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HEAVY AND LIGHT MESONS IN THE FRAME OF EFFECTIVE QCD-INSPIRED MODELS

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TABLE OF CONTENTS

INTRODUCTION

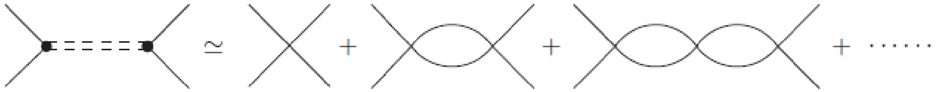
- A complete understanding of the full range of hadronic spectra from the lightest to the heaviest remains challenge.

- Direct calculations of hadron properties from the first principles of QCD still encounter some technical and conceptual difficulties.

- The use of different approaches is determined by the flavour structure of mesons:
  - for light flavors, the behaviour governed by chiral symmetry and requires fulfillment of low-energy theorems in QCD;
  - for heavy quarks, the strong dynamics is more simple, since they behave like classical particles

MESONS IN THE FRAMEWORK OF THE NJL-LIKE MODELS

The effective interaction resulting from the exchange of a π meson can be obtained as an infinite sum of loops in the random-phase approximation (RPA)



$$= i\gamma_5 T_i \frac{-ig^2_{\pi qq}}{k^2 - m^2_\pi} i\gamma_5 T_j$$

Summing up all the terms on the right-hand side, we obtain

$$i\gamma_5 T_i \frac{2iG}{1 - 2G\Pi_{ps}(k^2)} i\gamma_5 T_j$$

The mass of π mesons is related to the pole of Eq. above, which is the solution of the following equation:

$$1 - 2G\Pi_{ps}(k^2) = 0 \qquad \text{where} \qquad -i\Pi_{ps}(k^2) = -\int \frac{\mathrm{d}^4 p}{(2\pi)^4} \mathrm{Tr}\left[i\gamma_5 T_i iS(k+p) i\gamma_5 T_j iS(p)\right]$$

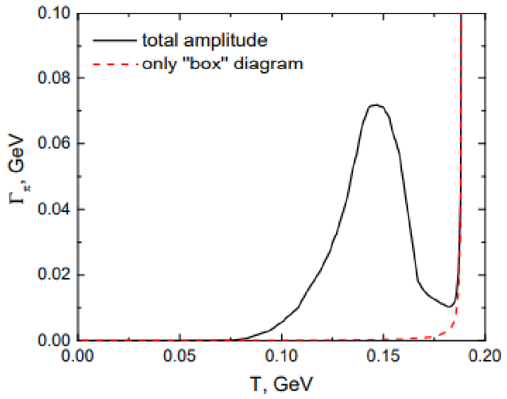
Furthermore, the coupling strength between π meson and quarks $g_{\pi qq}$ can be obtained as

$$g^2_{\pi qq} = \left[\frac{\partial \Pi_{ps}(k^2)}{\partial k^2}\right]^{-1} \Bigg|_{k^2 = m^2_\pi}$$

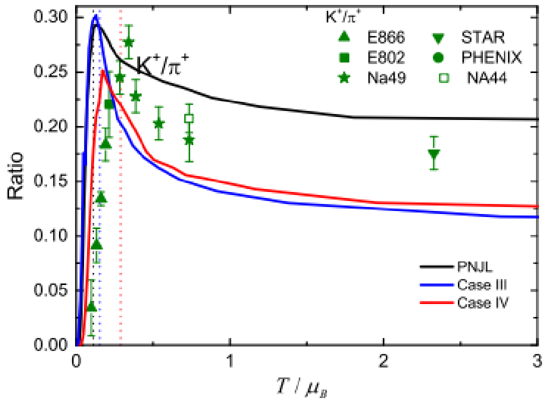
MESONS IN THE FRAMEWORK OF THE NJL-LIKE MODELS

Three free parameters: $m_0 = 5.5\ MeV, \Lambda = 0.639\ GeV, G = 5.227\ GeV^{-2}$

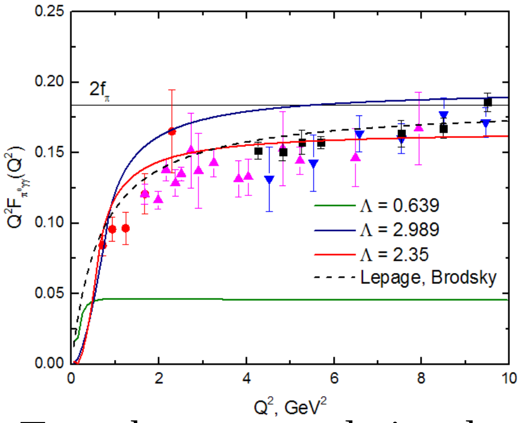
Finite T is introduces via Matsubara formalism



The pion damping width.



“Horn” behaviour in the K/pi ratio



Two-photon neutral pion decay and pion transition form factor

[AF, Yu.K.,IJMP A 37,2250135 (2022)]

[AF, YuK et al Phys.Part.Nucl. 52 (2021) 4, 609, Eur.Phys.J.ST 229 (2020) 22-23]

NJL-LIKE MODELS:

Advantages of the models:

- the model describes the mechanism of the spontaneous chiral symmetry breaking
- is a good model for description of the thermodynamical properties of the hot and dense matter (finite T is introduces via imaginary time formalism, Matsubara summation);
- model of confinement and phase transitions using the Polyakov loop extension
- the magnetic fields as well as rotation can be introduced

Disadvantages of the models:

- description of the hadron decay processes requires difficult calculations due to appearing the loop integrals with a multiple poles
- the spectra of mesons is limited due to limitation of flavours [arXiv:1504.03799, 9608223v2]

THE QUARK MODEL WITH SEPARABLE INTERACTION

We start from the Bethe-Salpeter equation in the ladder approximation:

$$\Gamma_H(q,P) = -\frac{4}{3}\int \frac{d^4p}{(2\pi)^4} D(q-p)\gamma_\alpha S_1(p_1)\Gamma_H(p,P)S_2(p_2)\gamma_\alpha$$

Considering the interaction kernel in separable form $D(q-p) = D_0\varphi(q^2)\varphi(p^2)$

with 4-dimentional form-factor function in Gaussian form $\varphi(q^2) = e^{-q^2/\Lambda_H^2}$

and vertex function rewritten as $\Gamma_H(p,P) = N_H\,\varphi(p^2)\,\gamma_H$

the dressed quark propagator is considered in the Euclidean space $S_i(p_i) = \frac{1}{i(p_i\cdot\gamma)+m_i}$

The meson-quark coupling constants N_H are determined by the condition:

$$1 = N_c \frac{P_\mu}{2P^2}\frac{\partial}{\partial P_\mu}\int \frac{d^4p}{(2\pi)^4}\mathrm{tr}\,\{\Gamma_H(p,P)S_1(p_1)\Gamma_H(q,P)S_2(p_2)\}$$

SYSTEM OF EQUATIONS

The meson-quark coupling constants N_H are determined by the conditions:

For pseudoscalars:

$$
\begin{aligned}
1 \quad = \quad & -i\frac{N_c N_{ps}^2}{2P^2} P^\mu \int \frac{dp}{(2\pi)^4} \varphi^2(p^2) \cdot \\
& \cdot \{ b_1 \mathrm{tr}\left[i\gamma_5 S_1(p_1)\gamma_\mu S_1(p_1) i\gamma_5 S_2(p_2) \right] + b_2 \mathrm{tr}\left[S_1(p_1) i\gamma_5 S_2(p_2)\gamma_\mu S_2(p_2) i\gamma_5 \right] \}
\end{aligned}
$$

For vector mesons:

$$
\begin{aligned}
1 \quad = \quad & -i\frac{N_c N_{\mathrm{v}}^2}{6P^2} P^\mu \int \frac{dp}{(2\pi)^4} \varphi^2(p^2)\epsilon^{\rho\sigma} \cdot \\
& \cdot \{ b_1 \mathrm{tr}\left[\gamma_\rho S_1(p_1)\gamma_\mu S_1(p_1)\gamma_\sigma S_2(p_2) \right] + b_2 \mathrm{tr}\left[S_1(p_1)\gamma_\rho S_2(p_2)\gamma_\mu S_2(p_2) i\gamma_\sigma \right] \}
\end{aligned}
$$

The meson weak decay constant can be obtained from the matrix element of the axial current and in integral form has a view:

$$
P^\mu f_P \quad = \quad N_c N_P \int \frac{dp}{(2\pi)^4} \varphi(p^2) \mathrm{tr}\{(\mathrm{i}\gamma_5)\mathrm{S}_1(\gamma_\mu\gamma_5)\mathrm{S}_2\}.
$$

$$
f_{\mathrm{v}} M_{\mathrm{v}} \epsilon^{\mu\nu} \quad = \quad N_c N_{\mathrm{v}} \int \frac{dp}{(2\pi)^4} \varphi(p^2)\epsilon^{\mu\rho} \mathrm{tr}\{\gamma_\rho \mathrm{S}_1(\mathrm{p}_1)\gamma_\nu \mathrm{S}_2(\mathrm{p}_2)\}.
$$

INTEGRATION TECHNIQUES

The one-loop two-point integral

$$I(P^2) = \int \frac{dp}{\pi^2} F(p^2) \frac{1}{[p_1^2 + m_1^2]\,[p_2^2 + m_2^2]},$$

after applying the Feynman parametrization all integrals can be presented as

$$I(P^2) = \int_0^1 d\alpha \int_0^\infty dt \frac{t}{(1+t)^2} [F(z_0)]$$

with $z_0 = tD + \frac{t}{1+t} R^2$ $\quad D = \sum_i \alpha_i (q_i^2 + m_i^2) - R^2$ $\quad R = \sum_i \alpha_i q_i$

Finally all matrix elements can be presented presented as combination of simple integrals of type

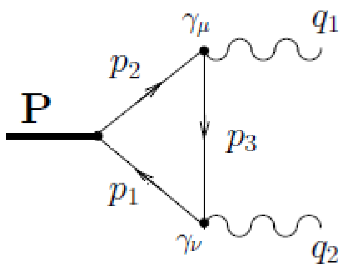
$$I(a_0 \ldots a_n, m, n, F) = \int_0^1 \{d\alpha_i\} \prod_i \alpha_i^{a_i} \int_0^\infty dt \frac{t^m}{(1+t)^n} [F(z_0)].$$

LIGHT MESONS

- Definition of parameters of the model:
  - Pion photo-decay
  - Rho-meson: hadronic and radiative decays

- Pion transition form factor

THE $\pi^0 \to \gamma\gamma$ DECAY



$$T^{\mu\nu}(q_1,q_2) = N_c N_p \int \frac{dp}{(2\pi)^4} \varphi(p^2) \mathrm{tr}\{i\gamma_5 S_i(p_2)\gamma_\mu S_i(p_3)\gamma_\nu S_i(p_1)\}.$$

The second amplitude $T^{(2)}$can be written by the replacement $q_1 \leftrightarrow q_2$

Using the pseudoscalar vertex function $\Gamma_H = i\gamma_5 g_{Hqq}$

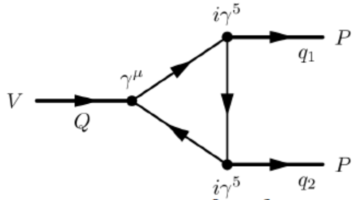
$$\begin{aligned} T(q_1,q_2) &= i\epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu \epsilon_2^\nu q_1^\alpha q_2^\beta \left(N_c Q_q^2\right) \frac{mN_\pi}{4\pi^2} I(Q,q_1,q_2) = \\ &= i\epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu \epsilon_2^\nu q_1^\alpha q_2^\beta \left(N_c Q_q^2\right) G_{\pi\gamma\gamma}(Q,q_1,q_2), \end{aligned}$$

$Q_q = (e_u^2 - e_d^2)$ with $e_u = 2/3e$ and $e_d = -1/3e$

$$g_{\pi\gamma\gamma} = G_{\pi\gamma\gamma}(M_\pi^2,0,0) \simeq \frac{mN_\pi}{4\pi^2\Lambda_\pi^2} I(M_\pi^2,0,0)$$

$$\boxed{\Gamma(\pi^0 \to \gamma\gamma) = \frac{M_\pi^3}{64\pi}(4\pi\alpha)^2 g_{\pi\gamma\gamma}^2}$$

ρ −MESON



$$T^\mu(q_1, q_2) = N_c N_\rho N_\pi \int \frac{dp}{(2\pi)^4} \varphi(p^2)$$
$$\mathrm{tr}\{\gamma_\mu S_2(p_2) i\gamma_5 S_3(p_3) i\gamma_5 S_1(p_1)\}$$

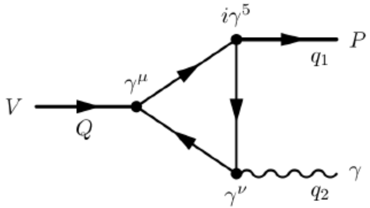
with form factor $\varphi(p^2) = \varphi_\rho(p^2)\varphi_\pi^2(p^2)$

The matrix element splits into two terms:

$$T^\mu(p_1, p_2) = (p_1 - p_2)^\mu f^+(t) + (p_1 + p_2)^\mu f^-(t)$$

with $f^-(t = M_\rho^2) = 0,\ \frac{1}{2} f^+(t = M_\rho^2) = g_{\rho\pi\pi}$

$$\Gamma_{\rho\pi\pi} = \frac{1}{6\pi} \frac{k^3}{M_\rho^2} g_{\rho\pi\pi}^2$$



$$T^{\mu\nu}(q_1, q_2) = i\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu(Q)\epsilon_2^{*\nu}(q_2)\cdot$$
$$\cdot Q^\alpha q_2^\beta \,(N_c Q_q e)\, \frac{m N_p N_v}{4\pi^2} I(Q, q_1, q_2)$$

$$\Gamma_{VP\gamma} = \frac{1}{3}\alpha k^3 g_{VP\gamma}^2$$

$$I(Q^2, q_1^2, q_2^2) = \int \frac{d^4p}{\pi^2} \varphi\left(\frac{p^2}{\Lambda^2}\right) \frac{\{1, p^\mu, p^{\mu\nu}, p^{\mu\nu\rho}\}}{(p_1^2 + m_1^2)(p_2^2 + m_2^2)(p_3^2 + m_3^2)}$$

PARAMETERS OF THE MODEL

Parameters of the model: masses of quarks (m_q, m_c, m_b), parameters Λ_H

Physical data used for parameter fitting: $m_\pi = 0.139, m_\rho = 0.77\ GeV$

Basic model parameters and constants for light mesons:

$m_{u(d)}$, GeV	Λ_π, GeV	N_π	Λ_ρ, GeV	N_ρ
0.223	1.14	3.724	0.75	3.612

	π	Exp.	ρ	Exp.
f_H, GeV	0.131	0.131	0.2	0.2
$\Gamma_{\pi\gamma\gamma}$, eV	7.76	7.82 [1]	-	-
$\Gamma_{\rho\pi\pi}$, GeV	-	-	0.151	0.150 [2]
$\Gamma_{\rho\pi\gamma}$, keV	-	-	71.8	84.2 [3]

[[1] PRD **33**, (1986) 3199–3202 [2] Phys. Rev. Lett. **106**, (2011) 162303 [3] PoS LATTICE2018 (2018) 065]

THE $\gamma^* \to \pi^0\gamma$ TRANSITION FORM FACTOR

$$T^{\mu\nu}(q_1,q_2) = N_c N_p \int \frac{dp}{(2\pi)^4}\varphi(p^2)\mathrm{tr}\{i\gamma_5 S_i(p_2)\gamma_\mu S_i(p_3)\gamma_\nu S_i(p_1)\}.$$

The second amplitude $T^{(2)}$can be written by the replacement $q_1 \leftrightarrow q_2$

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$$\begin{aligned} T(q_1,q_2) &= i\epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu \epsilon_2^\nu q_1^\alpha q_2^\beta \left(N_c Q_q^2\right) \frac{mN_\pi}{4\pi^2} I(Q,q_1,q_2) = \\ &= i\epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu \epsilon_2^\nu q_1^\alpha q_2^\beta \left(N_c Q_q^2\right) G_{\pi\gamma\gamma}(Q,q_1,q_2), \end{aligned}$$

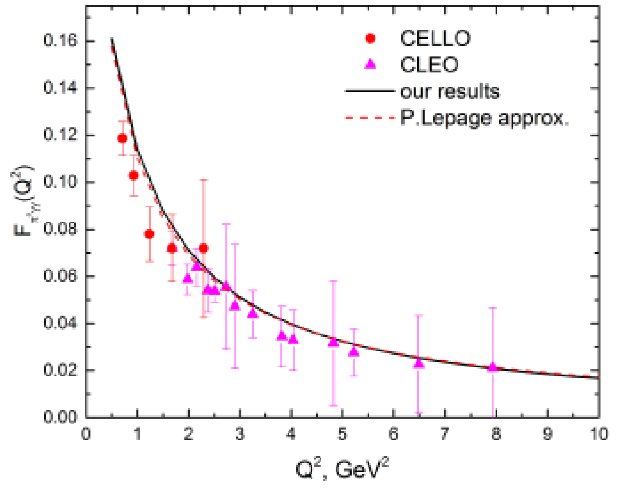
$Q_q = (e_u^2 - e_d^2)$ with $e_u = 2/3e$ and $e_d = -1/3e$ for the pion

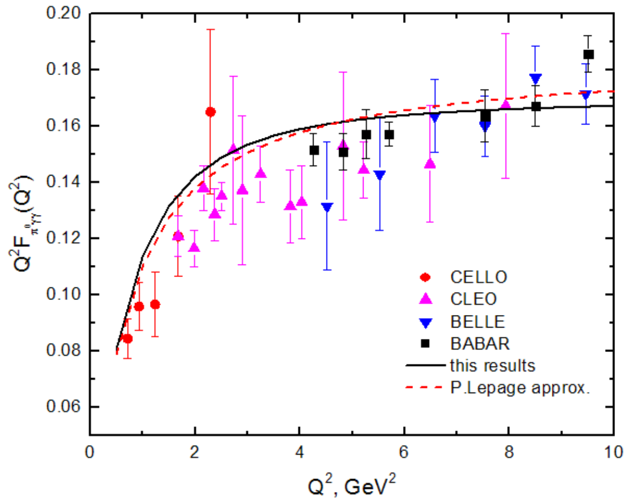
$$F_{\pi\gamma}(Q^2) = e^2 G_{\pi\gamma\gamma}(M_\pi^2, Q^2, 0)$$

THE $\gamma^* \to \pi^0\gamma$ TRANSITION FORM FACTOR

Sold line corresponds to the S. Brodsky and P. Lepage prediction within non-perturbative QCD
[PRD 24, (1981) 1808-1817]

$$F_{\pi\gamma}(Q^2) = \frac{2f_\pi}{8\pi^2 f_\pi^2 + Q^2},$$





Exp. data: CLEO PRD 57 (1998) 33-54, CELLO Z. Phys. 49, (1991) 401, Belle PRD86 (2012) 092007, BaBar PRD 80 (2009) 052002

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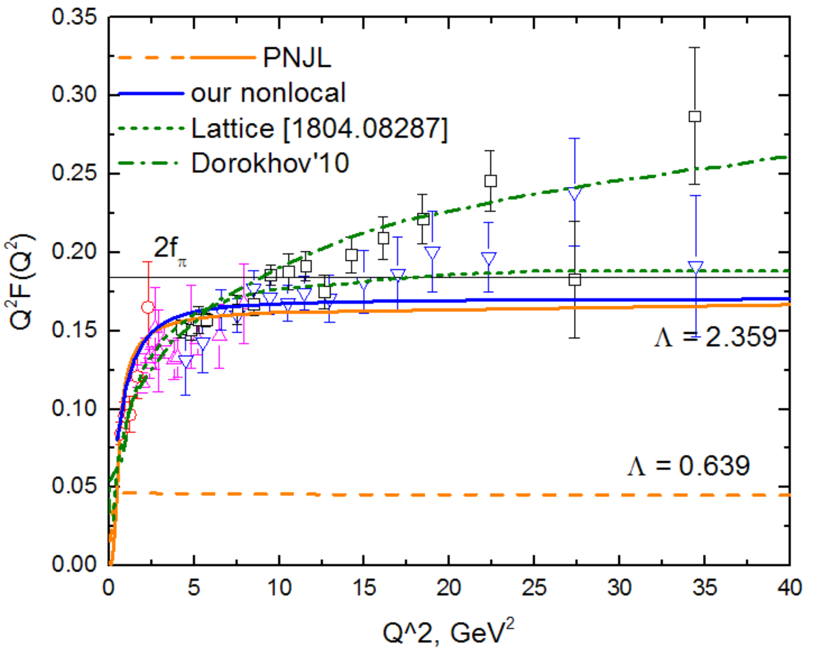
CONCLUSION - I

- perturbative QCD/Lattice QCD at high Q does not reproduce the behavior presented by BaBar data.

- there exist couple of models (A.E. Dorokhov, arXiv:0905.4577; A.V. Radyushkin, arXiv:0906.0323) which show logarithmic behavior of TFF at high momenta

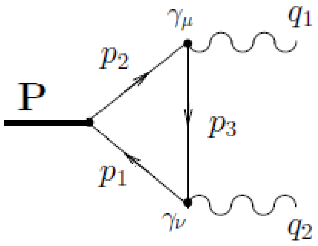
- non-perturbative QCD well works at low Q



HEAVY MESONS

- $\eta_c(\eta_b) \to \gamma\gamma$ decays and transition form factors
- ${}^{J}/_{\psi}, \Upsilon, D^*$ radiative decays

THE $\gamma^* \to H\gamma$ FORM FACTOR



$$T^{\mu\nu}(q_1, q_2) = N_c N_p \int \frac{dp}{(2\pi)^4} \varphi(p^2) \mathrm{tr}\{i\gamma_5 S_i(p_2)\gamma_\mu S_i(p_3)\gamma_\nu S_i(p_1)\}.$$

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$Q_c = 2/3e$, $Q_b = -1/3$e for heavy pseudoscalars

$$g_{H\gamma\gamma} = G_{H\gamma\gamma}(M_H^2, 0, 0) \simeq \frac{mN_H}{4\pi^2\Lambda_H^2} I(M_H^2, 0, 0)$$

$$\Gamma(H \to \gamma\gamma) = \frac{M_H^3}{64\pi}(4\pi\alpha)^2 g_{H\gamma\gamma}^2$$

$$F_{H\gamma}(Q^2) = e^2 G_{H\gamma\gamma}(M_H^2, Q^2, 0)$$

TRANSITION FORM FACTORS η_c, η_b

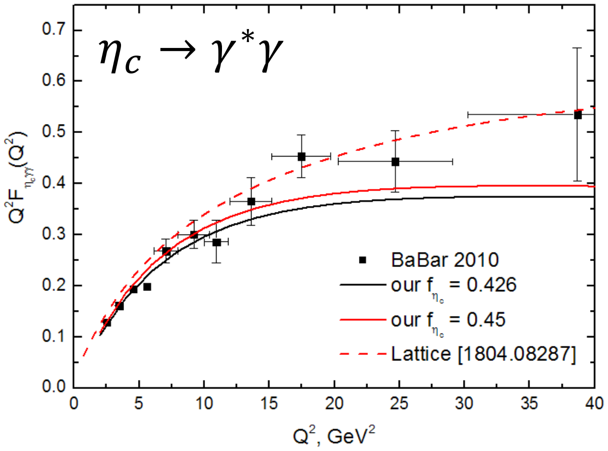
Model parameters and constants for heavy pseudoscalars: $m_c = 1.6$, $m_b = 4.77$

	$\Lambda_H[GeV]$	N_H	$M_H[GeV]$	f_H(our)	f_H(refs)
η_c	2.775	3.546	2.985	0.426	0.42
η_b	2.81	8.568	9.39	0.715	0.705

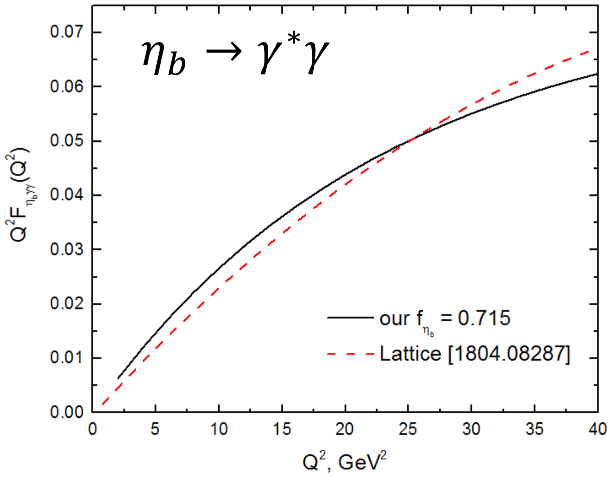
[arXiv: 9703364[hep-ph]]

	$\Gamma_{H\gamma\gamma}$ keV (our)	$\Gamma_{H\gamma\gamma}$ keV (refs)	PDG
η_c	5.03	4.88[1] - 6.788 [2]	5.1
η_b	0.18	0.17 - 0.69 [3]	-

[1]1804.08287 [hep-ph], [2] 2305.06231 [hep-lat]] [3][hep-ph/0609268]



[BaBar], PRD 81 (2010) 052010



Phys. Lett. B 413 (1997) 410-415

RADIATIVE DECAY OF HEAVY QUARCONIA $J/\psi \to \eta_c\gamma,\ \Upsilon \to \eta_b\gamma$

Model parameters and constants for heavy mesons: $m_c = 1.6$, $m_b = 4.77$

	$\Lambda_H[GeV]$	N_H	$M_H[GeV]$	$f_H(our)$	$f_h(refs)$
J/ψ	2.03	3.238	3.075	0.435	0.339
Υ	4.22	3.489	9.405	0.705	

Radiative decay width:

	our	refs.
$J/\psi \to \eta_c\gamma$	2.28 keV	1.83-2.49 keV (Lattice QCD)
$\Upsilon \to \eta_b\gamma$	25.3 eV	5.8 [1], 45[2]

CLEO experiment: $\Gamma_{J/\psi\eta_c\gamma}$= 1,83 keV [arXiv:0805.0252]
KEDR experiment: ($\Gamma_{J/\psi\eta_c\gamma}$= 2,17 keV) [arXiv:1002.2071]

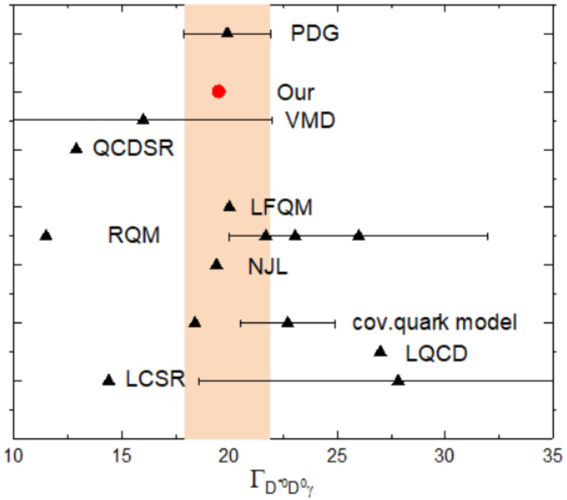
[[1]arXiv:0210381; [2] arXiv:1208.2855]

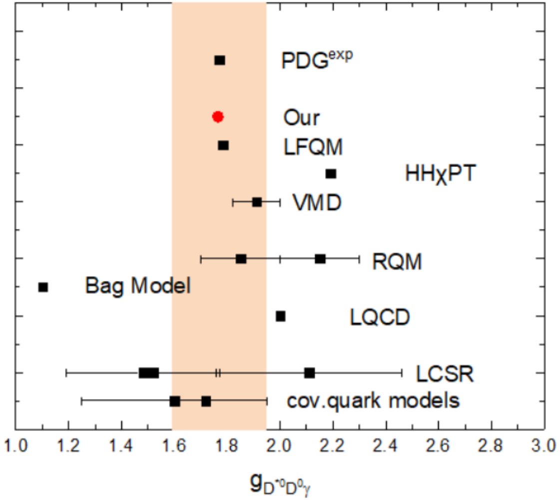
RADIATIVE DECAY OF MESONS WITH OPEN CHARM $D^* \to D\gamma$

Model parameters and constants for heavy mesons: $m_c = 1.6$, $m_u = 0.223$

	M_H, GeV	Λ_H, GeV	N_H	f_H (our)	f_H (refs)
D^*	2.031	1.29	2.916	0.24	0.223
D	1.889	1.42	4.325	0.213	0.203

Radiative decay width:





CONCLUSION-II

- This work presents an effective quark model with nonlocal interactions to provide a unified description of both light and heavy mesons. A consistent set of model parameters is established across the mass spectrum.

- All the parameters are fitted via observables such as the meson mass and decay constants.

- The model is applied to calculate physical processes such as the decays of pseudoscalar mesons into two photons and the radiative decays of vector mesons. The results are in good agreement with available experimental data and consistent with other theoretical models.

THANK YOU FOR ATTENTION