NNLO QCD corrections to SIDIS process

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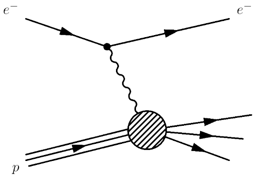
Content

Semi-inclusive DIS

DIS: $l(k_l) + H(P) \to l(k'_l) + X$

SIDIS: $l(k_l) + H(P) \to l(k'_l) + H'(P_H) + X'$



- In DIS, only the scattered lepton is detected while the remnants of the shattered nucleon are ignored i.e inclusive.
- In SIDIS, in addition to the scattered lepton one of the final-state hadron is also detected.
- In inclusive DIS, if one of the hadron is tagged with a specific momenta we get SIDIS i.e. with extra constrain on the phase space of out going particles.

Motivation

- Semi Inclusive Deep Inelastic Scattering (SIDIS) helps to study hadron structure and hadron fragmentation phenomena.
- With increase in experimental precision, one demands precise calculation from theoretical side as well.
- Till date there is no explicit calculation on full next-to-next-to leading order (NNLO) QCD correction to SIDIS process[1], however results in threshold limit are known[2].
- We calculated NNLO corrections in QCD strong coupling constant for SIDIS process using Feynman diagrammatic approach.
- Precision measurement of this process can have great impact on determination of various hadronic observables at upcoming EIC[3].

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[1] NLO by Alteralli et.al. in 1979

[2] W. Vogelsang et.al.{arXiv:2109.00847}

[3] I. Borsa et.al.{arXiv:1902.10663}

Semi-inclusive DIS

DIS: $l(k_l) + H(P) \rightarrow l(k'_l) + X$

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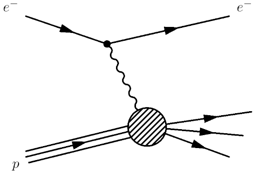


- In inclusive DIS, if one of the hadron is tagged with a specific momenta we get SIDIS i.e. with extra constrain on the phase space of out going particles.
- We have calculated NNLO corrections in QCD to differential SIDIS hadronic cross section.
- We consider photon(γ^*) as the mediator of interaction between lepton and the hadron.

Hadronic Cross section

Differential Hadronic cross section for
$e^-(k_l) + H(P) \to e^-(k_l') + H'(P_H) + X'$ is written as,

$$\frac{d^2\sigma_{e^-H}}{dE_l' d\Omega dz} = \frac{E_l'}{E_l}\frac{\alpha_e^2}{Q^4} L^{\mu\nu}(k_l, k'_l, q) W_{\mu\nu}(q, P, P_H).$$

Using the property of Lorentz invariance, current conservation and symmetry properties (like parity), $W_{\mu\nu}$ tensor can be parametrized in terms of structure functions F_1 and F_2.

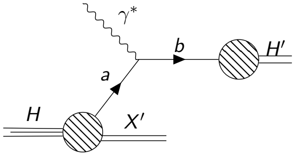
$$W^{\mu\nu} = F_1 \underbrace{\left[-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right]}_{T_1^{\mu\nu}} + F_2 \underbrace{\left[\frac{1}{P.q}(P^\mu - \frac{P.q}{q^2}q^\mu)(P^\nu - \frac{P.q}{q^2}q^\nu)\right]}_{T_2^{\mu\nu}}$$

Now we'll focus on calculation of these SFs F_I.

Structure functions

These F_I are Lorentz invariants which cannot be calculated in perturbation theory. We'll use Parton model to calculate them.
In parton model, we express F_I, $\{I = 1, 2\}$ as,

$$F_I = x^{I-1} \sum_{a,b} \int_x^1 \frac{dx_1}{x_1} f_a(x_1, \mu_F^2) \int_z^1 \frac{dz_1}{z_1} D_b(z_1, \mu_F^2) \times \mathcal{F}_{I,ab}\left(\frac{x}{x_1}, \frac{z}{z_1}, Q^2, \mu_F^2\right).$$



- $f_a dx_1$: The probability of finding a parton of type 'a' which carries a momentum fraction x_1 of the parent hadron H.
- $D_b dz_1$: The probability that a parton of type 'b' will fragment into hadron H' which carries a momentum fraction z_1 of the parton.
- $\mathcal{F}_{I,ab}$ are the finite coefficient functions (CFs) that can be computed perturbatively, it is related to partonic cross section.

Kinematics

Hadronic Kinematics :

P : Initial hadron momenta

k_l : Initial e^- momenta

q : Virtual photon momenta

q^2 : $-Q^2 < 0$

P_H : Momenta of hadron H'

y : $\frac{P.q}{P.k_l}$, Fractional energy loss by e^-

x : $\frac{Q^2}{2P.q}$, Bjorken-x, z : $\frac{P.P_H}{P.q}$

Partonic Kinematics :

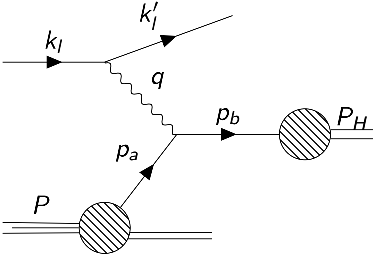
p_a : parton 'a' momenta

p_b : Tagged parton momenta 'b'

x_1 : $\frac{p_a}{P}$, z_1 : $\frac{P_H}{p_b}$

x' : $\frac{x}{x_1} = \frac{Q^2}{2p_a.q}$, z' : $\frac{z}{z_1} = \frac{p_a.p_b}{p_a.q}$

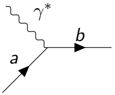
k_i : Momentum of real radiations



Coefficient functions

Computation of CFs ($\mathcal{F}_{I,ab}$) starts from the parton level cross section denoted by $\hat{\sigma}_{I,ab}$, where we defined,

$$\hat{\sigma}_{I,ab} = \frac{\mathcal{P}_I^{\mu\nu}}{4\pi} \int \mathrm{dPS}_{X'+b} \overline{\Sigma} |M_{ab}|^2_{\mu\nu} \, \delta\Big(\frac{z}{z_1} - \frac{p_a \cdot p_b}{p_a \cdot q}\Big)$$



here, $\mathcal{P}_I^{\mu\nu}$ are the projectors to project out CFs and $|M_{ab}|^2$ is the squared amplitude for the process $a(p_a) + \gamma^*(q) \to$ "b" $(p_b) + X'$.

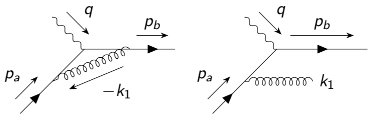
$$\mathcal{P}_1^{\mu\nu} = \frac{1}{(D-2)}\big(T_1^{\mu\nu} + 2x T_2^{\mu\nu}\big)$$

$$\mathcal{P}_2^{\mu\nu} = \frac{2x}{(D-2)x_1}\big(T_1^{\mu\nu} + 2x(D-1)\,T_2^{\mu\nu}\big).$$

Partonic Cross section

Computation of partonic cross section involves amplitude calculation of feynman diagrams order by order. Beyond leading order PCS gets contribution involving loops diagrams as well as real emission diagrams which gives divergent integrals. Types of divergences:

- UV divergence: Due to high momentum in the loop integral, the integral diverges, which are removed by renormalization of the strong coupling at μ_R scale.
- IR divergence: Present due to massless particles, further two types- soft and collinear.



using Dimensional Regularization($D = 4 + \epsilon$),
$a_s(\mu_R^2) = \frac{\alpha_s(\mu_R^2)}{4\pi}$

$$\frac{1}{(p_a - k_1)^2} = \frac{-1}{2p_a^{(0)}k_1^{(0)}(1 - cos\theta)}$$

$$k_1^{(0)} \to 0(soft) \quad \text{or} \quad \theta \to 0(collinear)$$

Partonic Cross section

- Infrared divergences cancels among virtual and real emission processes, except for the collinear divergences related to the a and b partons in the initial state and the final fragmentation state.
- These divergences can be factored out into Altarelli-Parisi (AP) kernels (mass factorisation) at μ_F scale,

$$\frac{\hat{\sigma}_{I,ab}(\epsilon)}{x'^{I-1}} = \Gamma_{c\leftarrow a}(\mu_F^2,\epsilon) \otimes \mathcal{F}_{I,cd}(\mu_F^2,\epsilon) \tilde{\otimes} \tilde{\Gamma}_{b\leftarrow d}(\mu_F^2,\epsilon), \qquad [f\otimes g](x) = \int_x^1 \frac{dt}{t} f(t) g(\frac{x}{t})$$

here, $\Gamma_{c\leftarrow a}(\mu_F^2,\epsilon)$ and $\tilde{\Gamma}_{b\leftarrow d}(\mu_F^2,\epsilon)$ are kernels (divergent) corresponding to initial and final leg respectively.

$$\Gamma_{a\leftarrow b}(x',\mu_F^2,\epsilon) = \delta_{ab}\delta(1-x') + \sum_{i=1}^{\infty} a_s^i(\mu_F^2)\Gamma^{(i)}_{a\leftarrow b}(x',\epsilon),$$

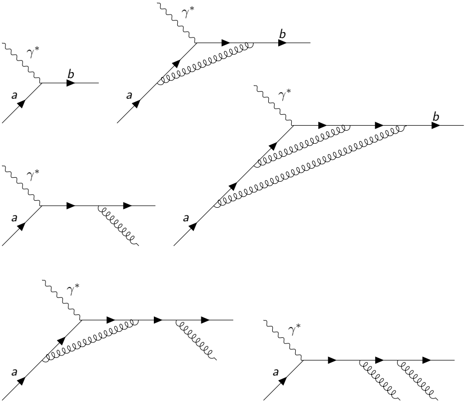
$$\tilde{\Gamma}_{a\leftarrow b}(z',\mu_F^2,\epsilon) = \delta_{ab}\delta(1-z') + \sum_{i=1}^{\infty} a_s^i(\mu_F^2)\tilde{\Gamma}^{(i)}_{a\leftarrow b}(z',\epsilon).$$

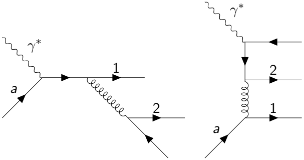
SIDIS subprocesses

LO		$\gamma^* q \to q$
NLO	1 Loop:(V)	$\gamma^* q \to q$
		$\gamma^* q \to q + g$
		$\gamma^* g \to q + \overline{q}$
	2 Loop:(VV)	$\gamma^* q \to q$
NNLO	1 Loop:(RV)	$\gamma^* q \to q + g$
		$\gamma^* q \to q + g + g$
		$\gamma^* q \to q + q_i + \overline{q}_i$
	1 Loop:(RV)	$\gamma^* g \to q + \overline{q}$
		$\gamma^* g \to q + \overline{q} + g$

SIDIS subprocesses: Sample diagrams

Quark initiated:





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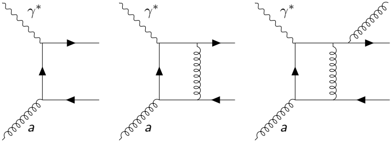


Computation Steps: Flow chart

- Generation of set of Feynman diagrams using 'QGRAF', which gives partonic level diagrams in symbolic form.
- Convert the output of QGRAF in 'FORM' form to get amplitude for individual diagrams using in-house codes.
- Used FORM extensively to do symbolic calculation like Lorentz contractions, Dirac algebra, handling Gell-Mann matrices.
- Used FORM for calculation of the $\mathcal{F}_1$ and $\mathcal{F}_2$.

To get the cross section we have to perform Loop integrations and Phase space integrations (done in D=$4+\epsilon$, dimensional regularisation).

Loop Integrals

Using, the fact that the integral of a total derivative vanishes within DR and the property of scaleless integral, one gets Integration-by-parts identities to write Loop integrals in terms of the basis of integrals called Master Integrals (MIs).

$$\int d^D l \frac{\partial}{\partial l^\mu}\left[\frac{l^\mu, p^\mu}{D_1^{\nu_1} D_2^{\nu_2} \ldots D_n^{\nu_n}}\right] = 0$$

Following steps are performed,

- Choose Integral families,
- Map the loop integrals (large set) onto these Integral families by shifting of momenta ('Reduze').
- Then reduce these integrals into basis of integrals i.e. MIs, we used 'LiteRed' package which produces IBP reduction rules.
- Solve MIs.

Phase Space Integrals

3-Body Phase Space, $p_a + q \to$ "p_b" $+ k_1 + k_2$,

$$\int [dPS]_3 =$$

$$\frac{1}{(2\pi)^{2D-3}} \int d^D k_1 \int d^D k_2 \int d^D p_b \delta(k_1^2)\delta(k_2^2)\delta(p_b^2)\delta^D(p_a + q - p_b - k_1 - k_2)\delta(z' - \frac{p_a.p_b}{p_a.q})$$

To solve the Phase space integrals, we used Reverse Unitarity method for converting phase-space integral into loop integral and performing reduction to get set of Master Integrals.

$$\delta(p^2 - m^2) \to \frac{i}{p^2 - m^2} - \underbrace{c.c.}_{\text{can be almost forget}}$$

We get total '21' MIs in phase space calculation. Results of these MI's are not there in literature, so we solved them using different methods. The calculation of MIs will be elaborated in next talk by Vaibhav.

Cross section Calculation

- Expanding the results of master integrals up to sufficient order in ϵ.
- Then substituted them into cross section calculation to obtain the divergent partonic cross section (in $\epsilon \to 0$).
- To separate the singularities at $x' \to 1$ or $z' \to 1$, we use Plus distribution. For eg.,

$$\frac{(1-x)^\epsilon}{1-x} = \frac{1}{\epsilon}\delta(1-x) + \sum_{i=0}^{\infty} \frac{\epsilon^i}{i!}\left[\frac{\ln^i(1-x)}{1-x}\right]_+$$
$$= \frac{1}{\epsilon}\delta(1-x) + \sum_{i=0}^{\infty} \frac{\epsilon^i}{i!} D_i(x)$$

$$\int_0^1 dx f(x)[g(x)]_+ = \int_0^1 dx \big(f(x) - f(1)\big) g(x)$$

Double distribution,

$$\int_0^1 dx \int_0^1 dz \frac{f(x,z)}{[1-x]_+[1-z]_+} =$$
$$\int_0^1 dx \int_0^1 dz \frac{f(x,z) - f(1,z) - f(x,1) + f(1,1)}{(1-x)(1-z)}$$

Cross section Calculation

We also encountered $(z' - x')^{a\epsilon - b}$, $(1 - z' - x')^{c\epsilon - d}$ which were taken care by partial fractioning and used theta function to separate different sectors.

- According to the Feynman $+i\varepsilon$ prescription of propagators, we added imaginary part to the scaling variables accordingly, $x' \equiv x' - i\varepsilon$ and $z' \equiv z' - i\varepsilon$.
- Using property of Heaviside theta function i.e. $\theta(y) + \theta(1 - y) = 1$, we can write, for eg.,

$$\left(\frac{z' - x'}{1 - x'}\right)^{\epsilon} = \left|\frac{z' - x'}{1 - x'}\right|^{\epsilon} \Big(\theta(z' - x') + (-1 + i\varepsilon)^{\epsilon}\, \theta(x' - z')\Big)$$

$$\left(\frac{1 - z'}{1 - z' - x'}\right)^{\epsilon} = \left|\frac{1 - z'}{1 - z' - x'}\right|^{\epsilon} \Big(\theta(1 - z' - x') + (-1 - i\varepsilon)^{\epsilon}\, \theta(z' + x' - 1)\Big)$$

Cross section Calculation

- Pole structure of the cross section is $\frac{1}{\epsilon^{2L}}$.
- Collect all the results of subprocesses and do the coupling constant renormalization to get UV finite result.
- After adding all the renormalized subprocesses, we observed the cancellation of $\frac{1}{\epsilon^4}$, $\frac{1}{\epsilon^3}$ at NNLO and $\frac{1}{\epsilon^2}$ at NLO.
- The left-over pole terms due to initial state and final state collinear singularities vanishes after mass factorisation procedure.

Mass Factorization

After coupling constant renormalization at scale μ_F, we can write mass factorisation, for eg. at NLO for $a, b = q$ process:

$$(\frac{1}{\mu_F^2})^{\frac{\epsilon}{2}} \hat{\sigma}^{(1)}_{1,qq} = \delta(1-x') \otimes \mathcal{F}^{(1)}_{1,qq} \tilde{\otimes} \delta(1-z') + \delta(1-x') \otimes \mathcal{F}^{(0)}_{1,qq} \tilde{\otimes} \tilde{\Gamma}^{(1)}_{qq} + \Gamma^{(1)}_{qq} \otimes \mathcal{F}^{(0)}_{1,qq} \tilde{\otimes} \delta(1-z'),$$

at NNLO for $a, b = q$ process:

$$(\frac{1}{\mu_F^2})^{\epsilon} \hat{\sigma}^{(2)}_{1,qq} + \frac{2\beta_0}{\epsilon} (\frac{1}{\mu_F^2})^{\frac{\epsilon}{2}} \hat{\sigma}^{(1)}_{1,qq} = \delta(1-x') \otimes \mathcal{F}^{(2)}_{1,qq} \tilde{\otimes} \delta(1-z') + \delta(1-x') \otimes \mathcal{F}^{(1)}_{1,qb'} \tilde{\otimes} \tilde{\Gamma}^{(1)}_{qb'} + \delta(1-x') \otimes \mathcal{F}^{(0)}_{1,qq} \tilde{\otimes} \tilde{\Gamma}^{(2)}_{qq} + \Gamma^{(1)}_{a'q} \otimes \mathcal{F}^{(0)}_{1,a'b'} \tilde{\otimes} \tilde{\Gamma}^{(1)}_{qb'} + \Gamma^{(1)}_{a'q} \otimes \mathcal{F}^{(1)}_{a'q} \tilde{\otimes} \delta(1-z') + \Gamma^{(2)}_{qq} \otimes \mathcal{F}^{(0)}_{1,qq} \tilde{\otimes} \delta(1-z').$$

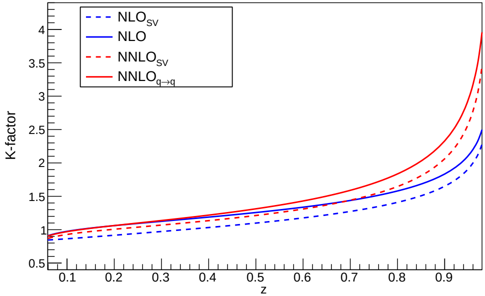
Checks and Results

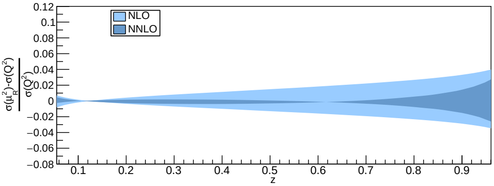
- We calculated NNLO QCD corrections to SIDIS process, which requires evaluation of new master integrals.
- We checked the result of MIs using different methods (brute force method and differential equation method).
- Since Infrared structure is universal, after mass factorization we get finite result.
- We agreed SV Limit of our finite partonic cross section results (in $\mathcal{F}_1$) i.e. terms containing $\{\delta(1-x'), D_i(x')\} \times \{\delta(1-z'), D_j(z')\}$ against result known[2].
- Inclusive results are obtain by integrating z', we agreed with N_c^2 part of the known results in literature[4].

[2] W. Vogelsang et.al.{arXiv:2109.00847}

[4] W.L. van Neerven and E.B. Zijlstra, Phys.Lett.B 272,127(1991).

Plot





We used test realistic model distributions[5]:

$$xq(x,\mu_F^2) = 0.6x^{-0.3}(1-x)^{3.5}(1+5.0x^{0.8}),$$
$$xg(x,\mu_F^2) = 1.6x^{-0.3}(1-x)^{4.5}(1-0.6x^{0.3}).$$

[5] S. Moch et.al.{arXiv:0404111}

Conclusion and Future Plans

- We calculated NNLO QCD corrections to SIDIS process using Feynman diagrammatic approach.
- We calculated 21 new master integrals using two different approaches.
- New NNLO results display a moderate increase of the K-factor.
- Noticed significant reduction in dependence of renormalization scales.
- To calculate polarized SIDIS cross section.
- To develop Threshold resummation framework for SIDIS.

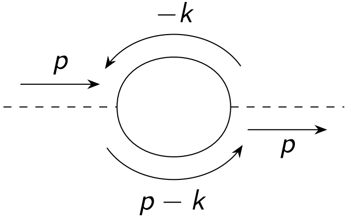
Thank you for listening :)

Back up slides:

Partonic Cross section

Computation of partonic cross section involves amplitude calculation of feynman diagrams order by order, Beyond Leading order PCS gets contribution involving loops diagrams as well as real emissions which gives divergent integrals. Types of divergences:

- UV Divergence: Due to high momentum (or energy) the loop integral divergences, eg.



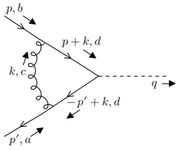
$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m^2 + i\epsilon)((p-k)^2 - m^2 + i\epsilon)}$$

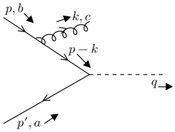
Above one is Logarithmic divergent by naive power counting.

Divergences in Partonic Cross section

IR divergence: Due to presence of massless particles,

- Soft Divergence: This occurs when one of the propagator corresponds to massless particle the divergences occurs due to low momentum OR when massless particles are involved (real) and their momentum is approaching zero then also divergences occurs in phase space integration.
- Collinear Divergence: When massless particle emits(radiates) another massless particle and the 3-Momentum of both these particles becomes parallel (we say they become collinear to each other), develop this kind of divergences (also called mass divergence).





$$\frac{1}{(p-k)^2} = \frac{-1}{2p^{(0)}k^{(0)}(1-\cos\theta)}$$

$k^{(0)} \to 0$ (soft) or $\theta \to 0$ (collinear)

Perturbative expansion

Coefficient functions ($\mathcal{F}_{I,ab}$) can be computed in perturbative expansion in powers of strong coupling $a_s(\mu_F^2)$,

$$\mathcal{F}_{I,ab} = \sum_{i=0}^{\infty} a_s^i(\mu_F^2)\mathcal{F}_{I,ab}^{(i)}(\mu_F^2)$$

Also, UV finite and IR divergent partonic cross section can be written as,

$$\hat{\sigma}_{I,ab} = \hat{\sigma}_{I,ab}^{(0)} + \left(\frac{1}{\mu_F^2}\right)^{\frac{\epsilon}{2}} a_s\ \hat{\sigma}_{I,ab}^{(1)} + \left(\frac{1}{\mu_F^2}\right)^{\frac{\epsilon}{2}} a_s^2\ \hat{\sigma}_{I,ab}^{(1)} + \left(\frac{1}{\mu_F^2}\right)^{\epsilon} a_s^2\ \hat{\sigma}_{I,ab}^{(2)} + O(a_s^3)$$

IBP

Lets Consider One Loop Vacuum massive Integral,

$$I_a = \int \frac{d^D k}{(k^2 - m^2)^a}$$

Using IBP identity,i.e.

$$\int d^D k \frac{\partial}{\partial k^\mu} [\frac{k^\mu}{(k^2 - m^2)^a}] = 0$$

we get,

$$(D - 2a)I_a - 2am^2 I_{a+1} = 0$$

$$I_a = \frac{(D - 2a + 2)}{2(a - 1)m^2} I_{a-1}$$

So any integral with $a > 1$ can be expressed recursively in terms of one integral I_1 (MI)