Jet production at the LHC

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Outline

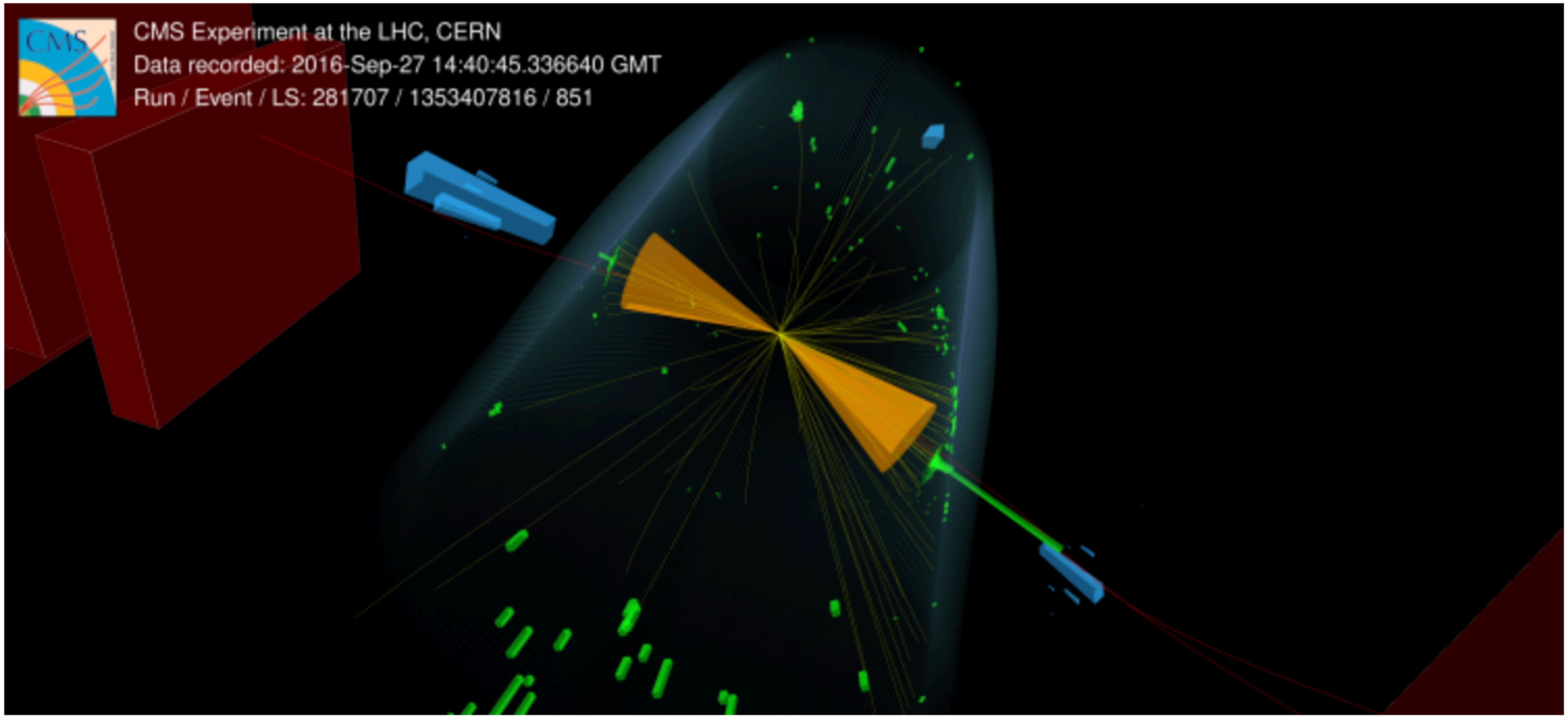
- Introduction
- Colour projection techniques
- Jet calculations
- Jet algorithm
- Fixed order results
- Summary

What are jets?

- Jets are spray of particles or collimated beam of hadrons (mesons and baryons) produced in the high energy collisions.
- They are observed in electron positron annihilations to quark-pair.
- This is very much similar to muon-pair creation, but a careful study inferred the existence of three colours that the quarks can have.
- The three jet events successfully describe the presence of gauge boson mediator responsible for strong interactions, gluons.
- Now they are produced in multitude at the LHC, can be used for various signals (e.g. top decays).
- They can also be used for BSM searches (like mono-jet events).

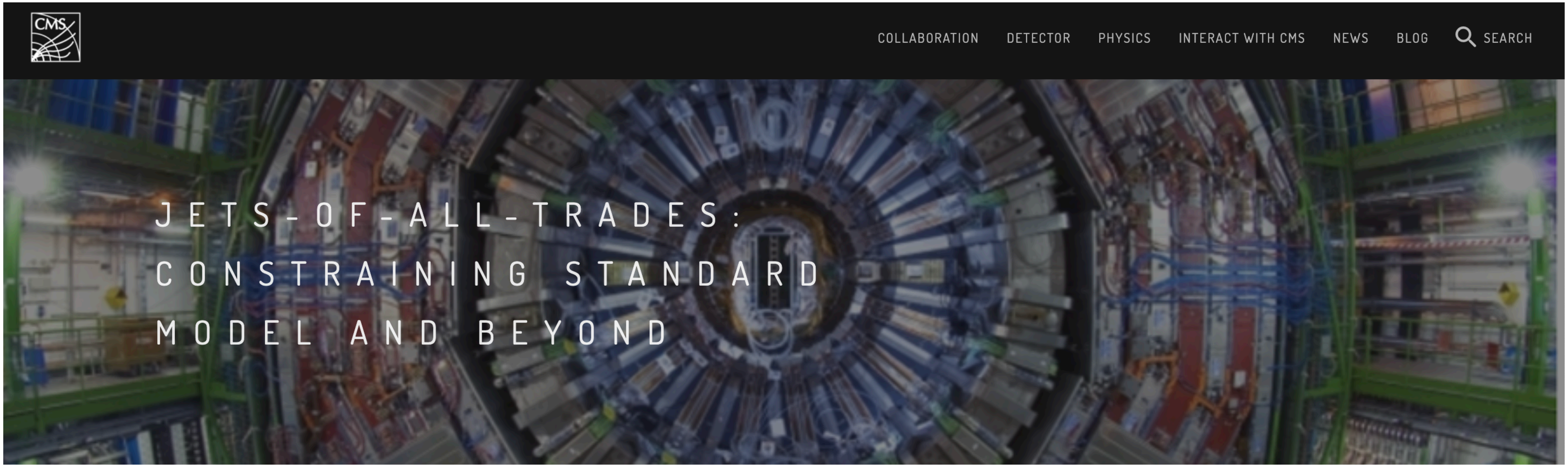
Jets@CMS (2016)

By CMS Collaboration



For the first time, CMS physicists extract the fundamental parameters of QCD together with constraints on the New Physics.

Hadroproduction of jets



- In hadron collisions, jets are produced in many ways not just in quark annihilations. Many parton channels are possible.

$$
\begin{aligned}
q_j\bar{q}_j &\to q_j\bar{q}_j\,, \quad q_j\bar{q}_j \to q_k\bar{q}_k\,, \quad q_j\bar{q}_k \to q_j\bar{q}_k\,, \quad q_jq_j \to q_jq_j\,, \quad q_jq_k \to q_jq_k\,, \\
q\bar{q} &\to gg\,, \quad gg \to q\bar{q}\,, \quad qg \to qg\,, \quad gg \to gg\,.
\end{aligned}
$$

Jet calculations : Colour projection technique

- Consider a 2 —> 2 scattering process (four parton scattering).

- The basic idea: The matrix element in the colour space can be thought of as a vector in the colour space spanned by some colour basis elements.

- To resolve the respective components, one can simply project these basis elements on to the given matrix element for a given four parton scattering.

- The number of basis elements changes with the parton scattering under consideration.

- Then these colour projected amplitudes can then be represented in the matrix form.

Quark quark scattering

Consider a four quark scattering process, represented by their colours: $i\, j \to k\, l$

$$c_1 = \delta_{ik}\, \delta_{jl}\,, \quad c_2 = T^c_{ki}\, T^c_{jl}$$

$|C_I>= \{c_1, c_2\}$ **Orthogonal basis**

$S_{IJ} = |C_I >< C_J|$ **Soft matrix**

$p_i = c_i / S_{ii}$ $\quad |P_I >= \{p_1, p_2\}$ **Orthonormal basis**

$|H_I >= \mathcal{M}|P_I >$ and $< H_I| =< P_I|\mathcal{M}^*$ **Colour projection / Colour decomposition**

Soft and hard functions

$$S^{q\bar{q}\to q\bar{q}} = \begin{bmatrix} N_c^2 & 0 \\ 0 & (N_c^2-1)/4 \end{bmatrix}$$

Soft matrix

$$H_{IJ} = |H_I >< H_J|$$

Hard matrix

$$H^{q_j\bar{q}_j\to q_j\bar{q}_j} = \alpha_s^2 \begin{bmatrix} H_{11}^{q_j\bar{q}_j\to q_j\bar{q}_j} & H_{12}^{q_j\bar{q}_j\to q_j\bar{q}_j} \\ H_{12}^{q_j\bar{q}_j\to q_j\bar{q}_j} & H_{22}^{q_j\bar{q}_j\to q_j\bar{q}_j} \end{bmatrix}$$

$$H_{11}^{q_j\bar{q}_j\to q_j\bar{q}_j} = \frac{2C_F^2}{N_c^4}\frac{(t^2+u^2)}{s^2},$$

$$H_{12}^{q_j\bar{q}_j\to q_j\bar{q}_j} = \frac{2C_F}{N_c^3}\left[-\frac{(t^2+u^2)}{N_c s^2}+\frac{u^2}{st}\right],$$

$$H_{22}^{q_j\bar{q}_j\to q_j\bar{q}_j} = \frac{1}{N_c^2}\left[\frac{2}{N_c^2}\frac{(t^2+u^2)}{s^2}+2\frac{(s^2+u^2)}{t^2}-\frac{4}{N_c}\frac{u^2}{st}\right]$$

$$|M|^2 = S_{IJ}H_{JI} = Tr[S.H]$$

Born level

$$\sigma^B_{q_j\bar{q}_j\to q_j\bar{q}_j} = \alpha_s^2\frac{(N_c^2-1)}{2N_c^2\, s}\left[\frac{t^2+u^2}{s^2}+\frac{s^2+u^2}{t^2}-\frac{2}{N_c}\frac{u^2}{st}\right]$$

N. Kidonakis, G. Oderda and G. Sterman
Nucl. Phys. B. 531 (1998)

Different quark flavours $q_j\bar{q}_j \to q_k\bar{q}_k$

$$H^{q_j\bar{q}_j\to q_k\bar{q}_k} = \alpha_s^2 \begin{bmatrix} (C_F^2/N_c^2)h^{q_j\bar{q}_j\to q_k\bar{q}_k} & -(C_F/N_c^2)h^{q_j\bar{q}_j\to q_k\bar{q}_k} \\ -(C_F/N_c^2)h^{q_j\bar{q}_j\to q_k\bar{q}_k} & h^{q_j\bar{q}_j\to q_k\bar{q}_k}/N_c^2 \end{bmatrix}$$

$$s = (p_a + p_b)^2\,, \quad t = (p_a - p_J)^2\,, \quad u = (p_b - p_J)^2$$

$$s_4 \equiv s + t + u$$

$$h^{q_j\bar{q}_j\to q_k\bar{q}_k} = \frac{2}{N_c^2}\frac{(t^2+u^2)}{s^2}$$

$s_4 = 0$ (At threshold)

$$\sigma^B_{q_j\bar{q}_j\to q_k\bar{q}_k}\delta(s_4) = \alpha_s^2\frac{(N_c^2-1)}{2N_c^2\, s}\frac{(t^2+u^2)}{s^2}\delta(s_4)$$

For qg —> qg channel

$$c_1 = \delta_{ik}\,\delta_{jl}, \quad c_2 = d^{jlc}\,T^c_{ki} \quad c_3 = i\,f^{jlc}\,T^c_{ki}$$

$$S^{qg\to qg} = \begin{bmatrix} N_c(N_c^2-1) & 0 & 0 \\ 0 & (N_c^2-4)(N_c^2-1)/(2N_c) & 0 \\ 0 & 0 & N_c(N_c^2-1)/2 \end{bmatrix}$$

$$H^{qg\to qg} = \alpha_s^2 \begin{bmatrix} H_{11}^{qg\to qg} & H_{12}^{qg\to qg} & H_{13}^{qg\to qg} \\ H_{12}^{qg\to qg} & H_{22}^{qg\to qg} & H_{23}^{qg\to qg} \\ H_{13}^{qg\to qg} & H_{23}^{qg\to qg} & H_{33}^{qg\to qg} \end{bmatrix}$$

$$\begin{aligned}
H_{11}^{qg\to qg} &= -\frac{1}{2N_c^3(N_c^2-1)}\left(\frac{t^2}{su}-2\right), \\
H_{12}^{qg\to qg} &= N_c H_{11}^{qg\to qg}, \\
H_{22}^{qg\to qg} &= N_c^2 H_{11}^{qg\to qg}, \\
H_{13}^{qg\to qg} &= \frac{1}{N_c^2(N_c^2-1)}\left[-1-\frac{2s}{t}+\frac{u}{2s}-\frac{s}{2u}\right], \\
H_{23}^{qg\to qg} &= N_c H_{13}^{qg\to qg}, \\
H_{33}^{qg\to qg} &= \frac{1}{N_c(N_c^2-1)}\left[3-\frac{4su}{t^2}-\frac{t^2}{2su}\right].
\end{aligned}$$

$$\sigma^B_{qg\to qg}\delta(s_4) = \alpha_s^2\frac{1}{s}\left[2-\frac{1}{N_c^2}-\frac{(N_c^2-1)}{2N_c^2}\frac{t^2}{su}-2\frac{su}{t^2}\right]\delta(s_4)$$

For gg—> gg channel

$$
\begin{aligned}
c_1 &= \frac{i}{4}[f^{ijm}d^{klm} \; - \; d^{ijm}f^{klm}] \; \delta_{ik} \; \delta_{jl}, \\
c_2 &= \frac{i}{4}[f^{ijm}d^{klm} \; + \; d^{ijm}f^{klm}], \\
c_3 &= \frac{i}{4}[f^{ikm}d^{jlm} \; + \; d^{ikm}f^{jlm}], \\
c_4 &= \frac{1}{8}\delta_{ik}\delta_{jl}, \\
c_5 &= \frac{3}{5}d^{ikn} \; d^{jln}, \\
c_6 &= \frac{1}{3}f^{ikn} \; f^{jln}, \\
c_7 &= \frac{1}{2}\left(\delta_{ij}\delta_{kl} - \delta il \delta_{jk}\right) - \frac{1}{3}f^{ikn} \; f^{jln}, \\
c_8 &= \frac{1}{2}\left(\delta_{ij}\delta_{kl} + \delta il \delta_{jk}\right) - \frac{1}{8}\delta_{ik}\delta_{jl} - \frac{3}{5}d^{ikn} \; d^{jln}
\end{aligned}
$$

For gg—> gg channel

$$S_{8\times 8} = \begin{bmatrix} G_{3\times 3} & 0_{3\times 5} \\ 0_{5\times 3} & G_{5\times 5} \end{bmatrix}$$

where $G_{3\times 3} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ and $G_{5\times 5} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 0 & 27 \end{bmatrix}$

N. Kidonakis, G. Oderda and G. Sterman

Nucl. Phys. B. 531 (1998)

For gg—> gg channel

$$H^{gg\to gg} = \alpha_s^2 \begin{bmatrix} 0_{3\times 3} & 0_{3\times 5} \\ 0_{5\times 3} & H^{gg\to gg}_{5\times 5} \end{bmatrix}$$

$$H^{gg\to gg}_{5\times 5} = \begin{bmatrix} H^{gg\to gg}_{11} & H^{gg\to gg}_{12} & H^{gg\to gg}_{13} & 0 & H^{gg\to gg}_{15} \\ H^{gg\to gg}_{12} & H^{gg\to gg}_{22} & H^{gg\to gg}_{23} & 0 & H^{gg\to gg}_{25} \\ H^{gg\to gg}_{13} & H^{gg\to gg}_{23} & H^{gg\to gg}_{33} & 0 & H^{gg\to gg}_{35} \\ 0 & 0 & 0 & 0 & 0 \\ H^{gg\to gg}_{15} & H^{gg\to gg}_{25} & H^{gg\to gg}_{35} & 0 & H^{gg\to gg}_{55} \end{bmatrix}$$

$$\begin{aligned}
H^{gg\to gg}_{11} &= \frac{9}{16}\left(1 - \frac{tu}{s^2} - \frac{st}{u^2} + \frac{t^2}{su}\right), \\
H^{gg\to gg}_{12} &= \frac{1}{2} H^{gg\to gg}_{11}, \\
H^{gg\to gg}_{13} &= \frac{9}{32}\left(\frac{st}{u^2} - \frac{tu}{s^2} + \frac{u^2}{st} - \frac{s^2}{tu}\right), \\
H^{gg\to gg}_{15} &= -\frac{1}{3} H^{gg\to gg}_{11}, \quad H^{gg\to gg}_{22} = \frac{1}{4} H^{gg\to gg}_{11}, \\
H^{gg\to gg}_{23} &= \frac{1}{2} H^{gg\to gg}_{13}, \quad H^{gg\to gg}_{25} = -\frac{1}{6} H^{gg\to gg}_{11}, \\
H^{gg\to gg}_{33} &= \frac{27}{64} - \frac{9}{16}\left(\frac{su}{t^2} + \frac{tu}{4s^2} + \frac{st}{4u^2}\right) + \frac{9}{32}\left(\frac{u^2}{st} + \frac{s^2}{tu} - \frac{t^2}{2su}\right) \\
H^{gg\to gg}_{35} &= -\frac{1}{3} H^{gg\to gg}_{13}, \quad H^{gg\to gg}_{55} = \frac{1}{9} H^{gg\to gg}_{11}.
\end{aligned}$$

$$\sigma^B_{gg\to gg}\delta(s_4) = \alpha_s^2 \frac{1}{s}\left[\frac{27}{2} - \frac{9}{2}\left(\frac{su}{t^2} + \frac{tu}{s^2} + \frac{st}{u^2}\right)\right]\delta(s_4)$$

What about higher orders?

Threshold corrections as the first attempt

Thanks to the Factorisation theorem

$$
\begin{aligned}
d\hat{\sigma}_{12\,\rightarrow\,34} \quad = \quad & \exp\left\{\sum_{a=1,2} \mathcal{J}_a^I\right\} \times \exp\left\{\sum_{b=3,4} \mathcal{J}_b^I\right\} \\
& \times \exp\left[2\sum_{a=1,2}\int_{\mu_F}^{p_T} \frac{d\mu}{\mu}\gamma_a[\alpha_s(\mu^2)]\right] \times \exp\left[4\int_{\mu_R}^{p_T} \frac{d\mu}{\mu}\beta(\alpha_s(\mu^2))\right] \\
& \times \mathrm{Trace}\left\{H(\alpha_s(\mu_R^2))\ \bar{P}\exp\left[\int_{p_T}^{p_T/N} \frac{d\mu}{\mu}\Gamma_S^\dagger(\alpha_s(\mu^2))\right]\right. \\
& \left.\times\ S(\alpha_s(p_T^2/N^2))\ P\exp\left[\int_{p_T}^{p_T/N} \frac{d\mu}{\mu}\Gamma_S(\alpha_s(\mu^2))\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
H(x) &= H^{(0)}(x) + \frac{\alpha_s}{\pi} H^{(1)}(x) \\
S(x) &= S^{(0)}(x) + \frac{\alpha_s}{\pi} S^{(1)}(x)
\end{aligned}
$$

$$
\mu\frac{d}{d\mu}S_I = (\Gamma_S)_{JI}\left(\alpha_s, \frac{\beta_i.\beta_j|n|^2}{|\beta_i.n||\beta_j.n}\right) S_J
$$

Soft anomalous dimension

Initial and final state jet functions

$$\mathcal{J}_a^I = -2\int_{\mu_F}^{2p_a.\zeta}\frac{d\mu}{\mu}C_a\frac{\alpha_s(\mu^2)}{\pi}\ln N_a$$
$$- \int_0^1 dz\frac{z^{N_a-1}}{1-z}\left[\int_{(1-z)^2}^{1}\frac{d\lambda}{\lambda}A^{(f_a)}[\alpha_s(\lambda(2p_a.\zeta)^2)] + \frac{1}{2}\nu^{f_i}[\alpha_s((1-z)^2(2p_a.\zeta)^2)]\right]$$

$$\mathcal{J}_a^F = \int_0^1 dz\frac{z^{N-1}}{1-z}\left[\int_{(1-z)^2}^{(1-z)}\frac{d\lambda}{\lambda}A^{(f_a)}[\alpha_s(\lambda(p_T^2))]\right.$$
$$\left.+ B_a^{(1)}[\alpha_s((1-z)p_T^2)] + B_a^{(2)}[\alpha_s((1-z)^2p_T^2)]\right]$$

Different subprocesses

$$
\begin{aligned}
q(p_1) + q'(p_2) &\to q(p_3) + q'(p_4)\,, \\
q(p_1) + \bar{q}(p_2) &\to q'(p_3) + \bar{q}'(p_4)\,, \\
q(p_1) + \bar{q}(p_2) &\to q(p_3) + \bar{q}(p_4)\,, \\
q(p_1) + q(p_2) &\to q(p_3) + q(p_4)\,, \\
q(p_1) + \bar{q}'(p_2) &\to q(p_3) + \bar{q}'(p_4)\,, \\
q(p_1) + \bar{q}(p_2) &\to g(p_3) + g(p_4)\,, \\
q(p_1) + g(p_2) &\to q(p_3) + g(p_4)\,, \\
g(p_1) + g(p_2) &\to q(p_3) + \bar{q}(p_4)\,, \\
g(p_1) + g(p_2) &\to g(p_3) + g(p_4)\,.
\end{aligned}
$$

Threshold corrections

$$s^2 \frac{d^2\hat{\sigma}}{dtdu} = \frac{\alpha_s}{\pi}\sigma^{(0)} \left\{ c_3 \left[\frac{\ln(s_4/p_T^2)}{s_4} \right]_+ + c_2 \left[\frac{1}{s_4} \right]_+ + c_1 \delta(s_4) \right\}$$

$$s^2 \frac{d^2\hat{\sigma}}{dtdu} = \left(\frac{\alpha_s}{\pi}\right)^2 \sigma^{(0)} \left\{ b_3 \left[\frac{\ln^3(s_4/p_T^2)}{s_4} \right]_+ + b_2 \left[\frac{\ln^2(s_4/p_T^2)}{s_4} \right]_+ + b_1 \left[\frac{\ln(s_4/p_T^2)}{s_4} \right]_+ \right\}$$

- These threshold corrections estimate the size of the logarithmic corrections at higher orders.
- The regular terms are not captured in these corrections and explicit computation of the full FO results are required for this.

Fixed order corrections

- We have two broad class methods to compute the NLO FO corrections.
- Phase space slicing : The soft and collinear singular regions are sliced using small cut-off parameters. The calculation is carried out analytically in these regions in $d = 4 - 2\epsilon$ dimensions.
- Subtraction methods : Dipole subtraction terms are added to the real matrix elements to cancel the singularities point-by-point in the phase space.
- The IR singularities cancel between the real and virtual corrections.
- The remaining finite contributions can be integrated out numerically using standard phase space generators.

Jet algorithms

- Partons in the final stage do fragment into hadrons and hence can not be detected as they are.
- Jet algorithm is precisely a way of defining the observable that is consistent with the experimental measurements (that involve the cone size defined as)$\Delta R = \sqrt{(\eta_2 - \eta_1)^2 + (\phi_2 - \phi_1)^2}$
- Infra-red safety has to be ensured while implementing the jet algorithm.
- Different jet algorithms
- Cone Algorithms
- Sequential Recombination methods (kT-class of algorithms)

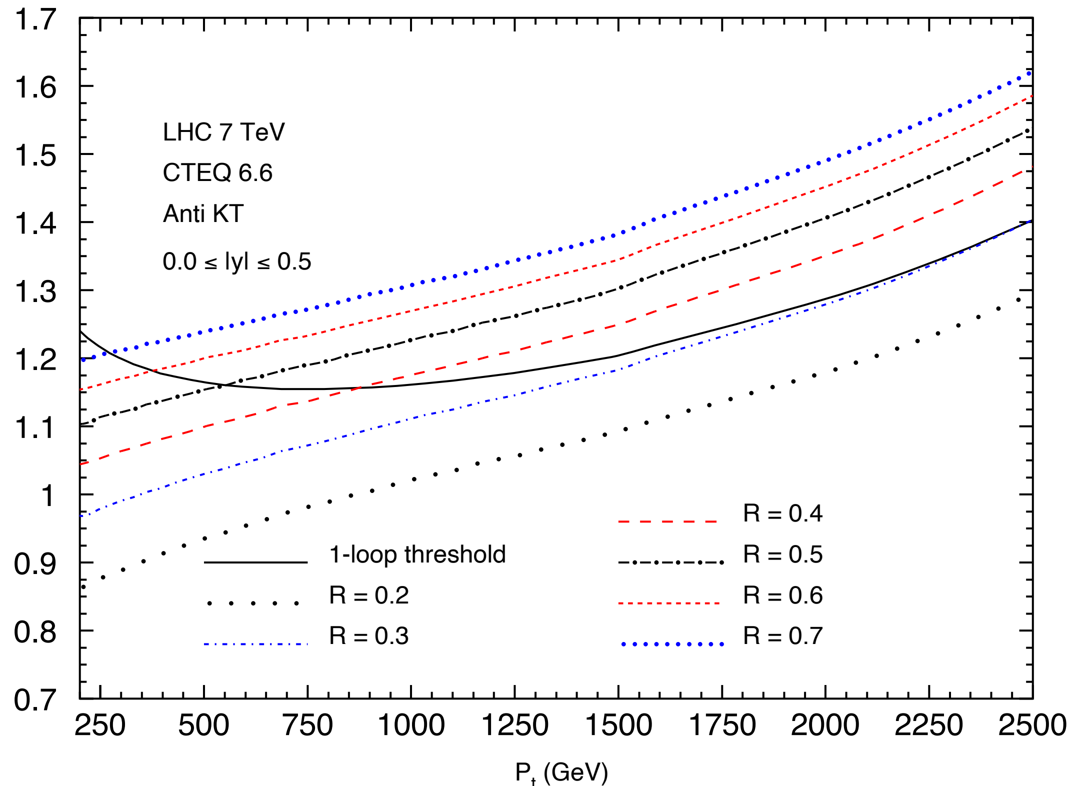
Sequential Recombination methods

- For each pair of particles i, j work out the k_t distance
- $$d_{ij} = \min(k_{ti}^{2p}, k_{tj}^{2p})\, \Delta R_{ij}^2 / R^2 \,,$$
  $$d_{iB} = k_{ti}^{2p} \,.$$
- Find the minimum d_{min} of all d_{ij} and d_{iB}
- If d_{min} is a d_{ij}, then merge the particles i and j into a single jet by summing their four-momenta (E-scheme recombination).
- If d_{min} is a d_{iB}, then declare it as a final jet and remove it from the list.

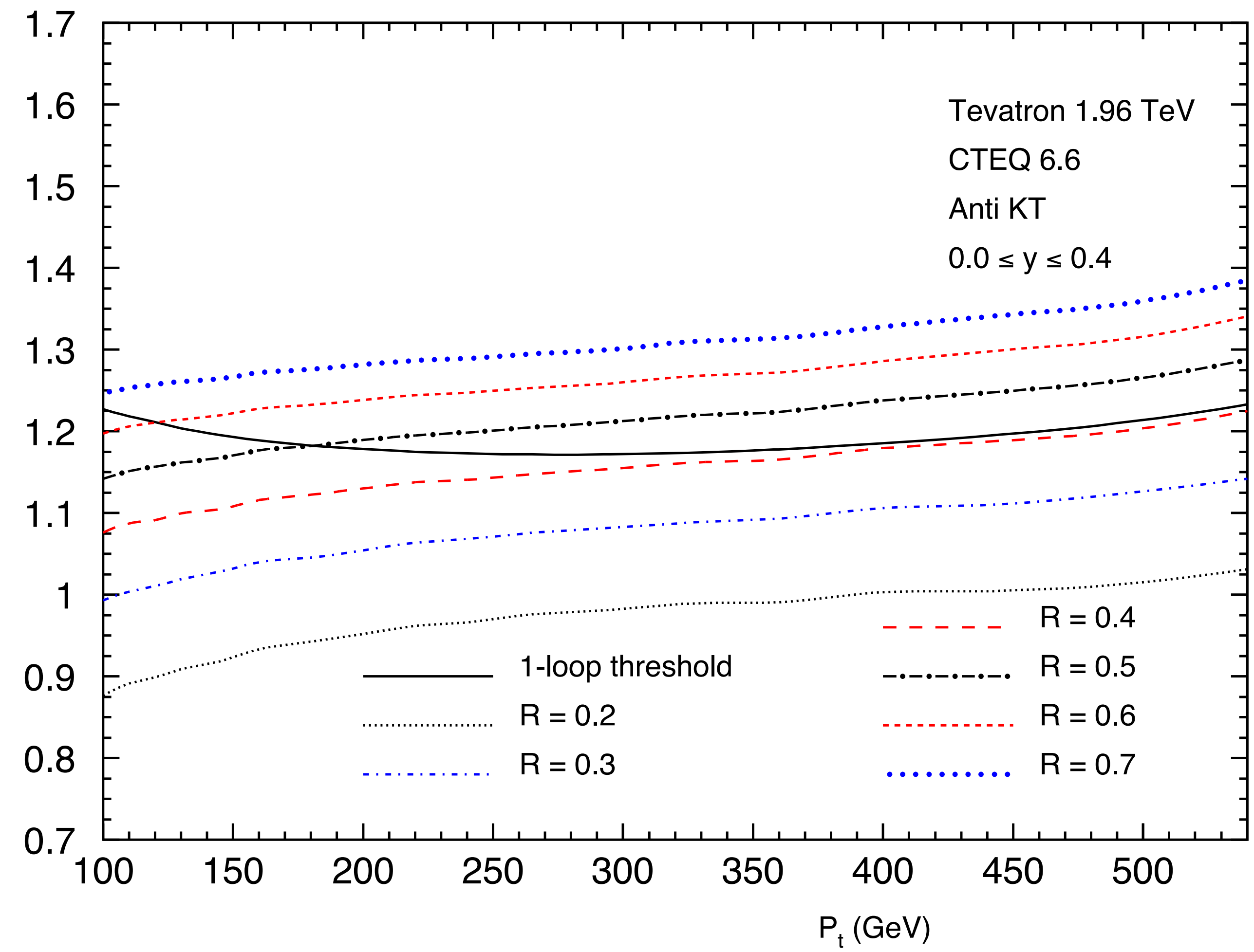
FastJet package : For implementing the jet algorithm

M. Cacciari, G. P. Salam and G. Soyez (2011)

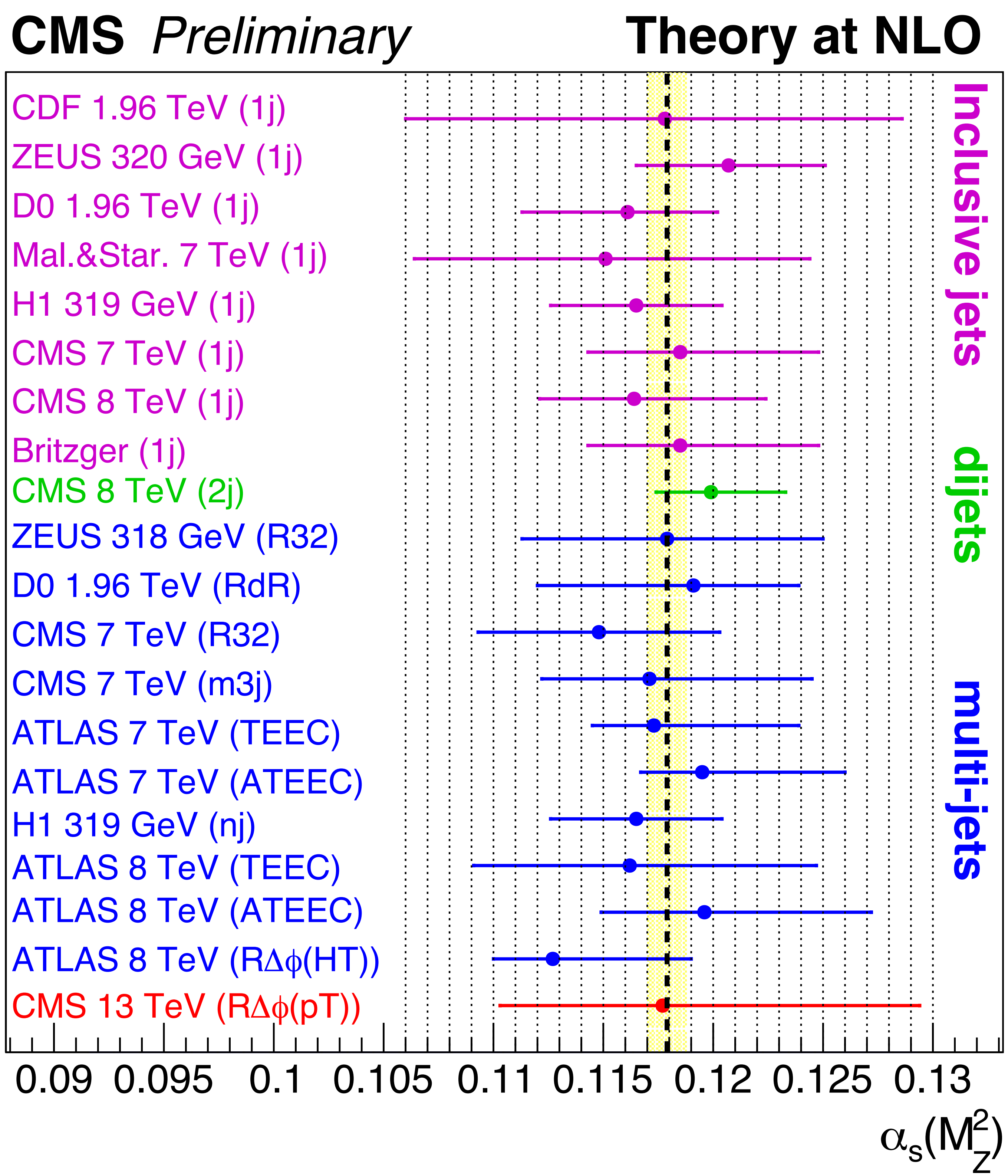
Threshold vs Fixed order results (at NLO)



MC Kumar and S Moch; Phys. Lett. B730(2014) 122



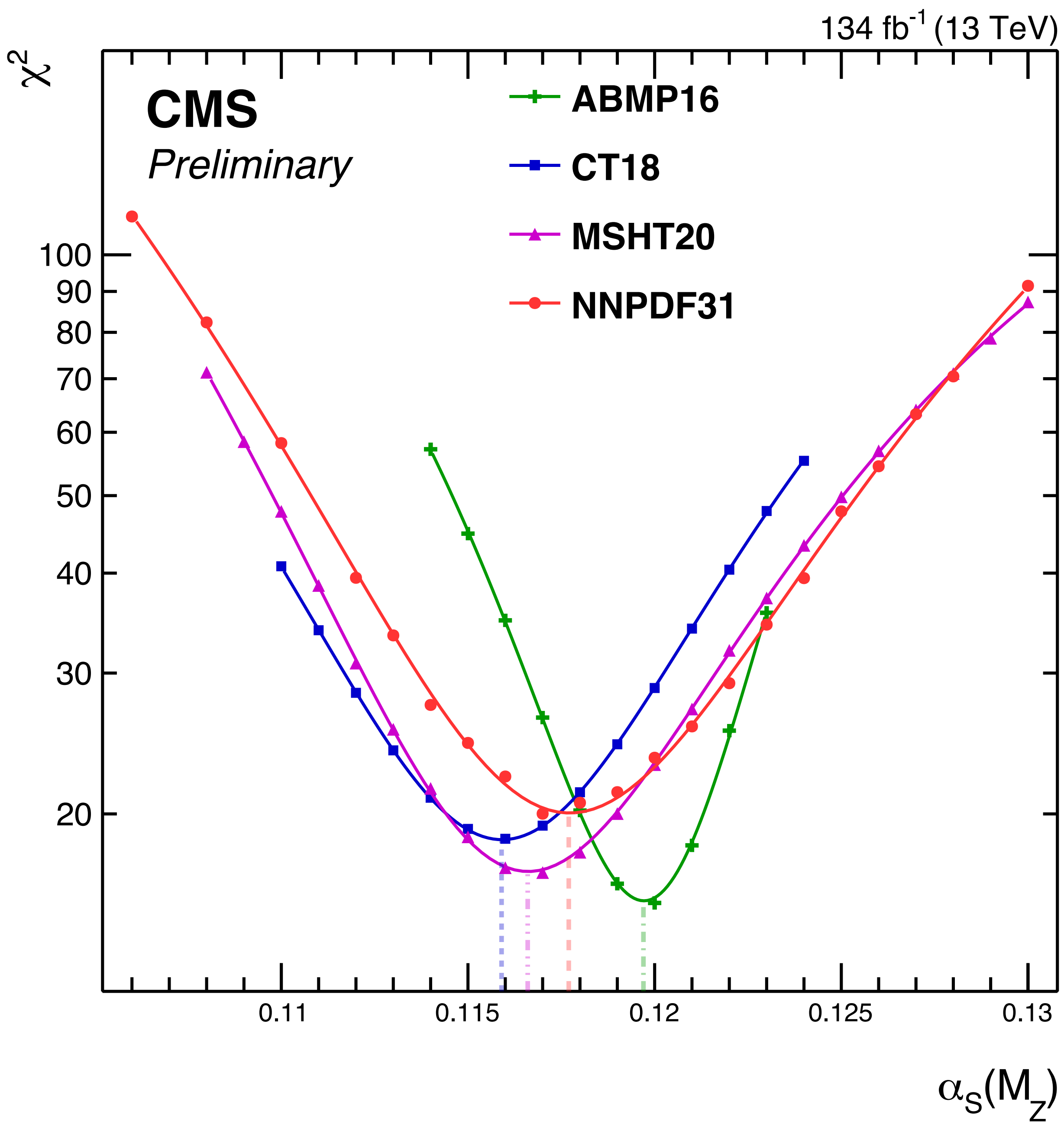
CMS Physics Analysis Summary (2023)



World average : Dashed line
Its uncertainty (yellow band)

CMS PAS SMP-22-005

α_s measurement from angular correlations



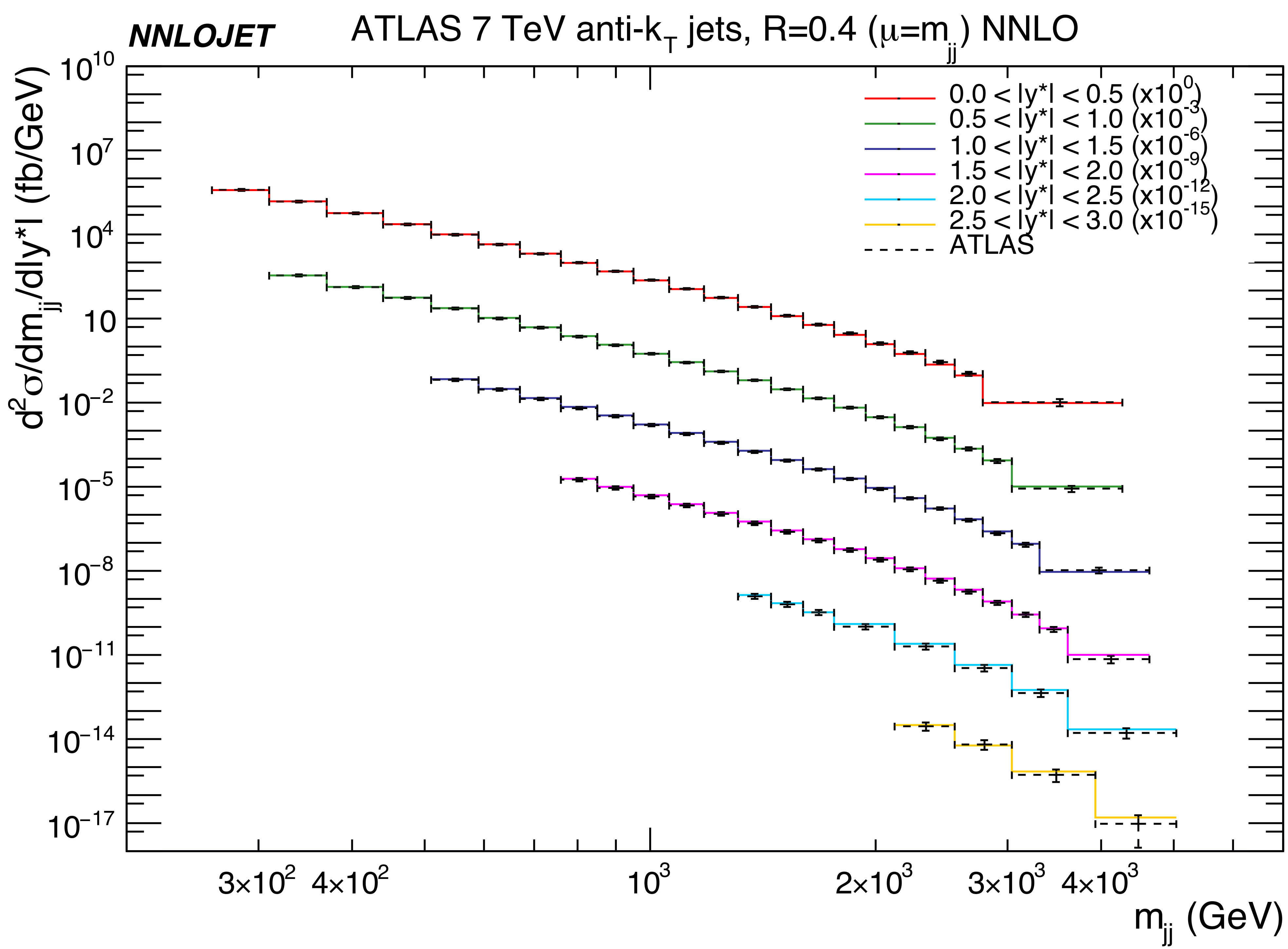
$$R_{\Delta\phi}(p_T) = \frac{\sum_{n=0}^{\infty} nN(p_T, n)}{\sum_{n=0}^{\infty} N(p_T, n)}$$

CMS Physics Analysis Summary (2023)

CMS PAS SMP-22-005

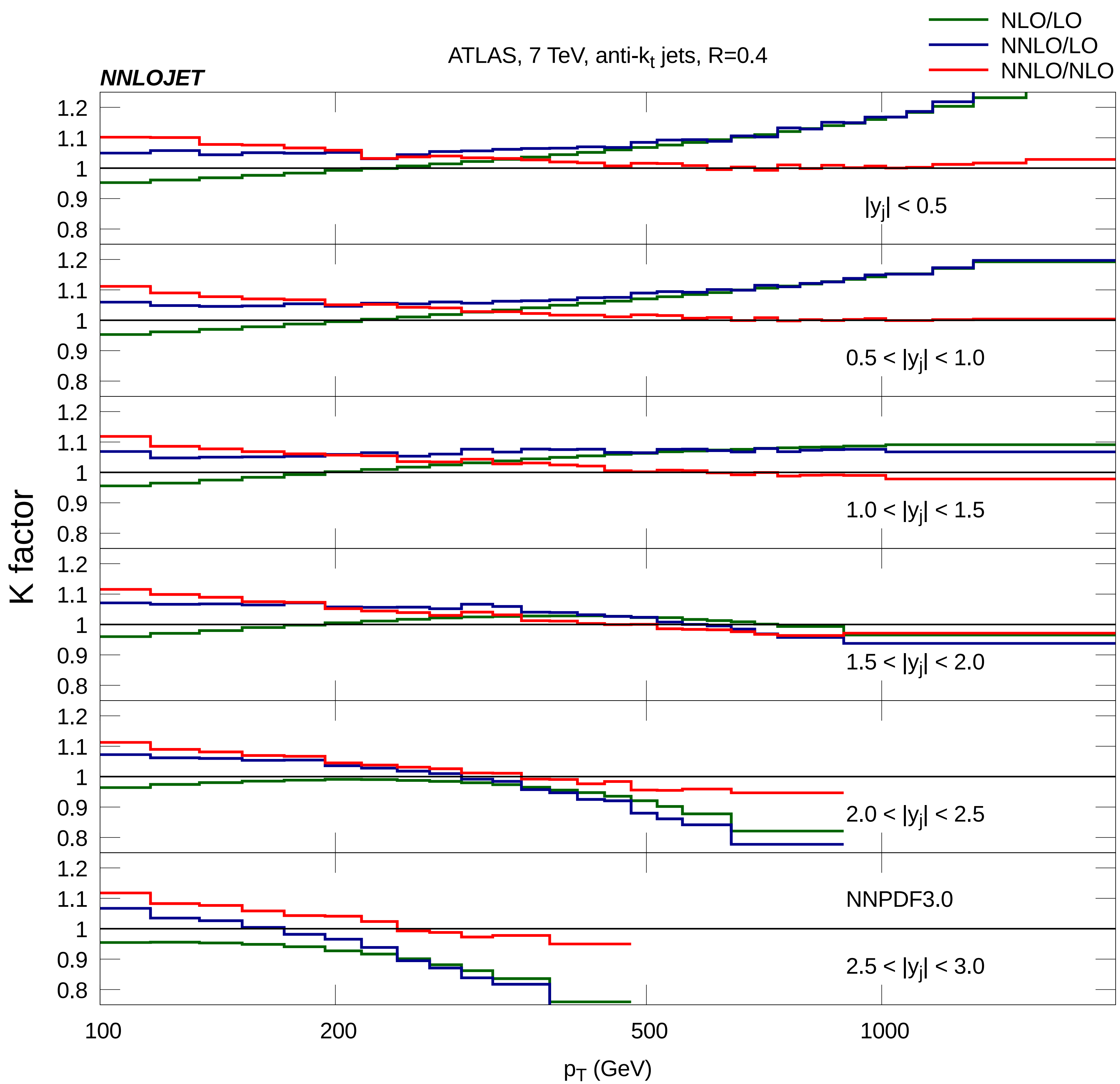
NLO PDF sets

Jet production@NNLO



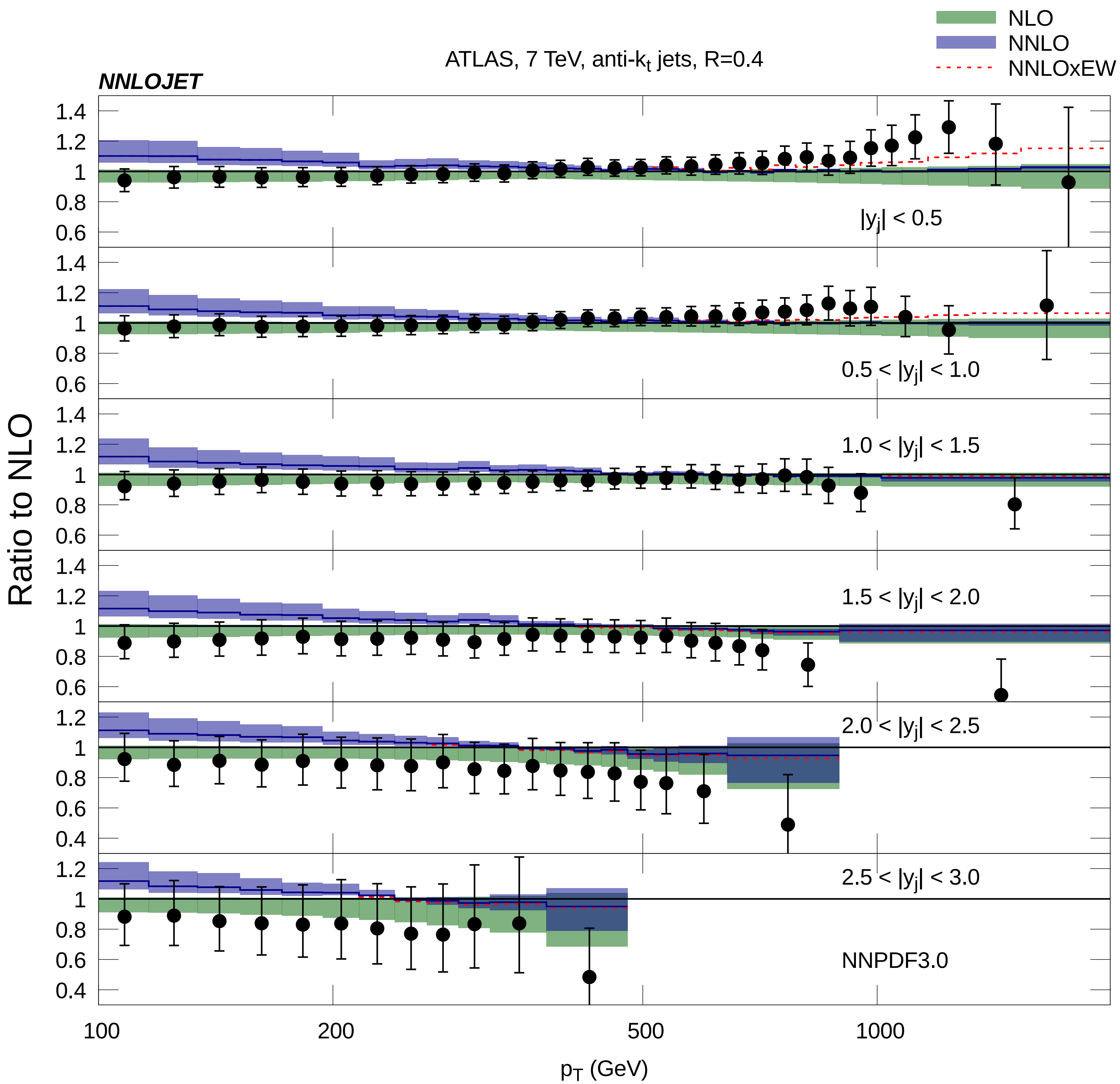
E.W.N. Glover et. al. (2016)

Inclusive jet production@NNLO



E.W.N. Glover et. al. (2017)

Inclusive jet production@NNLO



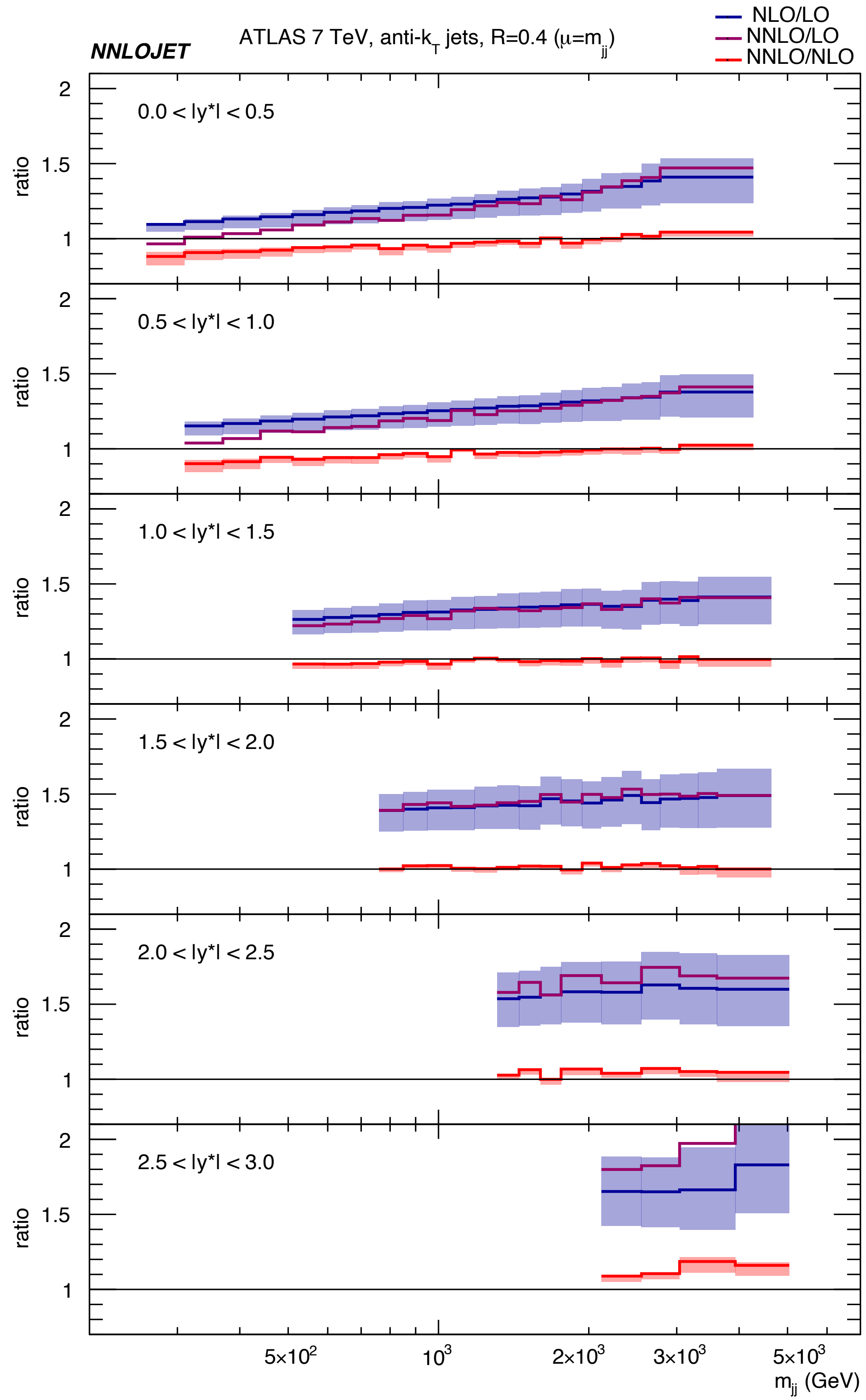
E.W.N. Glover et. al. (2017)

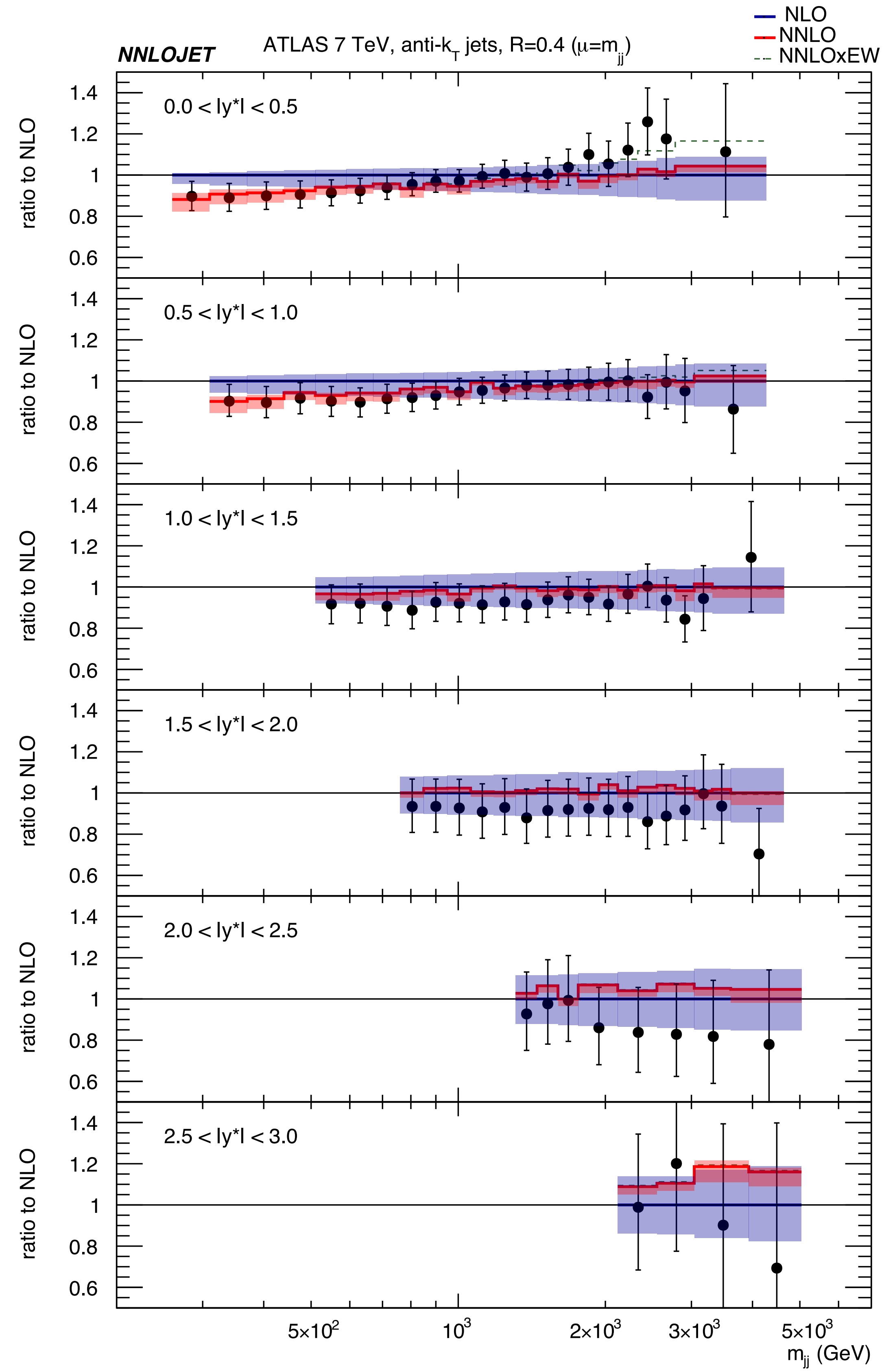
EW corrections :

−12% and −10%

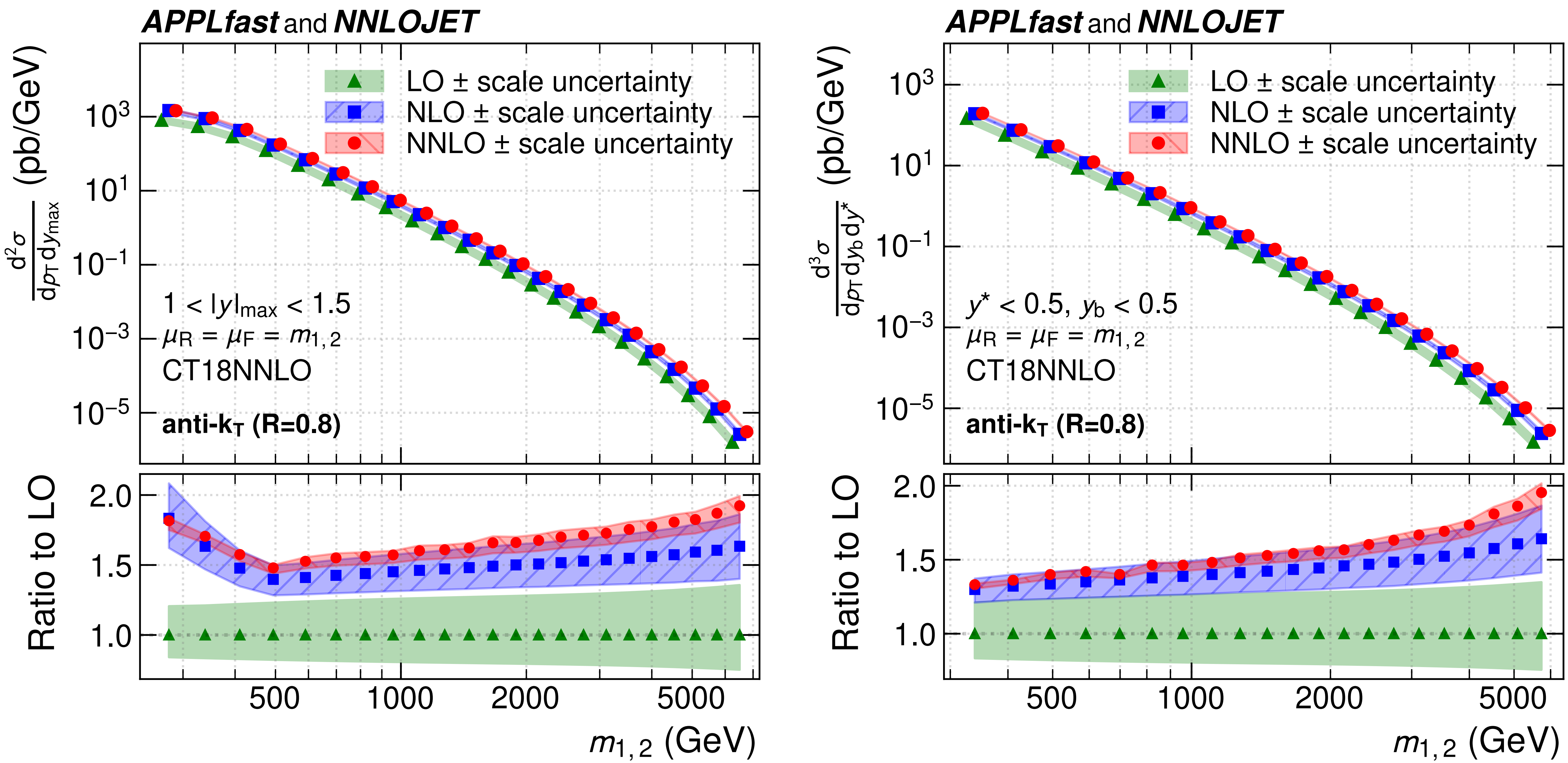
Diet production@NNLO

E.W.N. Glover et. al. (2017)



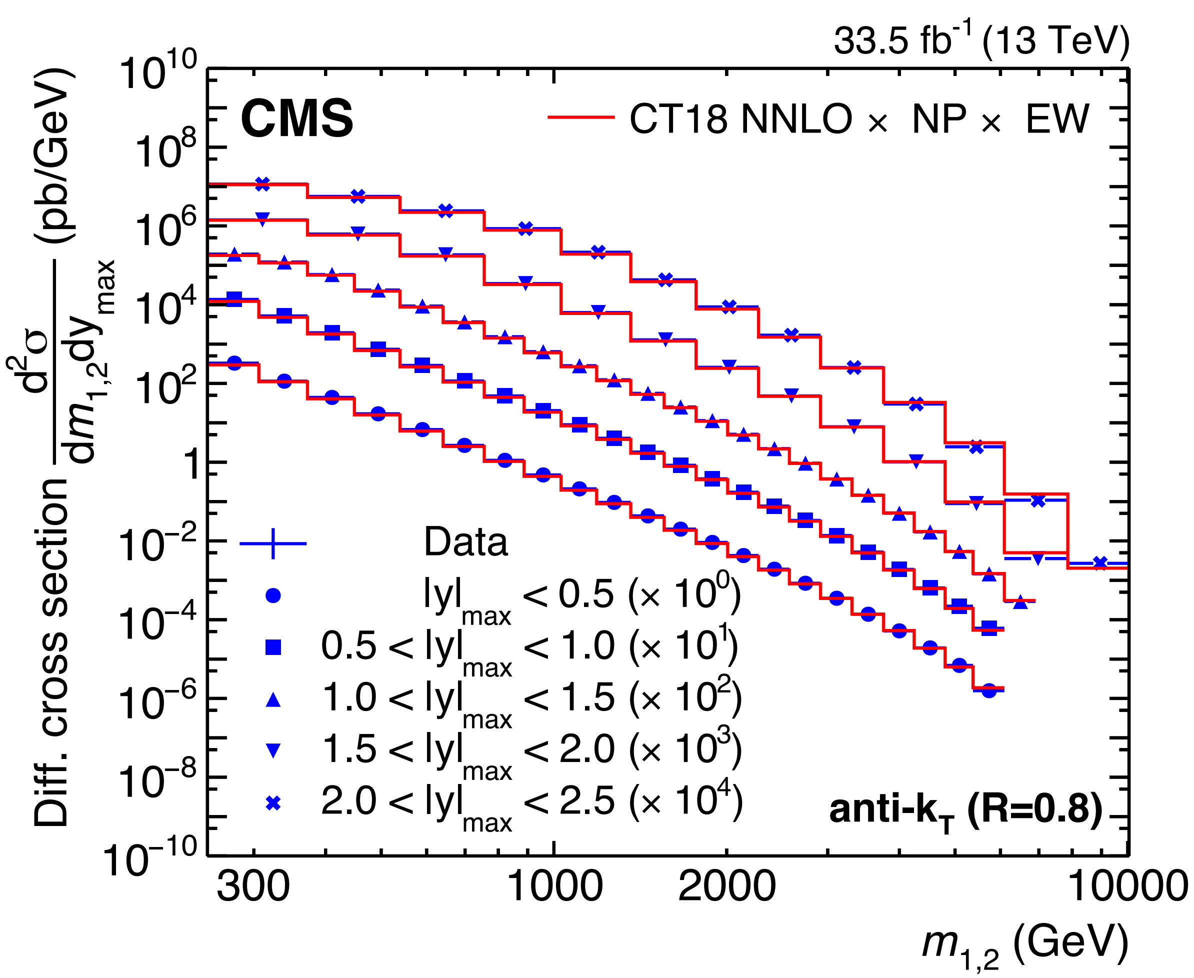


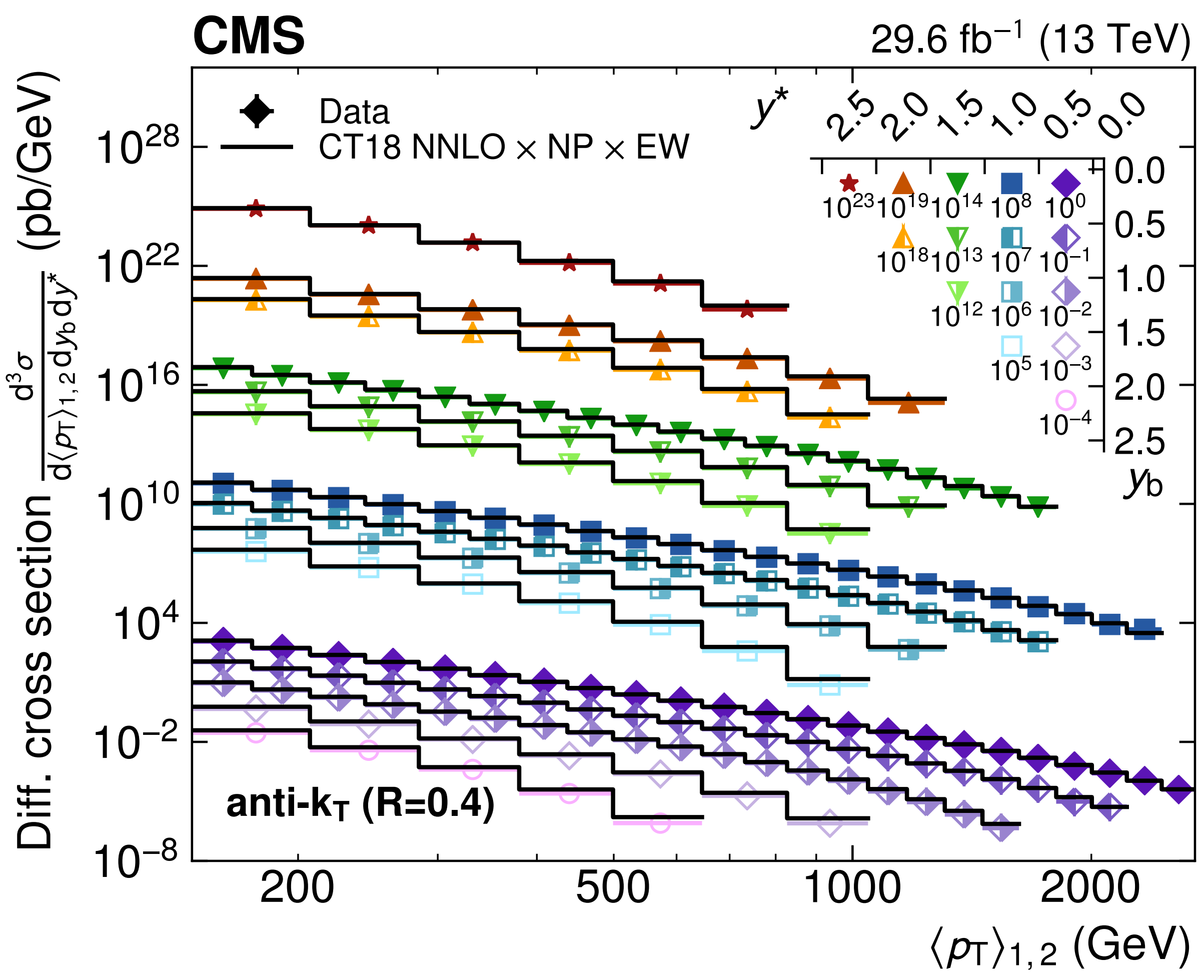
Diet production@NNLO for LHC13



Based on the Fast grids

Jets at CMS@13TeV



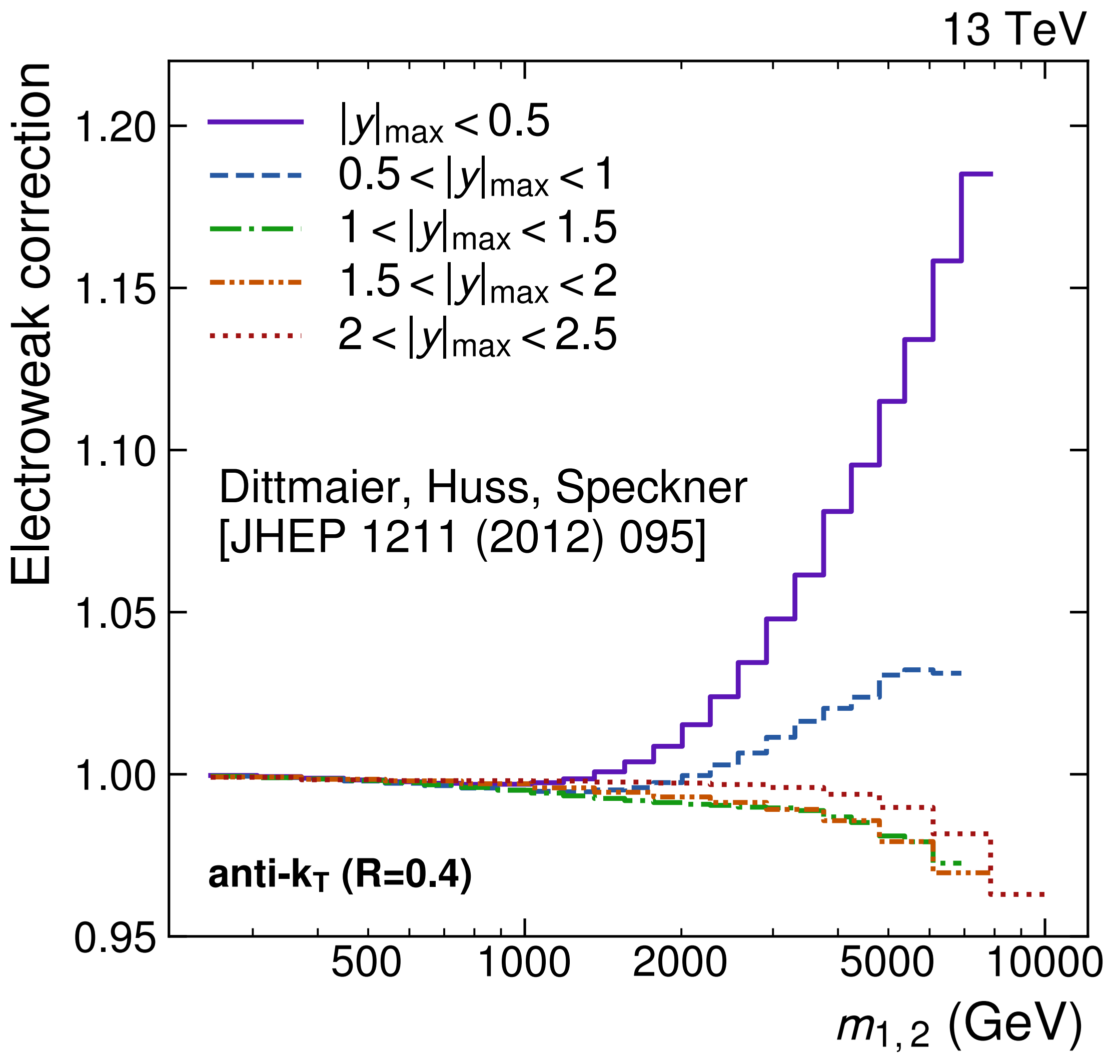


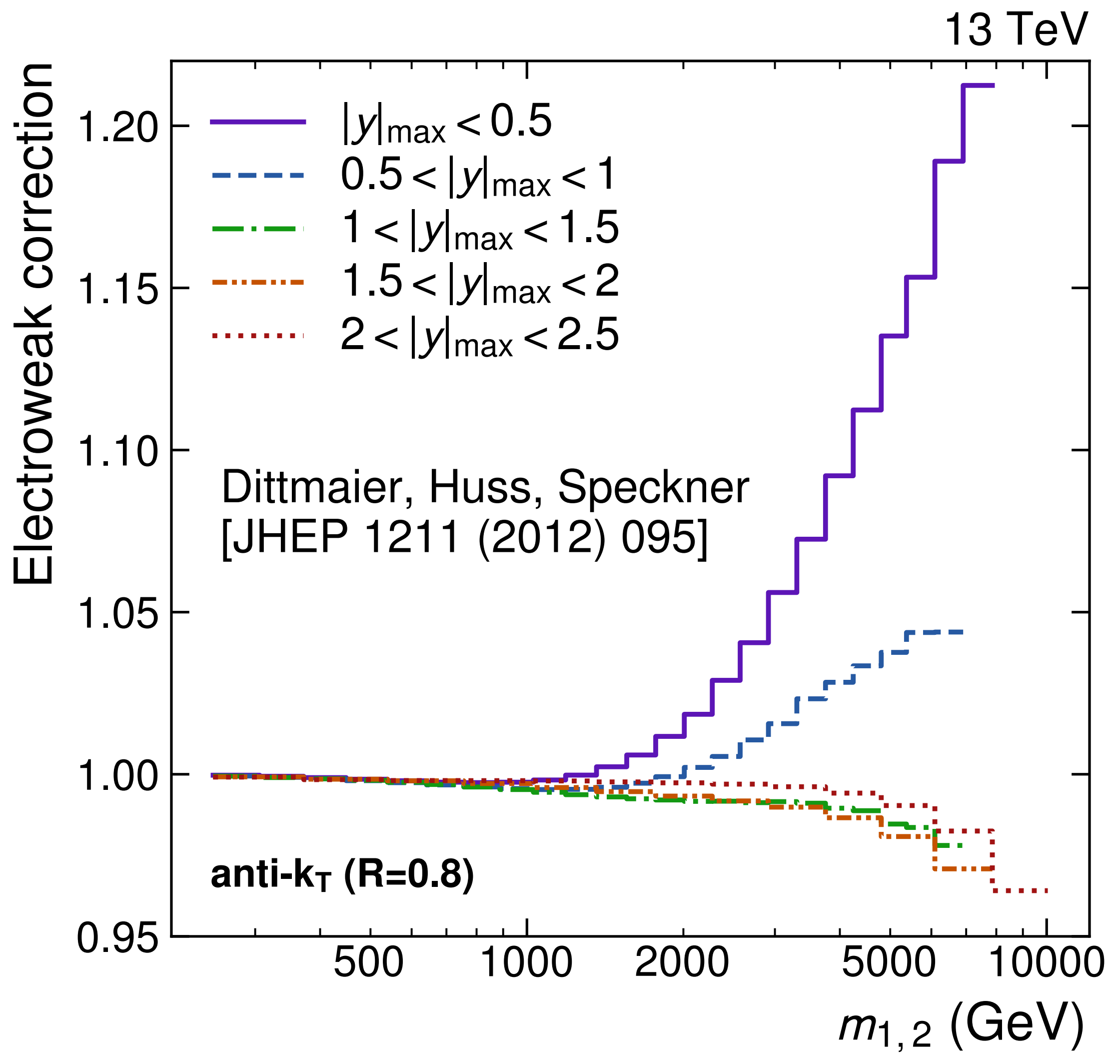
$$y^* = |y_1 - y_2|/2$$

$$y_b = |y_1 + y_2|/2$$

CERN-EP-2023-257
2023/12/29

EW corrections





EW corrections are important for central rapidity and high invariant mass region.
As high as 20%.

Elsewhere they can be ignored.

Outlook

- With the advent of high energy colliders, we have huge jet production cross sections at hadron colliders.
- Differential distributions are available now at NNLO
- Attempts towards N3LO calculation are on-going.
- Extremely tedious and time consuming calculations.
- Can be useful for the measurement of the strong coupling constant and the parton distribution functions from the experimental data.

Thank You