Threshold enhanced cross sections for colorless productions

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Advanced School & Workshop on Multiloop Scattering Amplitude

Based on: Phys.Rev.D 107 (2023) 3, 034038 (arXiv:2210.17534)

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Outlines

Introduction

- The Standard Model (SM) describes the physics of fundamental particles and their interactions.
- This is an exciting era of particle physics with high luminosity and high energy LHC.
- The SM prediction has been established through various collider experiments.
- Though it is a very successful theory, it is not the fundamental theory.
- There are some open problems (Higgs mass hierarchy problem, Dark matter and Dark energy problem etc) that can not be addressed by the SM.
- These new physics signals can either be seen from the heavy resonance or from the deviation of the SM signal.
- In both cases, one needs precise experimental as well as theoretical results.

Important Processes

- The DY process plays a key role in the measurement of parton distribution functions (PDFs) at the LHC.
- The DY process has clean final-state signature, which makes it an ideal candidate for luminosity measurements.
- The DY process has been used for detector calibration.
- VH Production process has importance in the observation of the Higgs decay $H \to b\bar{b}$
- This associated Higgs production is useful in probing the Higgs coupling to the weak gauge boson.
- If the coupling is deviating from the Standard Model (SM) prediction, then we can probe new physics (NP) from the coupling of the Higgs boson with the gauge boson.

Introduction

Drell-Yan process in Hadron level at the LHC

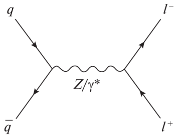
$$p\;p \rightarrow leptons + X$$

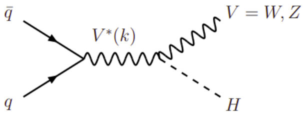
Higgs Strahlung process in Hadron level at the LHC

$$p\;p \rightarrow V H + X$$

DY and Higgs Strahlung process in Parton level at the LHC

$$q\bar{q} \rightarrow DY \qquad\qquad q\bar{q} \rightarrow VH$$





Hadron level Cross-section

Hadron level cross-section can be factorized as

$$Q^2 \frac{d\sigma}{dQ^2} = \sum_{a,b=q,\overline{q},g} \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \; f_a(x_1,\mu_F^2) \; f_b(x_2,\mu_F^2) \int_0^1 \mathrm{d}z \; \hat{\sigma}_{ab}(z,Q^2,\mu_F^2)\delta(\tau - z x_1 x_2),$$

The hadronic and partonic threshold variables τ and z are defined as

$$\tau = \frac{Q^2}{S}, \qquad z = \frac{Q^2}{\hat{s}}.$$

They are thus related by $\tau = x_1 x_2 z$. $f_a(x_1)$, $f_b(x_2)$ are the partonic distribution function (PDF) for the incoming partons.

Higgs Strahlung process in $q\bar{q}$-channel

For Drell-Yan process, the partonic cross section can be written as

$$\frac{d\hat{\sigma}}{dQ^2}(ab \to DY) = \hat{\sigma}(ab \to V^*) \frac{d\Gamma(V^* \to DY)}{dQ^2}$$

For VH production, the partonic cross section can be written as

$$\frac{d\hat{\sigma}}{dQ^2}(ab \to VH) = \hat{\sigma}(ab \to V^*) \frac{d\Gamma(V^* \to VH)}{dQ^2}$$

$$\hat{\sigma}_{ab}(z, Q^2, \mu_F) = \sigma^{(0)} \Big(\Delta_{ab}(z, \mu_F) \Big),$$

The general structure of n-th order perturbative partonic coefficient is

$$\Delta_{ab}^{(n)}(z; Q^2/\mu_R^2; Q^2/\mu_F^2) = C_\delta^{(n)} \delta(1-z) + \sum_{i=0}^{2n-1} A_i^{(n)} \mathcal{D}_i(z) + C_{reg}^{(n)} R_{ab}^{(n)}(z)$$

$\mathcal{D}_i = \left[\frac{\ln^i(1-z)}{1-z}\right]_+$ $\quad R_{ab}^{(n)}(z) =$ Regular Piece $\quad z = \frac{Q^2}{\hat{s}}$ $\quad \tau = \frac{Q^2}{S}$

Threshold Resummation

The n-th order partonic coefficient term is

$$\Delta_{ab}^{(n)}(z;Q^2/\mu_R^2;Q^2/\mu_F^2) = C_\delta^{(n)}\delta(1-z) + \sum_{i=0}^{2n-1} A_i^{(n)}\mathcal{D}_i(z) + C_{reg}^{(n)} R_{ab}^{(n)}(z)$$

$\mathcal{D}_i = \left[\frac{\ln^i(1-z)}{1-z}\right]_+$ $\quad R_{ab}^{(n)}(z) =$ Regular Piece $\quad z = \frac{Q^2}{\hat{s}}$ $\quad \tau = \frac{Q^2}{S}$

$$\int dx f(x)[g(x)]_+ = \int dx g(x)(f(x) - f(1))$$

When $z \to 1$ this $\mathcal{D}_i$ becomes large.

$n = 1$ $A_1^{(1)}\mathcal{D}_1(LL), A_0^{(1)}\mathcal{D}_0(NLL)$

$n = 2$ $A_3^{(2)}\mathcal{D}_3(LL), A_2^{(2)}\mathcal{D}_2(NLL), A_1^{(2)}\mathcal{D}_1(NNLL), A_0^{(2)}\mathcal{D}_0(N^3LL)$

$n = 3$ $A_5^{(3)}\mathcal{D}_5(LL), A_4^{(3)}\mathcal{D}_4(NLL), A_3^{(3)}\mathcal{D}_3(NNLL), A_2^{(3)}\mathcal{D}_2(N^3LL), A_1^{(3)}\mathcal{D}_1 N^4LL, \ldots$

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Threshold Resummation in Mellin Space

- The Mellin transformation with respect to τ is defined as
  $\sigma_N(Q^2) = \int_0^1 d\tau \tau^{N-1} \sigma(s, Q^2)$ $\qquad z \to 1 \equiv N \to \infty$

$$\Delta^{(res)}_{ab,N}(a_S(\mu_R^2), Q^2/\mu_R^2; Q^2/\mu_F^2) = g_0^{ab}(a_S(\mu_R^2), Q^2/\mu_R^2; Q^2/\mu_F^2) \times \exp\{\mathcal{G}_N(a_S(\mu_R^2), \ln N; Q^2/\mu_R^2, Q^2/\mu_F^2)\}.$$

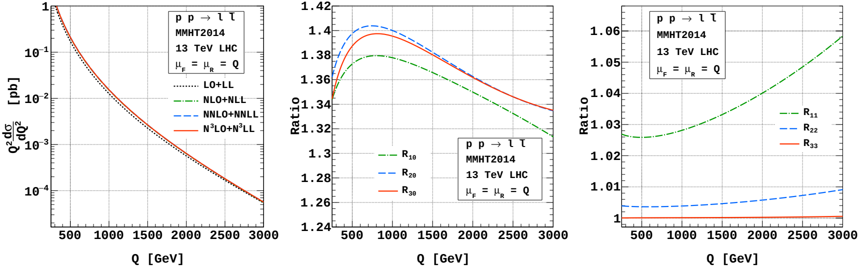
$$\mathcal{G}_N = \ln N g_N^{(1)}(\omega) + g_N^{(2)}(\omega, \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}) + a_S(\mu_R^2) g_N^{(3)}(\omega, \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}) + \ldots$$

$$g_0 = 1 + a_S \; g_{01} + a_S^2 \; g_{02} + a_S^3 \; g_{03} \,, \quad \omega = b_0 a_S(\mu_R^2) \ln N$$

- Mellin inversion and matching

$$\sigma^{\mathrm{N^nLO+N^nLL}} = \sigma^{\mathrm{N^nLO}} + \sigma^{(0)} \sum_{a,b \in \{q,\bar{q}\}} \int_{c-i\infty}^{c+i\infty} \frac{\mathrm{d}N}{2\pi i} \tau^{-N} f_{a,N}(\mu_F) f_{b,N}(\mu_F) \times \left(\hat{\sigma}_N^{\mathrm{N^nLL}} - \hat{\sigma}_N^{\mathrm{N^nLL}} \Big|_{\mathrm{tr}} \right)$$

Invariant mass and K-factor for neutral Drell-Yan (nDY)



(b) Das, CD, Kumar, Samanta 2023

$$K_{N^nLO} = \frac{(d\sigma/dQ)_{N^nLO}}{(d\sigma/dQ)_{LO}}$$

$$R_{ij} = \frac{(d\sigma/dQ)_{N^iLO+N^iLL}}{(d\sigma/dQ)_{N^jLO}}$$

Resum K-factor at $Q = 3000\text{GeV}$

$R_{10} = 1.31$

$R_{20} = 1.335$

$R_{30} = 1.335$

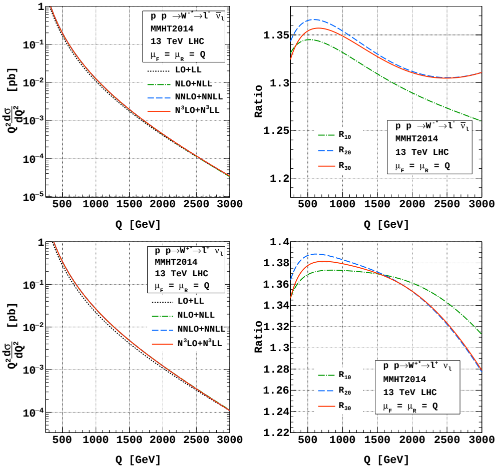
Resum K-factor at $Q = 3000\text{GeV}$

$R_{11} = 1.058$

$R_{22} = 1.010$

$R_{33} = 1.001$

Invariant mass and K-factor for charged Drell-Yan (cDY)



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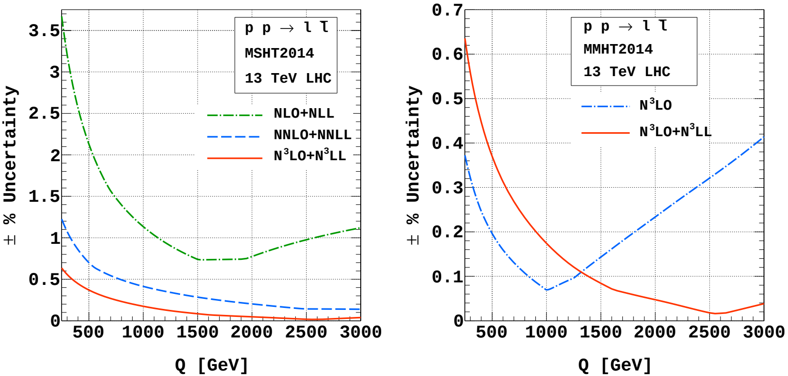
7-points Scale variation for DY



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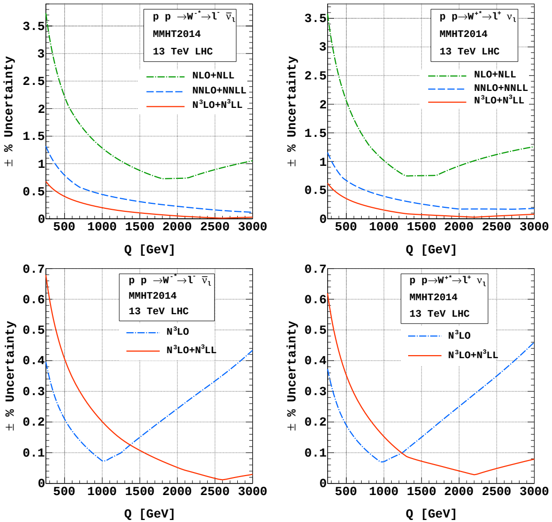
7-points Scale variation for charged DY



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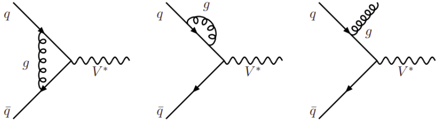
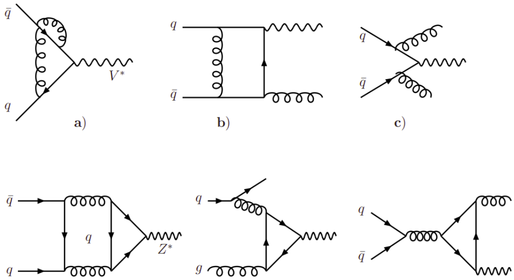
Higgs Strahlung process in $q\bar{q}$-channel



(e) NLO diagrams for $q\bar{q} \to V^*$

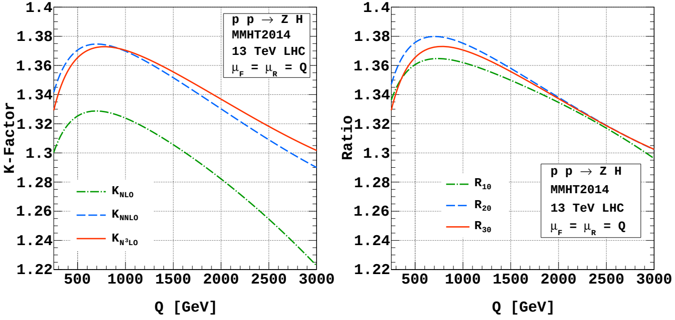


(f) NNLO diagrams for $q\bar{q} \to Z^*$

ZH production

- LO 0.6199 pb at 13 TeV LHC.
- NLO in QCD [Tao Han, et al., 1991] gives 26.12% correction.
- NNLO in QCD [O.Brein, et al., 2012] gives 2.63% correction.
- N^3LO_{sv} in QCD [M. C. Kumar, et al., 2015] gives 0.11% correction.
- N^3LO in QCD [J. Baglio, et al., 2022] gives -0.73% correction.
- NLO in EW [A. Denner, et al., 2012] gives -5.28% correction.

ZH Production for $q\bar{q}$-channel



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$$K_{N^nLO} = \frac{(d\sigma/dQ)_{N^nLO}}{(d\sigma/dQ)_{LO}}$$

$$R_{ij} = \frac{(d\sigma/dQ)_{N^iLO+N^iLL}}{(d\sigma/dQ)_{N^jLO}}$$

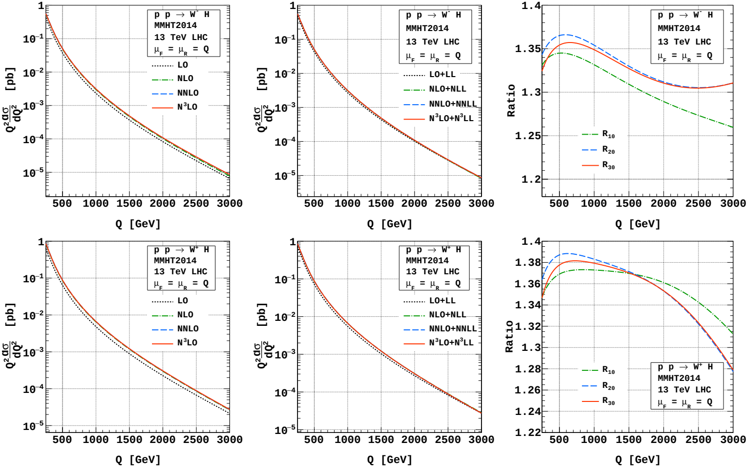
FO K-factor at $Q = 3000\text{GeV}$

$$K_{NLO} = 1.22$$
$$K_{NNLO} = 1.29$$
$$K_{N^3LO} = 1.30$$

Resummed K-factor at $Q = 3000\text{GeV}$

$$R_{10} = 1.295$$
$$R_{20} = 1.30$$
$$R_{30} = 1.30$$

Invariant mass and K-factor for WH Production



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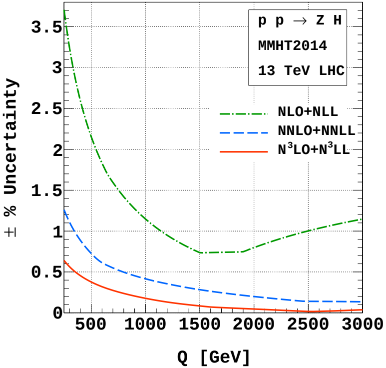
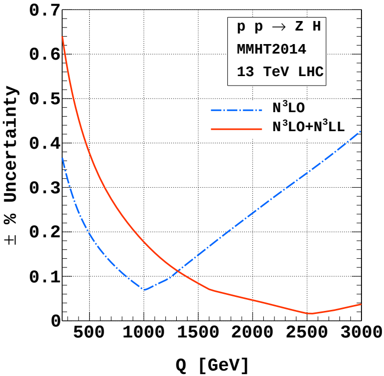
7-points Scale variation for ZH





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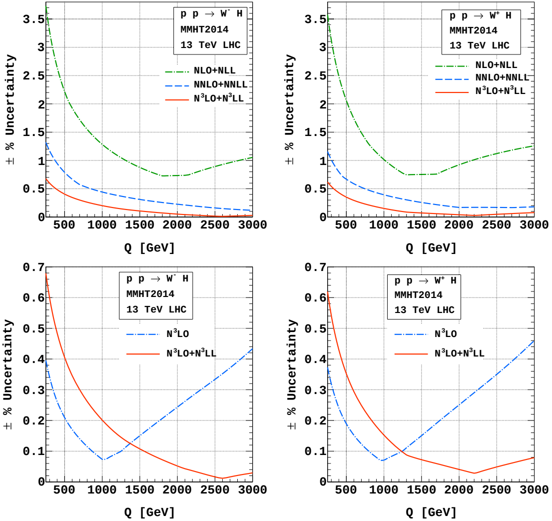
7-points Scale variation for WH



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FO and Matched results for ZH in different CM energy

$\sqrt{S}$ (TeV)	7.0	8.0	13.0	13.6	100.0
LO	$0.2363 \pm 0.36\%$	$0.2908 \pm 1.00\%$	$0.5934 \pm 3.81\%$	$0.6324 \pm 4.06\%$	$8.1105 \pm 13.57\%$
NLO	$0.3164 \pm 1.55\%$	$0.3878 \pm 1.50\%$	$0.7754 \pm 1.36\%$	$0.8245 \pm 1.36\%$	$9.1445 \pm 4.40\%$
NNLO	$0.3280 \pm 0.41\%$	$0.4017 \pm 0.37\%$	$0.8005 \pm 0.35\%$	$0.8508 \pm 0.36\%$	$9.1215 \pm 0.94\%$
N^3LO	$0.3266 \pm 0.25\%$	$0.3996 \pm 0.27\%$	$0.7943 \pm 0.32\%$	$0.8441 \pm 0.33\%$	$8.9790 \pm 0.49\%$
LO+LL	$0.2706 \pm 1.48\%$	$0.3319 \pm 1.45\%$	$0.6721 \pm 4.20\%$	$0.7158 \pm 4.44\%$	$9.0406 \pm 13.86\%$
NLO+NLL	$0.3260 \pm 4.34\%$	$0.3992 \pm 4.33\%$	$0.7966 \pm 4.31\%$	$0.8469 \pm 4.31\%$	$9.3550 \pm 5.39\%$
NNLO+NNLL	$0.3295 \pm 1.40\%$	$0.4034 \pm 1.43\%$	$0.8036 \pm 1.53\%$	$0.8542 \pm 1.53\%$	$9.1500 \pm 1.77\%$
N^3LO+N^3LL	$0.3266 \pm 0.46\%$	$0.3996 \pm 0.49\%$	$0.7943 \pm 0.57\%$	$0.8441 \pm 0.58\%$	$8.9795 \pm 0.75\%$

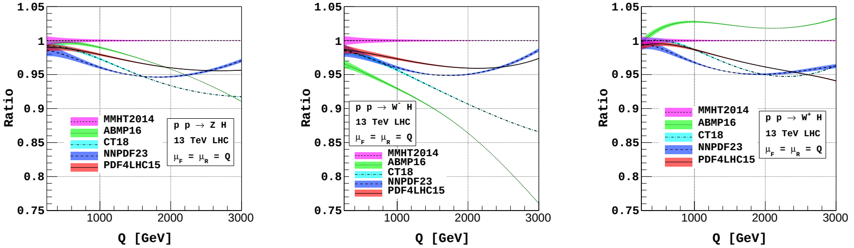
Full results for ZH in different CM energy

$$\sigma^{tot,ZH}_{\text{N}^3\text{LO}} = \sigma^{\text{DY,ZH}}_{\text{N}^3\text{LO}} + \sigma^{gg}(a_S^3) + \sigma^{\text{top}}(a_S^2) + \sigma^{b\bar{b}}$$

$$\sigma^{tot,ZH}_{\text{N}^3\text{LO+N}^3\text{LL}} = \sigma^{\text{DY,ZH}}_{\text{N}^3\text{LO+N}^3\text{LL}} + \sigma^{gg}(a_S^3) + \sigma^{\text{top}}(a_S^2) + \sigma^{b\bar{b}}$$

$\sqrt{S}$ (TeV)	$\sigma^{DY,ZH}_{\text{N}^3\text{LO}}$	$\sigma^{DY,ZH}_{\text{N}^3\text{LO+N}^3\text{LL}}$	$\sigma^{tot,ZH}_{\text{N}^3\text{LO}}$	$\sigma^{tot,ZH}_{\text{N}^3\text{LO+N}^3\text{LL}}$
7.0	$0.3266 \pm 0.25\%$	$0.3266 \pm 0.46\%$	$0.3534 \pm 1.25\%$	$0.3534 \pm 1.17\%$
8.0	$0.3996 \pm 0.27\%$	$0.3996 \pm 0.49\%$	$0.4373 \pm 1.36\%$	$0.4373 \pm 1.28\%$
13.0	$0.7943 \pm 0.32\%$	$0.7943 \pm 0.57\%$	$0.9112 \pm 1.79\%$	$0.9112 \pm 1.67\%$
13.6	$0.8441 \pm 0.33\%$	$0.8441 \pm 0.58\%$	$0.9728 \pm 1.86\%$	$0.9728 \pm 1.75\%$
100.0	$8.9790 \pm 0.49\%$	$8.9795 \pm 0.75\%$	$13.2674 \pm 4.57\%$	$13.2678 \pm 4.39\%$

PDF Variations



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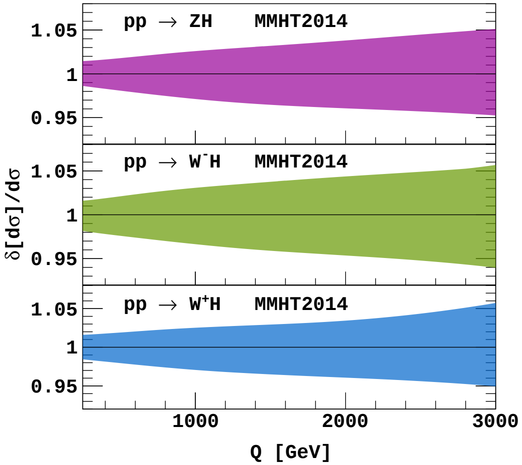
The intrinsic PDF variation



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Summary

- Threshold resummation for $q\bar{q}$-channel is shown up to $\mathrm{N^3LO{+}N^3LL}$ for total production cross-section of VH.
- Precise invariant mass distribution up to $\mathrm{N^3LO{+}N^3LL}$ for DY and VH production.
- The 7-point scale uncertainty reduced to less then 0.1% at high Q.
- Inclusive cross section for ZH production including $q\bar{q}$-channel up to $\mathrm{N^3LO{+}N^3LL}$, top-loops effect, gg-channel NLO in EFT and $b\bar{b} \to ZH$.

Thank you

Back Up Born

$$\sigma^{(0)}_{DY}(Q^2) = \frac{\pi}{n_c}\left[\mathcal{F}^{(0)}_{DY}(Q^2)\right], \qquad \text{with } DY \in \{nDY, cDY, ZH, WH\},$$

$$\sigma^{(0)}_{b\bar{b}H} = \frac{\pi m_b^2(\mu_R^2)\tau}{6M_H^2 v^2},$$

where,

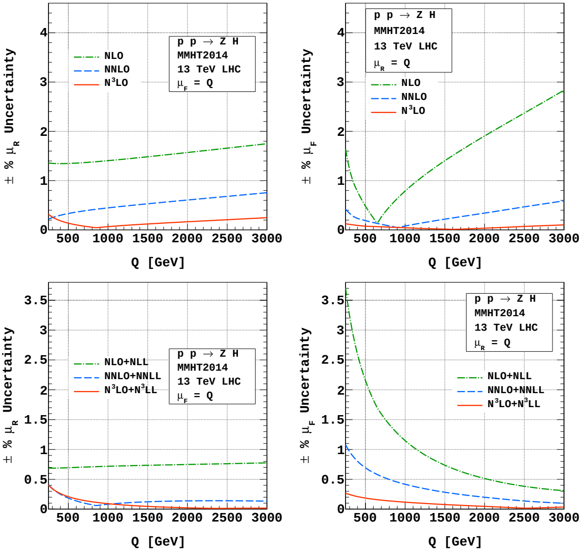
$$\mathcal{F}^{(0)}_{nDY}(Q^2) = \frac{4\alpha^2}{3S}\Bigg[Q_q^2 - \frac{2Q^2(Q^2-M_Z^2)}{\left((Q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2\right)c_w^2 s_w^2} Q_q g_e^V g_q^V$$
$$+ \frac{Q^4}{\left((Q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2\right)c_w^4 s_w^4}\left((g_e^V)^2+(g_e^A)^2\right)\left((g_q^V)^2+(g_q^A)^2\right)\Bigg],$$

$$\mathcal{F}^{(0)}_{cDY}(Q^2) = \frac{4\alpha^2}{3S}\left[\frac{Q^4|V_{qq'}|^2}{\left((Q^2-M_W^2)^2+M_W^2\Gamma_W^2\right)s_w^4}\left((g_e^{'V})^2+(g_e^{'A})^2\right)\left((g_q^{'V})^2+(g_q^{'A})^2\right)\right],$$

$$\mathcal{F}^{(0)}_{ZH}(Q^2) = \frac{\alpha^2}{S}\left[\frac{M_Z^2 Q^2 \lambda^{1/2}(Q^2,M_H^2,M_Z^2)\left(1+\frac{\lambda(Q^2,M_H^2,M_Z^2)}{12M_Z^2/Q^2}\right)}{((Q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2)c_w^4 s_w^4}\left((g_q^V)^2+(g_q^A)^2\right)\right],$$

$$\mathcal{F}^{(0)}_{WH}(Q^2) = \frac{\alpha^2}{S}\left[\frac{M_W^2 Q^2|V_{qq'}|^2 \lambda^{1/2}(Q^2,M_H^2,M_W^2)\left(1+\frac{\lambda(Q^2,M_H^2,M_W^2)}{12M_W^2/Q^2}\right)}{((Q^2-M_W^2)^2+M_W^2\Gamma_W^2)s_w^4}\left((g_q^{'V})^2+(g_q^{'A})^2\right)\right]$$

μ_R and μ_F Scale variation for ZH



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