Strong CP violation in dense matter: consequences for neutron stars

Hiranmaya Mishra
Institute of Physics Bhubaneswar, India

Nov 10, 2025

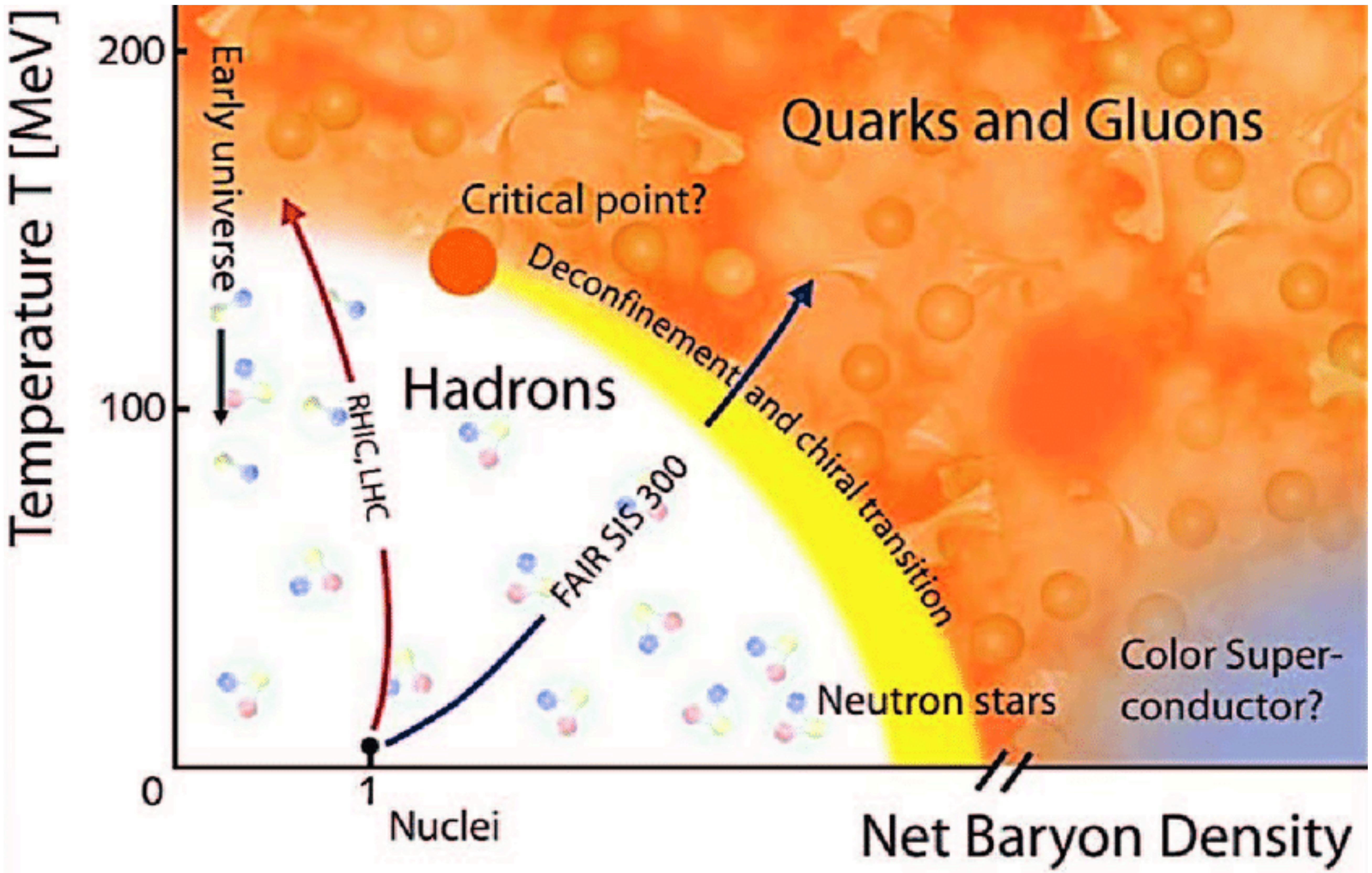
INDIA-JINR Workshop on Particle, Nuclear, Neutrino Physics and Astrophysics (10-12 Nov, 2024)

Based on: *Deepak Kumar*, ***HM***, Arxiv:2411.17828, A. Abhishek, A. Das, R. Mohapatra and **HM** Phys. Rev. D 103, 074003 (2021); B. Chatterjee, A. Mishra, **HM** Phys. Rev. D91, 034031 (015); PRD 85, 114008 (2012)

Outline

- Introduction
- Strong CP violation in hot and dense quark matter
- Hadron-quark phase transition with axions and neutron stars
  - Quark matter EOS including axions (Nambu--Jona-Lasinio model)
  - Relativistic mean field (RMF) EOS (hadronic matter)
  - Phase transition and mixed phase construct
- Non-radial oscillations in neutron stars (f and g modes)
- Axions and stability of neutron stars and f-mode oscillations
- Conclusions and outlook

- Sketch of QCD Phase diagram



Introduction

- Neutron stars (NS) are interesting astrophysical laboratories to study the behaviour of matter at extreme densities.
- NS are heavy ($M \sim 2M_\odot$) and compact $R \leq 13.5$ km.

  - PSR J1614-2230 $\rightarrow M = 1.97 \pm 0.04 M_\odot$ ApJ 832,167 (2016)
  - PSR J0348+0432 $\rightarrow M = 2.01 \pm 0.04 M_\odot$ Science 340,1233232
  - PSR J1810+1744 $\rightarrow M = 2.13 \pm 0.07 M_\odot$ ApJL 908 L46 (2021)

- Virgo and LIGO collaboration of binary neutron stars merger GW170817 opened up a new observational window on compact stars properties.

- Gravitational waves from the merging NS measure

  tidal deformablity, $Q_{ij} = -\lambda E_{ij}$, Dimensionless, $\Lambda = \frac{2}{3} k \left(\frac{R}{M}\right)^5$

- The tides accelerate the inspiral and produce the phase shifts in the GW compared to the case of two point masses.

$$\delta\phi \propto \Lambda \sim \left(\frac{R}{M}\right)^5$$

Introduction

- Tidal deformiblity constrains EOS at high density.
- It is possible, though not conclusive, that one or both components of the compact stars in the merger could be hybrid stars.

Nandi, Char Apj 2018, 857, 12

Phys. Rev. D 97, 084038, 2018

- A hybrid star with a quark matter core is indistinguishable from a canonical NS based on current observational status. (masquerade problem)

Alford *et . al .* ApJ 629, 969, 2005

- It has been suggested that the non-radial oscillation modes of NS can have the possibility of providing the compositional information about the matter in the interior of the NSs(hyperon core, quark core, mixed phase with quark and hadronic matter).

How do axions affect EOS and hence NR oscillations of neutron stars ?
(signatures)?

- Strong CP violation in hot and dense quark matter

CP violation in dense matter

- Consistent with Lorentz invariance and gauge invariance QCD Lagrangian can have the "θ" term

$$\mathcal{L}_\theta = \frac{\theta}{64\pi^2} g^2 F^a_{\mu\nu} \tilde{F}^{a\mu\nu}$$

non-vanishing $\theta \Rightarrow$ explicit breaks of P and CP

CP is broken spontaneously for $\theta = \pi$

- For vacuum: $\theta \sim 0$. (electric dipole moment of neutron $\Rightarrow \theta < 10^{-11}$)
- Why θ is small despite the fact that it is not forbidden by gauge invariance/Lorentz invariance? (strong CP problem)
- An elegant solution - (Peccei-Qunin Mechanism) a pseudo scalar field "a" exists and couples to non-trivial field configuration

$$\mathcal{L}_a \sim \frac{a}{f_a} F\tilde{F}$$

so that CP violation is driven by $(\theta + \frac{a}{f_a})$. In vacuum, the effective potential for a has a minimum such that

$$\langle \theta + \frac{a}{f_a} \rangle = 0$$

CP violation in dense matter

- Axion: a cold dark matter candidate

  How do the properties of axions vary in medium ?

  How do the axions affect the EOS of quark matter ?

  Consequences of axion interaction on neutron star oscillations.

- Axions, as prime DM candidate, may also influence neutron star properties due to their possible continual accumulation and by their gravitational capture during stellar formation.

- NS may contain substantial amount of DM in the form of axions.

- Changes in composition in NS matter affects EOS which enters as a crucial input for the structure of the star. (mass, radius, tidal deformability, oscillation modes)

CP violation in dense matter

- Even if CP is not violated for QCD vacuum, it is conceivable that it can be violated for QCD matter at finite temperature and/ densities.

- Presence of a non-vanishing θ leads to a rich vacuum structure.

- Studying CP violation in full QCD is difficult due to non-perturbative nature of θ term for arbitrary values of θ.

- We shall use a Nambu--Jona-Lasinio model (NJL model) which incorporates instanton induced interaction responsible for $U(1)_A$ breaking and also able to describe chiral symmetry breaking as well as coupling of quarks to axions.

- Quark matter with axions within Nambu--Jona-Lasinio model

Strong CP violation in dense matter (NJL model)

The Lagrangian for 3flavour NJL model including the CP violating effects through axion fields

$$\mathscr{L} = \bar{q}(i\gamma^\mu\partial_\mu - \hat{m})q + G_s \sum_{A=0}^{8} \left[(\bar{q}\lambda^A q)^2 + (\bar{q}i\gamma_5\lambda^A q)^2\right] + \mathscr{L}_{\text{KMT}} + \mathscr{L}_{\text{V}}$$

$$\theta = \frac{a}{f_a}$$

$$\mathscr{L}_{\text{KMT}} = -K\left[e^{i\theta}\det\{\bar{q}(1+\gamma^5)q\} + e^{-i\theta}\det\{\bar{q}(1-\gamma^5)q\}\right]$$

$$\mathscr{L}_{\text{V}} = -\sum_{A=0}^{8} G_{\text{v}} \left[\left(\bar{q}\gamma^\mu\lambda^A q\right)^2 + \left(\bar{q}\gamma^\mu\gamma^5\lambda^A q\right)^2\right]$$

where $q = (u \;\; d \;\; s)^{\text{T}}$, G_{v} is a free parameter introduced to include a repulsive vector interaction.

$\hat{m} = \text{diag}(m_u, m_d, m_s)$; $\lambda^A, A = (1...8)$ are Gellmann matrices

Kobayashi, Maskawa Prog. Theor. Phys. 44 (1970), 1422

G, 't Hooft, Phys. Rev. D 14 (1976), 3432

Strong CP violation in dense matter (NJL model)

$\mathscr{L}_{\text{KMT}}$ breaks $U(1)_{\text{Axial}}$

$$\partial_\mu J_5^\mu = 2im\bar{q}\gamma^5 q + 2iN_f K\left(e^{i\theta}\det_f \bar{q}(1+\gamma^5)q - \text{h.c.}\right)$$

Compare with the usual anomaly equation

$$\partial_\mu J_5^\mu = 2im\bar{q}\gamma^5 q + 2N_f \frac{g^2}{32\pi^2} F_{\mu\nu}^a \tilde{F}_a^{\mu\nu}$$

Effect of topological terms for the gluon field is simulated by the imaginary part of the determinant term in the quark sector

Thermodynamic potential/Effective potential ($\theta = a/f_a$)

The thermodynamic potential (negative of pressure) can be derived in terms of scalar and pseudo-scalar condensate as

$$I_s^i = -\langle \bar{q}q \rangle^i$$

$$\Omega(I_s^i, I_p^i, \theta, T, \mu) = \Omega_{\bar{q}q} + \sum_{i=u,d,s} 2G_s \left(I_s^{i2} + I_p^{i2} \right) + \Omega_{\text{det}} = -p$$

$$I_p^i = \langle \bar{q} i \gamma^5 q \rangle^i$$

$$\Omega_{\bar{q}q} = -\frac{2N_c}{(2\pi)^3} \sum_{i=u,d,s} \int d\mathbf{p} E^i(\mathbf{p}) - \frac{\mathbf{2N_c}}{(\mathbf{2}\pi)^{\mathbf{3}}} \sum_{\mathbf{i=u,d,s}} \int \mathbf{dp} \left[\log(\mathbf{1} + \mathbf{e}^{-\beta((\mathbf{E^i(p)} - \mu^{\mathbf{i}}))}) + \log(\mathbf{1} + \mathbf{e}^{-\beta(\mathbf{E^i(p)} + \mu^{\mathbf{i}})}) \right]$$

$$\Omega_{\text{det}} = 4K \left(\cos\theta \; I_s^u I_s^d I_s^s + \sin\theta \; I_p^u I_p^d I_p^s \right)$$
$$-4K \left(\cos\theta \left(I_s^u I_p^d I_p^s + I_s^d I_p^u I_p^s + I_s^s I_p^d I_p^u \right) + \sin\theta \left(I_p^u I_s^d I_s^s + I_p^d I_s^u I_s^s + I_p^s I_s^u I_s^d \right) \right)$$

Single particle quark energy $E^i(p) = \sqrt{p^2 + M^{i2}}$; the constituent mass of each flavour $M^i = \sqrt{M_s^{i2} + M_p^{i2}}$ depends upon both scalar condensates $I_s^i = -\langle \bar{q}q \rangle^i$ and pseudo scalar condensate $I_p^i = \langle \bar{q} i \gamma^5 q \rangle^i$ calculated ***self-consistently using mass gap equations***.

Mass gap equations and EOS

Mass gap equations for "up" quark ($i = u$):

$$M_s^u = m_u + 4G_s I_s^u + 2K\left(\cos\theta(I_s^d I_s^s - I_p^d I_p^s) - \sin\theta(I_p^d I_s^s + I_b^s I_s^d)\right)$$

$$M_p^u = 4G_s I_p^u - 2K\left(\cos\theta(I_s^d I_p^s + I_p^d I_s^s) + \sin\theta(I_s^d I_s^s - I_p^d I_p^s)\right)$$

$$I_s^i = -\langle \bar{q}q\rangle^i$$
$$I_p^i = \langle \bar{q}i\gamma^5 q\rangle^i$$
$$M^{u2} = \sqrt{M_s^{u2} + M_p^{u2}}$$

Quark matter EOS at $T = 0$

$$\Omega_{\bar{q}q} = -\frac{N_c}{\pi^2}\sum_i \left[\Lambda^4 H\left(\frac{M^i}{\Lambda}\right) - k_f^{i4} H\left(\frac{M^i}{k_f^i}\right)\right] - \sum_i \mu^i \rho^i .$$

$$H(z) = \frac{1}{2}(1+z^2)^{3/2} - \frac{z^2}{8}(1+z^2)^{1/2} - \frac{z^4}{8}\ln\left(\frac{1+\sqrt{1+z^2}}{z}\right)$$

Quark matter EOS : Nambu--Jona-Lasinio model

Bag constant is the difference in energy densities between perturbative vacuum and vacuum with condensates at $T = 0 = \mu = \theta$,

$$B = \Omega(I_s^i, I_p^i, \theta = 0 = T = \mu^i) - \Omega(I_s^i = 0 = I_p^i, \theta = 0 = T = \mu^i)$$

Pressure and energy density of quark matter in the NJL model are

$$P_{NJL}(\theta, I_s^i, I_p^i) = -\Omega(I_s^i, I_p^i, \theta, T = 0, \mu^i) + B \qquad \epsilon_{NJL}(\theta, I_s^i, I_p^i) = \sum_i \mu^i \rho^i - P_{NJL}(\theta, I_s^i, I_p^i)$$

⇒ this defines the EOS of quark matter in the NJL model.

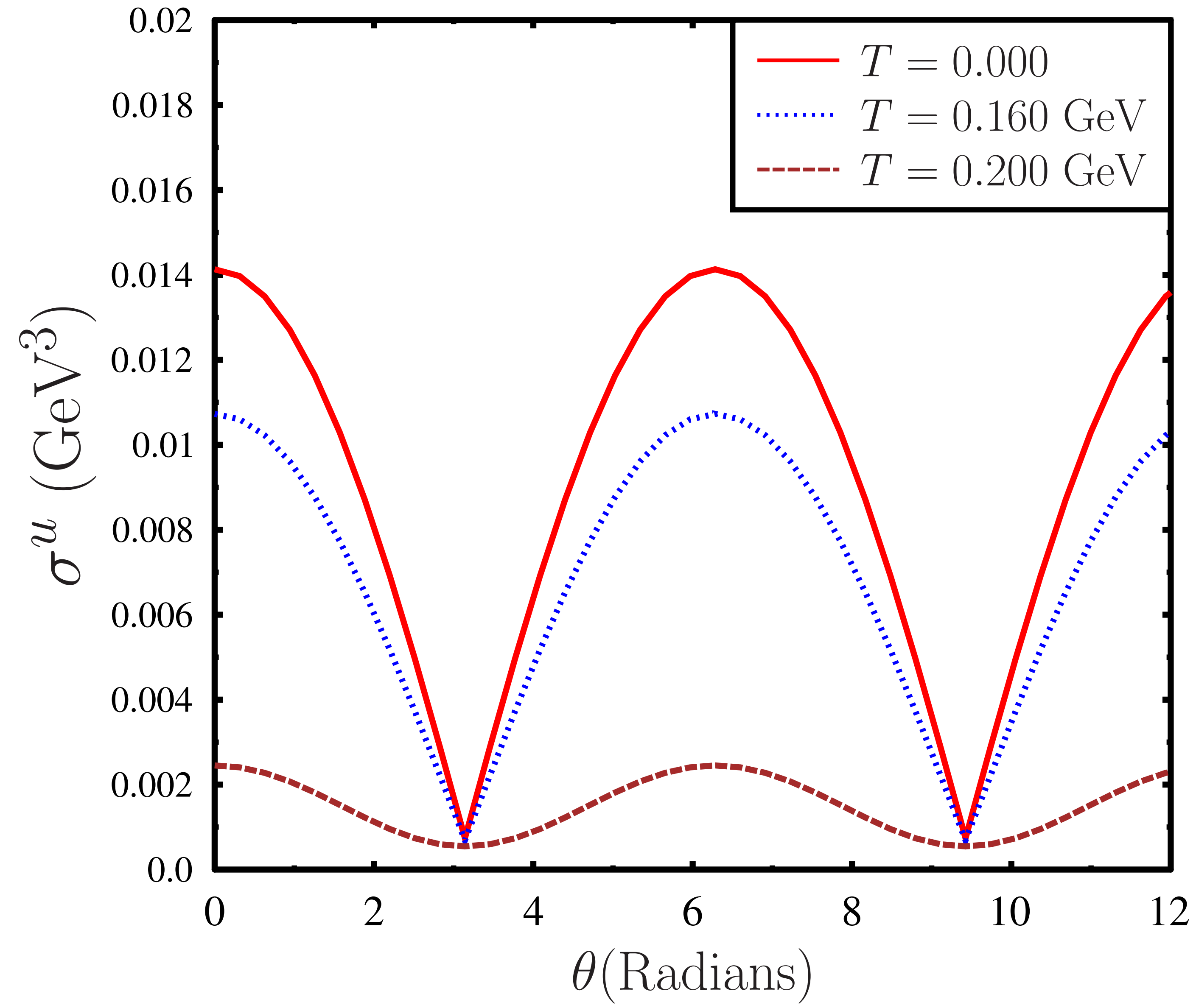
- Parameters on NJL model:

$$m_u = m_d = 5.5 \text{ MeV}, m_s = 140.7 \text{ MeV}, \Lambda = 602.3 \text{ MeV}, G_s\Lambda^2 = 1.835, K_s\Lambda^5 = 12.36$$

After choosing the current quark mass for up and down quarks, the remaining four parameters are chosen by fitting f_π, m_π, m_k and $m_{\eta'}$.

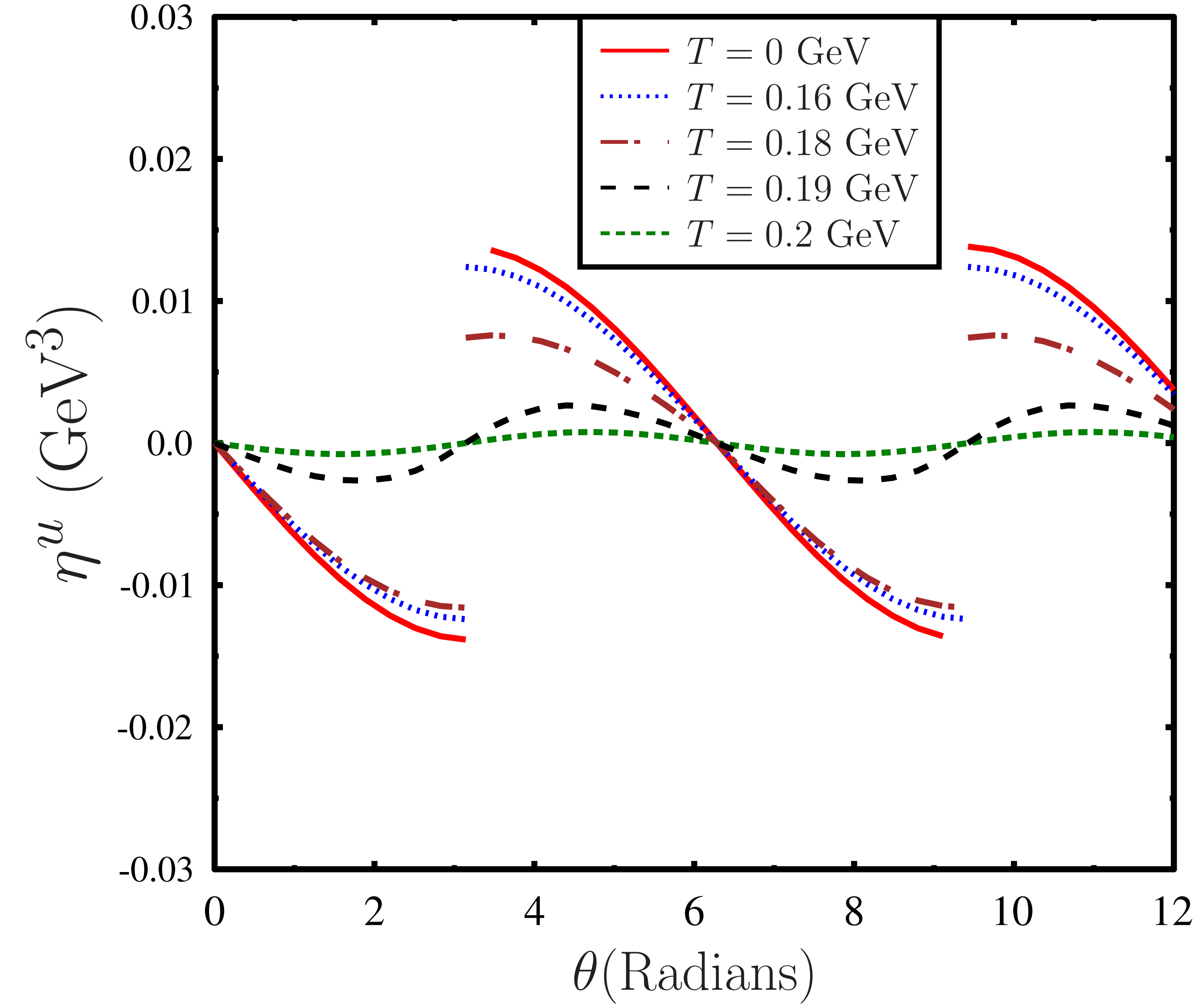
M. Buballa, Physics Reports 407, 205 (2005)

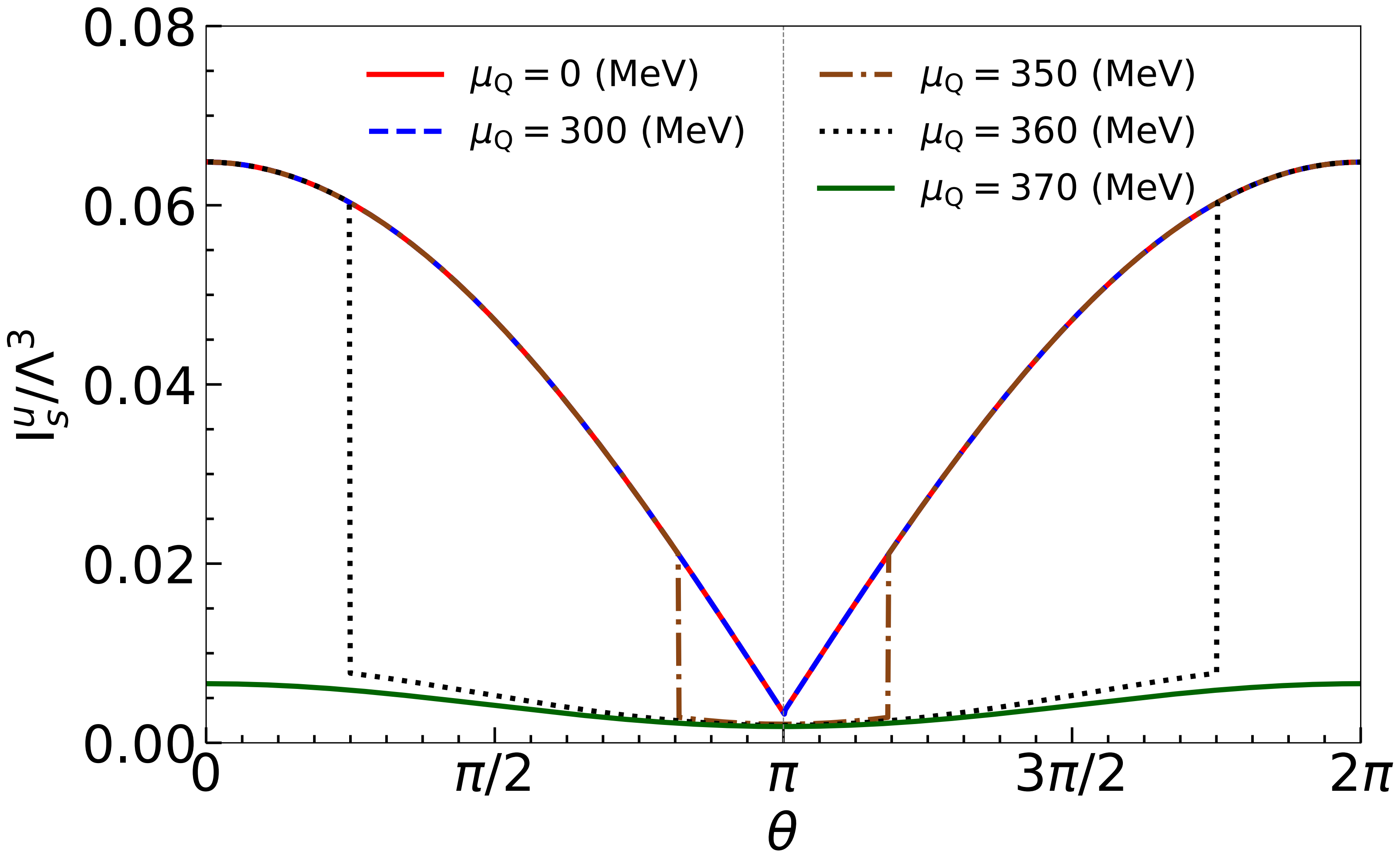
In medium condensates

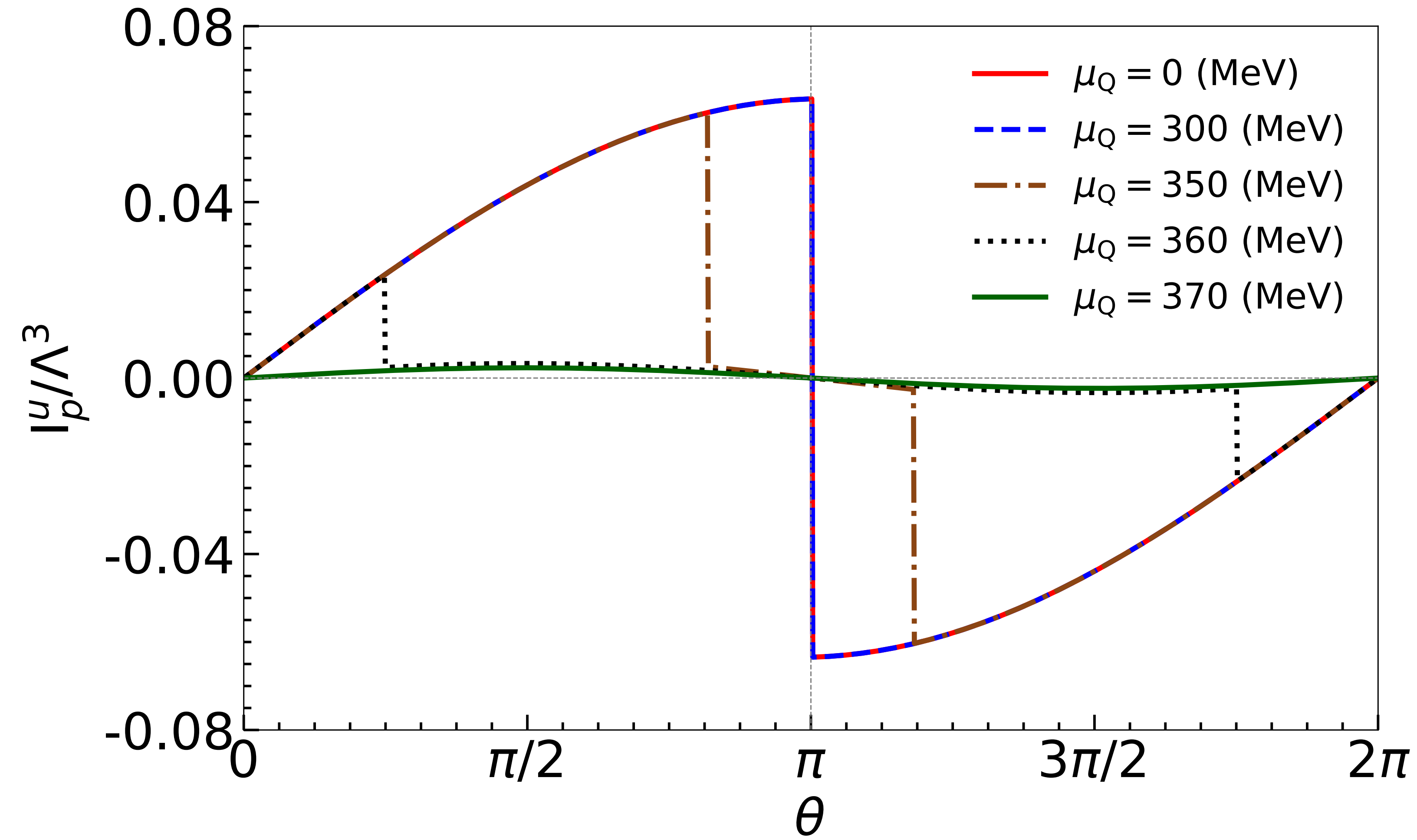


A. Abhishek, A. Das, R. Mohapatra and **HM**

Phys. Rev. D 103, 074003 (2021)



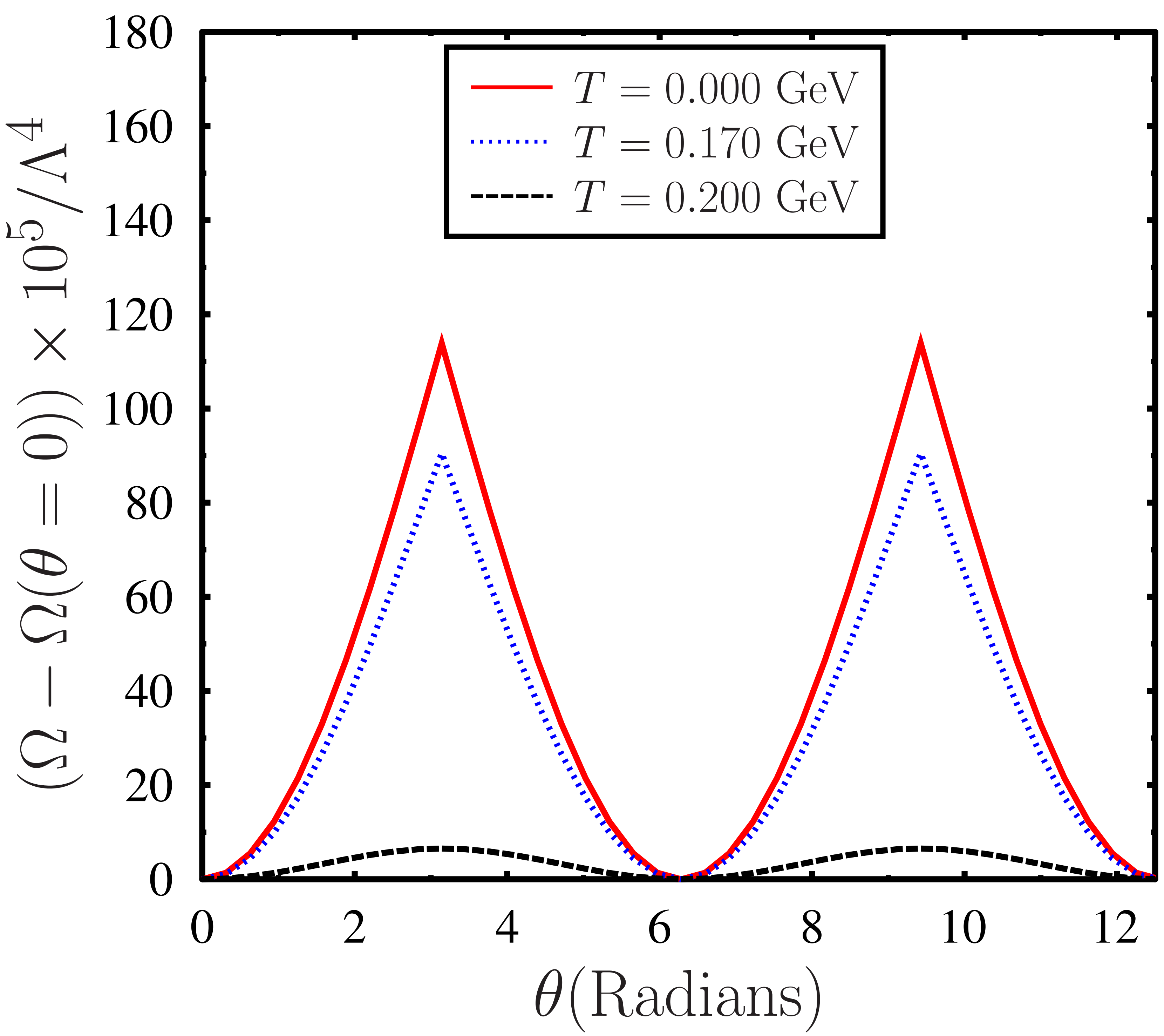




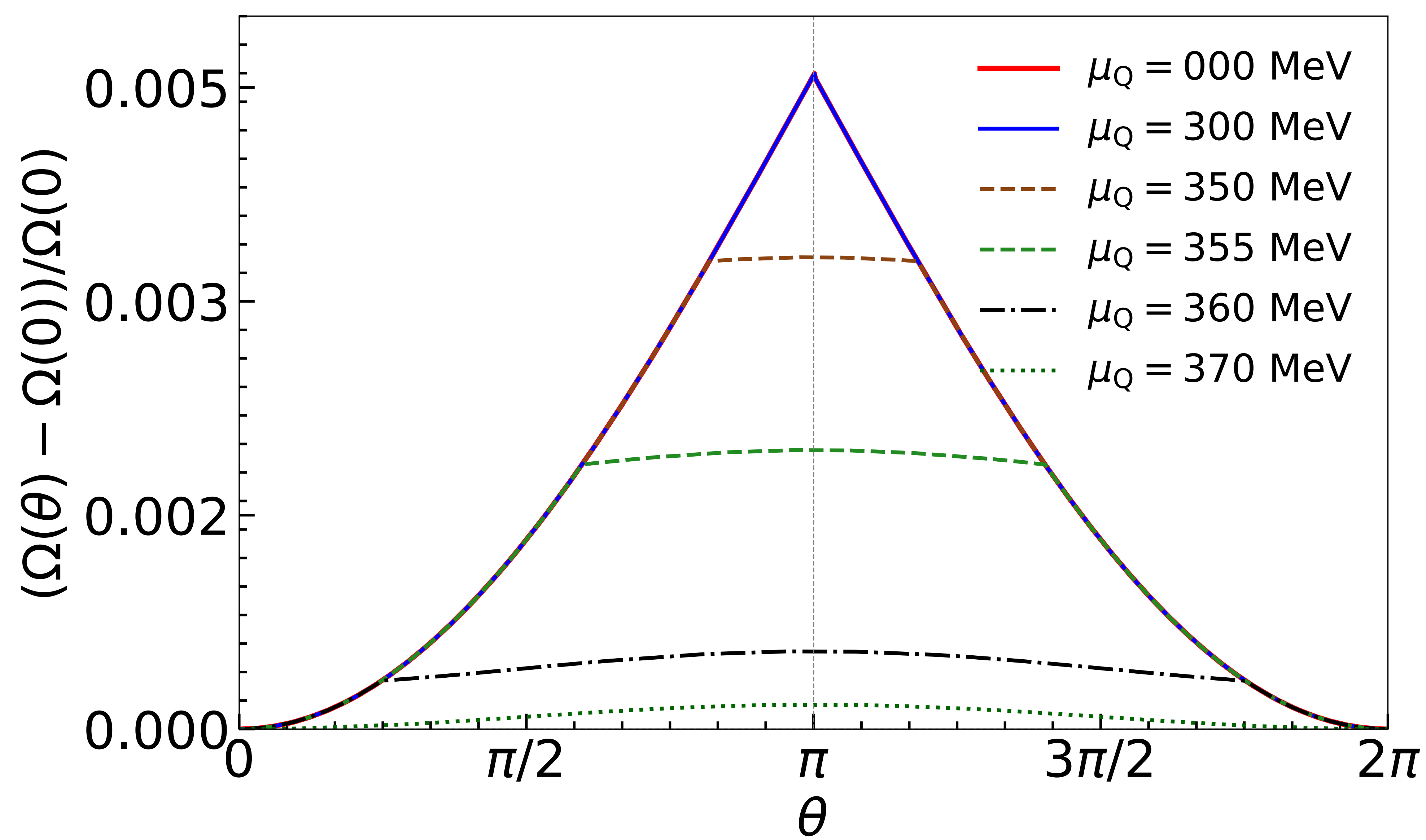
$$\sigma^u \equiv I_s^u \qquad \eta^u \equiv -I_p^u$$

*Deepak Kumar, **HM**,* Arxiv:2411.17828

Thermodynamic potential



A. Abhishek, A. Das, R. Mohapatra and **HM**
Phys. Rev. D 103, 074003 (2021)



Deepak Kumar, ***HM****,* Arxiv:2411.17828

Quark masses

Condensates as a function of $\theta = \frac{a}{f_a}$

$$I_s = -\langle \bar{\psi}\psi \rangle$$

$$I_p = \langle \bar{\psi} i\gamma_5 \psi \rangle$$

Constituent quark mass

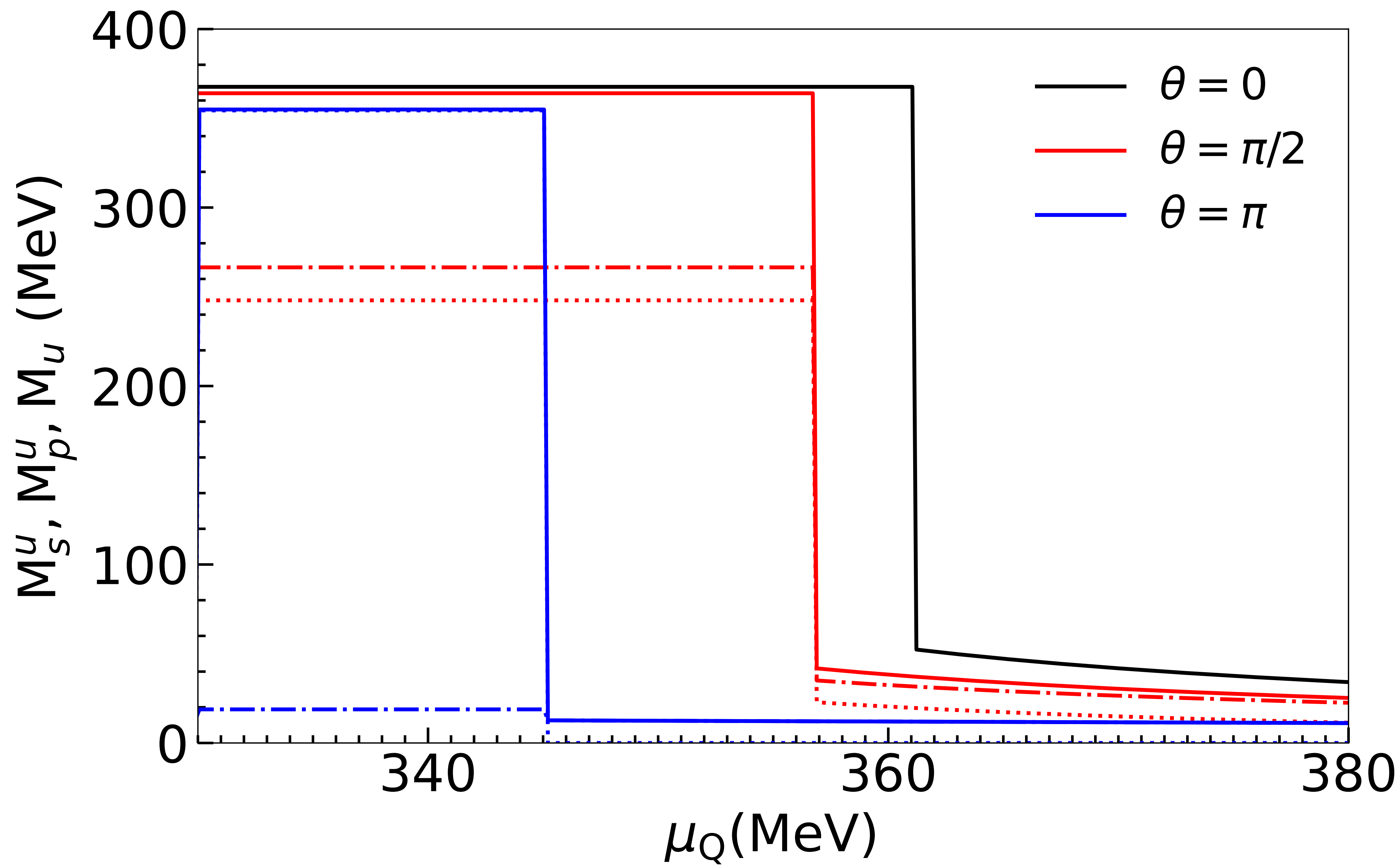
$$M^{i2} = M_s^{i2} + M_p^{i2}$$

With θ, the critical chemical potential decreases.

Gap equations:

$$M_s^u = m_u + 4G_s I_s^u + 2K \left(\cos\theta (I_s^d I_s^s - I_p^d I_p^s) - \sin\theta (I_p^d I_s^s + I_b^s I_s^d) \right)$$

$$M_p^u = 4G_s I_p^u - 2K \left(\cos\theta (I_p^d I_s^s + I_p^s I_s^d) - \sin\theta (I_p^d I_p^s - I_s^d I_s^s) \right)$$



- Hadronic matter within RMF model

Hadronic EOS : Relativistic mean field model

– Walecka's mean field model

- Relativistic nucleons interact through exchange of mesons
  - scalar meson exchange ⇒ ***Attraction***
  - vector meson exchange ⇒ ***Repulsion***

- Adjust the couplings and the masses so that B.E. per nucleon at saturation density is reproduced.
- More terms/parameters are introduced to describe other properties of nuclear matter.

$$
\begin{aligned}
\mathscr{L} = & \sum_{B \in \text{baryons}} \bar{\psi}_B (i\gamma^\mu \partial_\mu - m_B)\psi_B \\
& + \frac{1}{2}\left[\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2\right] - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^2 \\
& - \frac{1}{4}\mathbf{R}_{\mu\nu}\mathbf{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu - \frac{1}{4}\Phi_{\mu\nu}\Phi^{\mu\nu} + \frac{1}{2} m_\phi^2 \phi_\mu \phi^\mu \\
& - \sum_{B \in \text{baryons}} \bar{\psi}_B \gamma_\mu \left(g_{\omega B}\omega^\mu + g_{\rho B}\tau_B \cdot \rho^\mu + g_{\phi B}\phi^\mu \right) \psi_B \\
& - \frac{\kappa}{3!}(g_{\sigma N}\sigma)^3 - \frac{\lambda}{4!}(g_{\sigma N}\sigma)^4 + \Lambda_\omega g_{\rho N}^2 g_{\omega N}^2 \omega_\mu \rho^\mu + \frac{\xi_\omega}{4!}(g_{\omega N}^2 \omega_\mu \omega^\mu)^2 .
\end{aligned}
$$

$$\Omega^{\mu\nu} = \partial^\mu \omega^\nu - \partial^\nu \omega^\mu$$

$$\mathbf{R}^{\mu\nu} = \partial^\mu \rho^\nu - \partial^\nu \rho^\mu$$

$$\Phi^{\mu\nu} = \partial^\mu \phi^\nu - \partial^\nu \phi^\mu$$

Hadronic EOS : Relativistic mean field model

- The field equations of motion are solved at the mean field level $\langle\sigma\rangle = \sigma_0$, $\langle\omega_\mu\rangle = \omega_0\delta_{\mu 0}$, $\langle\rho_\mu^a\rangle = \delta_{\mu 0}\delta_3^a\rho_3^0$ and $\langle\phi_\mu\rangle = \phi_0\delta_{\mu 0}$.

– Effective chemical potential and effective mass

$$\mu_B^* = \mu_B - g_{\omega B}\omega_0 - g_{\rho B}I_{3B}\rho_{03} - g_{\phi B}\phi_0 \,.$$

$$m_B^* = m_B - g_{\sigma B}\sigma_0$$

– Energy density and pressure

$$\epsilon_{HP} = \frac{1}{\pi^2}\sum_{i\,\in\,\text{Baryon}} H(m^*/k_F^i) + U(\sigma_0, \omega_0, \rho_0, \phi_0)$$

Baryon kinetic [$H(m^*/k_F^i)$]

meson mean field [$U(\sigma_0, \omega_0, \rho_0, \phi_0)$]

$$U(\sigma_0, \omega_0, \rho_0, \phi_0) = \frac{1}{2}m_\sigma^2\sigma_0^2 + \frac{\kappa}{3!}(g_\sigma\sigma_0)^3 + \frac{\lambda}{4!}(g_\sigma\sigma_0)^4 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_3^{0\,2} + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{\xi_\omega}{8}(g_\omega\omega_0)^4 + 3\Lambda_\omega(g_\rho g_\omega\rho_0\omega_0)^2$$

where,

$$H(x) = \frac{1}{8}\left(\sqrt{1+x^2}(2+x^2) - x^4\ln\left(\frac{x+\sqrt{1+x^2}}{x}\right)\right)$$

$$p_{HP} = \sum_{i\,\in\,Baryons}\mu_i n_i + \epsilon_{HP}$$

⇒ this defines the EOS of hadronic matter in RMF model.

Hadronic EOS : Relativistic mean field model

- Parameters of the RMF model (NL3 parametrisation)

$m = 939$ MeV

$m_\sigma = 508.19$ MeV

$m_\omega = 782.501$ MeV

$m_\rho = 763$ MeV

$m_\phi = 1020$ MeV

$g_\sigma = 10.217$

$g_\omega = 12.868$

$g_\rho = 11.2766$

$\kappa = 3.85990$

$\lambda = -0.0159050$

$\Lambda_\omega = 0.03$

Horowitz and Piekarewicz

Phys.Rev.C 64 (2001) 062802

Meson-neucleon coupling are chosen to satisfy nuclear saturation properties.

From there hyperon-meson couplings are generated using quark counting rule respecting SU(6) symmetry.

Hadron masses in presence of axions

A constituent quark scaling hypothesis for in-medium hadron masses:

$$M_B(T,\mu_B,\theta) = M_B(T=0,\mu_B=0,\theta=0) + (3-N_s)\Delta M_Q(T,\mu_B,\theta) + N_s\Delta M_s(T,\mu_B,\theta)$$

where,

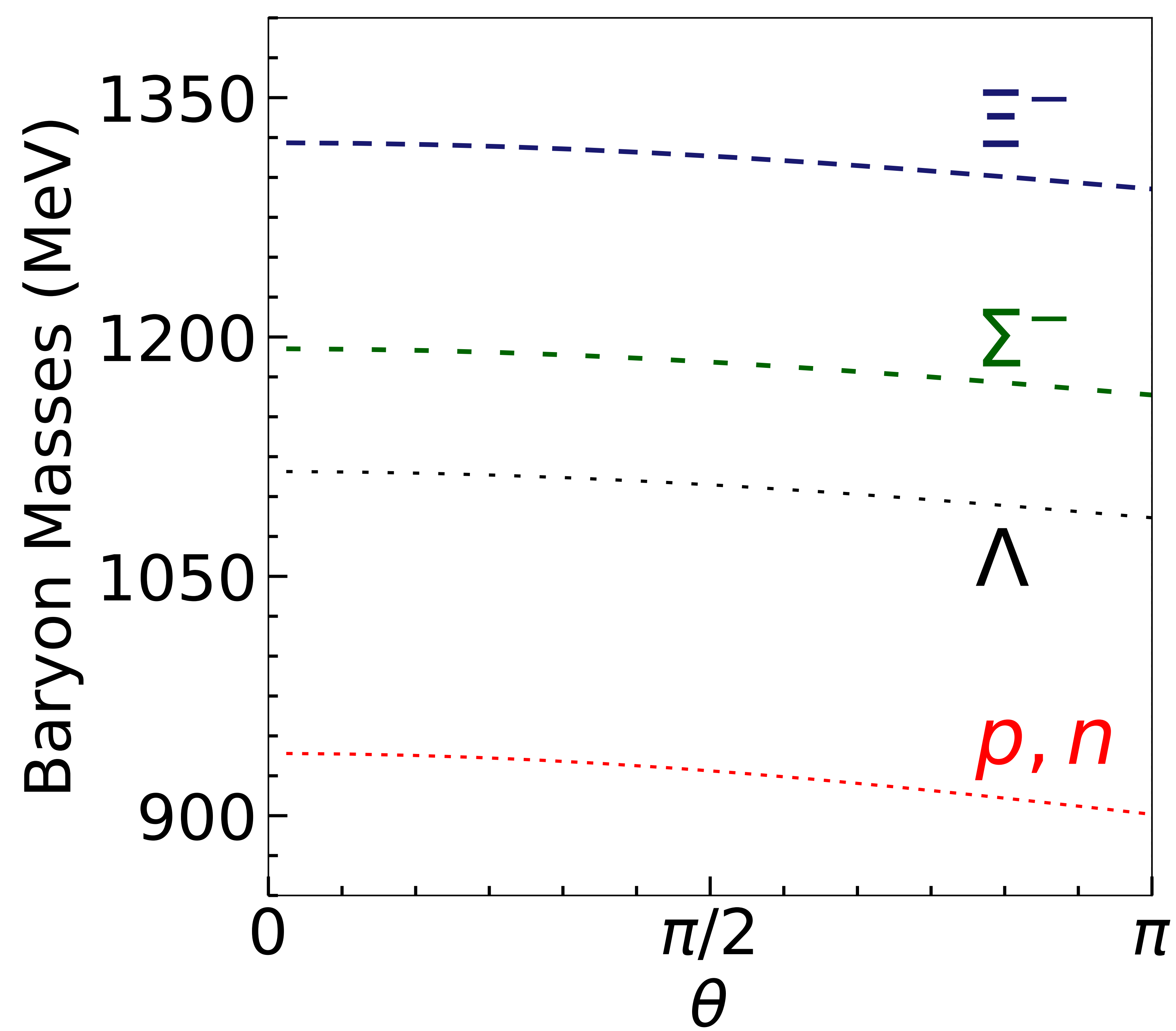
$$\Delta M_Q(T,\mu_B,\theta) = M_Q(T,\mu_B,\theta) - M_Q(T=0,\mu_B=0,\theta=0)$$

$$\Delta M_s(T,\mu_B,\theta) = M_s(T,\mu_B,\theta) - M_s(T=0,\mu_B=0,\theta=0)$$

Leupold, J. Phys. G, 32:2199 2218 (2006)

Jankowaski, Blaschke, Spalinksi Phys Rev D, 87:105018, 2013:

Kadam, HM Phys Rev C 93:025205, 2016



- Phase transition with Gibbs construct

Hadron-quark phase transition

✦ Gibbs construction of mixed phase:

Gibbs condition for equilibrium

$$p_{\rm MP}(\mu_B,\mu_E) = p_{\rm HP}(\mu_B,\mu_E) = p_{\rm QP}(\mu_B,\mu_E)$$

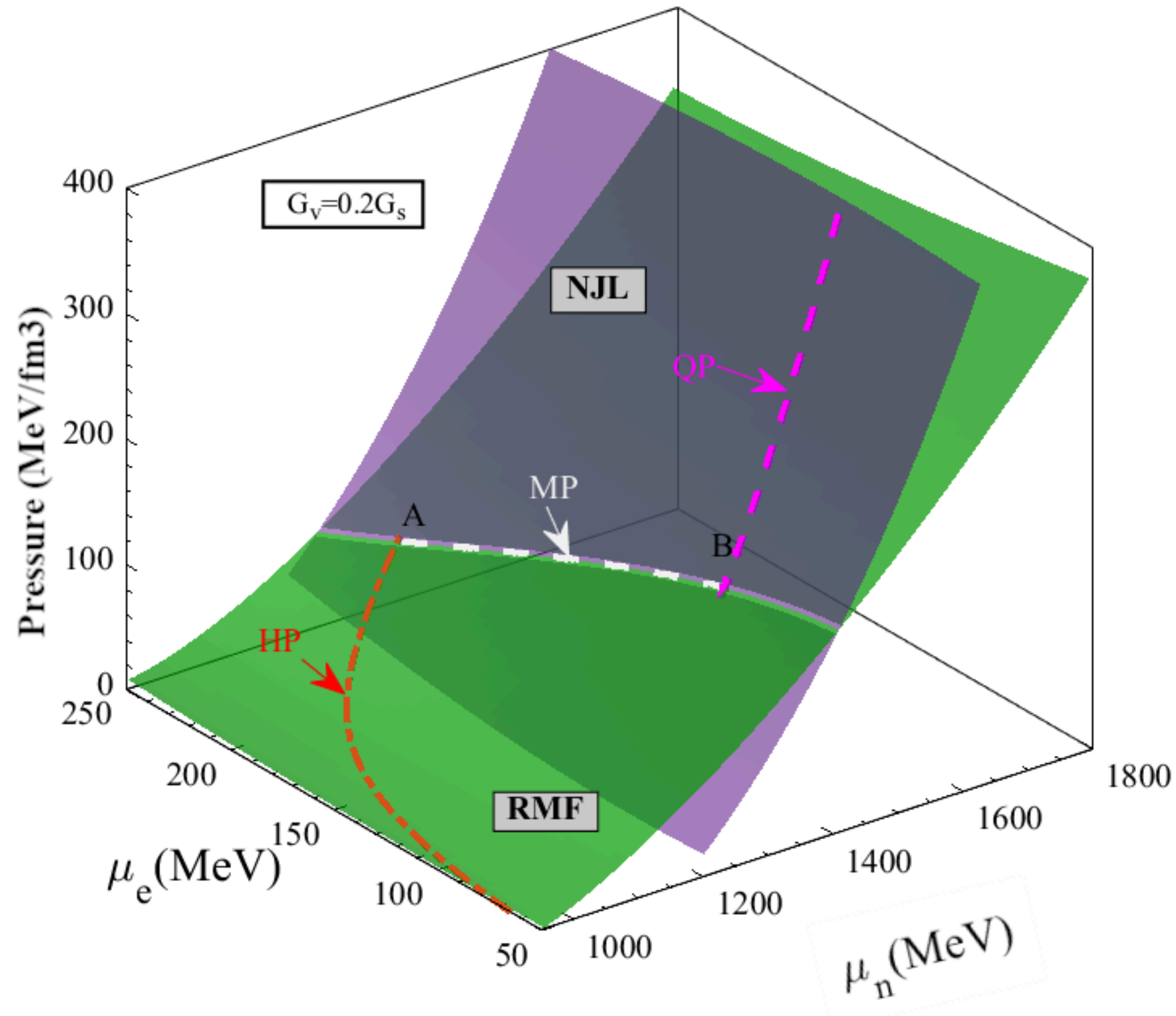
Global charge neutrality condition

$$\chi\rho_c^{\rm QP} + (1-\chi)\rho_c^{\rm HP} = 0$$

χ : Volume fraction of quark matter in mixed phase

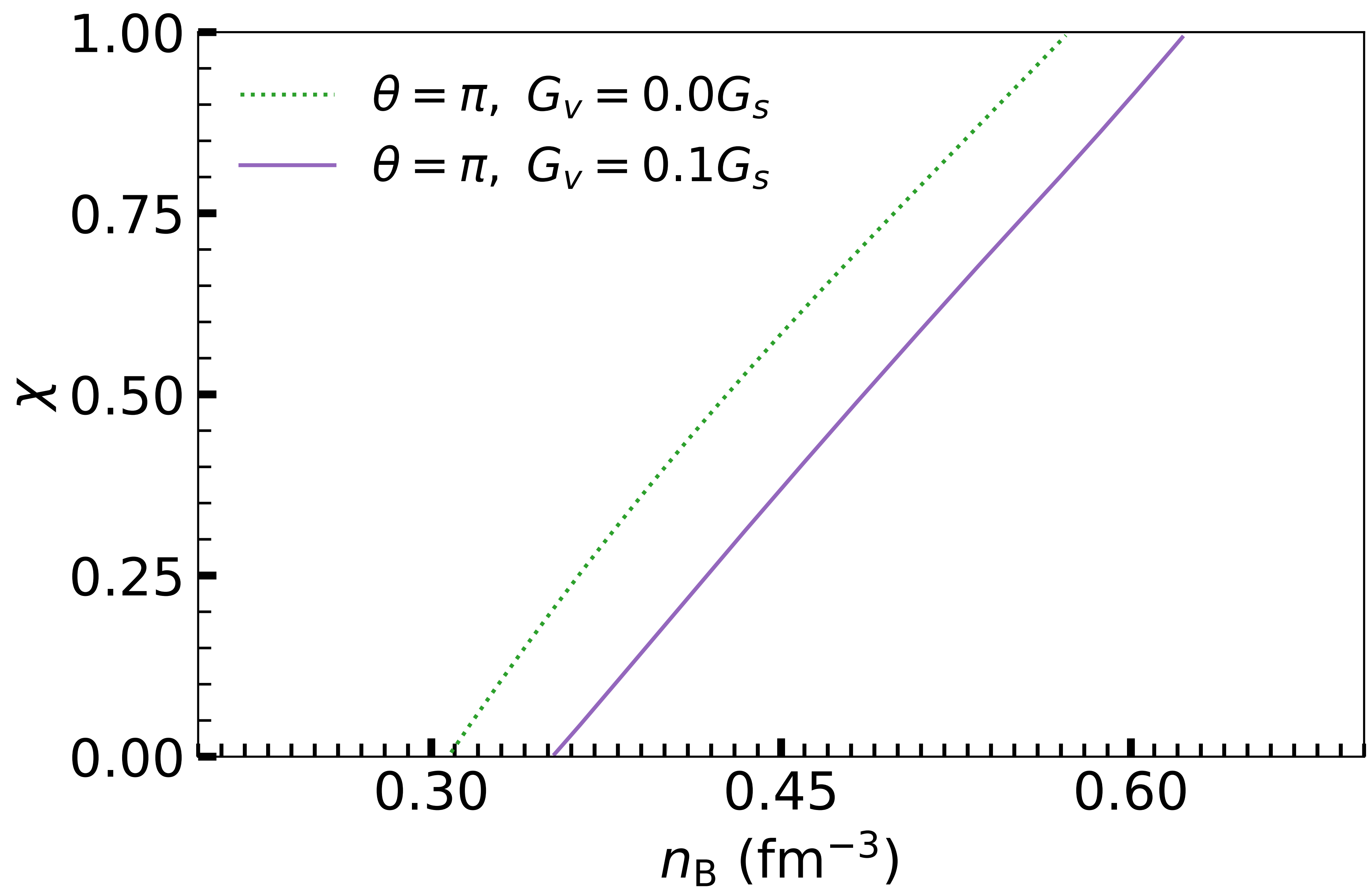
$$\epsilon_{\rm MP} = \chi\epsilon_{\rm QP} + (1-\chi)\epsilon_{\rm HP}$$

The mixed phase starts from $n_B = 0.3\ \mathrm{fm}^{-3}$ and ends at $n_B = 0.6\ \mathrm{fm}^{-3}$.

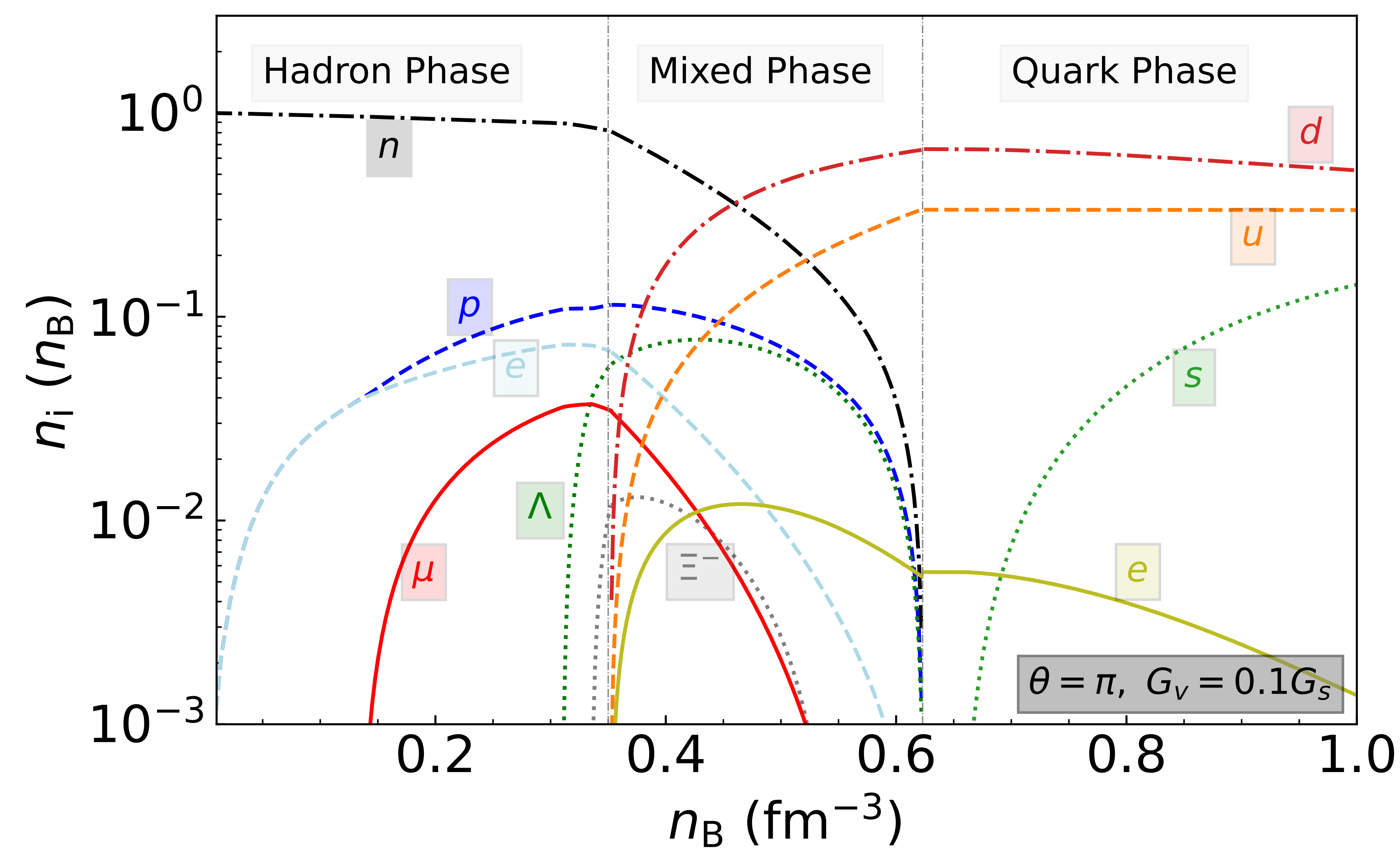


Hadron-quark phase transition

- Volume fraction of quark matter in mixed phase

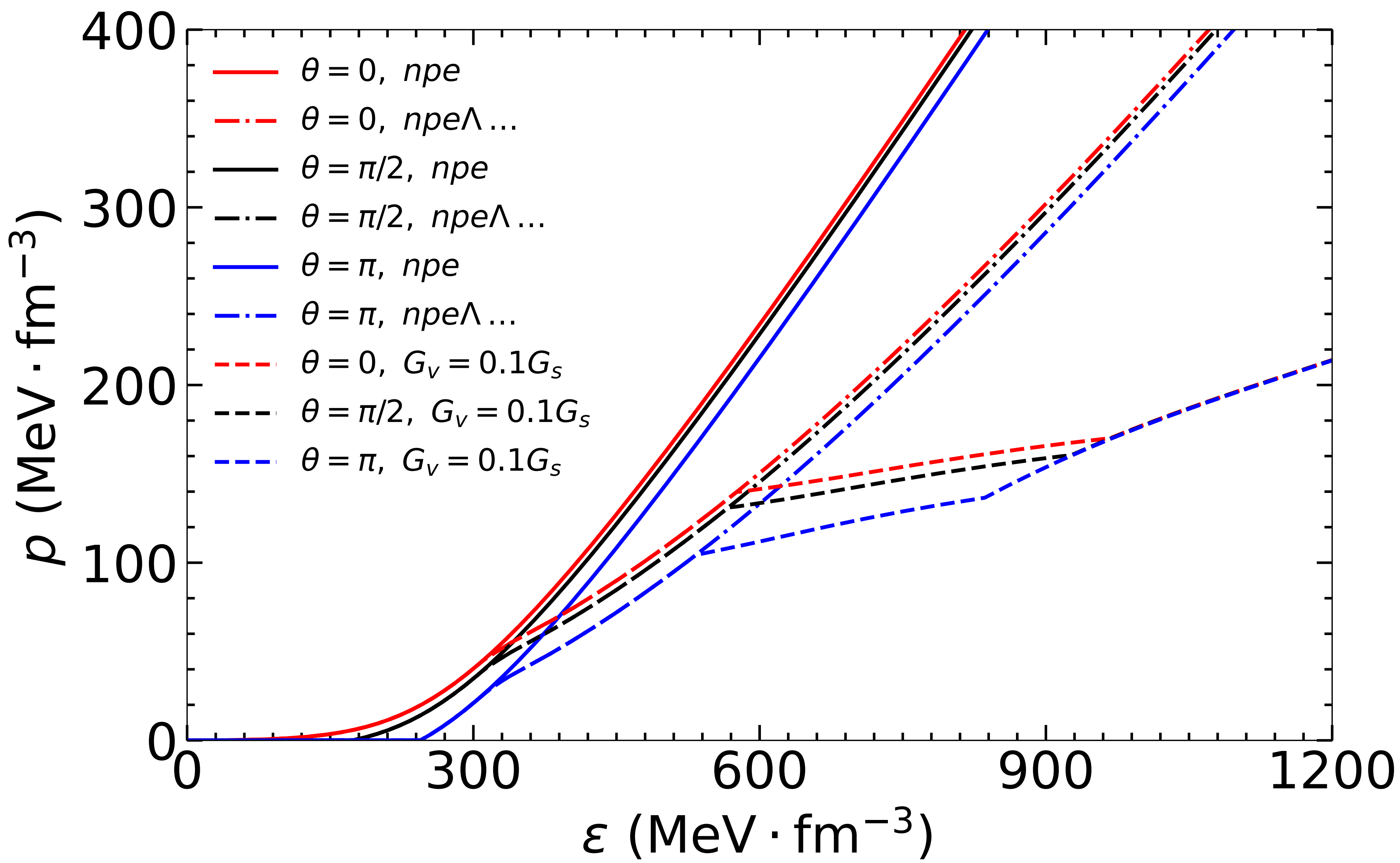


Volume fraction of quark matter in mixed phase as a function of baryon density.

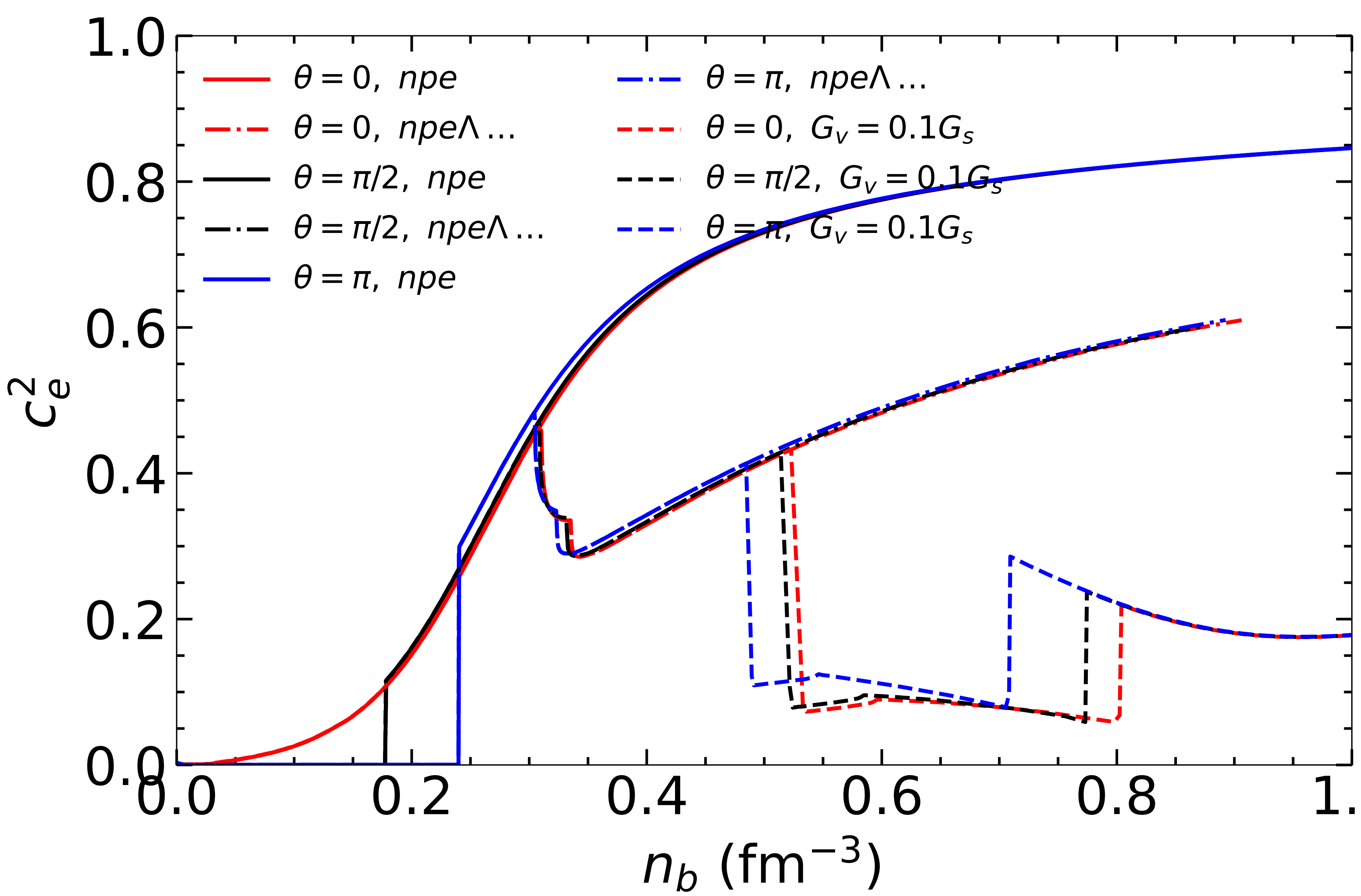


Particle fractions as a function of baryon density in different phases.

Equation of state and speed of sound



Equation of state with Gibbs construct.



Square of speed of sound in different phases.

- Neutron star structure and non-radial oscillations with axions

(*PI in the sky: neutron stars with exceptionally light QCD axions, arxiv:2410.21590*)

TOV equations and stellar structure

- Static spherically symmetric metric

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

- Tolman-Oppenheimer-Volkoff equations (Hydrostatic equilibrium between gravity and matter)

$$\frac{dP(r)}{dr} = -\frac{\left(\epsilon(r) + P(r)\right)\left(m(r) + 4\pi r^3 P(r)\right)}{r(r-2m(r))} \qquad e^{2\lambda(r)} = \left(1 - \frac{2m}{r}\right)^{-1},$$

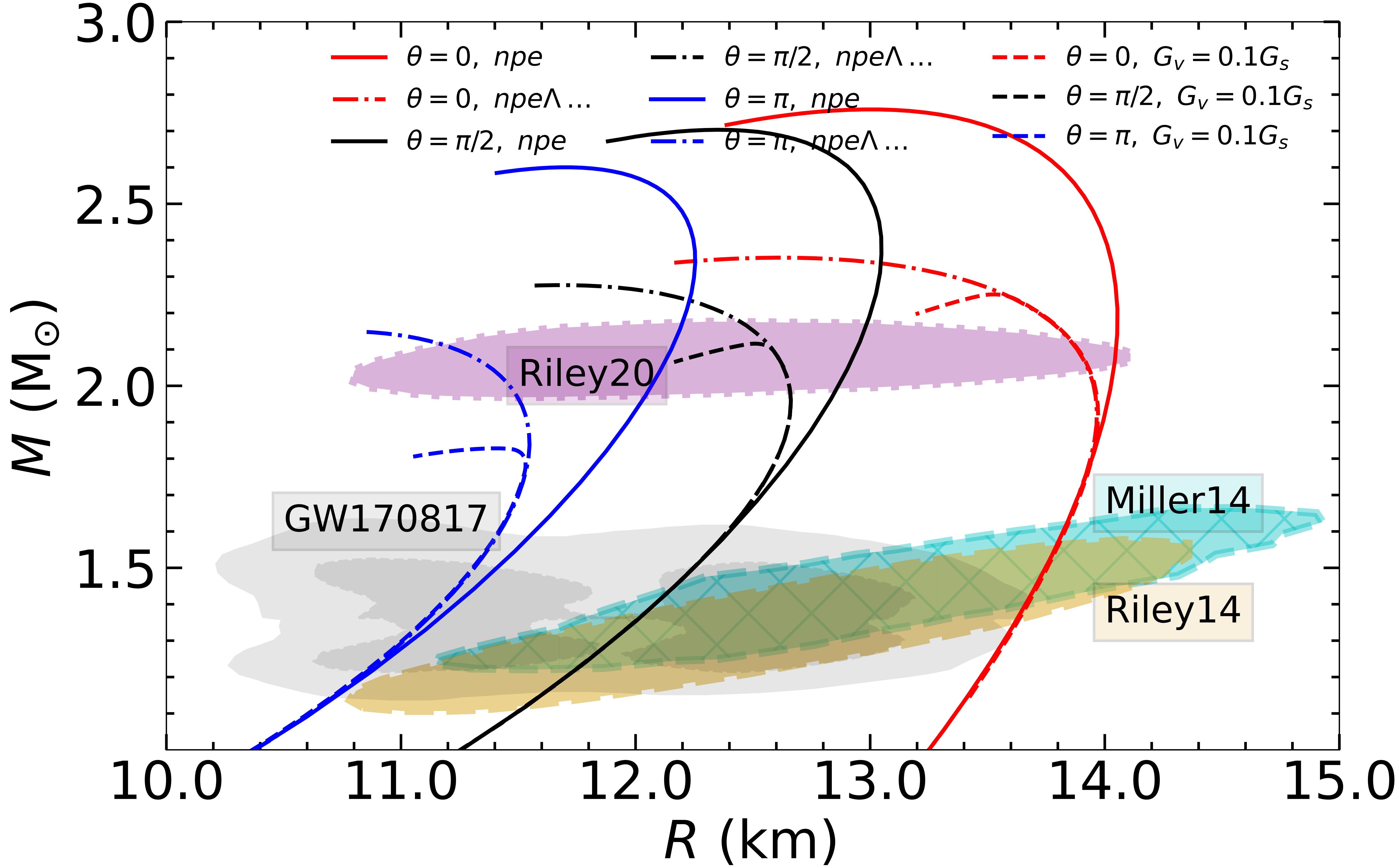
$$\frac{dm(r)}{dr} = 4\pi r^2 \epsilon(r), \qquad \frac{d\nu(r)}{dr} = \frac{m(r) + 4\pi r^3 p(r)}{r(r-2m(r))}$$

Units: $G = 1 = c$

Supplement with an EOS, $p(\epsilon)$ and solve with the boundary conditions

$$m(0) = 0, \quad P(0) = p_c \quad \text{and} \quad e^{2\nu(R)} = 1 - \frac{2m(R)}{R}$$

Mass-radius relations



Non-radial oscillations in compact stars

- Compact stars oscillate with characteristic frequencies when subjected to external perturbations. The perturbation modes can be characterised by spherical harmonics :
  - odd (axial) ⇒ toroidal deformation
  - even (polar) ⇒ spheroidal deformation
- The polar modes get related to emission of GWs
  - pressure mode, (p)
  - fundamental mode, (f)
  - gravity mode, (g)
- The perturbations can be in metric as well as the fluid element ⇒ the fluid and metric perturbations get coupled
- Relativistic Cowling approximation ⇒ neglect metric perturbations
- Hydrostatic equilibrium ⇒ gravitational force balanced by pressure gradients
- Perturbed from equilibrium ⇒ gravity pulls it back ⇒ oscillations

Non-radial oscillations in compact stars

- Fluid perturbations are described by the Lagrangian fluid displacement vector

$$\xi^i = e^{i\omega t}\left(e^{-\lambda(r)}Q(r), -Z(r)\ \partial_\theta,\ -Z(r)\sin^{-2}\theta\ \partial_\phi\right)r^{-2}Y_{lm}(\theta,\phi)$$

- Perturbed from equilibrium ⇒ gravity pulls it back ⇒ oscillations

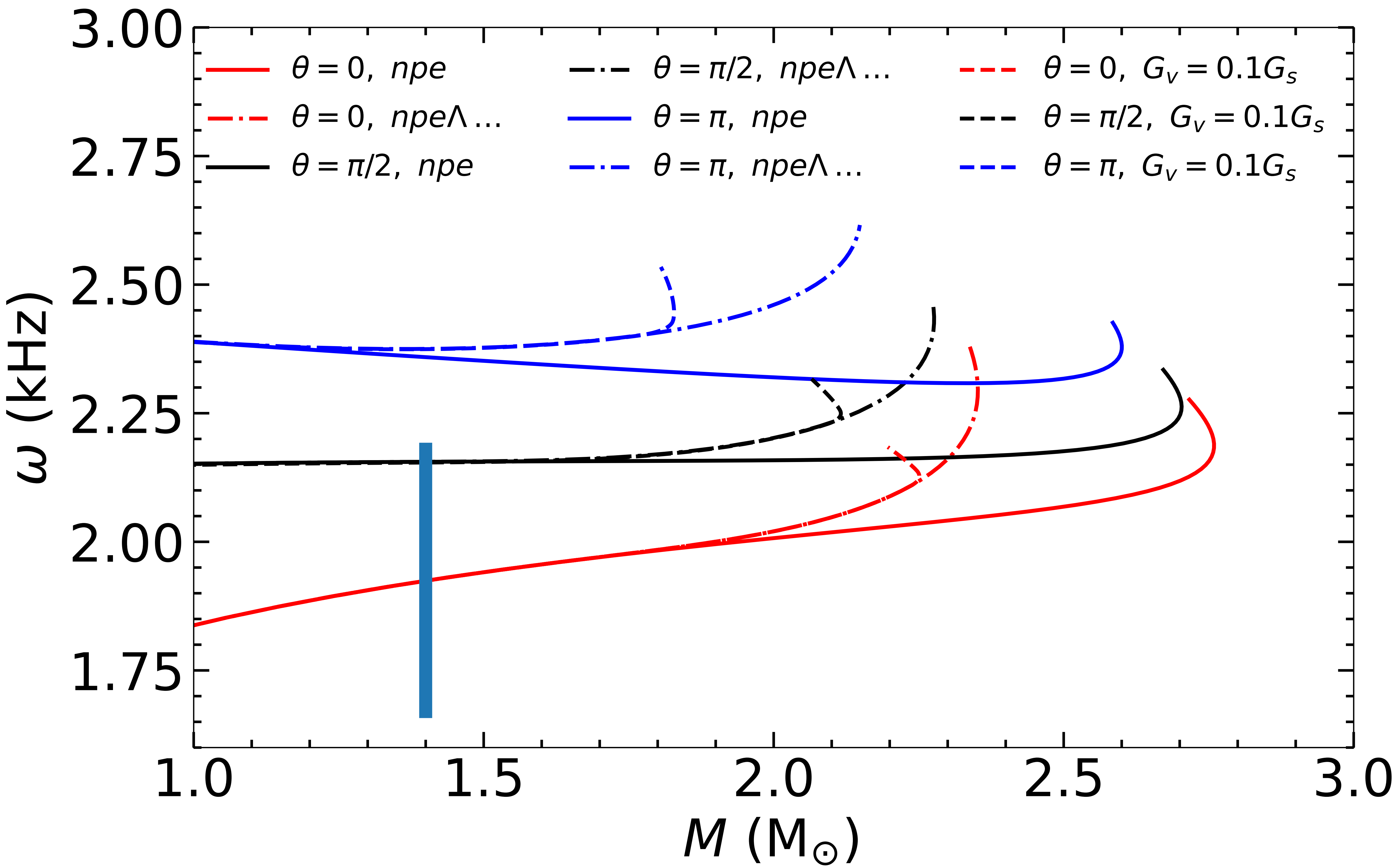
$$Q' - \frac{1}{c_s^2}\left[\omega^2 r^2 e^{\lambda-2\nu}Z + \nu' Q\right] + l(l+1)e^{\lambda}Z = 0.$$

$$Z' - 2\nu' Z + e^{\lambda}\frac{Q}{r^2} - \frac{\omega_{BV}^2 e^{-2\nu}}{\nu'\left(1-\frac{2m}{r}\right)}\left(Z + \nu' e^{-\lambda+2\nu}\frac{Q}{\omega^2 r^2}\right) = 0.$$

Brunt-Vaisala frequency $\omega_{BV}^2 = \nu'^2 e^{2\nu}\left(1-\frac{2m}{r}\right)\left(\frac{1}{c_e^2}-\frac{1}{c_s^2}\right).$

Mass, radius and *f*-mode frequency

Mass-frequency relations



Conclusion

- We have explored the possibility of hybrid star stability in presence of axions condensates.
- Non-vanishing θ leads to quark matter phase at a lower density compared to $\theta = 0$. This leads to a larger quark matter core for hybrid star.
- The axion condensate and vector interaction play complemental roles regarding the stability of hybrid stars.
- The *f*-mode oscillation frequency increases for stars with a quark core.
- It will be interesting to study other non-radial oscillation modes in presence of axions.

Thank you for your attention

Universal relations

UR between dimensionless frequencies $M\omega$ and the compactness C ($C = M/R$):

$$M\omega = a\left(\frac{M}{R}\right) + b$$

Parameter	a (kHz Km)	b (kHz Km)
Pradhan et al. (2021) [94]	197.30	-3.84
This work		
$\theta = 0$	199.56	-5.00
$\theta = \pi/2$	198.40	-5.76
$\theta = \pi$	197.57	-5.61

URs are not sensitive non-nucleonic degrees of freedom like hyperons, quark matter