\rho,0)$

Symmetry energy — $S_{sym}(\rho)$

$$X_0^{(n)} = 3^n \rho_0^n \left(\frac{\partial^n \epsilon(\rho,0)}{\partial \rho^n}\right)_{\rho_0}, \qquad n = 2,3,4;$$

$$X_{sym,0}^{(n)} = 3^n \rho_0^n \left(\frac{\partial^n S(\rho)}{\partial \rho^n}\right)_{\rho_0}, \qquad n = 1,2,3,4.$$

Mass of neutron star, $M = 2.01 \pm 0.04\ M_\odot$

Radius of Neutron star of Mass 1.4 $M_\odot$, $R_{1.4} = 11\ \text{km}$

Maximum mass, $M_{max} = 2.6 \pm 0.08\ M_\odot$

Nuclear matter at high density

- Theoretical description of infinite nuclear matter and finite nuclear properties is relying on models since the primordial developments of nuclear physics.
- All relativistic models are written in terms of parameters that are fitted to reproduce either bulk nuclear matter or finite nuclei properties.
  - ✓ Most models behave approximately the same as far as equations of state are concerned around saturation density.
  - ✓ These very same models are used to describe physics taking place at sub-saturation densities, and also at very high densities, such as neutron-star matter.
  - ✓ As a consequence, models that describe similar physics at saturation density yield very different results when used in the low- or high-density limits.

☑ Walecka's mean field model

- Relativistic nucleons interact through exchange of mesons
  - scalar meson exchange ⇒ *Attraction*
  - vector meson exchange ⇒ *Repulsion*
- Adjust the couplings and the masses so that B.E. per nucleon at saturation densities is reproduce.
- More terms can also be introduced to describe other properties of nuclear matter.

Formalism: RMF model

Walecka's mean field model

$$\mathcal{L} = \sum_i \bar{\Psi}_i \left(i\gamma_\mu \partial^\mu - m_i + g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu - g_\rho \gamma_\mu \vec{I}_i \vec{\rho}^\mu \right) \Psi_i + \mathcal{L}_{\text{mes}}$$

$$\mathcal{L}_{\text{mes}} = \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4}\vec{R}^{\mu\nu}\vec{R}_{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \vec{\rho}^\mu$$
$$-\frac{\kappa}{3!}(g_{\sigma N}\sigma)^3 - \frac{\lambda}{4!}(g_{\sigma N}\sigma)^4 + \frac{\xi}{4!}(g_{\omega N}^2 \omega_\mu \omega^\mu)^2 + \Lambda_{\omega\rho}(g_{\omega N}^2 \omega_\mu \omega^\mu)(g_{\rho N}^2 \rho_\mu \rho^\mu)$$

$$\epsilon = -\gamma \sum_{i=n,p,e} \int \frac{d^3\mathbf{k}}{(2\pi)^3} E_i^* \left(1 - f_-^i - f_+^i\right)$$
$$+\frac{1}{2} m_\sigma^2 \sigma_0^2 + \frac{1}{3}\kappa \sigma_0^3 + \frac{1}{4}\lambda \sigma_0^4 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_0^{3\,2} + \frac{\xi}{8}\left(g_{\omega N}\omega\right)^4 + 3\Lambda_{\omega\rho}\left(g_{\omega N} g_{\rho N} \omega_0 \rho_0^3\right)^2$$

and,
$$p = \sum_{i=n,p,e} \mu_i n_i - \epsilon$$

The predictive power of the RMF model crucially depends on the choice of these coupling constants, which are usually calibrated to reproduce nuclear saturation properties and neutron star observations.

Formalism: Bayesian inference

To systematically quantify uncertainties in the coupling constants and propagate them to macroscopic observables (mass–radius relations, tidal deformabilities, etc.), **Bayesian inference** offers a powerful framework.

☑ Bayesian analysis:

$$P(\theta\,|\,\mathbf{D}) = \frac{\mathscr{L}(\mathbf{D}\,|\,\theta)\mathbf{P}(\theta)}{\mathscr{Z}}$$

- where θ and $\mathbf{D}$ denote the set of model parameters and the fit data.
- $\mathbf{P}(\theta)$ is the prior for the model parameters and $\mathscr{Z}$ is the evidence.
- The $\mathbf{P}(\theta\,|\,\mathbf{D})$ is the joint posterior distribution of the parameters,
- $\mathscr{L}(\mathbf{D}\,|\,\theta)$ is the likelihood

Prior distribution: based on physical considerations (e.g., nuclear saturation constraints, symmetry energy bounds)

Likelihood: constructed using experimental and observational data (finite nuclei properties, astrophysical measurements such as neutron star masses and radii

Formalism: Bayesian inference

Prior distribution

$g_\sigma \in [8.0,12.0]$, $g_\omega \in [10.0,13.0]$, $g_\rho \in [8.0,14.0]$, $\kappa \in [3.0,8.5]$,
$\lambda \in [-0.05,0]$, $\xi \in [0,0.001]$, $\Lambda_{\omega\rho} \in [0,0.2]$.

Likelihood

$$\mathscr{L}(D|\boldsymbol{\theta}) \propto \exp\left[-\frac{1}{2}\chi^2(\boldsymbol{\theta})\right]$$

$$\chi^2(\boldsymbol{\theta}) = \sum_{i=1}^{N}\left[-\frac{X_i^{\text{model}}(\boldsymbol{\theta}) - X_i^{\text{exp}}}{\sigma_{X_i}}\right]^2$$

Nuclear saturation properties	Numerical values
E/A (MeV)	-16.5 ± 0.05
K_0 (MeV)	240 ± 20
J_0 (MeV)	32.5 ± 1.5
L_0 (MeV)	40 ± 20
$K_{\text{sym},0}$ (MeV)	-100 ± 50
M ($M_\odot$)	2.05 ± 0.04
$R_{1.40}$ (J0030 + 0451) (km)	12.35 ± 0.75
$R_{2.08}$ (J0740 + 6620) (km)	12.45 ± 0.65
$R_{1.36}$ (GW170817) (km)	11.45 ± 0.60
$\Lambda_{1.40}$ (GW170817)	190 ± 85
$M(M_\odot)$: (GW190814)	2.59 ± 0.08

Formalism: Bayesian inference

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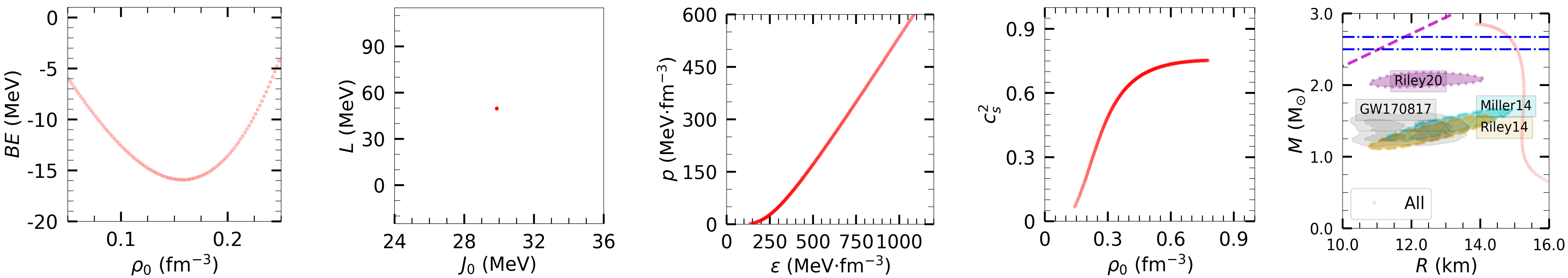
☑ For single parameter set **MCMC approach**



Results

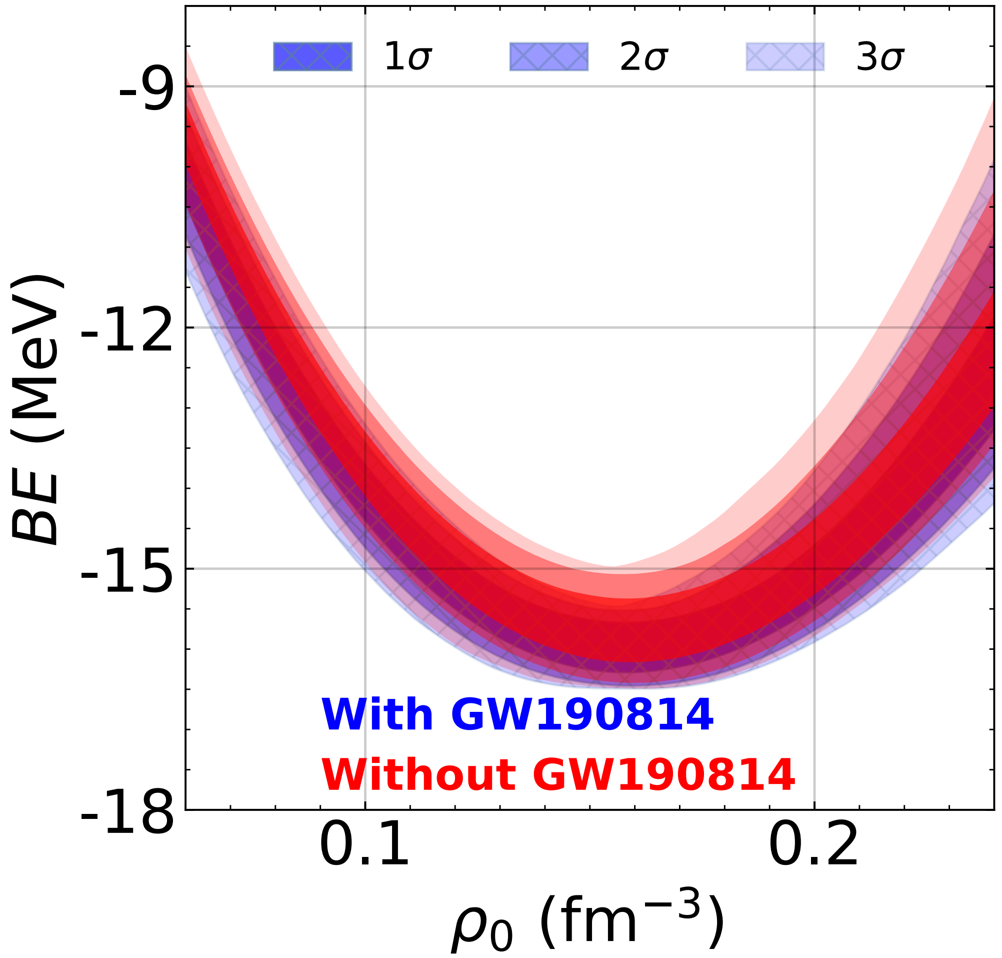
- ☑ Case I: when GW190814 is included
- ☑ Case II: when GW190814 is not included

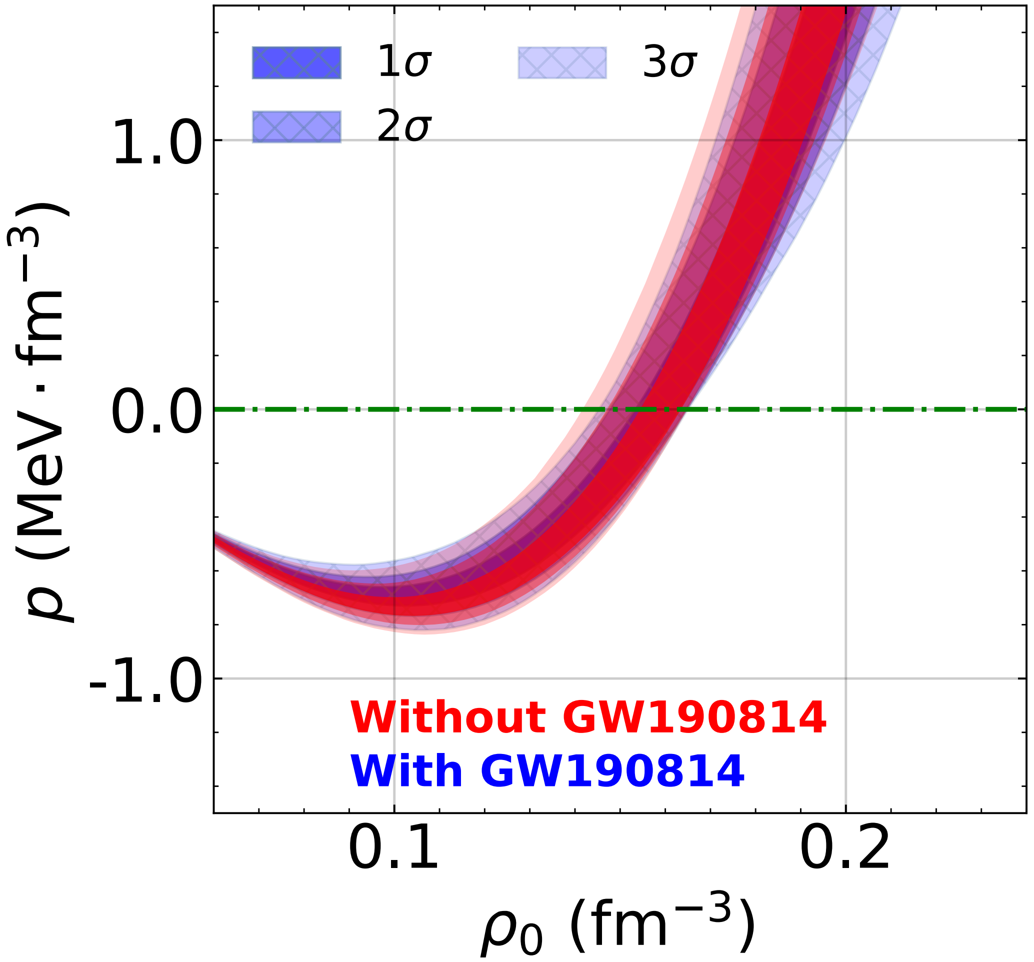
✓ Sampled parameter set: 60 Million

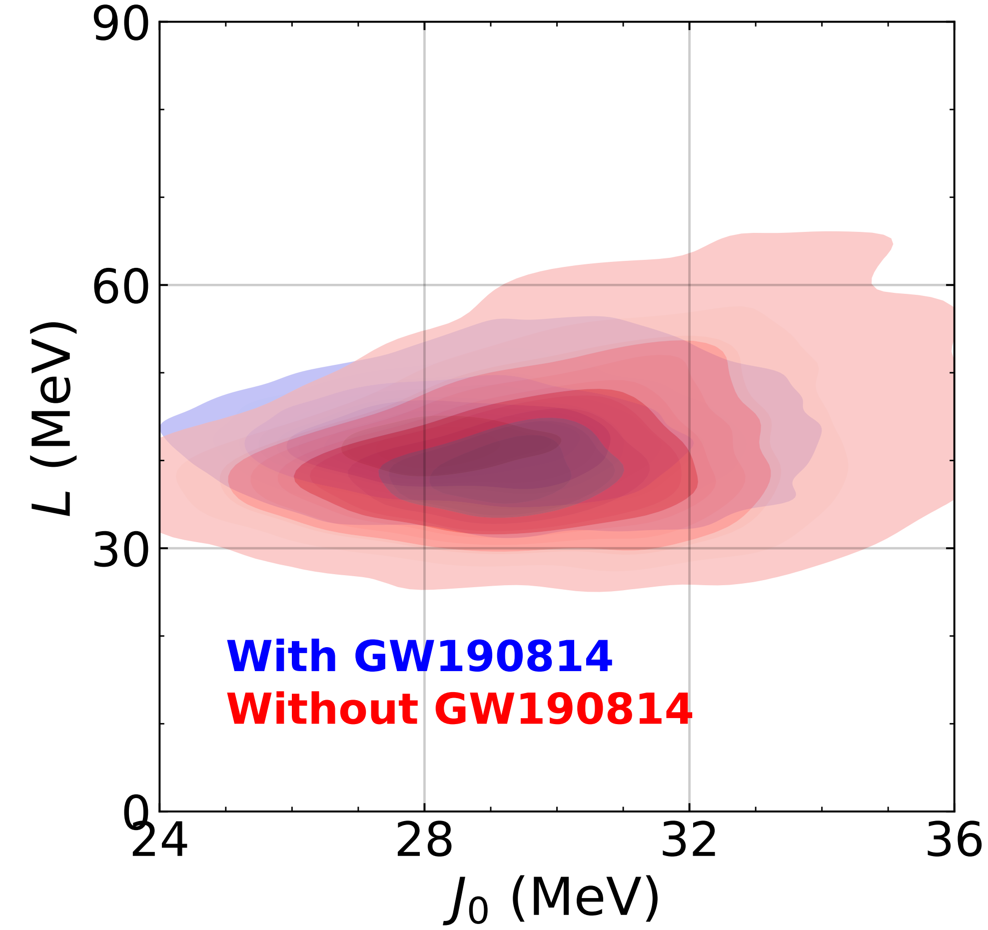
✓ Selected parameter sets: about 20k

$$\chi^2|_{\max} < 50 : \text{with GW190814 constraint}$$

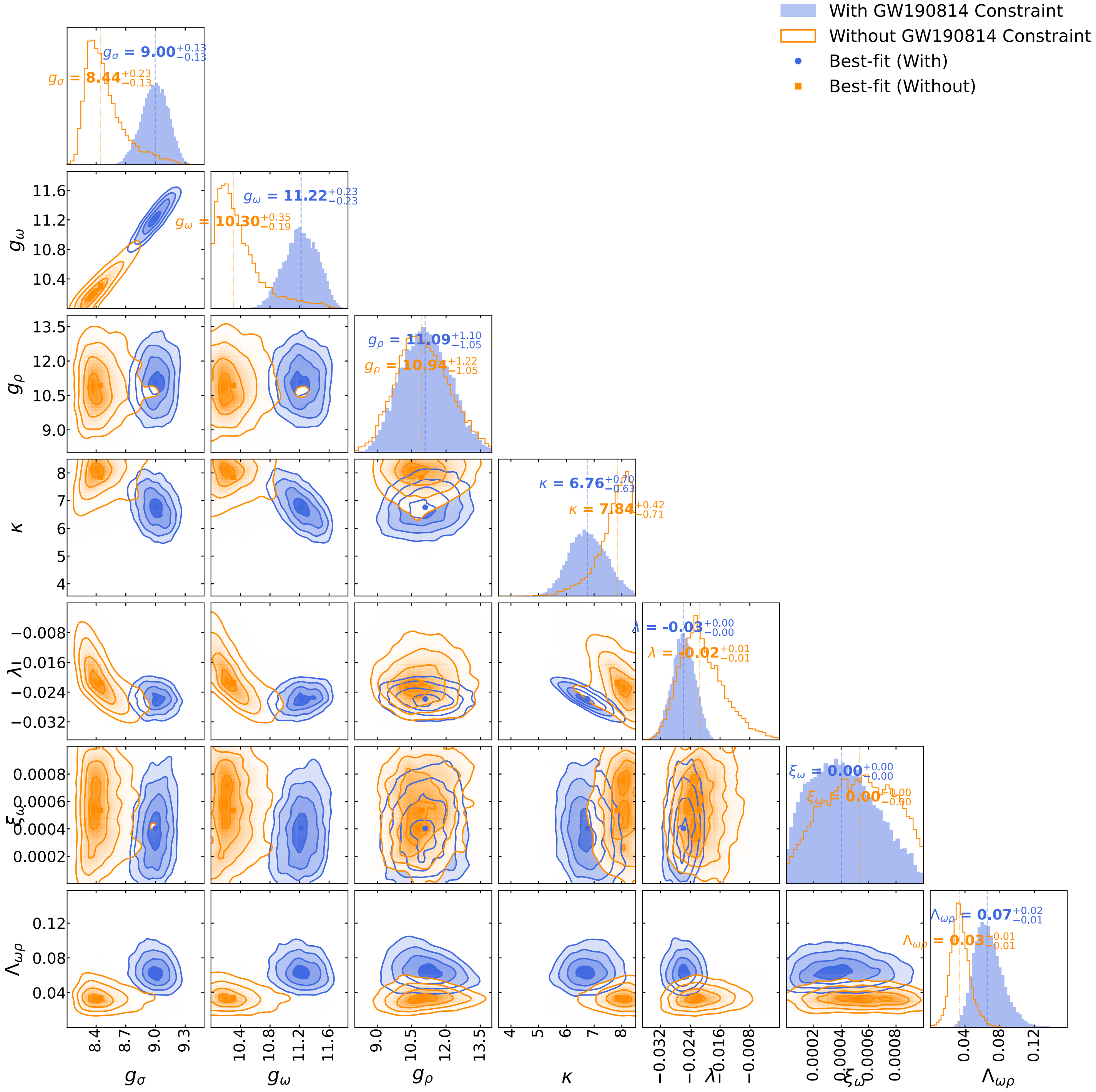
$$\chi^2|_{\max} < 50 : \text{without GW190814 constraint}$$



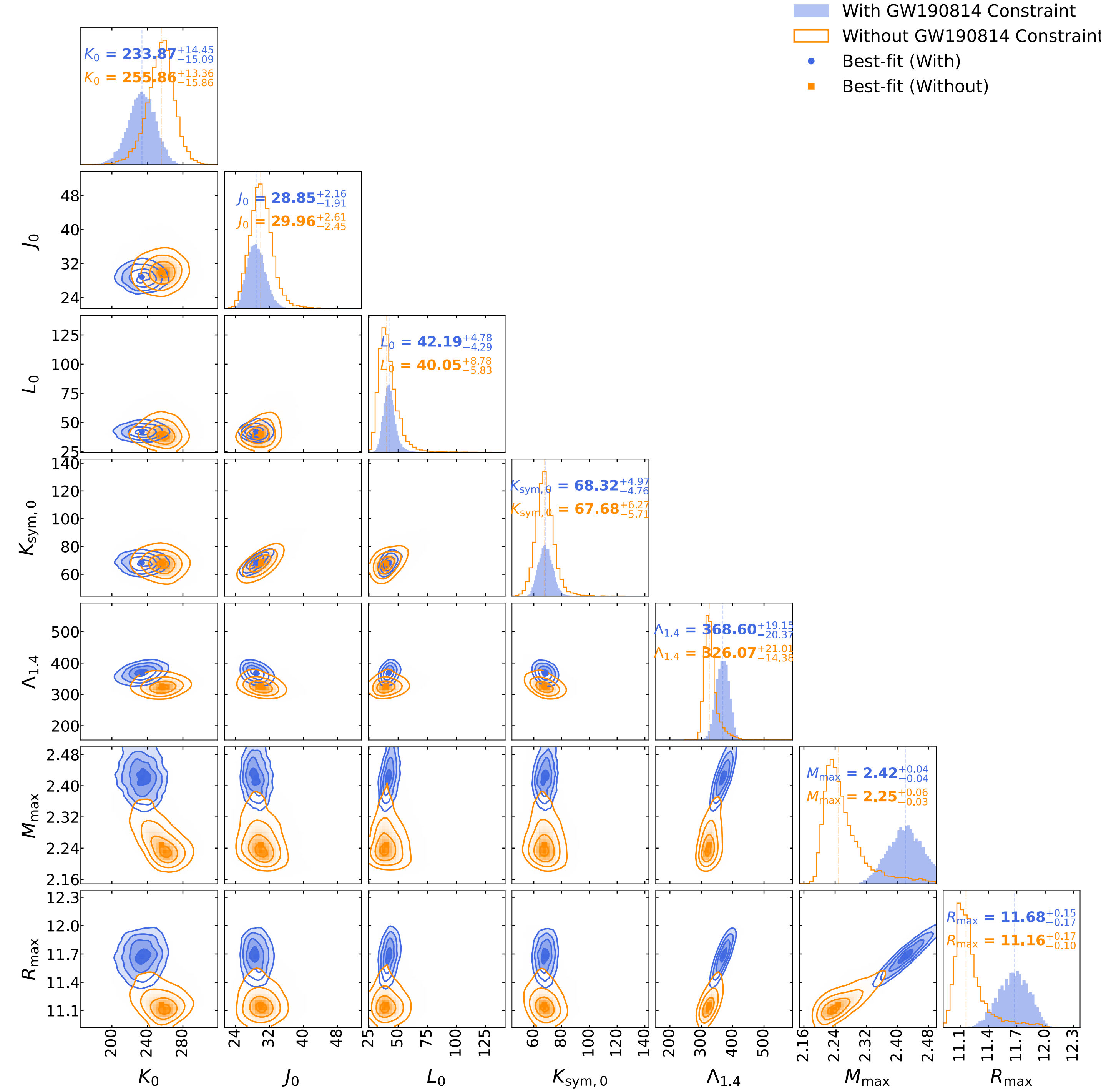




Results

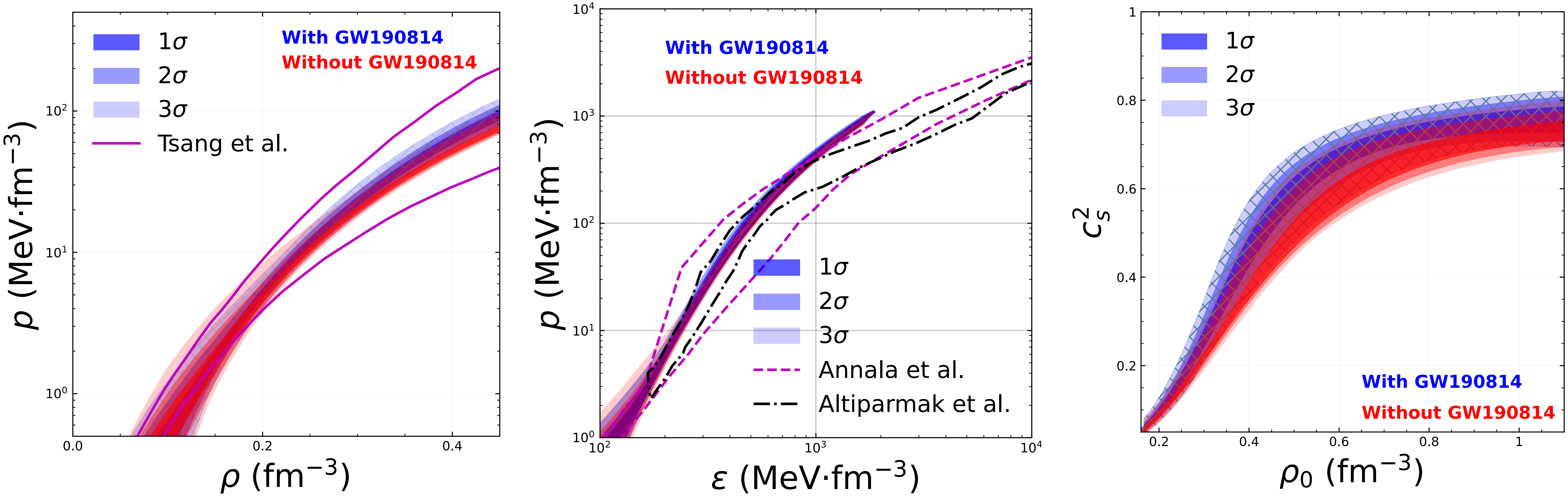


coupling constants



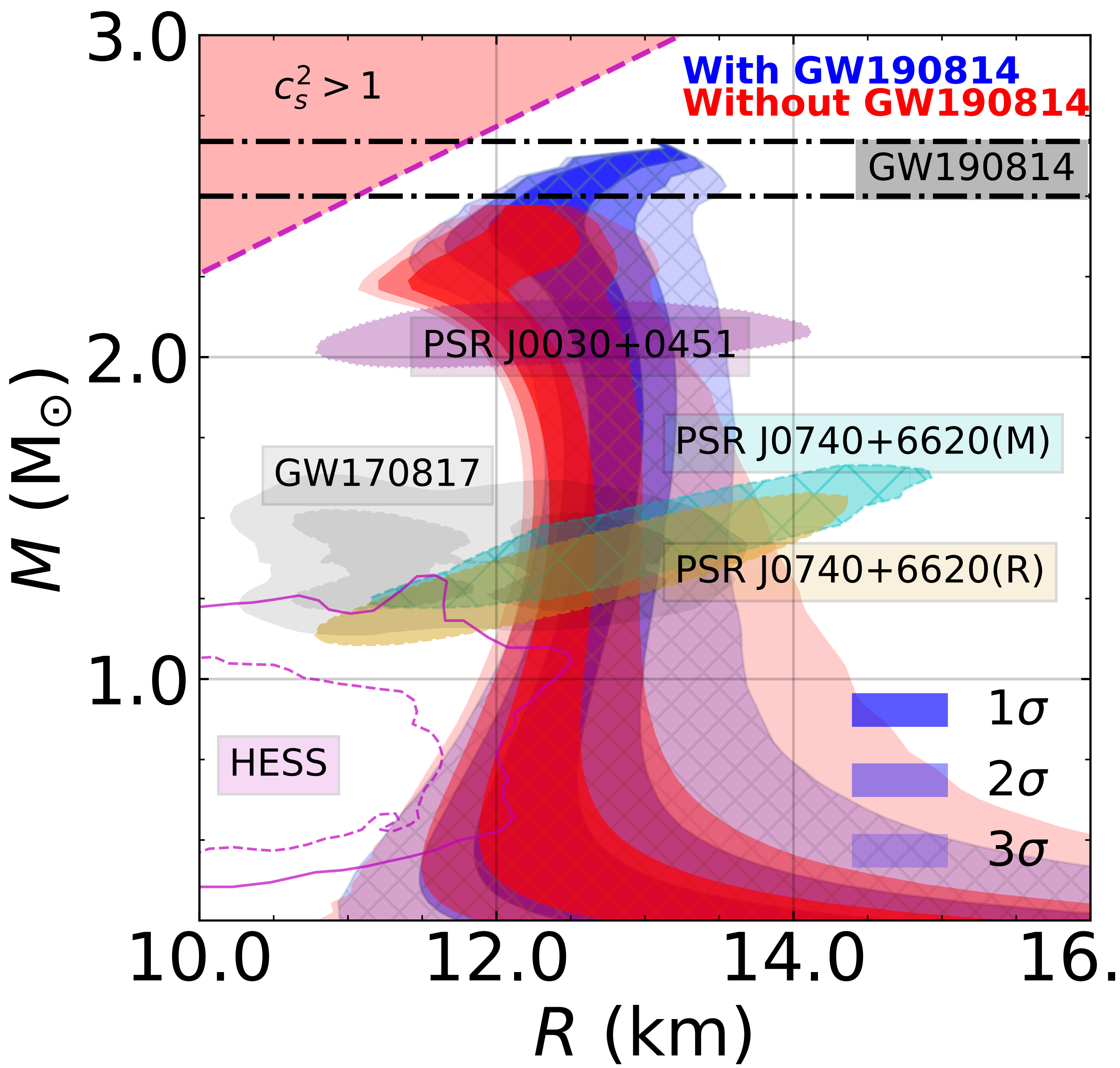
Nuclear saturation and NS properties

Results

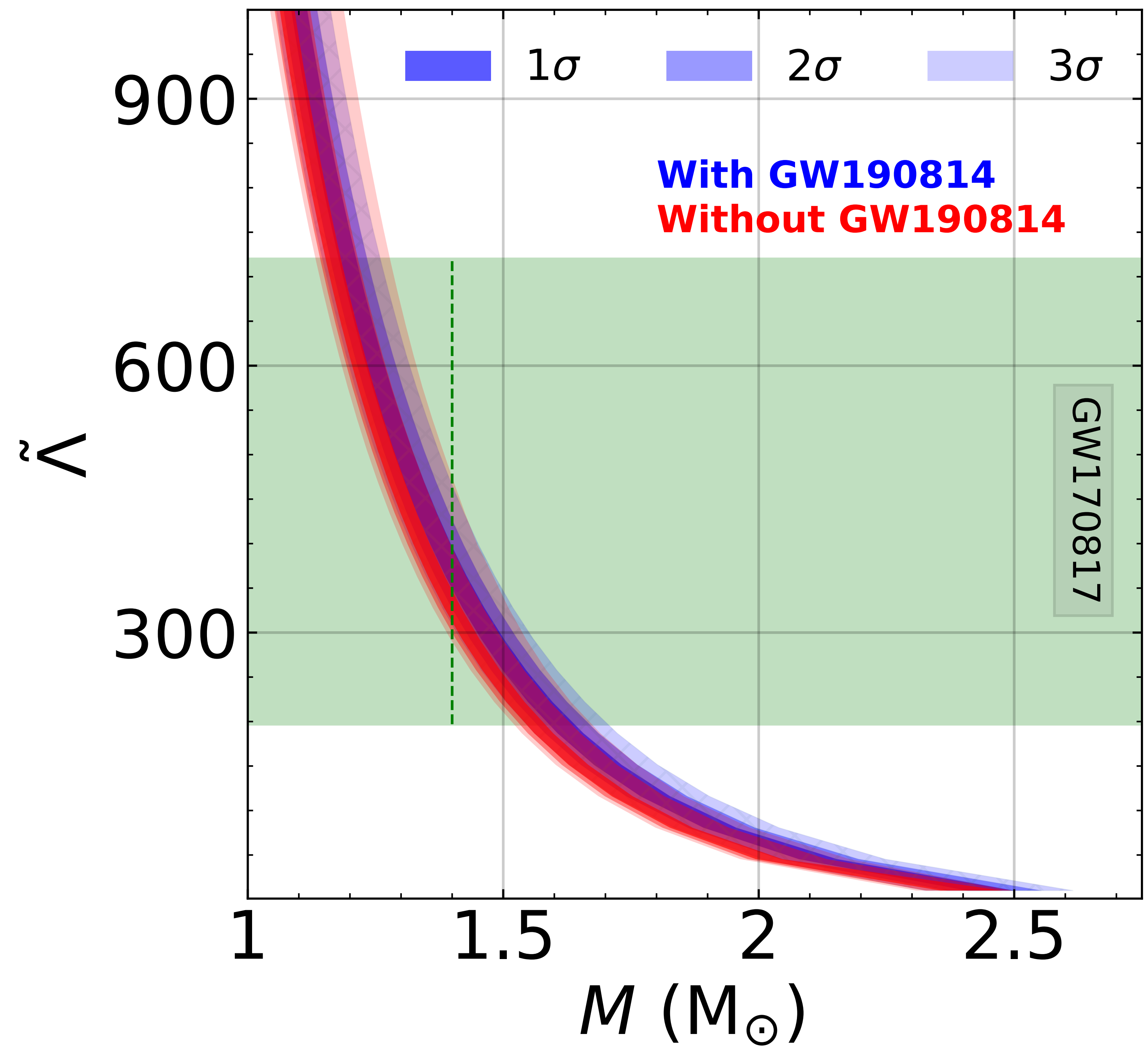


1σ
2σ
3σ
Tsang et al.
With GW190814
Without GW190814
p (MeV·fm^{-3})
ρ (fm^{-3})
Annala et al.
Altiparmak et al.
ε (MeV·fm^{-3})
c_s^2
ρ_0 (fm^{-3})

Results



3.0
2.0
1.0
$c_s^2 > 1$
With GW190814
Without GW190814
GW190814
PSR J0030+0451
PSR J0740+6620(M)
GW170817
PSR J0740+6620(R)
HESS
1σ
2σ
3σ
M (M⊙)
10.0
12.0
14.0
16.
R (km)



1σ
2σ
3σ
900
600
300
Λ̃
With GW190814
Without GW190814
GW170817
1
1.5
2
2.5
M (M⊙)

Summary

- We discussed the basics of neutron stars where neutron stars are natural laboratories to study high-density interaction.
- Nuclear matter is described within the relativistic mean field model.
- Bayesian approach is used to extract coupling parameters in such a way that nuclear saturation properties along with neutron stars observations must be satisfy.
- We have seen here that larger value of $\Lambda_{\omega\rho}$ gives larger mass of NS without changing saturation properties.

thank you your kind attention

The data of 20k equations of states can be obtained by email (deepak.kumar@iopb.res.in). We will upload the whole data on public repository like zenodo/git-hub after accepting the article.