Λ spin polarization from dissipative spin hydrodynamics

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Rotation and Polarization : Barnett effect

An initially unmagnetized body becomes magnetized under rotation, due to spin alignment induced by conservation of total angular momentum.

$$M = \chi\Omega/\gamma$$

where χ is the magnetic susceptibility, Ω is the angular velocity and γ is the gyromagnetic ratio.

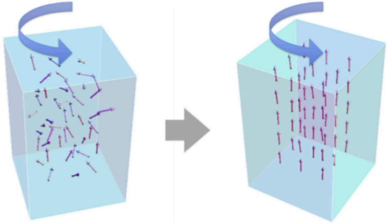


Image source : Front. Phys. 3:54 (2015)

Spin Polarization in lab. QGP

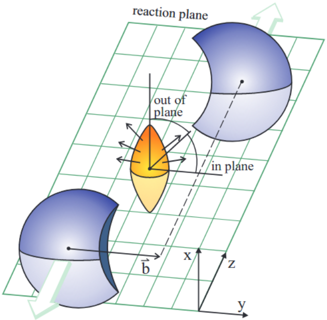
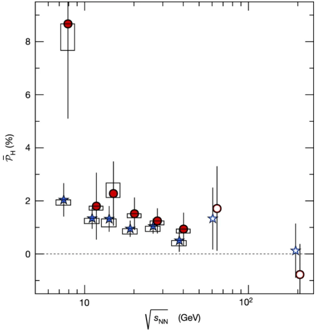


Image source - arXiv:0910.4114

- Relativistic collision of two heavy-ions creates rotating QGP.
- The initial orbital angular momentum (OAM) is, $L_0 = pb \simeq A\sqrt{s_{NN}}b/2$. For $\sqrt{s_{NN}} = 200$ GeV and $b = 5$ fm, $L_0 \sim 5 \times 10^5$.
- A fraction of L_0 is transferred to QGP fireball resulting in polarization of quarks due to spin-orbit coupling.
- Quark polarization is then transferred to hadrons, which is experimentally observed.

Experimental observation of Λ-polarization



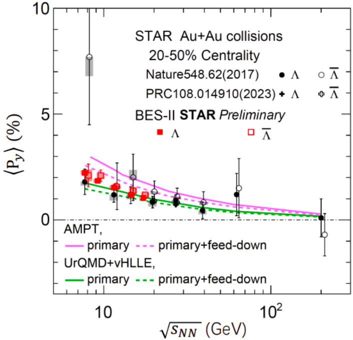


Image source: STAR Collaboration (left) Nature 548, 62-65 (2017) and (right) arXiv:2412.09897

Equilibrium approach based on Zubarev's formalism

AOP 338, 32 (2013); PLB 820, 136519 (2021) and PRL 127, 272302 (2021)

- In the Belinfante pseudogauge (spin tensor is 0), compute local equilibrium density operator ($\hat{\rho}_{\rm LE}$) by maximizing the entropy ($S = -{\rm tr}[\hat{\rho}\,\log\hat{\rho}]$) subject to constraints

$$n_\mu\ {\rm tr}[\hat{\rho}\,\hat{T}^{\mu\nu}] = n_\mu T^{\mu\nu} \qquad , \qquad n_\mu\ {\rm tr}[\hat{\rho}\hat{N}^{\mu}] = n_\mu N^{\mu}$$

  The procedure gives:

$$\hat{\rho}_{\rm LE} = \frac{1}{Z_{\rm LE}} \exp\left[-\int_\Sigma d\Sigma_\mu \left(\beta_\nu \hat{T}^{\mu\nu} - \zeta \hat{N}^{\mu}\right)\right] \text{ with } \beta_\nu = \frac{u_\nu}{T}\ ,\zeta = \frac{\mu}{T}$$

- Evaluate the mean value of a quantum operator as

$$O(x) = {\rm tr}[\hat{\rho}_{\rm LE}\hat{O}(x)].$$

Polarization of spin-1/2 particles in a fluid cell is

$$S_\mu(x,p) = -\frac{1}{8m}\epsilon_{\mu\rho\sigma\tau}(1-n_F)\varpi^{\rho\sigma}p^\tau + \mathcal{O}(\varpi^2)$$

where n_F is the Fermi-Dirac distribution and $\varpi_{\rho\sigma}$ is thermal vorticity defined as

$$\varpi_{\rho\sigma} = \frac{1}{2}(\partial_\sigma\beta_\rho - \partial_\rho\beta_\sigma) \qquad \text{with} \qquad \beta_\rho = \frac{u_\rho}{T}$$

Mean polarization vector is given by

$$S^\mu(p) = \frac{\int_\Sigma (d\Sigma.p)\, S^\mu(x,p) n_F(x,p)}{\int_\Sigma (d\Sigma.p)\, n_F(x,p)}$$

Input T, u^μ and their gradients from the numerical solution of relativistic hydrodynamics to compute the above expression.

Hydrodynamic simulation for global polarization

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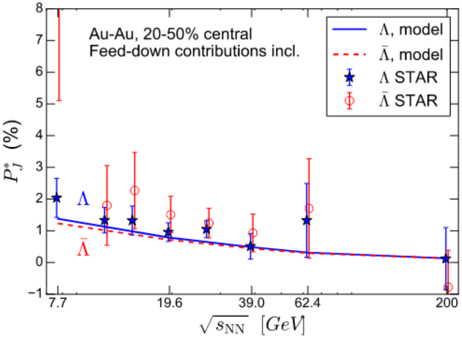
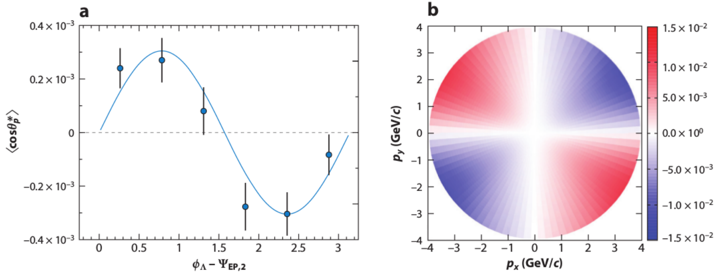
Vorticity in the QGP liquid and Λ polarization at the RHIC Beam Energy Scan

Iurii Karpenko[a,b], Francesco Becattini[a,c]



Spin sign puzzle

"Hydrodynamics predict a negative sign of the longitudinal component of the polarization vector."



Ann. Rev. Nucl. Part. Sci. 70 (2020) 395

Additional contribution from thermal shear, $\xi_{\mu\nu} = \frac{1}{2}(\partial_\sigma\beta_\rho + \partial_\rho\beta_\sigma)$, was derived such that

$$S^\mu(p) = S^\mu_\varpi(p) + S^\mu_\xi(p)$$

The sign of longitudinal polarization is still negative.

Isothermal approximation

At high energies $\mu_B \approx 0$. Hence, constant energy density implies constant T on Σ, so that

$$\hat{\rho}_{\rm LE} \sim \exp\left[-\frac{1}{T}\int_\Sigma d\Sigma_\mu \ \hat{T}^{\mu\nu} u_\nu\right]$$

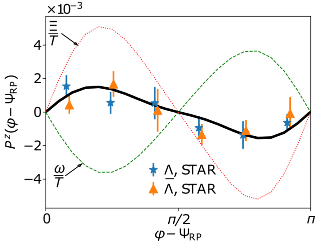
This gives the following formula for mean spin vector (PRL 127, 272302 (2021))

$$S^\mu(p) = -\epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int d\Sigma\cdot p \ n_F(1-n_F)\left[\omega_{\nu\rho} + 2\hat{t}_\nu \frac{p^\lambda}{p\cdot\hat{t}}\Xi_{\lambda\rho}\right]}{8 m_\Lambda T \int d\Sigma\cdot p \ n_F}$$

where $\hat{t} = (1,0,0,0)$ and

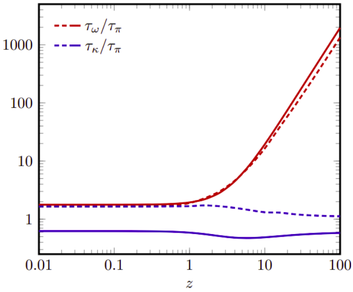
$$\omega_{\nu\rho} = \frac{1}{2}(\partial_\rho u_\nu - \partial_\nu u_\rho)$$

$$\Xi_{\nu\rho} = \frac{1}{2}(\partial_\rho u_\nu + \partial_\nu u_\rho)$$



Limitations of the equilibrium approach

- Assumes instantaneous equilibration of spin degrees of freedom. However, spin-relaxation time can become comparable to the lifetime of the QGP fireball.



Spin relaxation times for a scalar four-fermion interaction, $\mathcal{L}_{\text{int}} = G(\bar{\psi}\psi)^2$, plotted as function of $z = m/T$. Image source: Phys. Rev. Res. 6, 043103 (2024)

- The equilibrium formula works only with the isothermal approximation which may not be valid at lower collisions energies.

Relativistic spin hydrodynamics

- Spin relaxation time can become comparable to the lifetime of the fireball. In that case spin dynamics cannot be neglected.

- Spin dynamics is introduced by demanding conservation of total angular momentum

$$D_\mu J^{\mu,\alpha\beta} = 0$$

in addition to conservation of energy-momentum and charge

$$D_\mu T^{\mu\nu}(x) = 0 \quad , \quad D_\mu N^\mu(x) = 0.$$

- We have $J^{\mu,\alpha\beta} = L^{\mu,\alpha\beta} + S^{\mu,\alpha\beta}$, where $L^{\mu,\alpha\beta} = T^{\mu[\beta}x^{\alpha]}$. This gives

$$D_\mu S^{\mu,\alpha\beta} = T^{[\beta\alpha]} = T^{\beta\alpha} - T^{\alpha\beta}$$

where $T^{[\beta\alpha]}$ is the antisymmetric part of energy-momentum tensor.

Kinetic description of currents

- In kinetic theory, macroscopic quantities are obtained from moments of the single-particle distribution function

$$N^\mu = \int \frac{d^3p}{(2\pi)^3 p^0}\; p^\mu \; f(x,p)$$

$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3 p^0}\; p^\mu \; p^\nu \; f(x,p)$$

- Can we define an analogous expression for the spin current, $S^{\mu,\alpha\beta}$?

- This requires extending $f(x,p)$ to include spin degrees of freedom $\rightarrow$ spin-dependent distribution $f(x,p,\mathfrak{s})$.

$$S^{\mu,\alpha\beta} = \sigma \int d\Gamma \; k^\mu \; \Sigma_{\mathfrak{s}}^{\alpha\beta} \; f(x,p,\mathfrak{s})$$

where $\Sigma_{\mathfrak{s}}^{\mu\nu} = -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}k_\alpha \mathfrak{s}_\beta$ and $d\Gamma$ is the measure for extended phase space.

- The time evolution of the single-particle distribution function, $f(x, p, \mathfrak{s})$, is determined by the Boltzmann equation

$$k.\partial f(x, p, \mathfrak{s}) = C[f] = C_{\text{local}}[f] + C_{\text{nonlocal}}[f]$$

- Non-local collisions provide an exchange mechanism between spin and orbital angular momentum, facilitating spin equilibration.

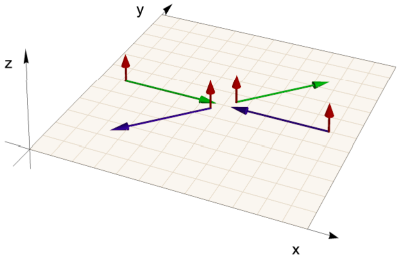


Image source:Prog. Part. Nucl. Phys. 108 (2019) 103709

- Equilibrium distribution is defined as

$$C_{\text{local}}[f_{\text{eq}}] = 0$$

- Local-equilibrium distribution function is

$$f_{\rm eq}(x,p,\mathfrak{s}) = \left[\exp\left(\frac{p.u}{T} - \frac{\mu}{T} - \frac{\sigma\hbar}{2}\,\Omega_{0,\mu\nu}\Sigma_{\mathfrak{s}}^{\mu\nu}\right) \pm 1\right]^{-1}$$

where $\Omega_{0,\mu\nu}$ is known as spin potential.

- Spin potential is a rank-2 antisymmetric tensor and can be decomposed as

$$\Omega_0^{\mu\nu} = u^\mu \kappa_0^\nu - u^\nu \kappa_0^\mu + \epsilon^{\mu\nu\alpha\beta} u_\alpha \omega_{0,\beta}$$

- Spin hydrodynamics is obtained by taking moments of the Boltzmann transport equation. Note that if $C_{\rm nonlocal}[f] = 0$, the ideal spin equation conserves total spin

$$D_\mu S_0^{\mu,\alpha\beta} = 0$$

Conservative spin equation is solved numerically in **SKS**, R. Ryblewski & W. Florkowski, PRC 111, 024907 (2025).

- For $C_{\mathrm{nonlocal}}[f] \neq 0$, the equations of ideal spin hydrodynamics are

$$D_\mu T_0^{(\mu\nu)} = 0 + \mathcal{O}(\hbar^2) \qquad , \qquad D_\lambda S_0^{\lambda,\mu\nu} = \frac{1}{\hbar} T_0^{[\nu\mu]} + \mathcal{O}(\hbar^2)$$

where the anti-symmetric part of energy-momentum tensor is

$$T_0^{[\mu\nu]} = \frac{\hbar\sigma}{2} \int d\Gamma\, \Sigma_{\mathfrak{s}}^{\mu\nu}\; C_{\mathrm{nonlocal}}[f_{\mathrm{eq}}]$$

- In small polarization limit, spin evolution decouples from the background.

Image source: https://pixabay.com/

Relativistic dissipative spin hydrodynamics

Derived in D. Wagner, PRD 111, 016008 (2025) using the moment expansion method. Numerically solved in Sapna, **SKS** and D. Wagner, arXiv:2503.22552 (accepted in PRC)

$$\tau_\omega \dot{\omega}_0^{\langle\mu\rangle} + \omega_0^\mu = -\frac{\omega_K^\mu}{T} + \delta_{\omega\omega}\omega_0^\mu\theta + \epsilon^{\mu\nu\alpha\beta}u_\nu\left(\ell_{\omega\kappa}\nabla_\alpha\kappa_{0,\beta} - \tau_\omega\dot{u}_\alpha\kappa_{0,\beta}\right)$$
$$+ \lambda_{\omega\omega}\sigma^{\mu\nu}\omega_{0,\nu} + \lambda_{\omega\mathfrak{t}}\mathfrak{t}^\mu{}_\nu\omega_K^\nu\,,$$

$$\tau_\kappa \dot{\kappa}_0^{\langle\mu\rangle} + \kappa_0^\mu = -\frac{\dot{u}^\mu}{T} + \delta_{\kappa\kappa}\kappa_0^\mu\theta + \epsilon^{\mu\nu\alpha\beta}u_\nu\left(\frac{\tau_\kappa}{2}\nabla_\alpha\omega_{0,\beta} + \tau_\kappa\dot{u}_\alpha\omega_{0,\beta}\right)$$
$$+ \ell_{\kappa\mathfrak{t}}\Delta^\mu_\lambda\nabla_\nu\mathfrak{t}^{\nu\lambda} + \tau_{\kappa\mathfrak{t}}\mathfrak{t}^{\mu\nu}\dot{u}_\nu + \left(\lambda_{\kappa\kappa}\sigma^{\mu\nu} + \frac{\tau_\kappa}{2}\omega_K^{\mu\nu}\right)\kappa_{0,\nu},$$

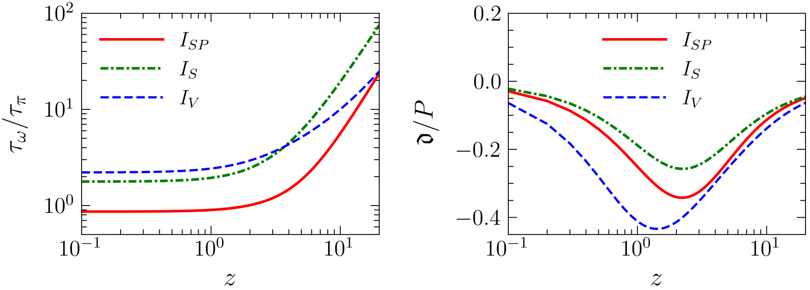
$$\tau_\mathfrak{t}\dot{\mathfrak{t}}^{\langle\mu\nu\rangle} + \mathfrak{t}^{\mu\nu} = \frac{\mathfrak{d}}{T}\sigma^{\mu\nu} + \delta_{\mathfrak{t}\mathfrak{t}}\mathfrak{t}^{\mu\nu}\theta + \lambda_{\mathfrak{t}\mathfrak{t}}\mathfrak{t}_\lambda{}^{\langle\mu}\sigma^{\nu\rangle\lambda} + \frac{5}{3}\tau_\mathfrak{t}\mathfrak{t}_\lambda{}^{\langle\mu}\omega_K^{\nu\rangle\lambda} + \ell_{\mathfrak{t}\kappa}\nabla^{\langle\mu}\kappa_0^{\nu\rangle}$$
$$+ \tau_{\mathfrak{t}\omega}\omega_K^{\langle\mu}\omega_0^{\nu\rangle} + \lambda_{\mathfrak{t}\omega}\sigma_\lambda{}^{\langle\mu}\epsilon^{\nu\rangle\lambda\alpha\beta}u_\alpha\omega_{0,\beta}.$$

In above equations, the symbols are as follows:

$$A^{\langle\mu\rangle} = \Delta^{\mu\nu}A_\nu, \quad \dot{A} = u\cdot\partial A, \quad \nabla^\mu = \Delta^{\mu\nu}\partial_\nu, \quad \sigma^{\mu\nu} = D^{\langle\mu}u^{\nu\rangle}$$

$$\omega_K^{\mu\nu} = \frac{1}{2}\Delta^\mu_\alpha\Delta^\nu_\beta D^{[\alpha}u^{\beta]} = \epsilon^{\mu\nu\alpha\beta}u_\alpha\omega_{K,\beta}$$

Transport coefficients τ_ω and $\mathfrak{d}$

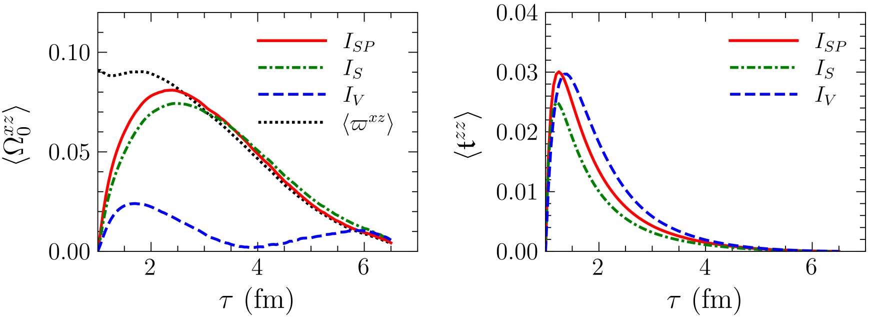


- The transport coefficients are computed for three interactions (i) Scalar (I_S): $\mathcal{L}_{\text{int,S}} = G(\bar{\psi}\psi)^2$, (ii) Scalar+Pseudoscalar (I_{SP}): $\mathcal{L}_{\text{int,SP}} = G[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2]$, and (iii) Vector ($I_V$): $\mathcal{L}_{\text{int,V}} = -G(\bar{\psi}\gamma^\mu\psi)^2$

- The transport coefficients are independent of G when computed as ratios.

- The only parameter of the model is the mass of particles, which we fix at 300 MeV. For Au+Au collision at 200 GeV with $b = 8.4$ fm, z increases from 1.2 to approximately 1.9.

Result: Time evolution of spin potential and spin-shear stress



arXiv:2503.22552

$$\langle A \rangle(\tau) = \frac{\int \mathrm{d}x\ \mathrm{d}y\ \mathrm{d}\eta_s\ A(\tau, x, y, \eta_s)\varepsilon(\tau, x, y, \eta_s)\Theta(\varepsilon - \varepsilon_{\mathrm{cut}})}{\int \mathrm{d}x\ \mathrm{d}y\ \mathrm{d}\eta_s\ \varepsilon(\tau, x, y, \eta_s)\Theta(\varepsilon - \varepsilon_{\mathrm{cut}})}$$

Spin Polarization

- The spin polarization is given by

$$\boxed{S^\mu(p) = S^\mu_\omega(p) + S^\mu_\kappa(p) + S^\mu_{\mathfrak{t}}(p)}$$

where

$$S^\mu_\omega(p) = \frac{1}{N(p)} \int \mathrm{d}\Sigma \cdot p \frac{u^\mu(\omega_0 \cdot p) - \omega_0^\mu(p \cdot u)}{2m_\Lambda} f_0 \widetilde{f_0},$$

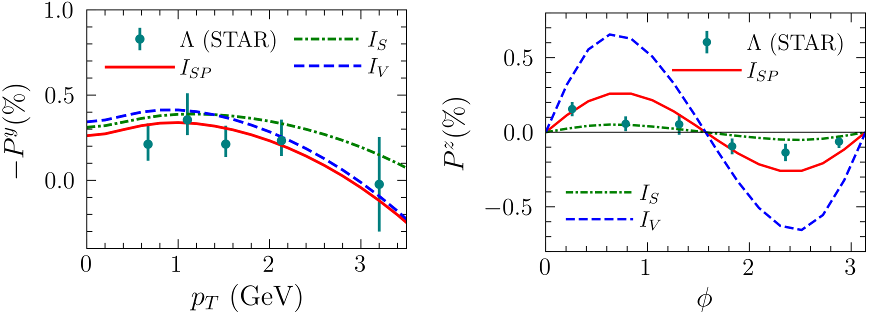
$$S^\mu_\kappa(p) = -\frac{1}{N(p)} \int \mathrm{d}\Sigma \cdot p \frac{\epsilon^{\mu\nu\rho\sigma} u_\nu p_\sigma}{2m_\Lambda} \kappa_{0,\rho} f_0 \widetilde{f_0}\ ,$$

$$S^\mu_{\mathfrak{t}}(p) = \frac{1}{N(p)} \int \mathrm{d}\Sigma \cdot p \frac{\epsilon^{\mu\nu\rho\sigma} u_\nu p^\lambda p_\sigma}{3T^2(\varepsilon + P)} \mathfrak{t}_{\rho\lambda} f_0 \widetilde{f_0}\ ,$$

and f_0 denotes the Fermi-Dirac distribution, $\widetilde{f_0} = 1 - f_0$, and $N(p) = 2\int \mathrm{d}\Sigma \cdot p\, f_0$.

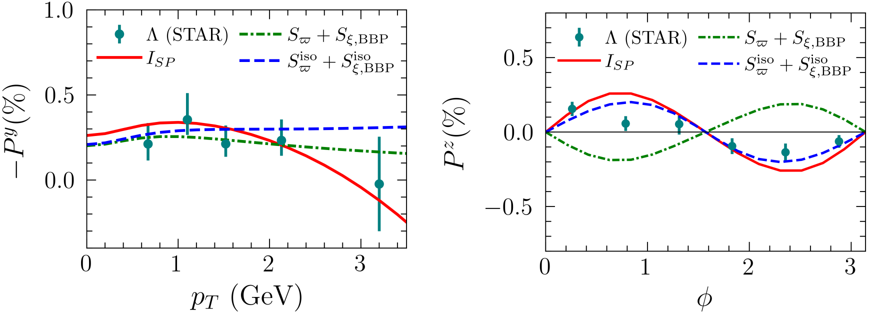
- Note that $S^\mu_{\mathfrak{t}}(p)$ is a dissipative correction to spin polarization.

Result: Sensitivity of spin polarization to interaction type



(left) y-component of spin polarization as function of transverse momentum p_T and (right) z-component of spin polarization as function of azimuthal angle ϕ. Results are shown for Λ hyperons from Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV in the 20-50% centrality, for three interaction types. Source: arXiv:2503.22552

Result: Comparison with the equilibrium approach



(left) y-component of spin polarization as function of transverse momentum p_T and (right) z-component of spin polarization as function of azimuthal angle ϕ. Results are shown for Λ hyperons from Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV in the 20-50% centrality. Also shown are results from the equilibrium approach, with and without the isothermal approximation. Source: arXiv:2503.22552

Summary

- We have numerically solved the equations of relativistic dissipative spin hydrodynamics.
- The system is initialized with zero spin current; the spin current is generated and transported through exchanges between orbital and spin angular momentum.
- Spin degrees of freedom can relax to their equilibrium values at sufficiently high temperatures.
- We applied the framework to describe the spin polarization of Λ hyperons in $\mathrm{Au}{+}\mathrm{Au}$ collisions at $\sqrt{s_{NN}} = 200$ GeV, finding good agreement with experimental data.
- The proposed framework is also applicable at lower collision energies and in small systems.