NLP corrections to H+jet

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Precision Physics

$$\sigma = \sum_{a,b} \int dx_1 \; f_{a/H_1}(x_1, \mu_F^2) \int dx_2 \; f_{b/H_2}(x_2, \mu_F^2) \frac{1}{2\hat{s} \; n_s(1) \; n_s(2) \; n_c(1) \; n_c(2)} \frac{1}{S} \int d\phi_n \left| A_{2\to n}^{(m)} \right|^2$$

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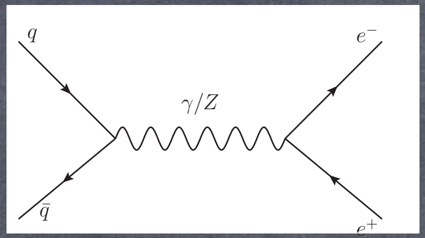

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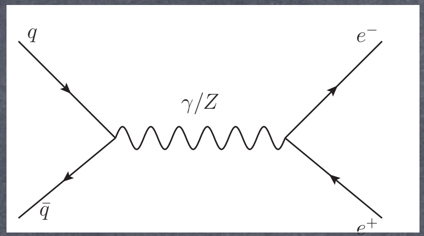
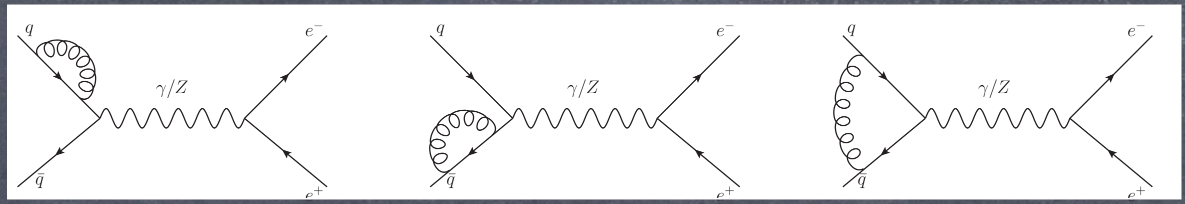


$$\frac{d\sigma^{\mathrm{V}}}{dQ^2} = \frac{\alpha_s}{2\pi} \, \sigma_B \, C_F \, D(\epsilon) \, \delta(1 - \frac{Q^2}{s}) \left[-\frac{4}{\epsilon^2} - \frac{6}{\epsilon} + \frac{4\pi^2}{3} - 16 + \mathcal{O}(\epsilon) \right]$$



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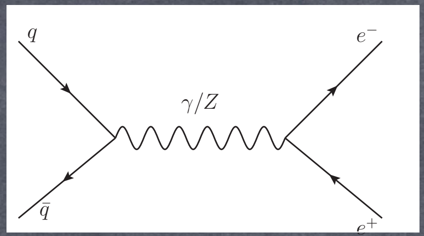

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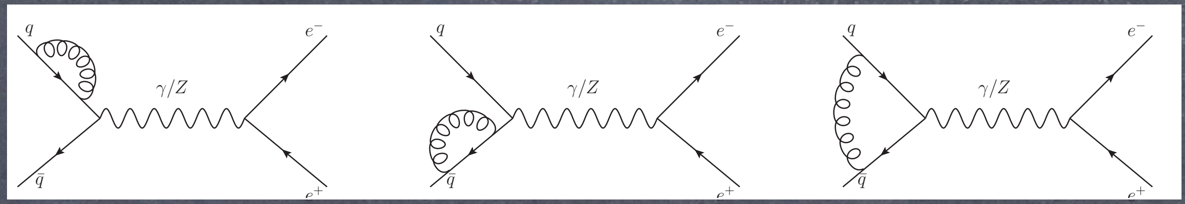
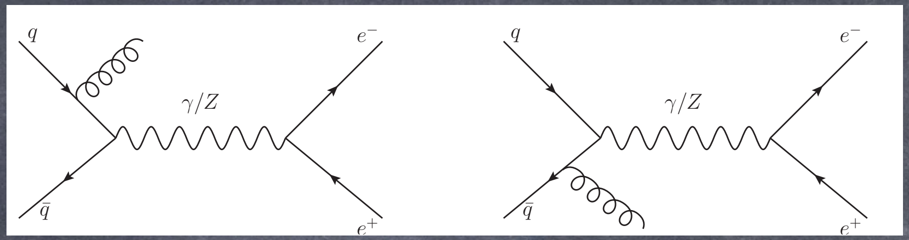


$$\frac{d\sigma^{\mathrm{R}}}{dQ^2} = \frac{\alpha_s}{2\pi} \sigma_B C_F D(\epsilon) \left[\frac{4}{\epsilon^2} \delta(1-z) - \frac{4}{\epsilon} \frac{1+z^2}{(1-z)_+} + 8(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - 4 \left(\frac{1+z^2}{1-z} \right) \ln(z) \right]$$



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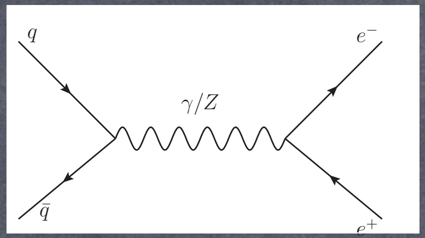
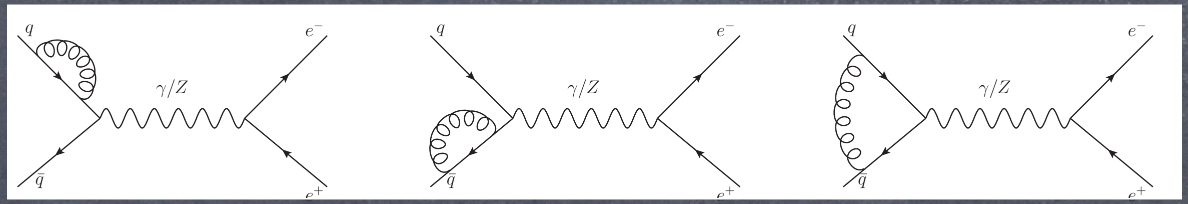
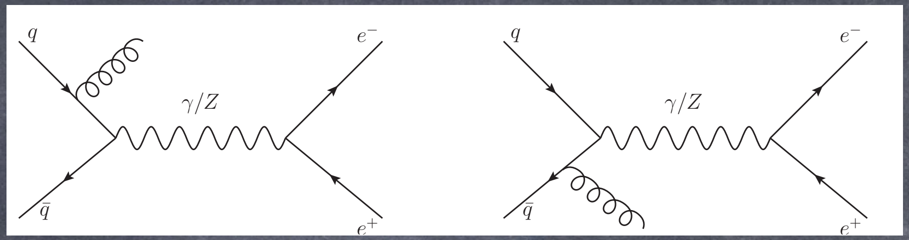


$$\frac{d\sigma^{\rm V}}{dQ^2} = \frac{\alpha_s}{2\pi} \; \sigma_B \; C_F \; D(\epsilon) \; \delta(1 - \frac{Q^2}{s}) \left[-\frac{4}{\epsilon^2} - \frac{6}{\epsilon} + \frac{4\pi^2}{3} - 16 + \mathcal{O}(\epsilon) \right]$$



$$\frac{d\sigma^{\rm R}}{dQ^2} = \frac{\alpha_s}{2\pi} \sigma_B \, C_F \, D(\epsilon) \left[\frac{4}{\epsilon^2} \delta(1-z) - \frac{4}{\epsilon} \frac{1+z^2}{(1-z)_+} + 8(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - 4 \left(\frac{1+z^2}{1-z} \right) \ln(z) \right]$$



$$Q^2 = zs$$

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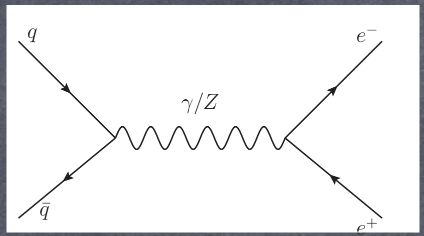

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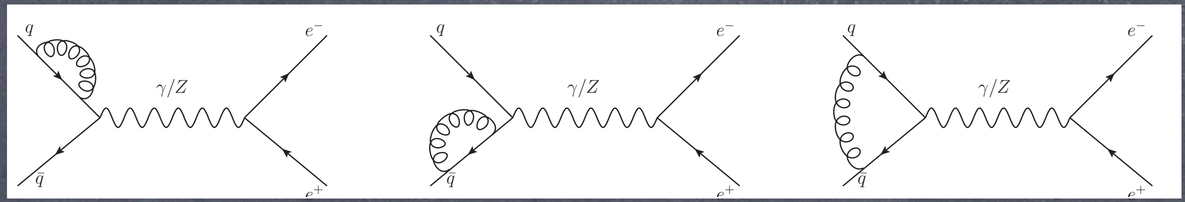

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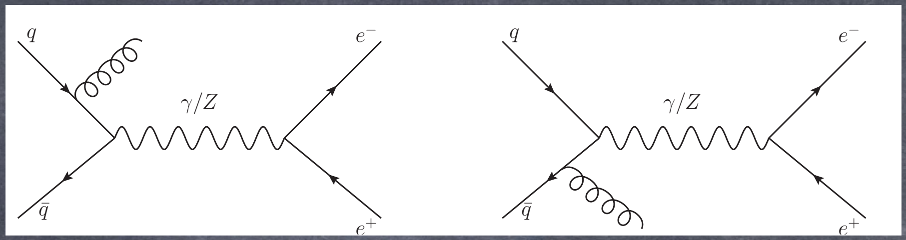

$$\frac{P_{qq}}{C_F} - \frac{3}{2} \delta(1 - z)$$

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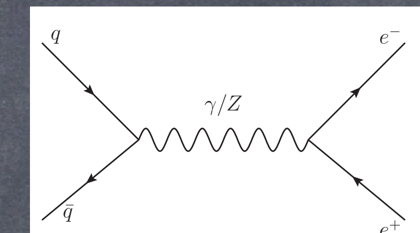

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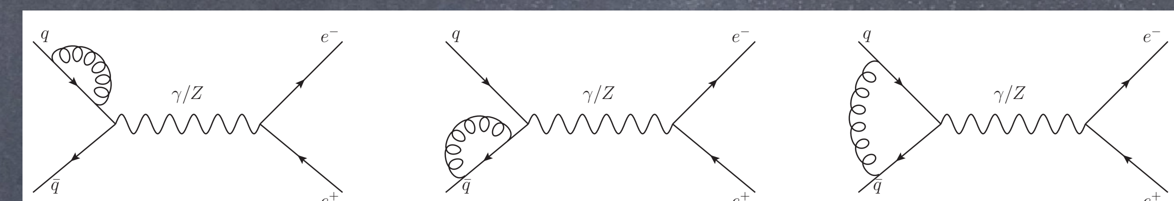

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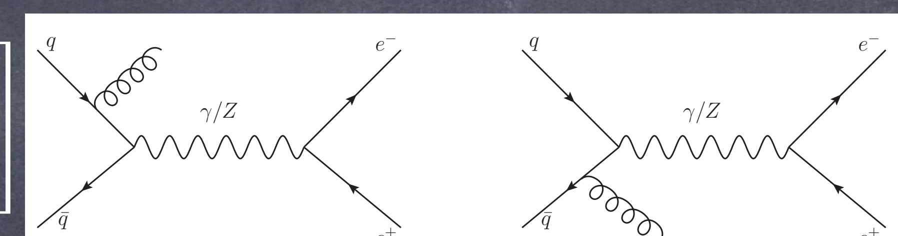

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NLP Logs

$$\frac{d\sigma}{dz} = \sigma^B \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} \left[c_{nm}^{(-1)} \left(\frac{\ln^m(1-z)}{1-z} \right)_+ + c_n^{(\delta)}\, \delta(1-z) + c_{nm}^{(0)} \ln^m(1-z) + \mathcal{O}(1-z) \right]$$

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- Well studied in the literature
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Coloured final states: γ+jet   Beekveld et al.

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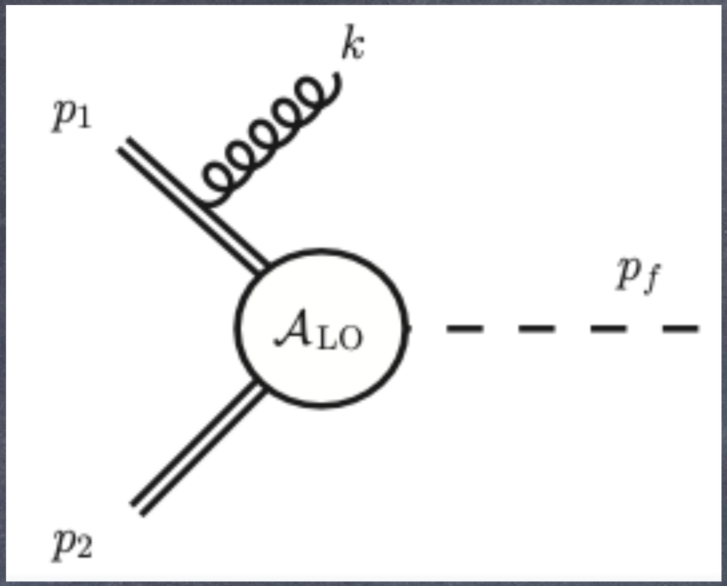
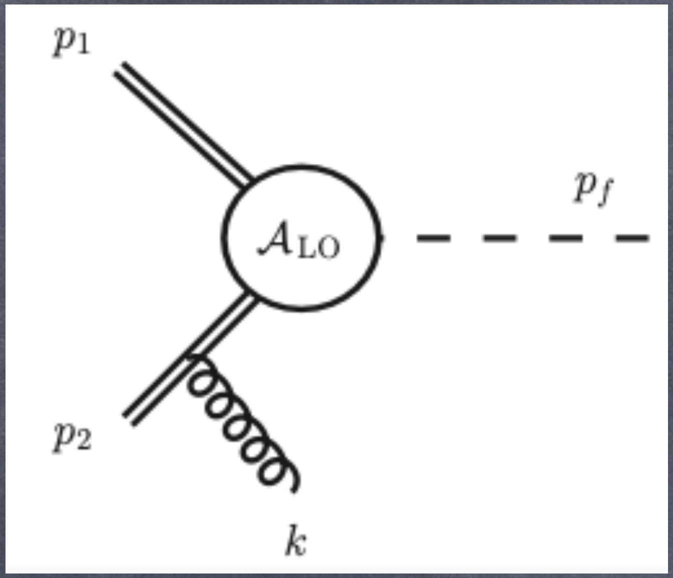
H+jet — Pal, Seth

Next-to-soft radiation





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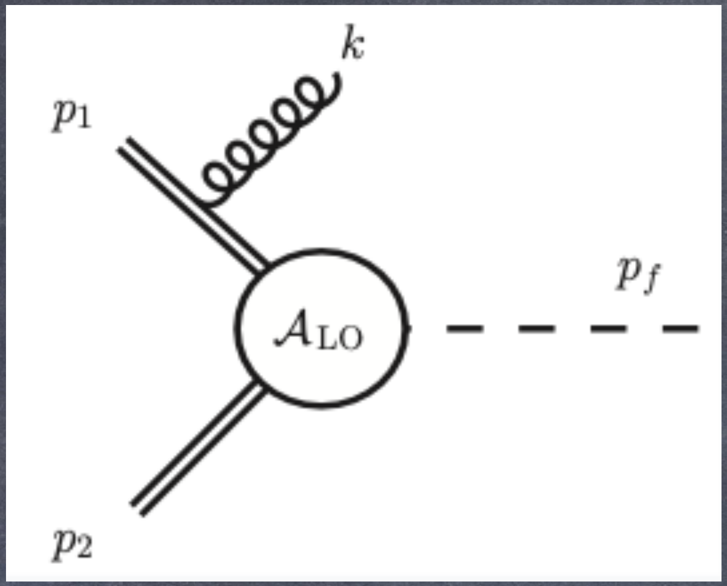

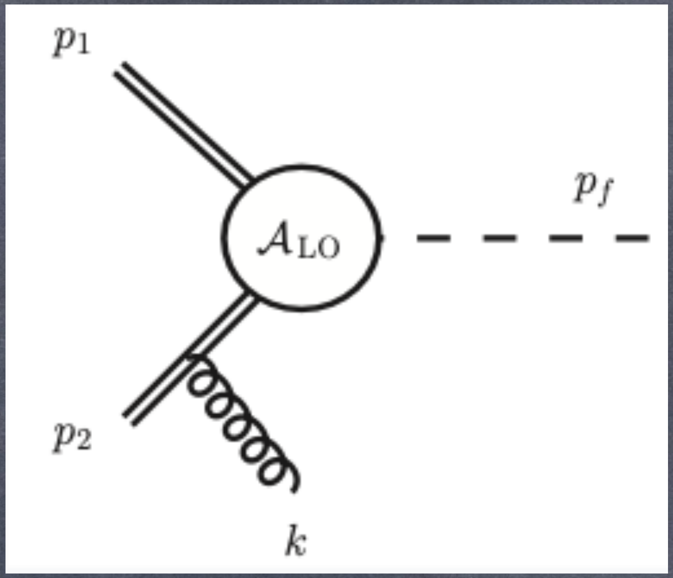

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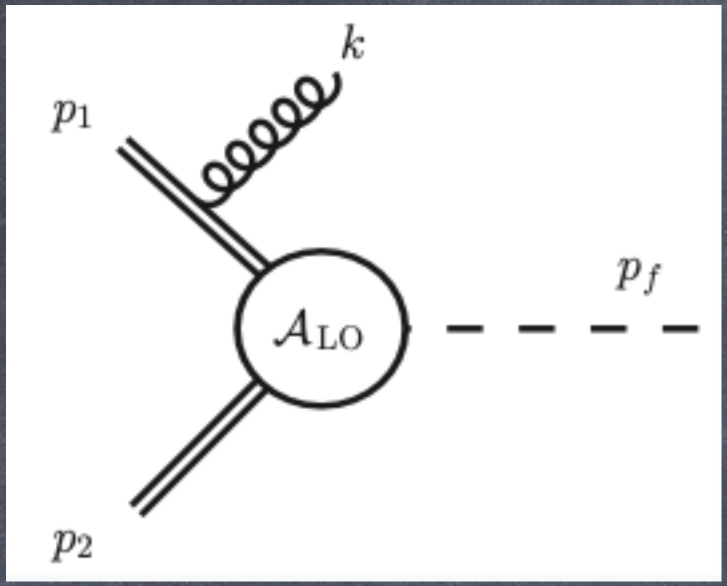

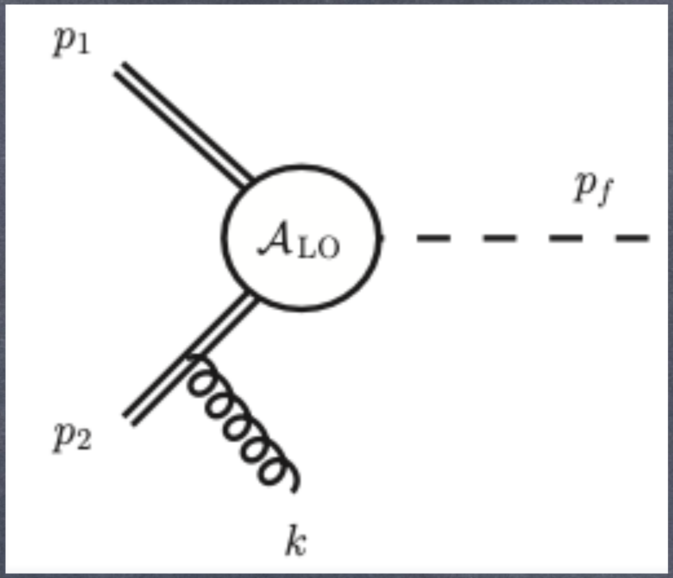

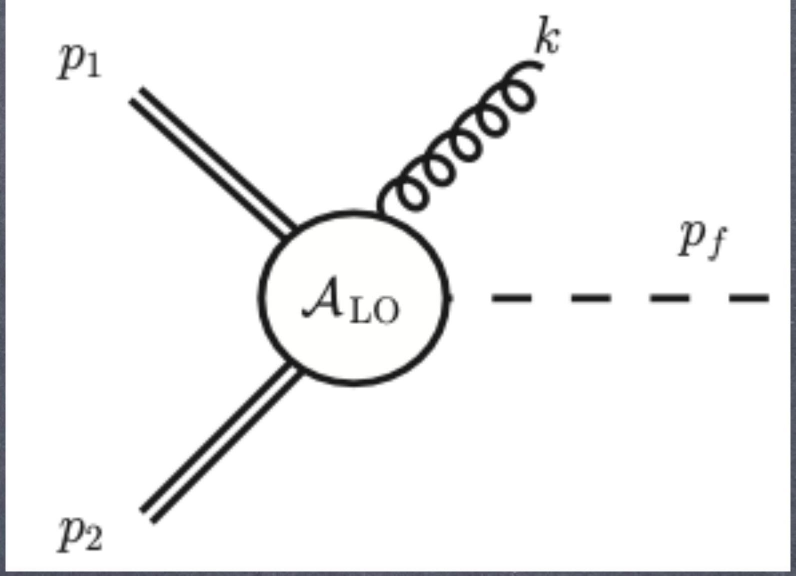

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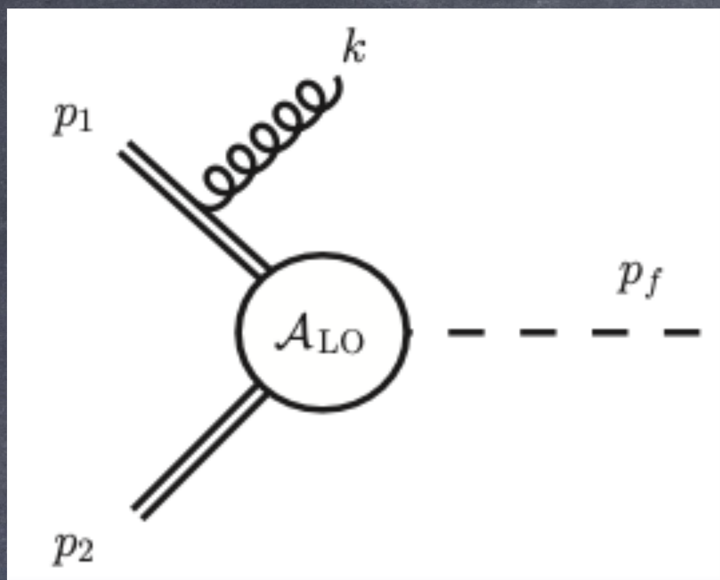

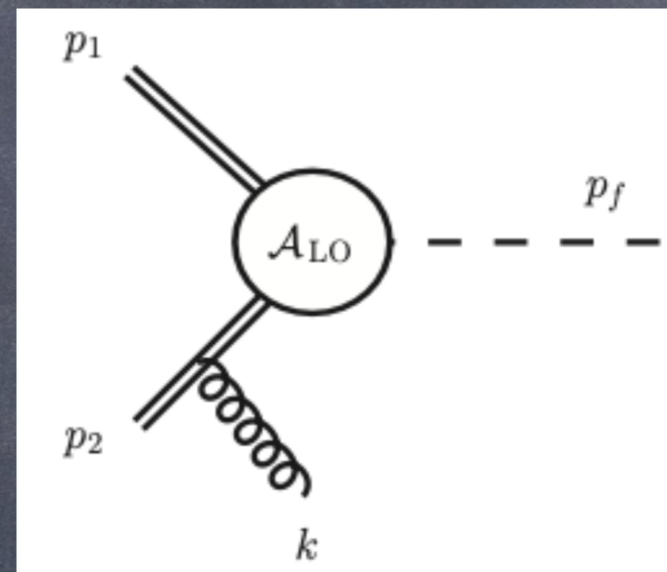

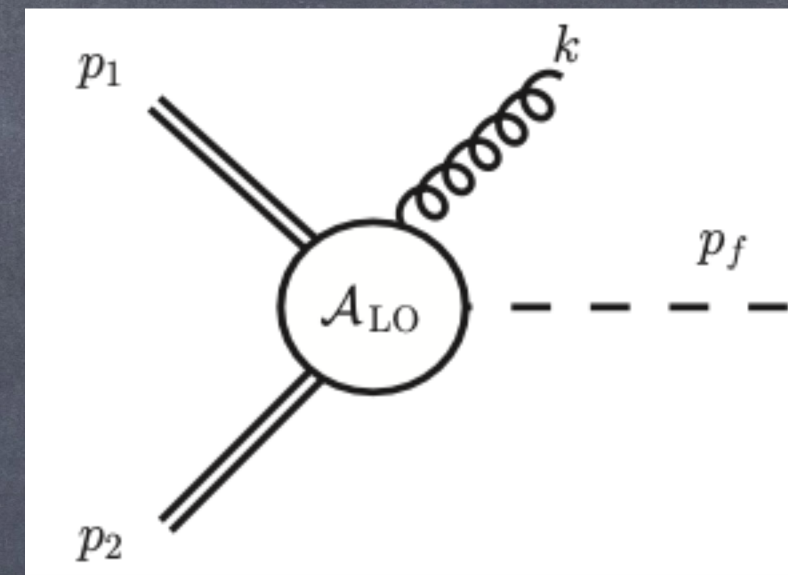

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Low; Burnell, Knoll; Del Duca

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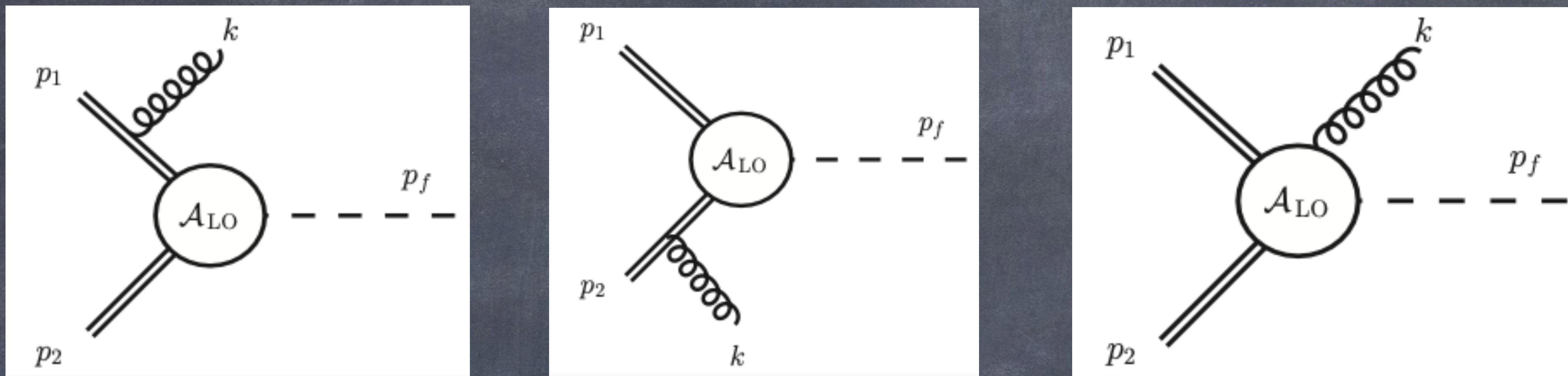

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Low; Burnell, Knoll; Del Duca

Bonocore, Laenen, Magnea, Vernazza, White

Squared Amplitude

$$\mathscr{A}^\sigma = \sum_{i=1}^{2} \mathbf{T}_i \Big(\frac{2p_i^\sigma - k^\sigma}{2p_i \,.\, k} - \frac{ik_\alpha \Sigma_i^{\sigma\alpha}}{p_i \,.\, k} - \frac{ik_\alpha L_i^{\sigma\alpha}}{p_i \,.\, k} \Big) \mathscr{A}_{\text{LO}}$$

$$\Sigma_{i,q}^{\sigma\alpha} = \frac{i}{4}[\gamma^\sigma, \gamma^\alpha]$$

$$\Sigma_{i,g}^{\sigma\alpha} = = i(g^{\rho\sigma} g^{\alpha\nu} - g^{\sigma\nu} g^{\alpha\rho})$$

$$L_i^{\sigma\alpha} = -i\left(p_i^\sigma \frac{\partial}{\partial p_i^\alpha} - p_i^\alpha \frac{\partial}{\partial p_i^\sigma}\right)$$

Squared Amplitude

$$\mathscr{A}^\sigma = \sum_{i=1}^{2} \mathbf{T}_i \Big(\frac{2p_i^\sigma - k^\sigma}{2p_i\,.\,k} - \frac{ik_\alpha \Sigma_i^{\sigma\alpha}}{p_i\,.\,k} - \frac{ik_\alpha L_i^{\sigma\alpha}}{p_i\,.\,k} \Big) \mathscr{A}_{\mathrm{LO}}$$

Scalar $\downarrow$ $\mathcal{O}\left(\frac{1}{k}\right) + \mathcal{O}(1)$

Spin $\downarrow$ $\mathcal{O}(1)$

Orbital $\downarrow$ $\mathcal{O}(1)$

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Scalar ↓ $\mathcal{O}\left(\frac{1}{k}\right) + \mathcal{O}(1)$

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$$\left| \mathcal{A}_{\rm NLP} \right|^2 = \left| \mathcal{A}_{\rm scalar} \right|^2 + 2 \, {\rm Re}(\mathcal{A}_{\rm spin} + \mathcal{A}_{\rm orbital})^\dagger \mathcal{A}_{\rm scalar}$$

$$= C \frac{2p_1 \, . \, p_2}{(p_1 \, . \, k)(p_2 \, . \, k)} \mathcal{A}_{\rm LO}^2(p_1 + \delta p_1, p_2 + \delta p_2)$$

$$\delta p_1 = -\frac{1}{2}\Big(\frac{p_2 \, . \, k}{p_1 \cdot p_2} p_1 - \frac{p_1 \, . \, k}{p_1 \cdot p_2} p_2 + k \Big), \qquad \delta p_2 = -\frac{1}{2}\Big(\frac{p_1 \, . \, k}{p_1 \cdot p_2} p_2 - \frac{p_2 \, . \, k}{p_1 \cdot p_2} p_1 + k \Big)$$

Del Duca, Laenen, Magnea, Vernazza, White

NLP Recipe

- ☐ Calculation of LO squared amplitude
- ☐ Amplitude symmetrisation
- ☐ Momentum parametrisation in a reference frame
- ☐ Shifting of momenta in squared amplitude
- ☐ Phase space integration
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$$\left(\frac{t}{u}\right) = \frac{1}{4}\left(\frac{2p_1 \cdot p_3}{2p_2 \cdot p_3} + \frac{2p_1 \cdot p_3}{2p_1 \cdot p_4} + \frac{2p_2 \cdot p_4}{2p_2 \cdot p_3} + \frac{2p_2 \cdot p_4}{2p_1 \cdot p_4}\right)$$

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$$\begin{aligned}
p_1 &= (E_1, 0, \cdots, 0, E_1) \\
p_2 &= (E_2, 0, \cdots, 0, p_3 \sin\psi, p_3 \cos\psi - E_1) \\
p_3 &= -(E_3, 0, \cdots, 0, p_3 \sin\psi, p_3 \cos\psi) \\
p_4 &= -\frac{\sqrt{s_{45}}}{2}(1, 0, \cdots, 0, \sin\theta_1 \sin\theta_2, \sin\theta_1 \cos\theta_2, \cos\theta_1) \\
p_5 &= -\frac{\sqrt{s_{45}}}{2}(1, 0, \cdots, 0, -\sin\theta_1 \sin\theta_2, -\sin\theta_1 \cos\theta_2, -\cos\theta_1)
\end{aligned}$$

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Non-trivial when analytic form of LO squared amplitude is complicated...

In the case of $m(1^-,2^+,3^+,4^+)$, we rewrite Eq. 22 in a more compact form:

$$m(1^-,2^+,3^+,4^+) = -\frac{\langle 1-|\not{p}_{\rm H}|3-\rangle^2[24]^2}{S_{124}S_{12}S_{14}} - \frac{\langle 1-|\not{p}_{\rm H}|4-\rangle^2[23]^2}{S_{123}S_{12}S_{23}}$$
$$-\frac{\langle 1-|\not{p}_{\rm H}|2-\rangle^2[34]^2}{S_{134}S_{14}S_{34}} + \frac{[24]}{[12][14]\langle 13\rangle}\left\{\frac{\langle 1-|\not{p}_{\rm H}|2-\rangle^2}{\langle 14\rangle\langle 34\rangle} + \frac{\langle 1-|\not{p}_{\rm H}|4-\rangle^2}{\langle 12\rangle\langle 23\rangle}\right\}. \quad (50)$$

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We then write

$$|m(1^-, 2^+, 3^+, 4^+)|^2 = \sum_{\rm i=1}^{5}\sum_{\rm j=1}^{\rm i}\frac{\rm n_{ij}}{\rm d_i d_j}, \quad (51)$$

where the independent terms are:

$$n_{11} = \frac{1}{4}S_{24}^2\{1(2{+}4)3(2{+}4)\}^2, \qquad n_{44} = \frac{1}{4}S_{12}S_{13}S_{24}S_{34}\{1(3{+}4)2(3{+}4)\}^2$$
$$n_{12} = \{1(2{+}4)324(2{+}3)\}^2 - 2S_{23}S_{24}(S_{12}S_{24}{+}S_{13}S_{34} + \{1243\})$$
$$\times(S_{12}S_{23}{+}S_{14}S_{34}{+}\{1234\})$$
$$n_{14} = -S_{24}\Big[\{1(2{+}4)342(3{+}4)\}\{1(2{+}4)312(3{+}4)\}$$
$$-\{1243\}(S_{12}S_{23}{+}S_{14}S_{34}{+}\{1234\})(S_{13}S_{23}{+}S_{14}S_{24}{+}\{1324\})\Big]$$
$$-4n_{24} = \{1(2{+}3)432(3{+}4)\}\{1(2{+}3)42\}\{132(3{+}4)\}$$
$$-\{1(2{+}3)432(3{+}4)\}(\{1234\}^2{-}4S_{12}S_{23}S_{34}S_{41})$$
$$-\Big[\{1(2{+}3)432(3{+}4)\}\{1234\}$$
$$+2S_{23}S_{34}(S_{12}\{1324\}{-}S_{13}\{1234\}{+}S_{14}\{1243\}{+}2S_{12}S_{14}S_{24})\Big]$$
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&\quad -\Big[\{1(2{+}3)432(3{+}4)\}\{1234\} \\
&\quad +2S_{23}S_{34}(S_{12}\{1324\}{-}S_{13}\{1234\}{+}S_{14}\{1243\}{+}2S_{12}S_{14}S_{24})\Big] \\
&\quad \times(\{1(2{+}3)42\}{-}\{132(3{+}4)\})
\end{aligned}$$

$$\begin{aligned}
n_{25} &= -\frac{1}{4}S_{23}\{1(2{+}3)4(2{+}3)\}^2\{1324\} \\
n_{45} &= -S_{13}S_{24}\Big[\{1(3{+}4)234(2{+}3)\}\{1(3{+}4)214(2{+}3)\} \\
&\qquad -\{1234\}(S_{13}S_{23}{+}S_{14}S_{24}{+}\{1324\})(S_{12}S_{24}{+}S_{13}S_{34}{+}\{1243\})\Big] \\
d_1 &= S_{12}S_{14}S_{124}, \qquad d_4 = S_{12}S_{13}S_{14}S_{34}. \qquad (52)
\end{aligned}$$

The remaining terms can be obtained by switching the momenta:

$$\begin{aligned}
n_{22} &= n_{11}(3{\leftrightarrow}4), \quad n_{33} = n_{11}(2{\leftrightarrow}3), \quad n_{55} = n_{44}(2{\leftrightarrow}4), \\
n_{13} &= n_{12}(2{\leftrightarrow}4), \quad n_{15} = n_{14}(2{\leftrightarrow}4), \quad n_{23} = n_{13}(3{\leftrightarrow}4), \\
n_{34} &= n_{25}(2{\leftrightarrow}4), \quad n_{35} = n_{24}(2{\leftrightarrow}4), \quad d_5 = d_4(2{\leftrightarrow}4), \\
d_2 &= d_1(1{\leftrightarrow}2, 3{\leftrightarrow}4), \quad d_3 = d_1(1{\leftrightarrow}4, 2{\leftrightarrow}3). \qquad (53)
\end{aligned}$$

Spinor Helicity

$$\gamma_{R/L} = \frac{1}{2}\left(1 \pm \gamma_5\right)$$

$$u_+(p) = \begin{pmatrix} \sqrt{p^+} \\ \sqrt{p^-}e^{i\phi_p} \\ 0 \\ 0 \end{pmatrix}$$

$$u_-(p) = \begin{pmatrix} 0 \\ 0 \\ \sqrt{p^-}e^{-i\phi_p} \\ -\sqrt{p^+} \end{pmatrix}$$

$$p^\pm = E \pm p^z\,,$$
$$e^{\pm i\phi_p} = \frac{p^x \pm ip^y}{\sqrt{(p^x)^2 + (p^y)^2}} = \frac{p^x \pm ip^y}{\sqrt{p^+p^-}}$$

$$u_+(p_i) = v_-(p_i) = |\, i+\rangle = |\, i\rangle$$
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$$\langle ij] = [ij\rangle = 0$$
$$\langle ij\rangle = -\langle ji\rangle$$
$$[ij] = -[ji]$$
$$\langle ii\rangle = [jj] = 0$$
$$\langle ij\rangle[ij] = -s_{ij}$$

$$[a\,|\,\gamma_\mu\,|\,b\rangle\langle c\,|\,\gamma^\mu\,|\,d] = 2\,[ad]\,\langle cb\rangle$$
$$\langle ab\rangle\langle cd\rangle = \langle ad\rangle\langle cb\rangle + \langle ac\rangle\langle bd\rangle$$
$$[a\,|\,\gamma_\mu\,|\,b\rangle\gamma^\mu = 2\left(|\,a]\langle b\,| + |\,b\rangle[a\,|\right)$$
$$\langle a\,|\,\gamma_\mu\,|\,b]\gamma^\mu = 2\left(|\,a\rangle[b\,| + |\,b]\langle a\,|\right)$$

Polarisations:

$$\varepsilon_\mu^+(k,q) = \frac{[k\,|\,\gamma_\mu\,|\,q\rangle}{\sqrt{2}\langle qk\rangle}\,, \qquad \not{\varepsilon}^+(k,q) = \frac{\sqrt{2}(|\,k]\langle q\,| + |\,q\rangle[k\,|)}{\langle qk\rangle}$$

$$\varepsilon_\mu^-(k,q) = \frac{\langle k\,|\,\gamma_\mu\,|\,q]}{\sqrt{2}[kq]}\,, \qquad \not{\varepsilon}^-(k,q) = \frac{\sqrt{2}(|\,k\rangle[q\,| + |\,q]\langle k\,|)}{[kq]}$$

Soft and Next-to-soft amplitudes

$$\mathscr{A} = \mathscr{A}_n\Big(\{|1\rangle, |1]\}, \ldots, \{|n\rangle, |n]\}\Big)$$

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Emission of a soft gluon with + helicity

$$p_s \to \lambda p_s\,, \qquad |s\rangle \to \lambda |s\rangle\,, |s] \to |s]$$

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$$\mathscr{A}_{n+1}\Big(\{\lambda|s\rangle, |s]\}, \{|1\rangle, |1]\}, \ldots, \{|n\rangle, |n]\}\Big) = \left(S^{(0)} + S^{(1)}\right)\mathscr{A}_n\Big(\{|1\rangle, |1]\}, \ldots, \{|n\rangle, |n]\}\Big)$$

$$S^{(0)} = \frac{\langle n1\rangle}{\langle s1\rangle\langle ns\rangle}$$

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Cachazo, Strominger

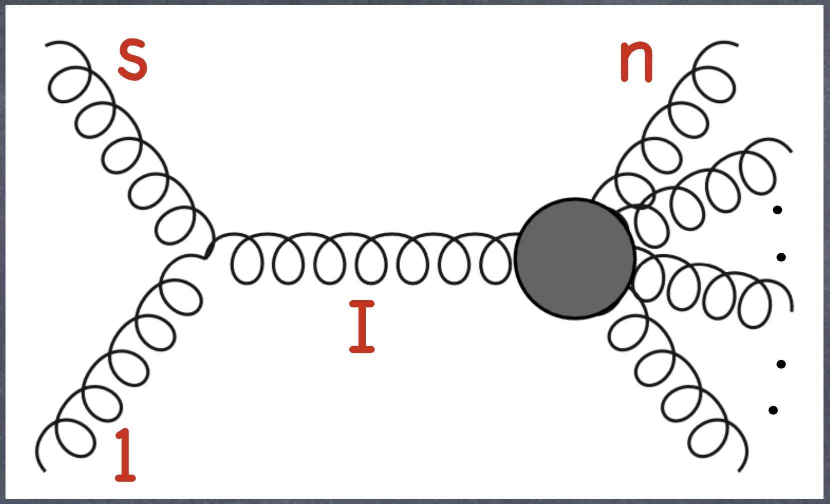
Casali

Mastrolia, bobadilla

$$S^{(1)} = \frac{1}{\langle s1\rangle}|s]\frac{\partial}{\partial|1]} - \frac{1}{\langle sn\rangle}|s]\frac{\partial}{\partial|n]}$$

On-shell recursion

Britto, Cachazo, Feng, Witten



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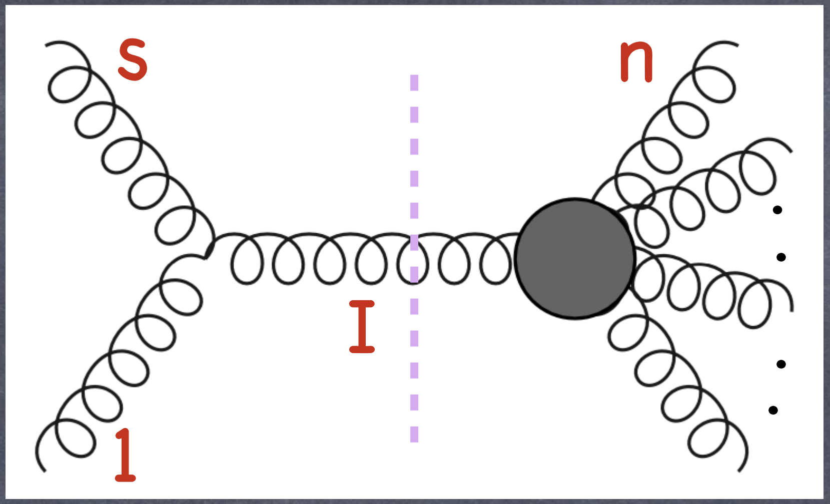

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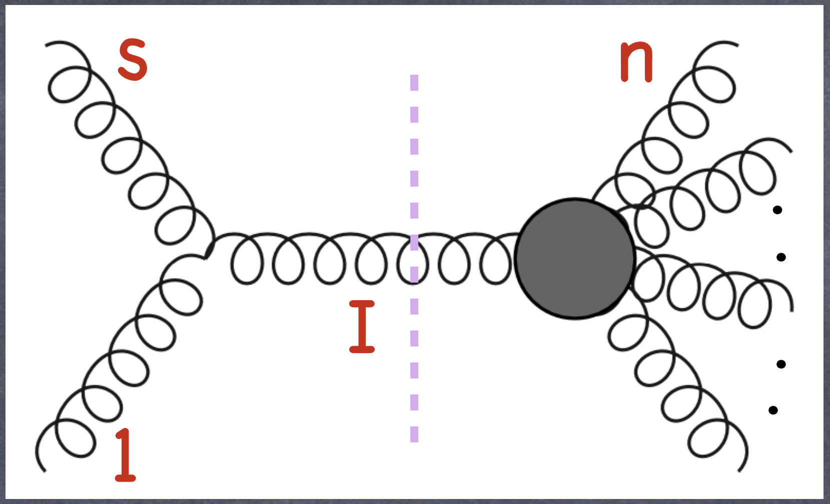

$$| \hat{s}\rangle = | s\rangle + z \, | n\rangle$$

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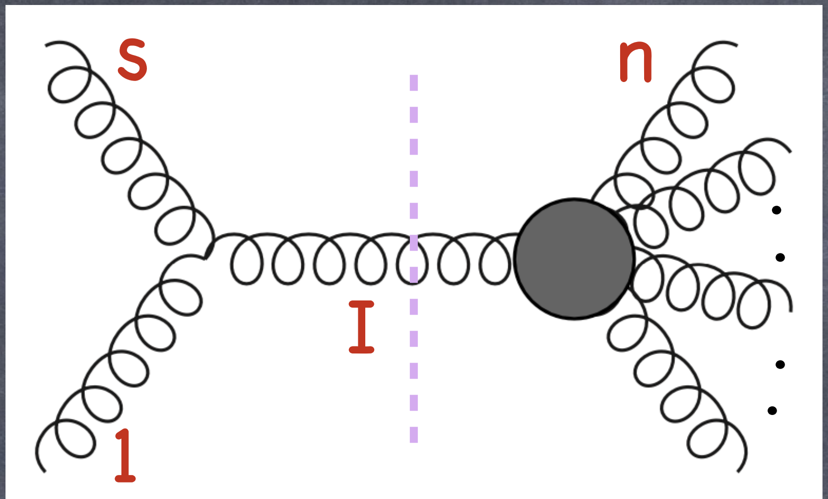


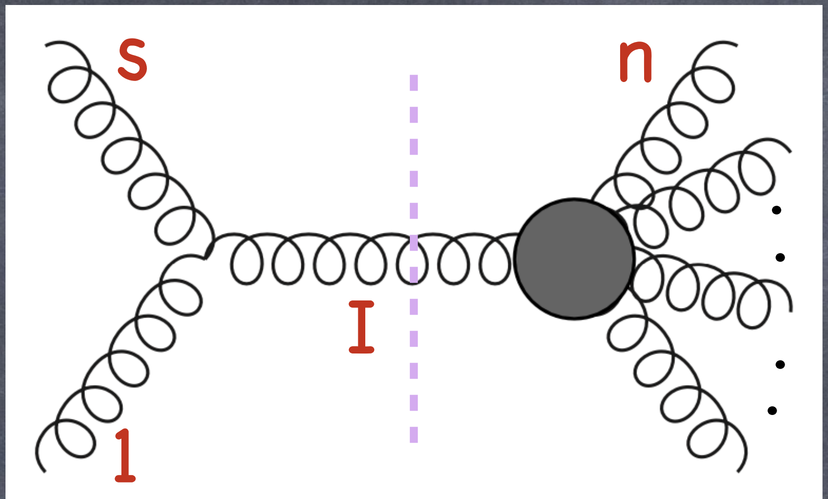
$$|\,\hat{s}\rangle = |\,s\rangle + z\,|\,n\rangle$$

$$|\,\hat{n}] = |\,n] - z\,|\,s]$$

$$p_I^2 = [s\,|\,1\,|\,\hat{s}\rangle = 0 \implies z = -\frac{\langle 1s\rangle}{\langle 1n\rangle}$$

On-shell recursion

Britto, Cachazo, Feng, Witten



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$$|I\rangle = |1\rangle$$

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On-shell recursion

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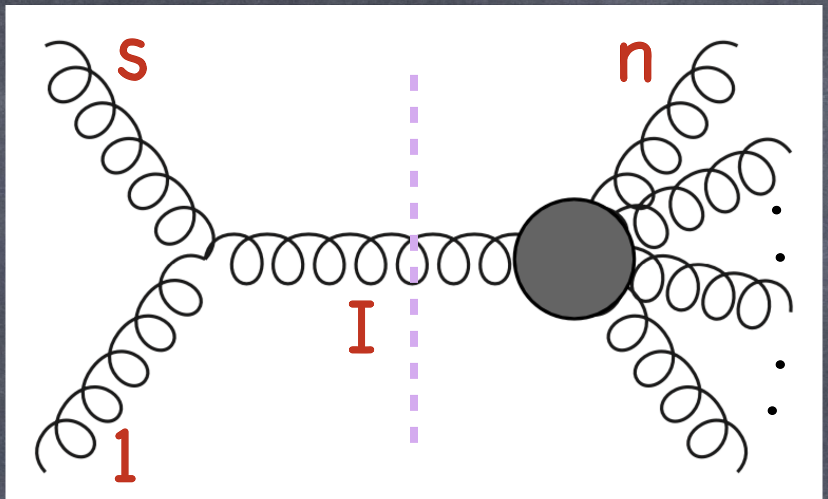

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BCFW

$$\mathscr{A}_{n+1}^{\text{LP+NLP}}\Big(\{\lambda \mid s\rangle, \mid s]\}, \{\mid 1\rangle, \mid 1]\}, \ldots, \{\mid n\rangle, \mid n]\}\Big)$$

$$= \frac{1}{\lambda^2}\frac{\langle 1n\rangle}{\langle 1s\rangle\langle ns\rangle} \times \mathscr{A}_n\Big(\{\mid 1\rangle, \mid 1']\}, \ldots, \{\mid n\rangle, \mid n']\}\Big)$$

$$\mid 1'] = \mid 1] + \Delta_s^{(1,n)} \mid s]$$

$$\mid n'] = \mid n] + \Delta_s^{(n,1)} \mid s]$$

$$\Delta_s^{(i,j)} = \lambda\frac{\langle js\rangle}{\langle ji\rangle}$$

Higgs-gluon amplitudes

$$\mathscr{L}_{\text{eff}} = -\frac{1}{4} G\, H\, F^{a}_{\mu\nu}\, F^{\mu\nu,a}$$

$$\mathscr{A}_n(p_i, h_i, c_i) = i\left(\frac{\alpha_s}{6\pi v}\right) g_s^{n-2} \sum_{\sigma \in \mathcal{S}_{n'}} \text{Tr}\left(T^{c_1} T^{c_2} \ldots T^{c_n}\right) \mathscr{A}_n^{\{c_i\}}\left(h_1\, h_2\, h_3 \ldots\, h_n; H\right)$$

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$$g(p_1) + g(p_2) \to H(-p_3) + g(-p_4)$$

$$\mathscr{A}^{124}_{+++} = \frac{m_H^4}{\langle 12\rangle\langle 24\rangle\langle 41\rangle}, \qquad \mathscr{A}^{124}_{-++} = \frac{[24]^3}{[12][14]}$$

Higgs-gluon amplitudes

$$g(p_1) + g(p_2) \to H(-p_3) + g(-p_4) + g(-p_5)$$

$$\mathcal{A}^{1245}_{++++} = \frac{m_H^4}{\langle 12\rangle\langle 24\rangle\langle 45\rangle\langle 51\rangle}$$

$$\mathcal{A}^{1245}_{--++} = -\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 24\rangle\langle 45\rangle\langle 51\rangle} - \frac{[45]^4}{[12][24][45][51]}$$

$$\mathcal{A}^{1245}_{-+++} = \frac{\langle 1\,|\,4+5\,|\,2]^3}{\langle 4\,|\,1\,|\,2]\langle 15\rangle\langle 45\rangle s_{145}} + \frac{[25][45]\langle 1\,|\,4+5\,|\,2]^2}{\langle 4\,|\,1\,|\,2]s_{15}s_{145}} + \frac{[24]\langle 1\,|\,2+4\,|\,5]^2}{\langle 24\rangle s_{12}s_{124}}$$
$$+ \frac{[25]\langle 1\,|\,2+4\,|\,5]^2}{\langle 14\rangle\langle 24\rangle[15]s_{12}} - \frac{[25]^2\langle 1\,|\,2+5\,|\,4]^2}{s_{12}s_{15}s_{125}}$$

$$\mathcal{A}(\{p_i, h_i, c_i\}) = i\left(\frac{\alpha_s}{6\pi v}\right) g_s^2 \Bigg[\left\{ \mathrm{Tr}\left(\mathrm{T}^{c_1}\mathrm{T}^{c_2}\mathrm{T}^{c_4}\mathrm{T}^{c_5}\right) + \left(\mathrm{T}^{c_1}\mathrm{T}^{c_5}\mathrm{T}^{c_4}\mathrm{T}^{c_2}\right) \right\} \mathcal{A}^{1245}_{h_1h_2h_4h_5}$$
$$+ \left\{ \mathrm{Tr}\left(\mathrm{T}^{c_1}\mathrm{T}^{c_4}\mathrm{T}^{c_5}\mathrm{T}^{c_2}\right) + \left(\mathrm{T}^{c_1}\mathrm{T}^{c_2}\mathrm{T}^{c_5}\mathrm{T}^{c_4}\right) \right\} \mathcal{A}^{1452}_{h_1h_2h_4h_5}$$
$$+ \left\{ \mathrm{Tr}\left(\mathrm{T}^{c_1}\mathrm{T}^{c_5}\mathrm{T}^{c_2}\mathrm{T}^{c_4}\right) + \left(\mathrm{T}^{c_1}\mathrm{T}^{c_4}\mathrm{T}^{c_2}\mathrm{T}^{c_5}\right) \right\} \mathcal{A}^{1524}_{h_1h_2h_4h_5} \Bigg]$$

$g(p_1) + g(p_2) + H(p_3) + g(p_4) \to 0$

$g(p_1) + g(p_2) + H(p_3) + g(p_4) + g(p_5) \to 0$

$g(p_1) + g(p_2) + H(p_3) + g(p_4) \to 0$

$g(p_1) + g(p_2) + H(p_3) + g(p_4) + g(p_5) \to 0$

- ☐ $\mathscr{D}_{12}$
- ☐ $\mathscr{D}_{14}$
- ☐ $\mathscr{D}_{24}$

$g(p_1) + g(p_2) + H(p_3) + g(p_4) \to 0$

$g(p_1) + g(p_2) + H(p_3) + g(p_4) + g(p_5) \to 0$

- ☐ $\mathscr{D}_{12}$
- ☐ $\mathscr{D}_{14}$
- ☐ $\mathscr{D}_{24}$

- ☐ $\mathscr{A}^{1245}_{h_1 h_2 h_4 h_5}$
- ☐ $\mathscr{A}^{1452}_{h_1 h_2 h_4 h_5}$
- ☐ $\mathscr{A}^{1524}_{h_1 h_2 h_4 h_5}$

$g(p_1) + g(p_2) + H(p_3) + g(p_4) \to 0$

$g(p_1) + g(p_2) + H(p_3) + g(p_4) + g(p_5) \to 0$

- ☐ $\mathscr{D}_{12}$
- ☐ $\mathscr{D}_{14}$ → ☐ $\mathscr{A}^{1245}_{h_1 h_2 h_4 h_5}$
- ☐ $\mathscr{D}_{24}$

- ☐ $\mathscr{A}^{1245}_{h_1 h_2 h_4 h_5}$
- ☐ $\mathscr{A}^{1452}_{h_1 h_2 h_4 h_5}$
- ☐ $\mathscr{A}^{1524}_{h_1 h_2 h_4 h_5}$

$g(p_1) + g(p_2) + H(p_3) + g(p_4) \to 0$

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- ☐ $\mathscr{D}_{12}$
- ☐ $\mathscr{D}_{14}$
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- ☐ $\mathscr{A}^{1245}_{h_1h_2h_4h_5}$
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- ☐ $\mathscr{A}^{1524}_{h_1h_2h_4h_5}$

$g(p_1) + g(p_2) + H(p_3) + g(p_4) \to 0$

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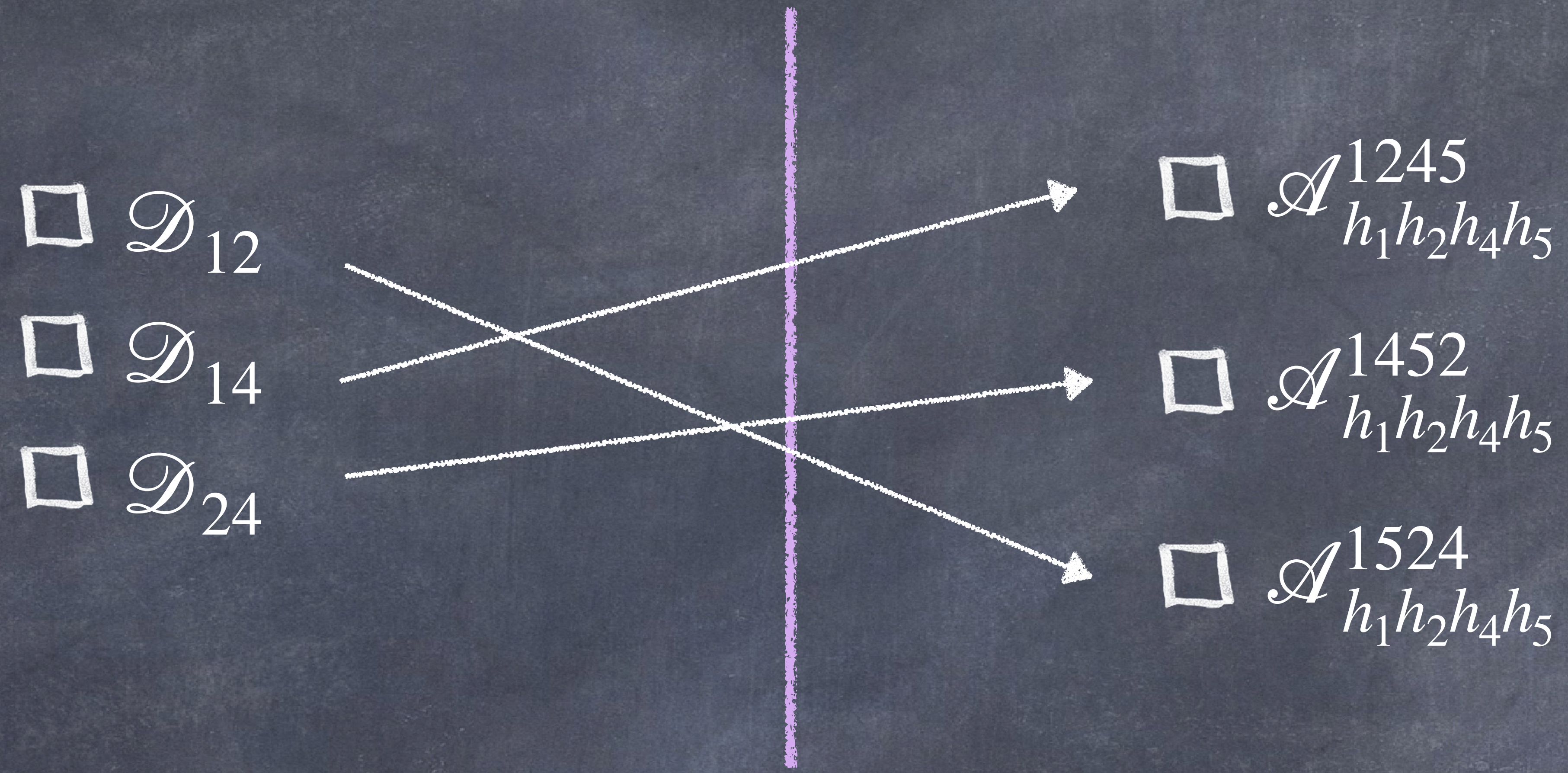
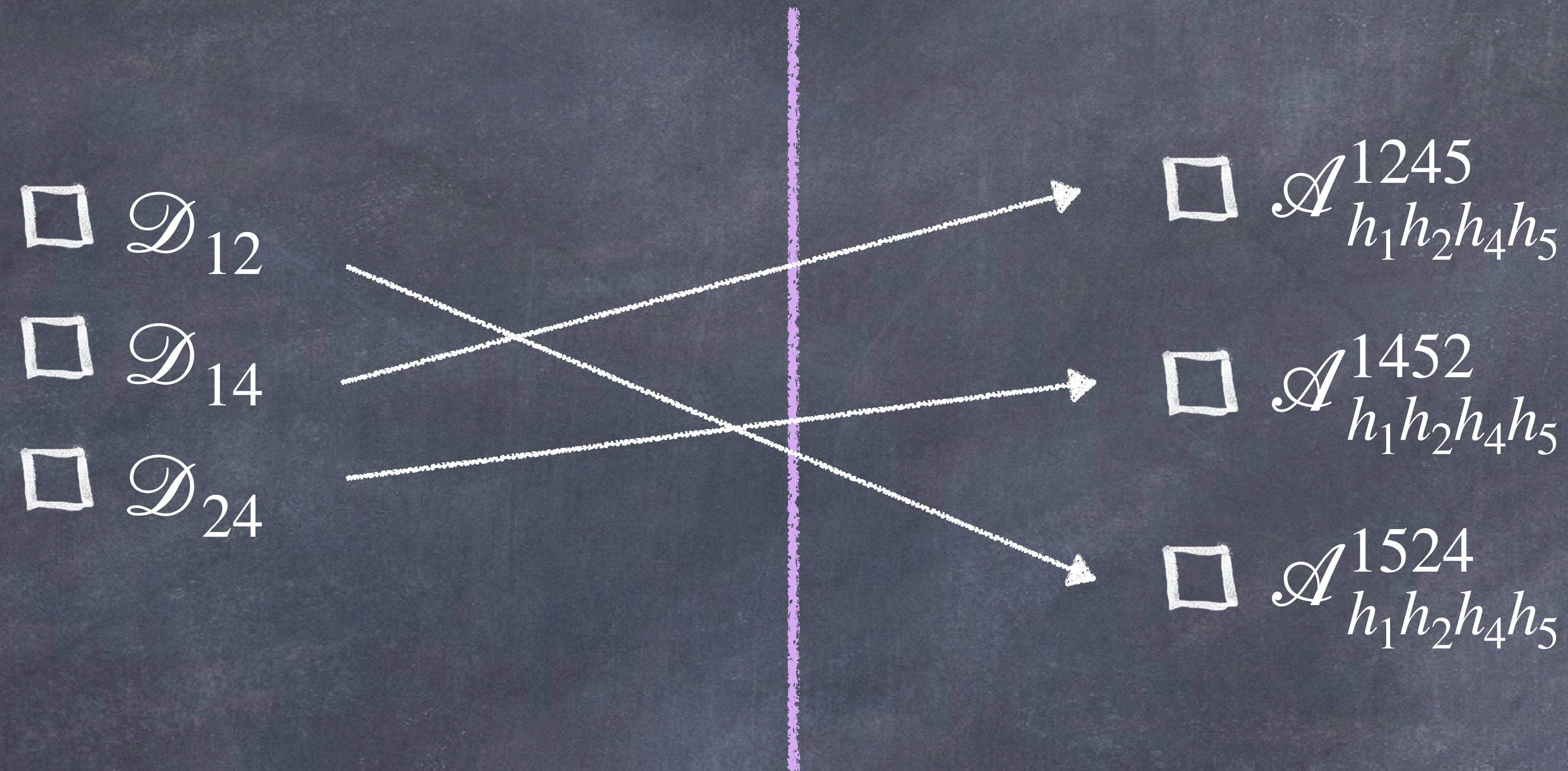


$g(p_1) + g(p_2) + H(p_3) + g(p_4) \to 0$

$g(p_1) + g(p_2) + H(p_3) + g(p_4) + g(p_5) \to 0$



- For every helicity config., No. of dipoles = No. of colour ordered amplitudes
- Connected by Eikonal factors, Shifting of spinors

Pal, Seth

New NLP Recipe

- Formation of dipoles
- Shifting of square spinors in "+" emission
- Shifting of angle spinors in "-" emission

New NLP Recipe

- Formation of dipoles
- Shifting of square spinors in "+" emission
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$$\mathscr{A}^{1245}_{h_1h_2h_4+}\Big|_{\text{LP+NLP}} = \frac{\langle 14\rangle}{\langle 15\rangle\langle 45\rangle}\mathscr{A}^{1'2\,4'}_{h_1h_2h_4}$$

$$|\,1'] = |\,1] + \frac{\langle 45\rangle}{\langle 41\rangle}\,|\,5]\,, \quad |\,4'] = |\,4] + \frac{\langle 15\rangle}{\langle 14\rangle}\,|\,5]$$

$$\mathscr{A}^{1452}_{h_1h_2h_4+}\Big|_{\text{LP+NLP}} = \frac{\langle 24\rangle}{\langle 25\rangle\langle 54\rangle}\mathscr{A}^{12'4'}_{h_1h_2h_4}$$

$$|\,2'] = |\,2] + \frac{\langle 45\rangle}{\langle 42\rangle}\,|\,5]\,, \quad |\,4'] = |\,4] + \frac{\langle 25\rangle}{\langle 24\rangle}\,|\,5]$$

$$\mathscr{A}^{1524}_{h_1h_2h_4+}\Big|_{\text{LP+NLP}} = \frac{\langle 12\rangle}{\langle 15\rangle\langle 52\rangle}\mathscr{A}^{1'2'4}_{h_1h_2h_4}$$

$$|\,1'] = |\,1] + \frac{\langle 25\rangle}{\langle 21\rangle}\,|\,5]\,, \quad |\,2'] = |\,2] + \frac{\langle 15\rangle}{\langle 12\rangle}\,|\,5]$$

No NLP from NMHV

$$\mathscr{A}^{124}_{+--} = -\frac{\langle 24\rangle^3}{\langle 12\rangle\langle 14\rangle}$$

No NLP from NMHV

$$\mathscr{A}^{124}_{+--} = -\frac{\langle 24\rangle^3}{\langle 12\rangle\langle 14\rangle}$$

Born	Helicity of extra emission	NLP
$\mathscr{A}_{+++}$	$+$ $-$	$\mathscr{A}_{++++}$ $\mathscr{A}_{+++-}$
$\mathscr{A}_{-++}$	$+$ $-$	$\mathscr{A}_{-+++}$ 0
$\mathscr{A}_{+-+}$	$+$ $-$	$\mathscr{A}_{+-++}$ 0
$\mathscr{A}_{++-}$	$+$ $-$	$\mathscr{A}_{++-+}$ 0

NLP amplitudes: ++++

$$\mathscr{A}^{1245}_{++++}\Big|_{\text{NLP}} = \frac{\langle 41\rangle}{\langle 45\rangle\langle 51\rangle}\,\frac{2\,(s_{15}+s_{25}+s_{45})}{(s_{12}+s_{14}+s_{24})}\,\mathscr{A}^{124}_{+++}$$

$$\mathscr{A}^{1524}_{++++}\Big|_{\text{NLP}} = \frac{\langle 12\rangle}{\langle 15\rangle\langle 52\rangle}\,\frac{2\,(s_{15}+s_{25}+s_{45})}{(s_{12}+s_{14}+s_{24})}\,\mathscr{A}^{124}_{+++}$$

$$\mathscr{A}^{1452}_{++++}\Big|_{\text{NLP}} = -\,\frac{\langle 42\rangle}{\langle 45\rangle\langle 52\rangle}\,\frac{2\,(s_{15}+s_{25}+s_{45})}{(s_{12}+s_{14}+s_{24})}\,\mathscr{A}^{124}_{+++}$$

NLP amplitudes: +++−

$$\mathscr{A}^{1245}_{+++-}\Big|_{\rm NLP} = \frac{[14]}{[15][45]}\left(-\frac{\langle 25\rangle[45]}{\langle 12\rangle[14]} - \frac{\langle 25\rangle[15]}{\langle 24\rangle[14]} - \frac{s_{15}}{s_{14}} - \frac{s_{45}}{s_{14}} + \frac{2\,(s_{15}+s_{25}+s_{45})}{(s_{12}+s_{14}+s_{24})}\right)\mathscr{A}^{124}_{+++}$$

$$\mathscr{A}^{1524}_{+++-}\Big|_{\rm NLP} = -\frac{[12]}{[15][25]}\left(\frac{\langle 45\rangle[15]}{\langle 24\rangle[12]} - \frac{\langle 45\rangle[25]}{\langle 14\rangle[12]} - \frac{s_{15}}{s_{12}} - \frac{s_{25}}{s_{12}} + \frac{2\,(s_{15}+s_{25}+s_{45})}{(s_{12}+s_{14}+s_{24})}\right)\mathscr{A}^{124}_{+++}$$

$$\mathscr{A}^{1452}_{+++-}\Big|_{\rm NLP} = -\frac{[24]}{[25][45]}\left(\frac{\langle 15\rangle[45]}{\langle 12\rangle[24]} - \frac{\langle 15\rangle[25]}{\langle 14\rangle[24]} - \frac{s_{25}}{s_{24}} - \frac{s_{45}}{s_{24}} + \frac{2\,(s_{15}+s_{25}+s_{45})}{(s_{12}+s_{14}+s_{24})}\right)\mathscr{A}^{124}_{+++}$$

NLP amplitudes: −+++

$$\mathscr{A}^{1245}_{-+++}\Big|_{\rm NLP} = \frac{\langle 14\rangle}{\langle 15\rangle\langle 45\rangle}\left(\frac{3\langle 15\rangle[25]}{\langle 14\rangle[24]} - \frac{\langle 45\rangle[25]}{\langle 14\rangle[12]} - \frac{s_{15}}{s_{14}} - \frac{s_{45}}{s_{14}}\right)\mathscr{A}^{124}_{-++}$$

$$\mathscr{A}^{1524}_{-+++}\Big|_{\rm NLP} = -\frac{\langle 12\rangle}{\langle 15\rangle\langle 25\rangle}\left(-\frac{\langle 25\rangle[45]}{\langle 12\rangle[14]} - \frac{3\langle 15\rangle[45]}{\langle 12\rangle[24]} - \frac{s_{15}}{s_{12}} - \frac{s_{25}}{s_{12}}\right)\mathscr{A}^{124}_{-++}$$

$$\mathscr{A}^{1452}_{-+++}\Big|_{\rm NLP} = -\frac{\langle 24\rangle}{\langle 45\rangle\langle 25\rangle}\left(\frac{\langle 45\rangle[15]}{\langle 24\rangle[12]} - \frac{\langle 25\rangle[15]}{\langle 24\rangle[14]} + \frac{3s_{25}}{s_{24}} + \frac{3s_{45}}{s_{24}}\right)\mathscr{A}^{124}_{-++}$$

NLP amplitudes: ++−+, +−++

$$\mathscr{A}^{1245}_{++-+}\Big|_{\text{NLP}} = \mathscr{A}^{1245}_{-+++}\Big|_{\text{NLP}} \{1 \leftrightarrow 4\}$$

$$\mathscr{A}^{1524}_{++-+}\Big|_{\text{NLP}} = \mathscr{A}^{1452}_{-+++}\Big|_{\text{NLP}} \{1 \leftrightarrow 4\}$$

$$\mathscr{A}^{1452}_{++-+}\Big|_{\text{NLP}} = \mathscr{A}^{1524}_{-+++}\Big|_{\text{NLP}} \{1 \leftrightarrow 4\}$$

$$\mathscr{A}^{1245}_{+-++}\Big|_{\text{NLP}} = \mathscr{A}^{1452}_{-+++}\Big|_{\text{NLP}} \{1 \leftrightarrow 2\}$$

$$\mathscr{A}^{1524}_{+-++}\Big|_{\text{NLP}} = \mathscr{A}^{1524}_{-+++}\Big|_{\text{NLP}} \{1 \leftrightarrow 2\}$$

$$\mathscr{A}^{1452}_{+-++}\Big|_{\text{NLP}} = \mathscr{A}^{1245}_{-+++}\Big|_{\text{NLP}} \{1 \leftrightarrow 2\}$$

Differential x-sec

$$\mathscr{A} = \mathscr{A}_{\mathrm{LP}} + \mathscr{A}_{\mathrm{NLP}}, \qquad \mathscr{A}^2 = \mathscr{A}^2_{\mathrm{LP}} + 2\,\mathrm{Re}\left(\mathscr{A}_{\mathrm{NLP}}\mathscr{A}^{\dagger}_{\mathrm{LP}}\right)$$

Differential x-sec

$$\mathscr{A} = \mathscr{A}_{\mathrm{LP}} + \mathscr{A}_{\mathrm{NLP}}\,, \qquad \mathscr{A}^2 = \mathscr{A}^2_{\mathrm{LP}} + 2\,\mathrm{Re}\left(\mathscr{A}_{\mathrm{NLP}}\mathscr{A}^{\dagger}_{\mathrm{LP}}\right)$$

$$\sum_{\text{colours}} |\mathscr{A}(\{p_i, h_i, c_i\})|^2 = \left[\left(\frac{\alpha_s}{6\pi v}\right) g_s^2\right]^2 (N^2-1)\left\{2N^2\left(\left|\mathscr{A}^{1245}\right|^2 + \left|\mathscr{A}^{1452}\right|^2 + \left|\mathscr{A}^{1524}\right|^2\right)\right.$$

$$\left. -4\frac{(N^2-3)}{N^2}\left|\mathscr{A}^{1245} + \mathscr{A}^{1452} + \mathscr{A}^{1524}\right|^2\right\}$$

Differential x-sec

$$\mathscr{A} = \mathscr{A}_{\rm LP} + \mathscr{A}_{\rm NLP}\,, \qquad \mathscr{A}^2 = \mathscr{A}^2_{\rm LP} + 2\,\mathrm{Re}\left(\mathscr{A}_{\rm NLP}\mathscr{A}^\dagger_{\rm LP}\right)$$

$$\sum_{\rm colours} |\mathscr{A}(\{p_i, h_i, c_i\})|^2 = \left[\left(\frac{\alpha_s}{6\pi v}\right) g_s^2\right]^2 (N^2-1)\left\{2N^2\left(\left|\mathscr{A}^{1245}\right|^2 + \left|\mathscr{A}^{1452}\right|^2 + \left|\mathscr{A}^{1524}\right|^2\right)\right.$$

$$\left. -4\frac{(N^2-3)}{N^2}\underbrace{\left|\mathscr{A}^{1245} + \mathscr{A}^{1452} + \mathscr{A}^{1524}\right|^2}_{\text{zero}}\right\}$$

Differential x-sec

$$\mathscr{A} = \mathscr{A}_{\rm LP} + \mathscr{A}_{\rm NLP}\,, \qquad \mathscr{A}^2 = \mathscr{A}^2_{\rm LP} + 2\,{\rm Re}\left(\mathscr{A}_{\rm NLP}\mathscr{A}^\dagger_{\rm LP}\right)$$

$$\sum_{\rm colours} |\mathscr{A}(\{p_i, h_i, c_i\})|^2 = \left[\left(\frac{\alpha_s}{6\pi v}\right) g_s^2\right]^2 (N^2-1)\left\{2N^2\left(\left|\mathscr{A}^{1245}\right|^2 + \left|\mathscr{A}^{1452}\right|^2 + \left|\mathscr{A}^{1524}\right|^2\right)\right.$$

$$\left. - 4\,\frac{(N^2-3)}{N^2}\left|\underbrace{\mathscr{A}^{1245} + \mathscr{A}^{1452} + \mathscr{A}^{1524}}_{\text{zero}}\right|^2\right\}$$

$$p_1 = (E_1, 0, \cdots, 0, E_1)$$
$$p_2 = (E_2, 0, \cdots, 0, p_3 \sin\psi, p_3\cos\psi - E_1)$$
$$p_3 = -(E_3, 0, \cdots, 0, p_3\sin\psi, p_3\cos\psi)$$
$$p_4 = -\frac{\sqrt{s_{45}}}{2}(1, 0, \cdots, 0, \sin\theta_1\sin\theta_2, \sin\theta_1\cos\theta_2, \cos\theta_1)$$
$$p_5 = -\frac{\sqrt{s_{45}}}{2}(1, 0, \cdots, 0, -\sin\theta_1\sin\theta_2, -\sin\theta_1\cos\theta_2, -\cos\theta_1)$$

$$s_{12}^2 \frac{d^2\sigma}{ds_{13}ds_{23}}\bigg|_{\rm NLP} = \mathscr{F}\left(\frac{s_{45}}{\bar{\mu}^2}\right)^{-\epsilon}\overline{\mathscr{A}^2_{\rm NLP}}$$

$$\overline{\mathscr{A}^2_{\rm NLP}} = \int_0^\pi d\theta_1\,(\sin\theta_1)^{1-2\epsilon}\int_0^\pi d\theta_2\,(\sin\theta_2)^{-2\epsilon}[\mathscr{A}^2]\,|_{\rm NLP}$$

NLP Logs

$$s_{12}^2 \frac{d^2\sigma_{++++}}{ds_{13}ds_{23}}\bigg|_{\text{NLP-LL}} = \mathcal{F}\left\{16\pi\left[s_{12}\left(\frac{1}{s_{13}}+\frac{1}{s_{23}}\right)+2\right]\log\left(\frac{s_{45}}{\bar{\mu}^2}\right)+16\pi\log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right)\right\}\frac{1}{m_H^2}\mathcal{A}^2_{+++}$$

$$s_{12}^2 \frac{d^2\sigma_{-+++}}{ds_{13}ds_{23}}\bigg|_{\text{NLP-LL}} = \mathcal{F}\left\{16\pi\left(\frac{1}{s_{13}}-\frac{1}{s_{23}}\right)\log\left(\frac{s_{45}}{\bar{\mu}^2}\right)+4\pi\left(\frac{3}{s_{13}}-\frac{1}{s_{23}}\right)\log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right)\right\}\mathcal{A}^2_{-++}$$

$$s_{12}^2 \frac{d^2\sigma_{++-+}}{ds_{13}ds_{23}}\bigg|_{\text{NLP-LL}} = \mathcal{F}\left\{16\pi\left(\frac{1}{s_{13}}+\frac{1}{s_{23}}\right)\log\left(\frac{s_{45}}{\bar{\mu}^2}\right)-4\pi\left(\frac{1}{s_{13}}+\frac{1}{s_{23}}\right)\log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right)\right\}\mathcal{A}^2_{++-}$$

$$s_{12}^2\, s_{12}^2 \frac{d^2\sigma_{+-++}}{ds_{13}ds_{23}}\bigg|_{\text{NLP-LL}} = \mathcal{F}\left\{16\pi\left(\frac{1}{s_{23}}-\frac{1}{s_{13}}\right)\log\left(\frac{s_{45}}{\bar{\mu}^2}\right)+4\pi\left(\frac{3}{s_{23}}-\frac{1}{s_{13}}\right)\log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right)\right\}\mathcal{A}^2_{+-+}$$

$$s_{12}^2 \frac{d^2\sigma_{+++-}}{ds_{13}ds_{23}}\bigg|_{\text{NLP-LL}} = \mathcal{F}\left\{-16\pi\left(\frac{1}{s_{13}}+\frac{1}{s_{23}}\right)\log\left(\frac{s_{45}}{\bar{\mu}^2}\right)-4\pi\left(\frac{1}{s_{13}}+\frac{1}{s_{23}}\right)\log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right)\right\}\mathcal{A}^2_{+++} + s_{12}^2 \frac{d^2\sigma_{++++}}{ds_{13}ds_{23}}\bigg|_{\text{NLP-LL}}$$

NLP Logs & Poles

$$s_{12}^2 \frac{d^2\sigma_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}} = \left\{ A \log\left(\frac{s_{45}}{\bar{\mu}^2}\right) + B \log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right)\right\} \mathscr{A}^2_{h_1h_2h_4}$$

$$- \frac{A}{\epsilon} \mathscr{A}^2_{h_1h_2h_4}$$

NLP Logs & Poles

$$s_{12}^2 \frac{d^2\sigma_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\bigg|_{\mathrm{NLP-LL}} = \left\{ A \log\left(\frac{s_{45}}{\bar{\mu}^2}\right) + B \log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right)\right\} \mathscr{A}^2_{h_1h_2h_4}$$

$$- \frac{A}{\epsilon}\mathscr{A}^2_{h_1h_2h_4}$$

Mass factorisation: $\frac{1}{\epsilon}\int_0^1 \frac{dx}{x}\frac{\alpha_s}{2\pi} P_{gg}(x,\epsilon)\, \hat{s}_{12}^2 \frac{d^2\sigma^{(0)}(\hat{s}_{12}, \hat{s}_{13}, s_{23})}{d\hat{s}_{13}ds_{23}}$

NLP logs & Poles

$$s_{12}^2 \frac{d^2\sigma_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\bigg|_{\text{NLP-LL}} = \left\{ A\log\left(\frac{s_{45}}{\bar{\mu}^2}\right) + B\log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right)\right\}\mathscr{A}^2_{h_1h_2h_4}$$

$$-\frac{A}{\epsilon}\mathscr{A}^2_{h_1h_2h_4}$$

Mass factorisation: $$\frac{1}{\epsilon}\int_0^1 \frac{dx}{x}\,\frac{\alpha_s}{2\pi}\,P_{gg}(x,\epsilon)\,\hat{s}_{12}^2\,\frac{d^2\sigma^{(0)}(\hat{s}_{12},\hat{s}_{13},s_{23})}{d\hat{s}_{13}ds_{23}}$$

$$P_{gg} = \frac{z}{1-z} + \frac{1-z}{z} + z(1-z)$$

NLP Logs & Poles

$$s_{12}^2 \frac{d^2\sigma_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\text{NLP-LL}} = \left\{ A \log\left(\frac{s_{45}}{\bar{\mu}^2}\right) + B \log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right) \right\} \mathcal{A}^2_{h_1h_2h_4}$$

$$- \frac{A}{\epsilon}\mathcal{A}^2_{h_1h_2h_4}$$

Mass factorisation: $\frac{1}{\epsilon}\int_0^1 \frac{dx}{x}\frac{\alpha_s}{2\pi} P_{gg}(x,\epsilon)\, \hat{s}_{12}^2 \frac{d^2\sigma^{(0)}(\hat{s}_{12}, \hat{s}_{13}, s_{23})}{d\hat{s}_{13}ds_{23}}$

$$P_{gg} = \frac{z}{1-z} + \frac{1-z}{z} + z(1-z)$$

$g \to gg$	$++$	$+-$	$-+$	$--$
$+$	$\frac{1}{z(1-z)}$	$\frac{z^3}{(1-z)}$	$\frac{(1-z)^3}{z}$	0
$-$	0	$\frac{(1-z)^3}{z}$	$\frac{z^3}{(1-z)}$	$\frac{1}{z(1-z)}$

Building Blocks

$$s_{12}^2 \frac{d^2\sigma_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}} = s_{12}^2 \frac{d^2\sigma^{1245}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}} + s_{12}^2 \frac{d^2\sigma^{1452}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}}$$

$$+ s_{12}^2 \frac{d^2\sigma^{1524}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}}$$

Building Blocks

$$\mathscr{D}_{14}$$

$$s_{12}^2 \frac{d^2\sigma_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}} = s_{12}^2 \frac{d^2\sigma^{1245}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}} + s_{12}^2 \frac{d^2\sigma^{1452}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}}$$

$$+ s_{12}^2 \frac{d^2\sigma^{1524}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}}$$

Building Blocks

$$\mathscr{D}_{14} \qquad \mathscr{D}_{24}$$

$$s_{12}^2 \frac{d^2\sigma_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\bigg|_{\mathrm{NLP-LL}} = s_{12}^2 \frac{d^2\sigma^{1245}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\bigg|_{\mathrm{NLP-LL}} + s_{12}^2 \frac{d^2\sigma^{1452}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\bigg|_{\mathrm{NLP-LL}}$$

$$+ s_{12}^2 \frac{d^2\sigma^{1524}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\bigg|_{\mathrm{NLP-LL}}$$

Building Blocks

$$s_{12}^2 \frac{d^2\sigma_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}} = \overbrace{s_{12}^2 \frac{d^2\sigma^{1245}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}}}^{\mathscr{D}_{14}} + \overbrace{s_{12}^2 \frac{d^2\sigma^{1452}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}}}^{\mathscr{D}_{24}}$$

$$+ s_{12}^2 \frac{d^2\sigma^{1524}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}} \quad \mathscr{D}_{12}$$

Building Blocks

$$s_{12}^2 \frac{d^2\sigma_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}} = \underbrace{s_{12}^2 \frac{d^2\sigma^{1245}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}}}_{\mathcal{D}_{14}} + \underbrace{s_{12}^2 \frac{d^2\sigma^{1452}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}}}_{\mathcal{D}_{24}}$$

$$+ \underbrace{s_{12}^2 \frac{d^2\sigma^{1524}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}}}_{\mathcal{D}_{12}}$$

$$s_{12}^2 \frac{d^2\sigma^{1245}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}} + s_{12}^2 \frac{d^2\sigma^{1452}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}} = s_{12}^2 \frac{d^2\sigma^{1524}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\mathrm{NLP-LL}}$$

Building Blocks

$$\mathcal{D}_{14} \qquad\qquad \mathcal{D}_{24}$$

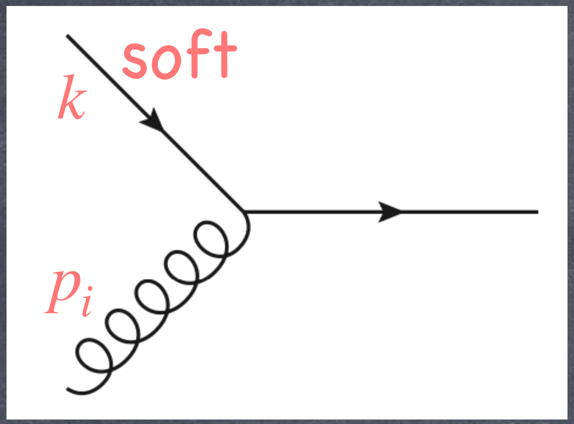
$$s_{12}^2 \frac{d^2\sigma_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\text{NLP-LL}} = s_{12}^2 \frac{d^2\sigma^{1245}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\text{NLP-LL}} + s_{12}^2 \frac{d^2\sigma^{1452}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\text{NLP-LL}}$$

$$+ s_{12}^2 \frac{d^2\sigma^{1524}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\text{NLP-LL}} \quad \mathcal{D}_{12}$$

$$s_{12}^2 \frac{d^2\sigma^{1245}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\text{NLP-LL}} + s_{12}^2 \frac{d^2\sigma^{1452}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\text{NLP-LL}} = s_{12}^2 \frac{d^2\sigma^{1524}_{h_1h_2h_4h_5}}{ds_{13}ds_{23}}\Bigg|_{\text{NLP-LL}}$$

Calculating only $\mathcal{D}_{12}$ is sufficient !

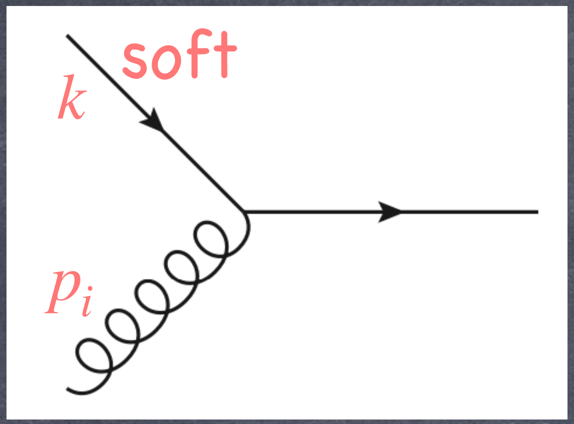
Soft Quarks



$$\mathcal{M} = \bar{u}(k)\frac{\gamma^\mu(\not{p}_i + \not{k})}{(p_i+k)^2}\varepsilon_\mu(p_i)\,\mathcal{M}_0$$

$$= \bar{u}(k)\frac{\not{\varepsilon}_i\ \not{p}_i}{2p_i\,.\,k}\,\mathcal{M}_0$$

Soft Quarks



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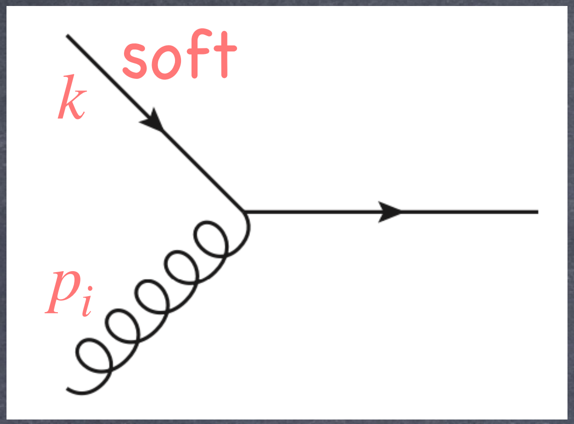
$$\mathcal{O}\left(\frac{1}{\sqrt{k}}\right)$$

Soft Quarks

Van Beekveld, Beenakker, Laenen, White



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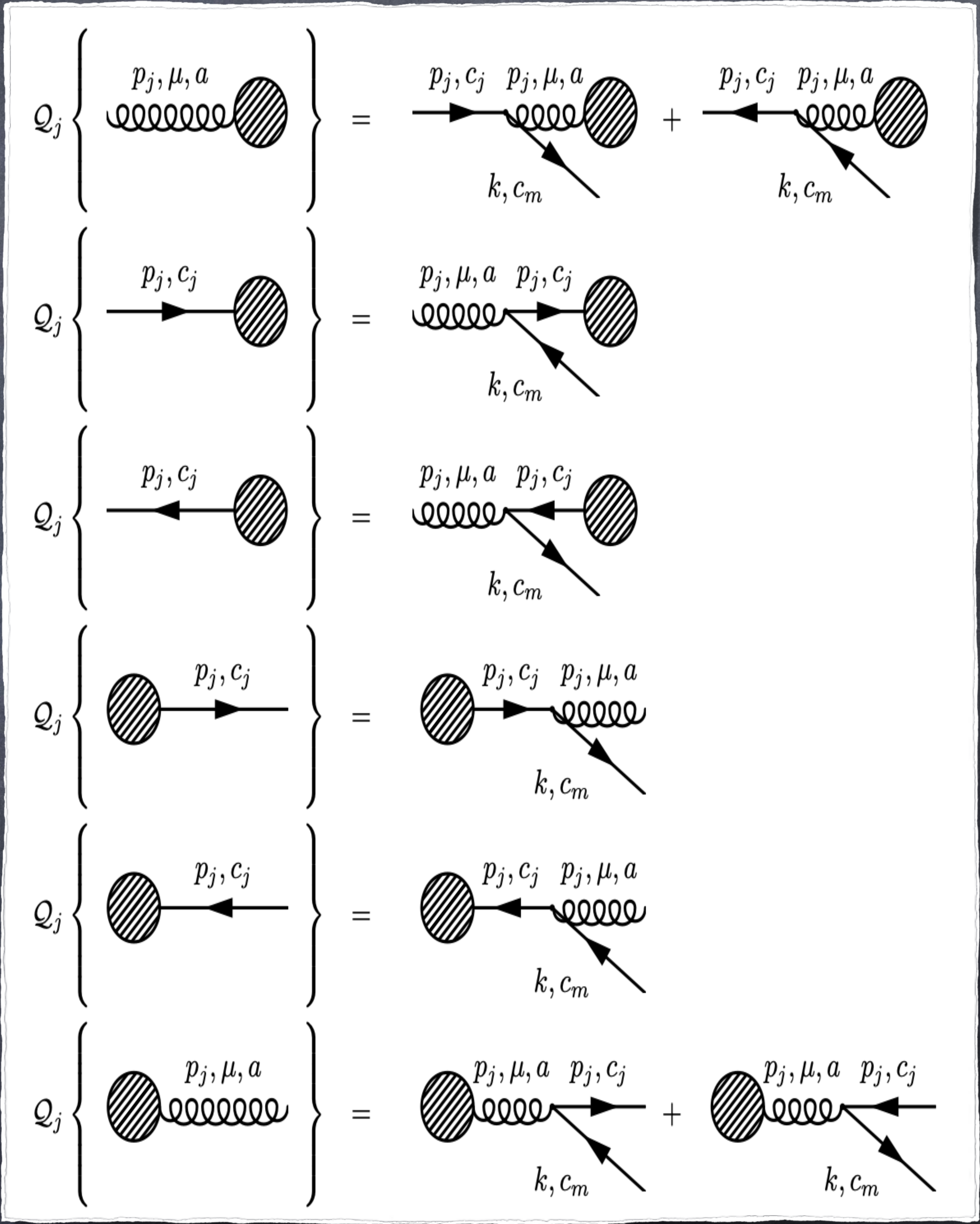
$$= \bar{u}(k)\frac{\not{\varepsilon}_i\ \not{p}_i}{2p_i\,.\,k}\,\mathcal{M}_0$$

$$\begin{aligned}
\mathcal{Q}_j\left(u(p_j)\right) &= t^a_{c_j c_m}\epsilon_\mu(p_j)\not{p}_j\gamma^\mu v(k)\,,\\
\mathcal{Q}_j\left(\bar{u}(p_j)\right) &= -t^a_{c_m c_j}\epsilon^*_\mu(p_j)\bar{u}(k)\gamma^\mu\not{p}_j\,,\\
\mathcal{Q}_j\left(v(p_j)\right) &= t^a_{c_j c_m}\epsilon^*_\mu(p_j)\not{p}_j\gamma^\mu v(k)\,,\\
\mathcal{Q}_j\left(\bar{v}(p_j)\right) &= -t^a_{c_m c_j}\epsilon_\mu(p_j)\bar{u}(k)\gamma^\mu\not{p}_j\,,\\
\mathcal{Q}_j\left(\epsilon_\mu(p_j)\right) &= -\left(t^a_{c_m c_j}\bar{u}(k)\gamma_\mu u(p_j) + t^a_{c_j c_m}\bar{v}(p_j)\gamma_\mu v(k)\right),\\
\mathcal{Q}_j\left(\epsilon^*_\mu(p_j)\right) &= t^a_{c_j c_m}\bar{u}(p_j)\gamma_\mu v(k) + t^a_{c_m c_j}\bar{u}(k)\gamma_\mu v(p_j)\,.
\end{aligned}$$



Soft Quarks

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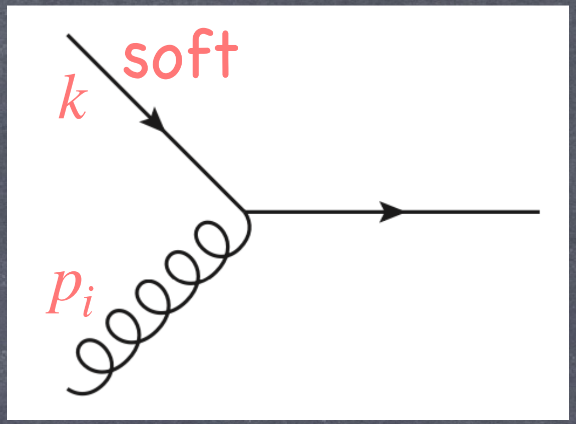

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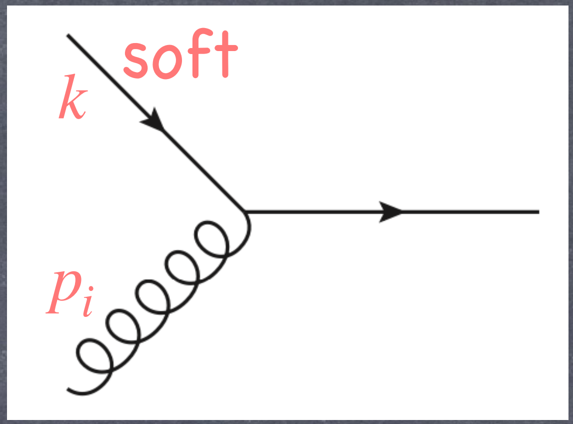

Recipe:

1. $q_s^+(p_1)\,g^+(p_2) \to \frac{1}{\langle 12\rangle}q^+(p_2)$

2. $q_s^+(p_1)\,g^-(p_2) \to 0$

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Soft Quarks: example

$$\mathscr{A}_{q_1^+\bar{q}_2^- g_4^+ g_5^-} = \frac{[14]^3}{[12][15][45]} - \frac{\langle 25\rangle^3}{\langle 12\rangle\langle 24\rangle\langle 45\rangle}$$

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$$= \frac{1}{\langle 12\rangle}\mathscr{A}_{g_2^- g_4^+ g_5^-}$$

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$$\mathscr{A}_{q_1^+\bar{q}_2^-g_4^+g_5^+} = \frac{[14](\langle 12\rangle[15] - \langle 24\rangle[45])^2}{s_{12}(s_{12}+s_{14}+s_{24})\langle 24\rangle} - \frac{\left(\frac{1}{s_{15}}+\frac{1}{s_{12}}\right)[15](\langle 12\rangle[14] + \langle 25\rangle[45])^2}{(s_{12}+s_{15}+s_{25})\langle 25\rangle} - \frac{(\langle 24\rangle[14] + \langle 25\rangle[15])^2}{\langle 24\rangle\langle 25\rangle\langle 45\rangle[12]}$$

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$$\mathcal{A}_{q_1^+\bar{q}_2^- g_4^+ g_5^+}\Big|_{\text{NLP}} = \frac{[45]^3}{\langle 12\rangle[24][25]} - \frac{[45]^2}{\langle 15\rangle[25]}$$

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Summary

- ☐ Shifting of spinors captures next-to-soft gluon contributions
- ☐ Coloured amplitudes = colour dipoles
- ☐ Starting point is non-radiative helicity amplitudes
- ☐ Reveals interesting features that are otherwise inaccessible
    - Vanishing of NMHV contributions at NLP
    - Twice the contribution of $\mathscr{D}_{12}$ can give the full result
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- ☐ Generic method - can be applied for processes with zero-jet or multi-jets

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Thank you !