Second Order Master Integrals to SIDIS

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Outline of the talk

- Introduction
- Processes relevant to our Cross-section at NNLO
- Calculation of Diagrams
- Calculation of Master Integrals
- Master Integrals
- Conclusions and Future directions

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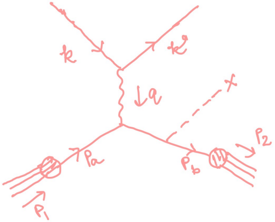
- Hadron production data tests our key concepts in QCD at high energies such as factorization, universality of splitting functions, and perturbative calculations.
- The Full NNLO calculation for semi-inclusive DIS will provide a significant result that will be helpful for the theoretical framework for precision studies of observables relevant to the Electron-Ion Collider(EIC).

Theoretical preliminaries



$$\frac{P_a}{P_1} = x' \quad \text{and} \quad \frac{P_2}{P_b} = z'$$

- We consider the semi-inclusive deep inelastic scattering process

$$h(P_1) + l(k) \longrightarrow l'(k') + h'(P_2) + X$$

On the Partonic level cross-section, we are considering processes like

$$q/g(P_a) + \gamma^*(q) \longrightarrow q/g(P_b) + X$$

where $P_a^2 = P_b^2$ =0 and q^2 =-Q^2. Our Kinematic variables are $x = \frac{Q^2}{2P_1.q} \Rightarrow x' = \frac{Q^2}{2P_a.q}$ and $z = \frac{P_1.P_2}{P_1.q} \Rightarrow z' = \frac{P_a.P_b}{P_a.q}$.

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- By imposing constraints on $W_{\mu\nu}$, like Lorentz covariance and gauge invariance, One can parametrize $W_{\mu\nu}$ as,

$$W_{\mu\nu} = W_1\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \frac{W_2}{P_1.q}\left(P_{1\mu} - \frac{P_1.q}{q^2}q_\mu\right)\left(P_{1\nu} - \frac{P_1.q}{q^2}q_\nu\right)$$

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- One can express Hadronic tensor into Partonic tensor by,

$$W_{\mu\nu} = \int dx_1 \hat{f}_a(x_1) \int dz_1 \hat{D}_b(z_1)[w_{\mu\nu}\left(\frac{x}{x_1}, \frac{z}{z_1}\right)]$$

where $\hat{f}_a$ is Parton distribution function and $\hat{D}_b$ is fragmentation function.

- The Above equation can be written as

$$W^{\mu\nu} = \int dx_1 \hat{f}_a(x_1) \int dz_1 \hat{D}_b(z_1) [(w_1 T_1^{\mu\nu} + w_2 x_1 T_2^{\mu\nu}) \left(\frac{x}{x_1}, \frac{z}{z_1}\right)]$$

$$W^{\mu\nu} = W_1 T_1^{\mu\nu} + W_2 T_2^{\mu\nu}$$

where $T_1^{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)$ and
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- One can calculate W_1 and W_2 by projecting out, $W_2 = P_2^{\mu\nu} W_{\mu\nu}$ and $W_1 = P_1^{\mu\nu} W_{\mu\nu}$.

- The Differential cross-section of the SIDIS can be expressed as

$$\frac{1}{xz}\frac{dW_1}{dz} = \int_x^1 \frac{dx_1}{x_1}(\hat{f}_a(x_1)) \int_z^1 \frac{dz_1}{z_1}\Big(\frac{\hat{D}_b(z_1)}{z_1}\Big)\Big\{\frac{1}{x'z'}\frac{d\hat{\sigma}^{(j)}_{w_1^{ab}}}{dz'}\Big\}$$

$$\frac{1}{xz}\frac{dW_2}{dz} = \int_x^1 \frac{dx_1}{x_1}(x_1\hat{f}_a(x_1)) \int_z^1 \frac{dz_1}{z_1}\Big(\frac{\hat{D}_b(z_1)}{z_1}\Big)\Big\{\frac{1}{x'z'}\frac{d\hat{\sigma}^{(j)}_{w_2^{ab}}}{dz'}\Big\}$$

$$\frac{1}{x'z'}\frac{d\hat{\sigma}_{w_i^{ab}}}{dz'} = \sum_{j=0}^{\infty} a_s^j(\mu_R^2)\left(\frac{1}{x'z'}\frac{d\hat{\sigma}^{(j)}_{w_i^{ab}}}{dz'}\right)$$

where $i = 1, 2$ and we are only considering up to NNLO. so $j = 0, 1, 2$ for LO, NLO, and NNLO respectively.

Processes relevant to our Cross-section at NNLO

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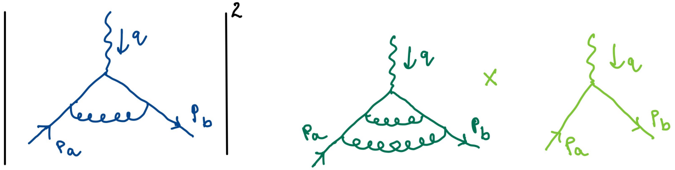
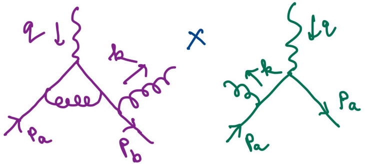
- Here are the relevant diagrams for our processes

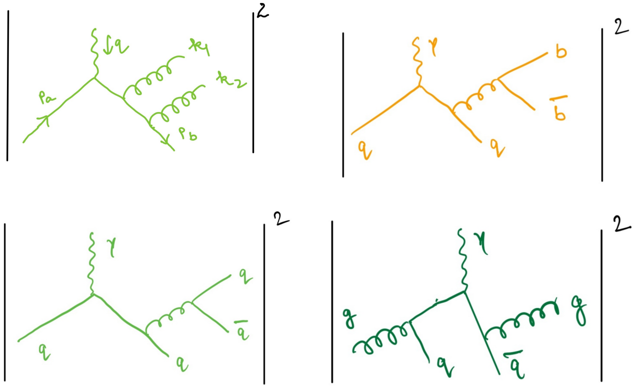
VV diagrams $q + \gamma^* \Rightarrow q(h)$,



RV diagrams $q + \gamma^* \Rightarrow q(h) + g$,



$$q+\gamma^* \Rightarrow q(h)+g+g,\; q+\gamma^* \Rightarrow q(h)+q+\bar{q},$$
$$q+\gamma^* \Rightarrow q(h)+b+\bar{b},\; g+\gamma^* \Rightarrow g(h)+q+\bar{q}.$$



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- To apply the Feynmann rules, perform $SU(n_c)$ color manipulation and d-dimensional Lorentz and spin algebra, we pass the resulting expression through various procedures based on $FORM$ and $Mathematica$.
- Using $Reduze$ package, we find appropriate loop momentum shifts for each Feynmann diagrams beyond tree level to classify them in one of the integral families.

- Before doing these routine procedures, We need to calculate phase space integrals using Reverse-Unitarity, where we will change all the delta functions into propagators. $\delta(p^2 - m^2) \rightarrow \frac{i}{p^2-m^2} - \underset{leave}{c.c.}$

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- The resulting expression contains Feynmann loop integrals which are reduced to a set of master integrals through $IBP\ identities$ with the help of LiteRed Package.

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- For the RV process, We have 2-pt, 3-pt, and 4-pt function diagrams. we introduced 3 families and got 7 MIs using LiteRed.One can derive results of these MIs by some variable manipulation of results of MIs of the DIS process.

Double real emission

- For RR process, we have many processes such as
  $q + \gamma^* \Rightarrow q + g + g, \quad q + \gamma^* \Rightarrow q + q + \bar{q},$
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  Any of these final state partons can hadronize.

- List of Propagators for RR process,
  $Pr1 = \frac{1}{(k_1 - p_1)^2}, \; Pr2 = \frac{1}{(k_1 - q)^2}, \; Pr3 = \frac{1}{(k_2 - p_1)^2}$

  $Pr4 = \frac{1}{(k_2 - q)^2}, \; Pr5 = \frac{1}{(k_1 - p_1 - q)^2}, \; Pr6 = \frac{1}{(k_2 - p_1 - q)^2}$

  $Pr7 = \frac{1}{(k_1 + k_2)^2}, \; Pr8 = \frac{1}{(k_1 + k_2 - p_1)^2}$

  • Below are the cut propagators.Except CPr4, all other cut propagators are coming from phase space delta functions . CPr4 is coming due to constraint delta function $\delta(z' - \frac{p_1.p_2}{p_1.q})$.

  $CPr1 = \frac{1}{(k_1)^2}, \quad CPr2 = \frac{1}{(k_2)^2}, \quad CPr3 = \frac{1}{(k_1 + k_2 - p_1 - q)^2}$

  $CPr4 = \frac{(zp - 1) * (p_1.q) + (k_1.p_1) + (k_2.p_1)}{(p_1.q)}$

Master Integrals -:

j[A01,1,1,1,1,0,0,0],j[A01,1,1,1,1,0,0,1],j[A01,1,1,1,1,1,1,1],

j[A02,1,1,1,1,1,1,1],j[A03,1,1,1,1,0,0,1],j[A03,1,1,1,1,1,1,1],

j[A05,1,1,1,1,1,1,1],j[A06,1,1,1,1,0,0,1],j[A06,1,1,1,1,0,0,2],

j[A06,1,1,1,1,0,1,1],j[A06,1,1,1,1,1,1,1],j[A07,1,1,1,1,1,1,1],

j[A08,1,1,1,1,0,1,1],j[A08,1,1,1,1,1,1,1],j[A09,1,1,1,1,0,1,1],

j[A09,1,1,1,1,1,1,1],j[A10,1,1,1,1,1,1,1],j[A12,1,1,1,1,1,1,1],

j[A13,1,1,1,1,1,1,1],j[A14,1,1,1,1,1,1,1],j[A15,1,1,1,1,1,1,1]

- To solve all RR processes we have introduced 15 families and doing IBP reduction we got 21 new MIs. We have solved these MIs using 2 different methods.

  (i) Conventional method of brute force by choosing the appropriate frame of reference.
  (ii) Differential equation method by calculating explicit Boundary conditions.

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  (i) Conventional method of brute force by choosing the appropriate frame of reference.
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  Method (i)

- We choose a suitable frame of Outgoing partons that are not tagged ($k_1 - k_2$ parton frame).

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- We have only $\int_0^\pi d\theta \int_0^\pi d\phi \frac{(\sin\theta)^{D-3}(\sin\phi)^{D-4}}{(a+b\cos\theta)(A+B\cos\theta+C\cos\phi\sin\theta)}$ as angular integration.

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- In the first 7 Integrals, $a = b$ and $A^2 = B^2 + C^2$. That's why we could solve these MIs using Brute force.

$$\int_0^\pi d\theta \int_0^\pi d\phi \frac{(\sin\theta)^{D-3}(\sin\phi)^{D-4}}{(1-\cos\theta)^i(1-\cos\psi\cos\theta - \sin\psi\cos\phi\sin\theta)^j}$$

$$= 2^{(1-i-j)}\pi \frac{\Gamma(\frac{D}{2}-1-j)\Gamma(\frac{D}{2}-1-i)}{\Gamma(D-2-i-j)} \frac{\Gamma(D-3)}{\Gamma(\frac{D}{2}-1)^2} F(i, j, \frac{D}{2}-1; \cos^2\frac{\psi}{2})$$

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- By using the same set of parametrization, we set linear differential equation system w.r.t both the kinematic variable in the form

$$\frac{d\overrightarrow{J}}{dx'} = M_{x'}(x', z', \epsilon)\overrightarrow{J}$$

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- Next, we calculated boundary conditions for these 21 Integrals explicitly and after solving these Integrals iteratively got full results for 21 MIs.

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- We used the Libra package, We went to canonical basis, where our differential system

became $\frac{d\overrightarrow{J}}{dx'} = M_{x'}(x',z',\epsilon)\overrightarrow{J} \quad \Rightarrow \quad \frac{d\overrightarrow{\tilde{J}}}{dx'} = \epsilon\widetilde{M_{x'}}(x',z')\overrightarrow{\tilde{J}}$

into ϵ-form. where $\widetilde{M_{x'}} = T^{-1}(M_{x'}T - \frac{dT}{dx'})$.

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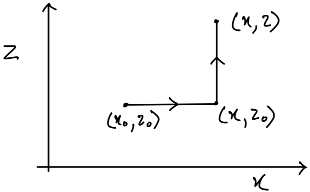
- path ordered diagram, we did line integration on this path



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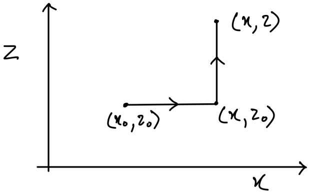

- We have calculated explicitly boundary conditions and expanded above order by order in ϵ and solved all the MIs iteratively.

- We get $J_i(x', z') = F_i(x', z', x_0 = 0, z_0 = 0) = F_i(x', z', \{J_i(0,0)\})$, since we are taking our starting point of path ordered integration is $(x_0 = 0, z_0 = 0)$.

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- To compute $J_i(1, 1)$, we calculated all the MIs in $(x' \to 1, z' \to 1)$ limit in $(k_1 - k_2)$ frame.
- After getting all the constants, we solved the full result $J_i(x', z') = F_i(x', z', \{B_i\})$.

- We got some Alphabets as arguments of iterative generalised PolyLog

$x', \quad (1-x'), \quad (1+x'), \quad z', \quad (1-z')$,

$r_i =$
$(x')^{1/2}, \quad (z')^{1/2}, \quad (z'-x'), \quad (z'+x'), \quad (1+x'z'),$
$((1-z')^2+4x'z')^{1/2}, \quad ((1+x')^2-4x'z')^{1/2}$,
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- we defined our set of GPL's can be written as

$$G(r_1,r_2,r_3,\lambda) = \int_0^{\lambda} \frac{d\lambda_1}{r_1(\lambda_1)} \int_0^{\lambda_1} \frac{d\lambda_2}{r_2(\lambda_2)} \int_0^{\lambda_2} \frac{d\lambda_3}{r_3(\lambda_3)}$$

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- We solved these GPL's numerically.

$\widetilde{M}_{x'}(x', z')$

$$
\left(\begin{array}{cccccccccccccccccccc}
\frac{ep}{(-1+xp)\,xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{ep}{(-1+xp)\,xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{ep}{(-1+xp)\,xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{ep}{(-1+xp)\,xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{2\,ep}{xp} & 0 & 0 & 0 & -\frac{ep}{2\,xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{16\,ep}{-1+xp} & \frac{ep\,(-1+zp)}{(-1+xp)\,(-xp+zp)} & 0 & 0 & -\frac{4\,ep}{xp-zp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{16\,ep}{xp-xp^2} & \frac{ep\,(-1+zp)}{(-1+xp)\,(xp-zp)} & 0 & 0 & \frac{4\,ep\,zp}{xp^2-xp\,zp} & 0 & -\frac{ep}{xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{8\,ep}{xp-xp^2} & \frac{ep\,(1+xp)}{4\,(-1+xp)\,xp} & 0 & 0 & 0 & 0 & 0 & -\frac{ep}{2\,xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{4\,ep}{(-1+xp)\,xp} & \frac{ep}{4-4\,xp} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{ep}{2\,xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{8\,ep\,(-1+xp)\,(-1+zp)\,\sqrt{zp}}{\sqrt{xp}\,(xp+zp+xp^2\,zp+xp\,zp^2)} & -\frac{ep\,\sqrt{zp}}{\sqrt{xp}\,(xp+zp)} & 0 & 0 & 0 & 0 & 0 & \frac{ep\,(-1+xp)\,(-1+zp)\,\sqrt{zp}}{\sqrt{xp}\,(xp+zp+xp^2\,zp+xp\,zp^2)} & 0 & -\frac{ep\,(xp+2\,zp+xp\,zp^2)}{2\,xp\,(xp+zp+xp^2\,zp+xp\,zp^2)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{16\,ep\,(xp+2\,zp+xp\,zp^2)}{xp\,(xp+zp+xp^2\,zp+xp\,zp^2)} & \frac{ep\,(-xp+zp)}{xp\,(xp+zp)} & 0 & 0 & 0 & 0 & 0 & \frac{2\,ep\,(-1+xp^2)\,zp}{xp\,(xp+zp+xp^2\,zp+xp\,zp^2)} & 0 & -\frac{ep\,(-1+xp)\,(-1+zp)\,\sqrt{zp}}{\sqrt{xp}\,(xp+zp+xp^2\,zp+xp\,zp^2)} & -\frac{ep}{xp+xp^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{16\,ep\,(-1+zp)}{(-1+xp)\,(xp-zp)} & -\frac{ep\,(-2+xp+zp)}{(-1+xp)\,(xp-zp)} & 0 & 0 & 0 & 0 & 0 & \frac{2\,ep}{xp-zp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{4\,ep\,(-1+xp)\,\sqrt{-1-xp^2+xp\,(-2+4\,zp)}}{xp\,(1+xp^2+xp\,(2-4\,zp))} & -\frac{ep\,(1+3\,xp)\,\sqrt{-1-xp^2+xp\,(-2+4\,zp)}}{4\,xp\,(1+xp^2+xp\,(2-4\,zp))} & 0 & 0 & -\frac{ep\,(-1+xp)\,\sqrt{-1-xp^2+xp\,(-2+4\,zp)}}{xp\,(1+xp^2+xp\,(2-4\,zp))} & 0 & 0 & \frac{ep\,(-1+xp)\,\sqrt{-1-xp^2+xp\,(-2+4\,zp)}}{2\,xp\,(1+xp^2+xp\,(2-4\,zp))} & 0 & 0 & 0 & 0 & -\frac{ep\,(1+xp-2\,zp)}{1+xp^2+xp\,(2-4\,zp)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{16\,ep}{xp} & \frac{ep}{xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2\,ep\,(-1+xp)\,\sqrt{-1-xp^2+xp\,(-2+4\,zp)}}{xp\,(1+xp^2+xp\,(2-4\,zp))} & -\frac{ep}{xp+xp^2} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{16\,ep\,(-1+zp)}{xp\,\sqrt{1+(-2+4\,xp)\,zp+zp^2}} & -\frac{ep\,(1+xp-zp+3\,xp\,zp)}{2\,(-1+xp)\,xp\,\sqrt{1+(-2+4\,xp)\,zp+zp^2}} & 0 & 0 & 0 & 0 & 0 & \frac{ep\,(-1+zp)}{xp\,\sqrt{1+(-2+4\,xp)\,zp+zp^2}} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{ep\,(-1+zp)^2}{xp\,(1+(-2+4\,xp)\,zp+zp^2)} & 0 & 0 & 0 & 0 & 0 \\
\frac{16\,ep}{xp-xp^2} & -\frac{ep}{2\,xp} & 0 & 0 & 0 & 0 & 0 & \frac{ep}{xp} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{ep\,(-1+zp)}{xp\,\sqrt{1+(-2+4\,xp)\,zp+zp^2}} & 0 & 0 & 0 & 0 & 0 \\
-\frac{16\,ep\,(xp-2\,zp+xp\,zp)}{(-1+xp)\,xp\,(xp-zp)} & \frac{ep\,(-zp+xp\,(-1+2\,zp))}{(-1+xp)\,xp\,(xp-zp)} & 0 & 0 & 0 & 0 & 0 & -\frac{2\,ep\,zp}{xp^2-xp\,zp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{ep}{xp} & 0 & 0 & 0 \\
\frac{16\,ep\,(-1+zp^2)}{xp+zp+xp^2\,zp+xp\,zp^2} & \frac{ep\,(xp^2+zp-3\,xp\,(1+zp))}{2\,(-1+xp)\,xp\,(xp+zp)} & 0 & 0 & 0 & 0 & 0 & \frac{ep\,(zp+xp^2\,zp+xp\,(-1+3\,zp^2))}{xp\,(xp+zp+xp^2\,zp+xp\,zp^2)} & \frac{2\,ep}{xp} & \frac{ep\,(1+xp)\,\sqrt{zp}\,(1+zp)}{\sqrt{xp}\,(xp+zp+xp^2\,zp+xp\,zp^2)} & 0 & 0 & 0 & 0 & -\frac{ep\,(1+zp)}{(-1+xp)\,\sqrt{1+(-2+4\,xp)\,zp+zp^2}} & 0 & 0 & \frac{ep}{(-1+xp)\,xp} & 0 & 0 \\
\frac{16\,ep\,zp}{xp^2-xp\,zp} & -\frac{ep\,(xp+zp)}{xp\,(xp-zp)} & 0 & 0 & 0 & 0 & 0 & \frac{2\,ep\,zp}{xp^2-xp\,zp} & -\frac{2\,ep}{xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{ep}{(-1+xp)\,xp} & 0 \\
0 & \frac{ep}{xp-zp} & 0 & 0 & \frac{4\,ep}{xp-zp} & 0 & 0 & -\frac{2\,ep}{xp} & -\frac{2\,ep}{xp} & 0 & 0 & 0 & -\frac{2\,ep\,(1+xp)\,\sqrt{-1-xp^2+xp\,(-2+4\,zp)}}{xp\,(1+xp^2+xp\,(2-4\,zp))} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{ep}{(-1+xp)\,xp}
\end{array}\right)
$$

$T(x', z')$

$$
\left(\begin{array}{cccccccccccccccccccc}
-\frac{(-1+xp)(-1+zp)}{xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{(1+ep)^2}{4\,ep^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{(1+ep)^2\,xp^2}{4\,ep^2\,(-1+xp)\,zp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{(1+ep)^2\,xp^2}{4\,ep^2\,(-1+xp)\,(-1+zp)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1+ep}{2\,ep} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2\,xp}{4\,ep^2\,(-1+zp)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2\,xp^2}{4\,ep^2\,(-1+zp)\,zp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2}{2\,ep^2} & -\frac{(1+ep)^2}{ep^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{2\,(1+ep)^2\,(-1+zp)}{zp} & \frac{(1+ep)^2\,(xp-zp)}{8\,zp} & 0 & 0 & 0 & 0 & 0 & \frac{1}{4}\,(1+ep)^2\,(1+xp) & \frac{(1+ep)^2\,(1+xp)\,(1+zp)}{4\,zp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{(1+ep)^2\,\sqrt{xp}}{4\,ep^2\,\sqrt{zp}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{(1+ep)^2\,xp^2}{4\,ep^2\,(1+xp)\,zp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2\,xp}{4\,ep^2\,(-1+zp)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2\,xp\,\sqrt{-1-xp^2+xp\,(-2+4\,zp)}}{2\,ep^2\,(1+xp^2+xp\,(2-4\,zp))} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2\,xp^2}{4\,ep^2\,(1+xp)\,(-1+zp)} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{(1+ep)^2\,xp}{2\,ep^2\,\sqrt{1+(-2+4\,xp)\,zp+zp^2}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2\,xp\,(-1+zp)}{4\,ep^2\,zp\,\sqrt{1+(-2+4\,xp)\,zp+zp^2}} & \frac{(1+ep)^2\,xp}{4\,ep^2\,zp} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2\,xp^2}{4\,ep^2\,(-1+zp)\,zp} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{(1+ep)^2\,xp^2\,(1+zp)}{4\,ep^2\,(-1+xp)\,zp\,\sqrt{1+(-2+4\,xp)\,zp+zp^2}} & 0 & 0 & -\frac{(1+ep)^2\,xp^2}{4\,ep^2\,(-1+xp)\,zp} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2\,xp^2}{4\,ep^2\,(-1+xp)\,zp} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{(1+ep)^2\,xp^2}{4\,ep^2\,(-1+xp)\,zp}
\end{array}\right)
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- We are going to do Mass factorization of our processes then we will have full results for SIDIS at NNLO.

Thank You !!!

UV and IR divergences

- Amplitude beyond LO has UV divergences. To cure this divergence, we need to first renormalize strong coupling constant through

$$\hat{a}_s S_\epsilon \left(\frac{1}{\mu^2}\right)^{\epsilon/2} = a_s(\mu_R^2)\left(\frac{1}{\mu_R^2}\right)^{\epsilon/2} Z_a(\mu_R^2)$$

where $Z_a(\mu_R^2) = 1 + a_s\left(\frac{2\beta_0}{\epsilon}\right) + a_s^2\left(\frac{4\beta_0^2}{\epsilon^2} + \frac{\beta_1}{\epsilon}\right) + O(a_s^3)$.

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- Divergences coming in Γ and $\tilde{\Gamma}$ will be totally absorbed in Splitting function $\hat{f}_a$ and Framentation function $\hat{D}_b$ respectively .

- At NNLO level, one can solve and find out that

$$\left(\frac{1}{\mu_F^2}\right)^{\epsilon}\hat{\triangle}^{(2)}_{qq} + \frac{2\beta_0}{\epsilon}\left(\frac{1}{\mu_F^2}\right)^{\epsilon/2}\hat{\triangle}^{(1)}_{qq} = \delta(1-x')\otimes\triangle^{(2)}_{qq}\otimes\frac{1}{z'}\delta(1-z')$$
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- After doing Mass factorization, we removed all the poles and got finite cross-section .