Effects of NLP Radiation on Angular Ordering

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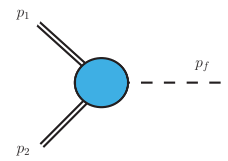
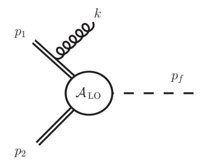
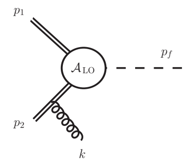
Threshold Variable



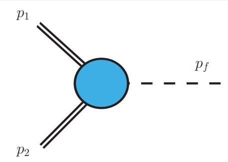


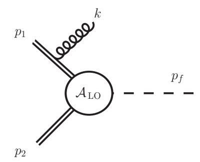


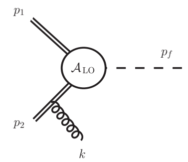
$$s = Q^2$$

$$s' = (p_1 + p_2 - k)^2 = zs = Q^2$$

Threshold Variable







$$s = Q^2$$

$$s' = (p_1 + p_2 - k)^2 = zs = Q^2$$

Threshold limit: $z \to 1$

What is NLP?

A general scattering at LO plus a radiation. For a generic threshold variable $(1-z)$,

$$\frac{d\sigma}{dz} = \sum_{n=0}^{\infty} \alpha_s^n \Bigg\{ \sum_{m=0}^{2n-1} C_{nm} \left(\frac{\log^m(1-z)}{(1-z)} \right)_+ + C_{nm} \log^m(1-z) + d_n \delta(1-z) \Bigg\} .$$

Leading Power Contributions

$$\frac{d\sigma}{dz} = \sum_{n=0}^{\infty} \alpha_s^n \Bigg\{ \sum_{m=0}^{2n-1} C_{nm} \underbrace{\left(\frac{\log^m(1-z)}{(1-z)}\right)_+}_{\text{LP}} + C_{nm} \log^m(1-z) + d_n \underbrace{\delta(1-z)}_{\text{LP}} \Bigg\}.$$

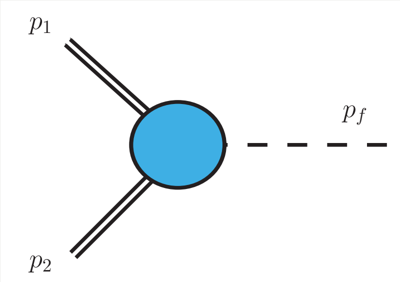
- Universal term. (process independent)
- Well studied in the Literature.
- Resummation methods are well known.

Next to Leading Power Contributions

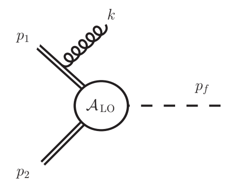
$$\frac{d\sigma}{dz} = \sum_{n=0}^{\infty} \alpha_s^n \left\{ \sum_{m=0}^{2n-1} C_{nm} \left(\frac{\log^m(1-z)}{(1-z)} \right)_+ + C_{nm} \underbrace{\log^m(1-z)}_{\text{NLP}} + d_n \delta(1-z) \right\}.$$

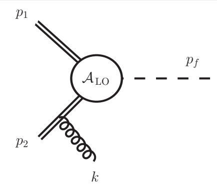
- Suppressed as compared to leading power.
- Singular in limit $z \to 1$
- Impact on numerical calculations.
- No general resummation method.

Colourless Final state: LO



Radiation of soft gluon (LP)

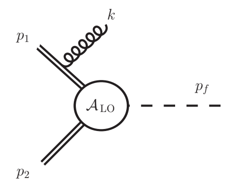


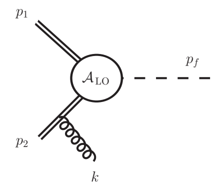


LP: $k = 0$

$$\mathcal{A}^\sigma = \sum_{i=1}^{2} \mathbf{T}_i \frac{p_i^\sigma}{p_i \cdot k} \mathcal{A}_{\text{LO}}$$

Radiation of soft gluon (LP)





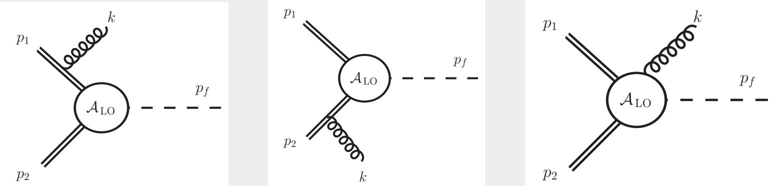
LP: $k = 0$

$$\mathcal{A}^\sigma = \sum_{i=1}^{2} \mathbf{T}_i \frac{p_i^\sigma}{p_i \cdot k} \mathcal{A}_{\text{LO}}$$

Singular: $k = 0$ or $k \,||\, p_i$.

Radiation of Next-to-soft gluon

$k \to 0$



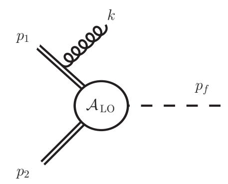
$$\mathcal{A}^{\sigma}_{\rm NLP} = \sum_{i=1}^{2} \mathbf{T}_i \left(\frac{2\, p_i^{\sigma} - k^{\sigma}}{2\, p_i \cdot k} - \frac{i k_{\alpha} \Sigma_i^{\sigma\alpha}}{p_i \cdot k} - \frac{i k_{\alpha} L_i^{\sigma\alpha}}{p_i \cdot k} \right) \otimes \mathcal{A}_{\rm LO}$$

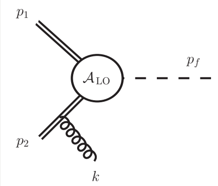
LKBD theorem: Low 1958, Knoll and Burnell 1968, Del-Duca 1990

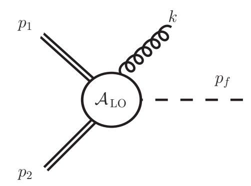
Alternative derivation: using factorization 1503.05156, 1610.06842

Radiation of Next-to-soft gluon

$k \to 0$







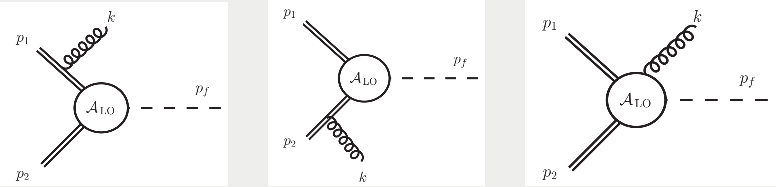
Scalar

$$\mathcal{A}^{\sigma}_{\rm NLP} = \sum_{i=1}^{2} \mathbf{T}_i \left(\frac{2\,p_i^{\sigma} - k^{\sigma}}{2\,p_i \cdot k} - \frac{i k_{\alpha} \Sigma_i^{\sigma\alpha}}{p_i \cdot k} - \frac{i k_{\alpha} L_i^{\sigma\alpha}}{p_i \cdot k} \right) \otimes \mathcal{A}_{\rm LO}$$

$$\mathcal{O}\left(\frac{1}{k}\right) + \mathcal{O}\left(1\right)$$

Radiation of Next-to-soft gluon

$k \to 0$



Spin

$$\mathcal{A}^{\sigma}_{\text{NLP}} = \sum_{i=1}^{2} \mathbf{T}_i \left(\frac{2\,p_i^{\sigma} - k^{\sigma}}{2\,p_i \cdot k} - \frac{i k_{\alpha} \Sigma_i^{\sigma\alpha}}{p_i \cdot k} - \frac{i k_{\alpha} L_i^{\sigma\alpha}}{p_i \cdot k} \right) \otimes \mathcal{A}_{\text{LO}}$$

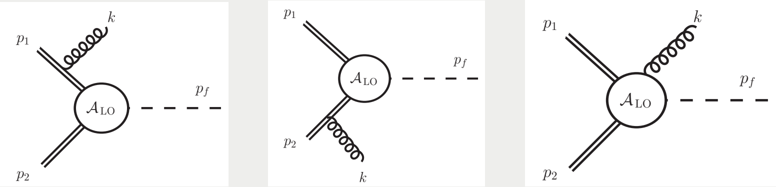
$$\mathcal{O}(1)$$

$$\Sigma_i^{\sigma\alpha} = \frac{i}{4}[\gamma^{\sigma}, \gamma^{\alpha}] = S^{\sigma\alpha}$$ Quarks

$$= i(g^{\rho\sigma} g^{\alpha\nu} - g^{\sigma\nu} g^{\alpha\rho}) = M^{\sigma\alpha,\rho\nu}$$ Gluons

Radiation of Next-to-soft gluon

$k \to 0$



Orbital

$$\mathcal{A}^{\sigma}_{\rm NLP} = \sum_{i=1}^{2} \mathbf{T}_i \left(\frac{2\,p_i^{\sigma} - k^{\sigma}}{2\,p_i \cdot k} - \frac{ik_{\alpha}\Sigma_i^{\sigma\alpha}}{p_i \cdot k} - \frac{ik_{\alpha}L_i^{\sigma\alpha}}{p_i \cdot k} \right) \otimes \mathcal{A}_{\rm LO}$$

$\mathcal{O}(1)$

$$L_i^{\sigma\alpha} = -i \left(p_i^{\sigma} \frac{\partial}{\partial p_i^{\alpha}} - p_i^{\alpha} \frac{\partial}{\partial p_i^{\sigma}} \right)$$

NLO amplitude at NLP

$$\mathcal{A}^{\sigma}_{\text{NLP}} = \sum_{i=1}^{2} \mathbf{T}_i \left(\frac{2\,p_i^{\sigma} - k^{\sigma}}{2\,p_i \cdot k} - \frac{ik_{\alpha}\Sigma_i^{\sigma\alpha}}{p_i \cdot k} - \frac{ik_{\alpha}L_i^{\sigma\alpha}}{p_i \cdot k} \right) \otimes \mathcal{A}_{\text{LO}}$$

$$\mathcal{A}^{\sigma}_{\text{NLP}} = \mathcal{A}^{\sigma}_{\text{scal}} + \mathcal{A}^{\sigma}_{\text{spin}} + \mathcal{A}^{\sigma}_{\text{orb}}$$

Amplitude Square

1706.04018

$$|\mathcal{A}^2_{\mathrm{NLP}}| = \mathcal{A}^2_{\mathrm{scal}} + 2\,\mathrm{Re}(\mathcal{A}_{\mathrm{spin}} + \mathcal{A}_{\mathrm{orb}})^\dagger \mathcal{A}_{\mathrm{scal}}$$

Eikonal

$$= C \; \frac{2\, p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)} \; \mathcal{A}^2_{\mathrm{LO}}(p_1 + \delta p_1, p_2 + \delta p_2)$$

$$\delta p_1 = -\frac{1}{2}\left(\frac{p_2 \cdot k}{p_1 \cdot p_2} p_1 - \frac{p_1 \cdot k}{p_1 \cdot p_2} p_2 + k\right) \qquad \delta p_2 = -\frac{1}{2}\left(\frac{p_1 \cdot k}{p_1 \cdot p_2} p_2 - \frac{p_2 \cdot k}{p_1 \cdot p_2} p_1 + k\right)$$

Why Angular Ordering?

Feature: Next-to-soft radiation does not change the behaviour of dipole radiation pattern.

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Dipole

$$\mathcal{A}^2_{\text{LP}} = C \; \frac{2\, p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)} \; \mathcal{A}^2_{\text{LO}}$$

Dipole

$$\mathcal{A}^2_{\text{NLP}} = C \; \frac{2\, p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)} \; \mathcal{A}^2_{\text{LO, shifts}}$$

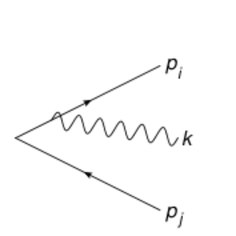
Why Angular Ordering?

Feature: Next-to-soft radiation does not change the behaviour of dipole radiation pattern.

$$\mathcal{A}^2_{\text{LP}} = C \underbrace{\frac{2\, p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)}}_{\text{Dipole}} \mathcal{A}^2_{\text{LO}} \qquad \mathcal{A}^2_{\text{NLP}} = C \underbrace{\frac{2\, p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)}}_{\text{Dipole}} \mathcal{A}^2_{\text{LO, shifts}}$$

- Useful for constructing Parton Showers.
- To understand the dynamics of next-to-soft gluon radiation.

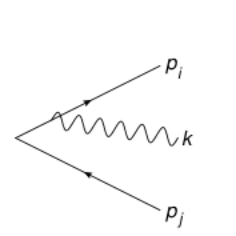
Angular Ordering



$$|\mathcal{M}_{\mathrm{LP}}|^2 \sim W_{ij}|\mathcal{M}^{(0)}|^2$$

$$W_{ij} = \frac{E_k^2 p_i \cdot p_j}{p_i \cdot k \, p_j \cdot k}$$

Angular Ordering



$$|\mathcal{M}_{\mathrm{LP}}|^2 \sim W_{ij}|\mathcal{M}^{(0)}|^2$$

$$W_{ij} = \frac{E_k^2 p_i \cdot p_j}{p_i \cdot k \, p_j \cdot k} \qquad W_{ij}^{[i]} = \frac{1}{2}\left(W_{ij} + \frac{1}{1-\cos\theta_{ik}} - \frac{1}{1-\cos\theta_{jk}}\right);$$

$$W_{ij}^{[j]} = \frac{1}{2}\left(W_{ij} + \frac{1}{1-\cos\theta_{jk}} - \frac{1}{1-\cos\theta_{ik}}\right).$$

Angular Ordering: NLP Contribution

$$
\begin{aligned}
p_i^\mu &= E_i(1, 0, 0, 1);\\
p_j^\mu &= E_j(1, \sin\theta_{ij}, 0, \cos\theta_{ij});\\
k^\mu &= E_k(1, \sin\theta_{ik}\cos\phi_{ik}, \sin\theta_{ik}\sin\phi_{ik}, \cos\theta_{ik}).
\end{aligned}
$$

$$
|\mathcal{M}_{\mathrm{NLP}}|^2 \sim W_{ij} f(s) - 2\left[\delta p_{i,j}\cdot p_j + \delta p_{j,i}\cdot p_i\right] W_{ij} f'(s)\,.
$$

Angular Ordering: NLP Contribution

$$
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|\mathcal{M}_{\mathrm{NLP}}|^2 \sim W_{ij} f(s) - 2\Big[\delta p_{i,j}\cdot p_j + \delta p_{j,i}\cdot p_i\Big] W_{ij} f'(s)\,.
$$

$$
f(s) = |\mathcal{M}^{(0)}|^2
$$

Angular Ordering

$$\int_0^{2\pi} \frac{d\phi_{ik}}{2\pi} W_{ij}^{[i]} = \frac{1}{2(1-\cos\theta_{ik})}\left[1 + \frac{(\cos\theta_{ik} - \cos\theta_{ij})}{|\cos\theta_{ik} - \cos\theta_{ij}|}\right]$$

Angular Ordering

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- Radiation is confined inside the cone: $\cos\theta_{ik} > \cos\theta_{ij}$, $\theta_{ij} > \theta_{ik}$

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- Radiation is confined inside the cone: $\cos\theta_{ik} > \cos\theta_{ij}$, $\theta_{ij} > \theta_{ik}$

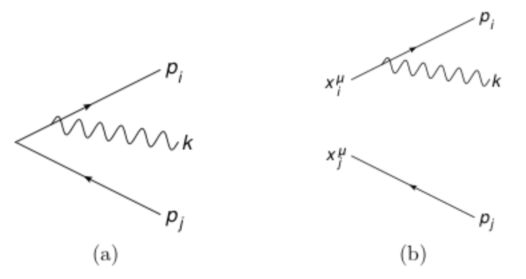
$$\widetilde{W}_{ij}^{[I]} = -\left(\frac{\delta p_{i,j}\cdot p_j}{E_k E_j} + \frac{\delta p_{j,i}\cdot p_i}{E_k E_i}\right) W_{ij}^{[I]},$$

$$\int_0^{2\pi} \frac{d\phi_{ik}}{2\pi} \widetilde{W}_{ij}^{[i]} = \frac{1}{2}\left[1 + \frac{\cos\theta_{ik} - \cos\theta_{ij}}{|\cos\theta_{ik} - \cos\theta_{ij}|}\right] + \sin^2\left(\frac{\theta_{ij}}{2}\right)\cot^2\left(\frac{\theta_{ik}}{2}\right).$$

- Radiation outside the cone.

Beekveld, Danish, Laenen, SP, Tripathi, White

Position Space Explanation



- Soft radiation can not see the separation between dipoles.
- Next-to-soft radiation can see the separation, results in breaking of angular ordering. Beekveld, Danish, Laenen, SP, Tripathi, White

Summary

- Next-to-soft radiation effects are important in phenomenological studies.
- Shifts in Born kinematics capture the effects of this radiation.
- Simple to use for colour-singlet processes
- Dipole feature of NLP radiation.
- Angular Ordering breaks down.
- More dipole feature: ?

Summary

- Next-to-soft radiation effects are important in phenomenological studies.
- Shifts in Born kinematics capture the effects of this radiation.
- Simple to use for colour-singlet processes
- Dipole feature of NLP radiation.
- Angular Ordering breaks down.
- More dipole feature: ?

Thank You!

Phase Space for Angular Ordering

$$p_i^\mu = E_i(1, 0, 0, 1);$$
$$p_j^\mu = E_j(1, \sin\theta_{ij}, 0, \cos\theta_{ij});$$
$$k^\mu = E_k(1, \sin\theta_{ik}\cos\phi_{ik}, \sin\theta_{ik}\sin\phi_{ik}, \cos\theta_{ik}).$$

$$|\mathcal{M}_{\mathrm{NLP}}|^2 \sim W_{ij} f(s) - 2\Big[\delta p_{i,j}\cdot p_j + \delta p_{j,i}\cdot p_i\Big] W_{ij} f'(s)\,.$$

$$f(s) = |\mathcal{M}^{(0)}|^2$$

Angular Ordering at NLP

$$\widetilde{W}_{ij}^{[l]} = -\left(\frac{\delta p_{i,j} \cdot p_j}{E_k E_j} + \frac{\delta p_{j,i} \cdot p_i}{E_k E_i}\right) W_{ij}^{[l]}, \tag{1}$$

such that

$$\widetilde{W}_{ij} = \widetilde{W}_{ij}^{[i]} + \widetilde{W}_{ij}^{[j]} = -\left(\frac{\delta p_{i,j} \cdot p_j}{E_k E_j} + \frac{\delta p_{j,i} \cdot p_i}{E_k E_i}\right) W_{ij}^{[l]} \tag{2}$$