UNIVERSITÀ DEGLI STUDI
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Precision electroweak physics at present and future colliders

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Introductory remarks

- There are big unanswered questions like dark matter, dark energy, matter-antimatter asymmetry;
  if the answer can be formulated according to a "particle paradigm", then we can search for such particles;
  direct searches are so far unsuccessful $\rightarrow$ we can formulate precision indirect tests and look for any BSM physics signs

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- A model (e.g. the SM) can be tested by checking how well it describes physical observables (i.e. xsecs and asymmetries)
  To this goal, we need the best predictions for the differential distributions, in order to make more significant the comparison

- Since every model has its own specific predictions (e.g. masses and couplings), we can test it at this level →
  we must devise a procedure to extract such parameters (pseudo-observables) from the data and
  then compare with the corresponding theoretical predictions

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  then compare with the corresponding theoretical predictions

- The possibility to parameterise our ignorance about BSM physics in the SMEFT language implies that we clarify
  how we test this model and how we determine fundamental parameters in this model

- The search for BSM signals benefits of a very precise understanding of the energy dependence of the observables
  One single deviation from the SM is not conclusive evidence of New Physics. (e.g. the CDF result for m_W) ;
  a systematic pattern of deviations from the SM, at different energies, would be a more significant signal

Outline of the talk

- The Precision Tests of the Standard Model of the strong and electroweak interactions

- The processes discussed here are an important set of "standard candles":
  - at hadron colliders the Drell-Yan process $pp \to l^+l^- + X$
  - at lepton colliders $e^+e^- \to \mu^+\mu^- + X$
  - in low energy experiments $e^-p \to e^-p$

- The precision achieved / expected in the measurement of the relevant observables allows a test of the SM at the quantum level $\longrightarrow$ status of the radiative corrections to the Drell-Yan processes

- The determination of SM parameters (masses, couplings) requires a discussion of the methodology adopted to fit the model to the data and to estimate the theoretical uncertainties
  $\longrightarrow$ the m_W and $\sin^2\theta^{\ell}_{eff}$ examples

- The challenge to extract indirect signs of BSM physics
  a "simplified" example: the determination in the SM of the running $\sin^2\hat{\theta}_{\overline{MS}}(\mu^2)$ at low and at large invariant masses

The precision tests of the SM

from the Fermi theory to the current best predictions of MW and $\sin^2\theta$ and beyond

From the Fermi theory of weak interactions to the discovery of W and Z

Fermi theory of β decay

muon decay $\mu^- \to \nu_\mu e^- \bar{\nu}_e$

$$\frac{1}{\tau_\mu} \to \Gamma_\mu \to G_\mu$$

QED corrections to Γ_μ necessary for precise determination of G_μ

computable in the Fermi theory (Kinoshita, Sirlin, 1959)

The independence of the QED corrections of the underlying model (Fermi theory vs SM) allows

- to define G_μ and to measure its value with high precision

$$G_\mu = 1.1663787(6)\ 10^{-5}\ \ \mathrm{GeV}^{-2}$$

- to establish a relation between G_μ and the SM parameters

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8 m_W^2} (1 + \Delta r)$$

The properties of physics at the EW scale
with sensitivity to the full SM and possibly to BSM via virtual corrections (Δr)
are related to a very well measured low-energy constant

From the Fermi theory of weak interactions to the discovery of W and Z

The SM predicts the existence of a new neutral current, different than the electromagnetic one
(Glashow 1961, Weinberg 1967, Salam 1968)

The observation of weak neutral current immediately allowed the estimate of the value of the weak mixing angle in the correct range
GARGAMELLE, Phys.Lett. 46B (1973) 138-140

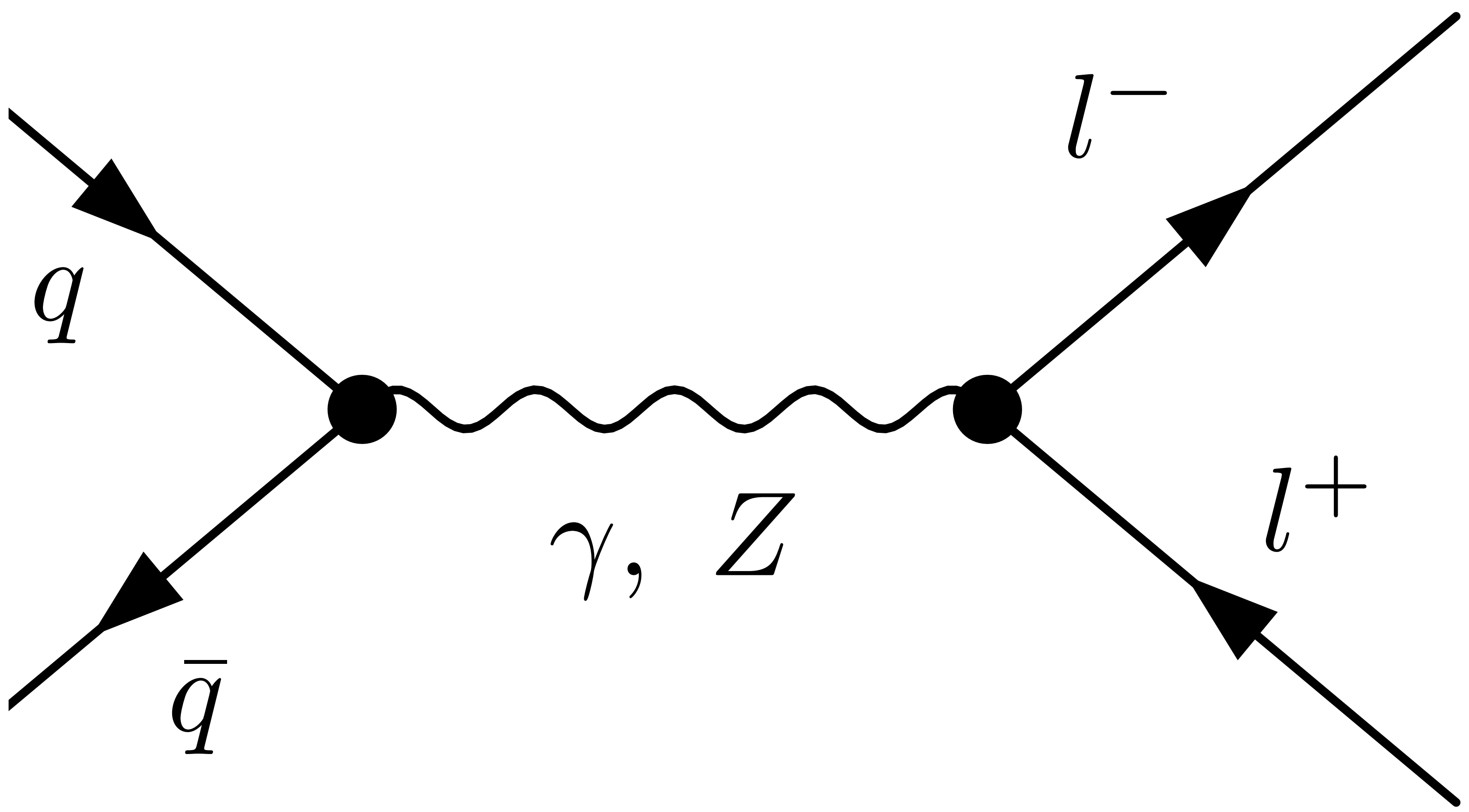
From the basic relation among the EW parameters it was immediately possible to estimate the order of magnitude of the mass of the weak bosons, in the 80 GeV range
(Antonelli, Maiani, 1981)

The discovery at the CERN SPPS of the W and Z bosons and the first determination of their masses allowed the planning of a new phase of precision studies accomplished with the construction of two e^+e^- colliders (SLC and LEP) running at the Z resonance

The precise determination of MZ and of the couplings of the Z boson to fermions
and in particular the value of the effective weak mixing angle
allowed to establish a framework for a test of the SM at the level of its quantum corrections

There is evidence of EW corrections beyond QED with 26 σ significance!
Full 1-loop and leading 2-loop radiative corrections are needed to describe the data
(indirect evidence of bosonic quantum effects, hints on the m_t and m_H values)

Scattering amplitudes and fundamental parameters



From the study of scattering processes we try to infer:

- the value of the masses of the intermediate particles (from the resonances, when measurable)
- the nature of the interaction between gauge bosons and matter fields;

  scalar, pseudo-scalar, vector, axial-vector,…

We try to define observables with well defined properties under Lorentz and discrete symmetries

this information is then translated into the structure and value of the couplings of the fundamental theory

The renormalisation of the SM and a framework for precision tests

- The Standard Model is a renormalizable gauge theory based on $SU(3) \times SU(2)_L \times U(1)_Y$
- The EW gauge sector of the SM lagrangian is assigned specifying (g, g', v, λ) in terms of 4 measurable inputs
- More observables can be computed and expressed in terms of the input parameters, including the available radiative corrections, at any order in perturbation theory
- The validity of the SM can be tested comparing these predictions with the corresponding experimental results

- The input choice $(g, g', v, \lambda) \leftrightarrow (\alpha, G_\mu, m_Z, m_H)$ minimises the parametric uncertainty of the predictions

$$
\begin{aligned}
\alpha(0) &= 1/137.035999139(31) \\
G_\mu &= 1.1663787(6) \times 10^{-5}\ \mathrm{GeV}^{-2} \\
m_Z &= 91.1876(21)\ \mathrm{GeV}/c^2 \\
m_H &= 125.09(24)\ \mathrm{GeV}/c^2
\end{aligned}
$$

- with these inputs, m_W and the weak mixing angle are predictions of the SM, to be tested against the experimental data

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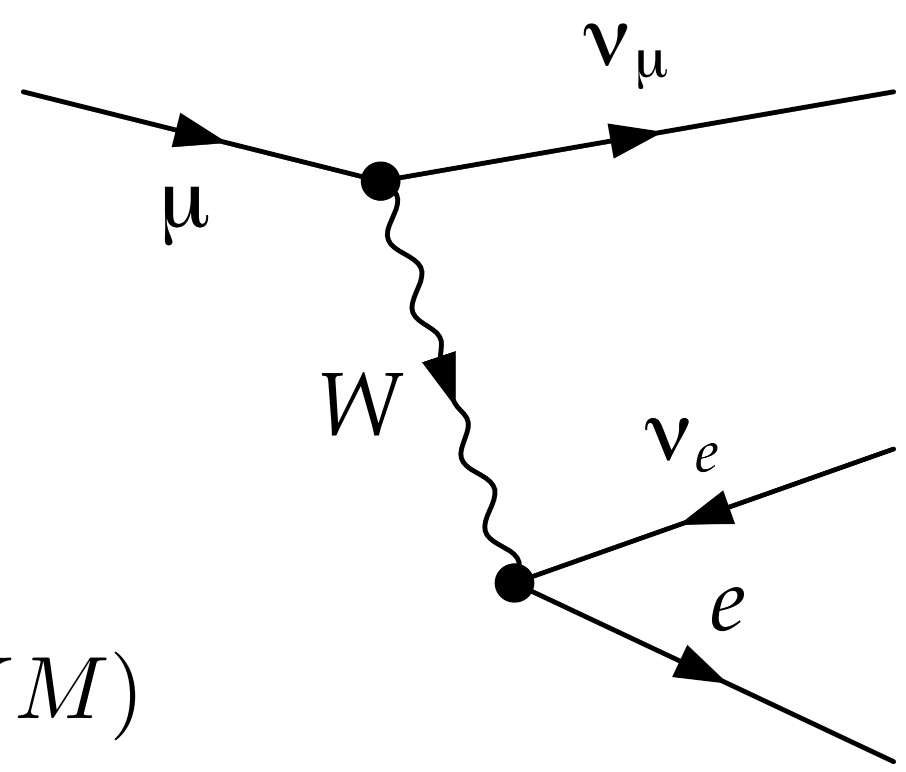
The W boson mass: theoretical prediction

$$\mathcal{L}_{SM} = \mathcal{L}_{SM}(\alpha,\, G_\mu,\, m_Z; m_H; m_f; CKM)$$

$\rightarrow$ we can compute m_W

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8 m_W^2}(1+\Delta r)$$



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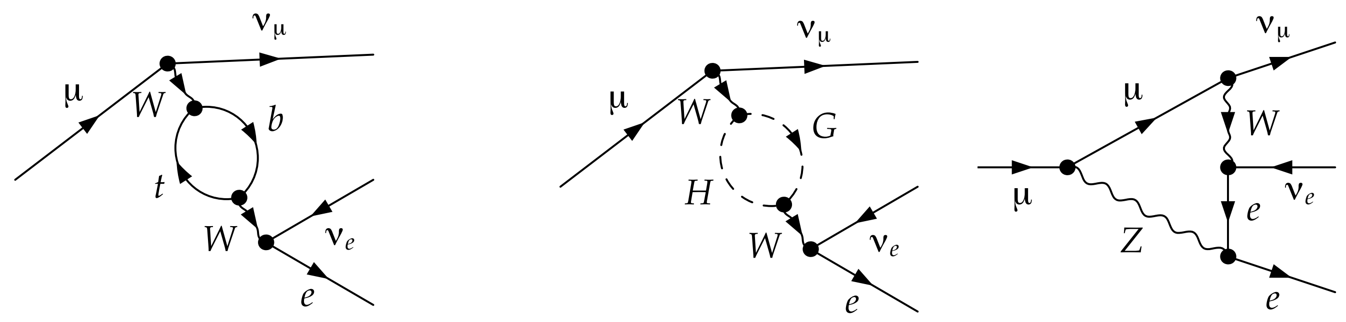

+ ....

$$\Delta r \;=\; \Delta r(\alpha, G_\mu, m_Z, m_H; m_f; CKM)$$

$$m_W^2 = \frac{m_Z^2}{2}\left(1 + \sqrt{1 - \frac{4\pi\alpha}{G_\mu \sqrt{2} m_Z^2}(1+\Delta r)}\right)$$

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The W boson mass: theoretical prediction

Sirlin, 1980, 1984; Marciano, Sirlin, 1980, 1981;
van der Bij, Veltman, 1984; Barbieri, Ciafaloni, Strumia 1993;
Djouadi, Verzegnassi 1987; Consoli, Hollik, Jegerlehner, 1989;
Chetyrkin, Kühn, Steinhauser, 1995;
Barbieri, Beccaria, Ciafaloni, Curci, Viceré, 1992, 1993; Fleischer, Tarasov, Jegerlehner, 1993;
Degrassi, Gambino, AV, 1996; Degrassi, Gambino, Sirlin, 1997;
Freitas, Hollik, Walter, Weiglein, 2000, 2003;
Awramik, Czakon, 2002; Awramik, Czakon, Onishchenko, Veretin, 2003; Onishchenko, Veretin, 2003

The best available prediction includes
the full 2-loop EW result, leading higher-order EW and QCD corrections,
resummation of reducible terms
Missing 3-loop and 4-loop terms needed to reduce the uncertainties.

$$m_W = w_0 + w_1 dH + w_2 dH^2 + w_3 dh + w_4 dt + w_5 dH dt + w_6 da_s + w_7 da^{(5)}$$

$$dt = [(M_t/173.34\,\text{GeV})^2 - 1]$$

$$da^{(5)} = [\Delta\alpha^{(5)}_{\text{had}}(m_Z^2)/0.02750 - 1]$$

$$dH = \ln\left(\frac{m_H}{125.15\,\text{GeV}}\right)$$

$$dh = [(m_H/125.15\ \text{GeV})^2 - 1]$$

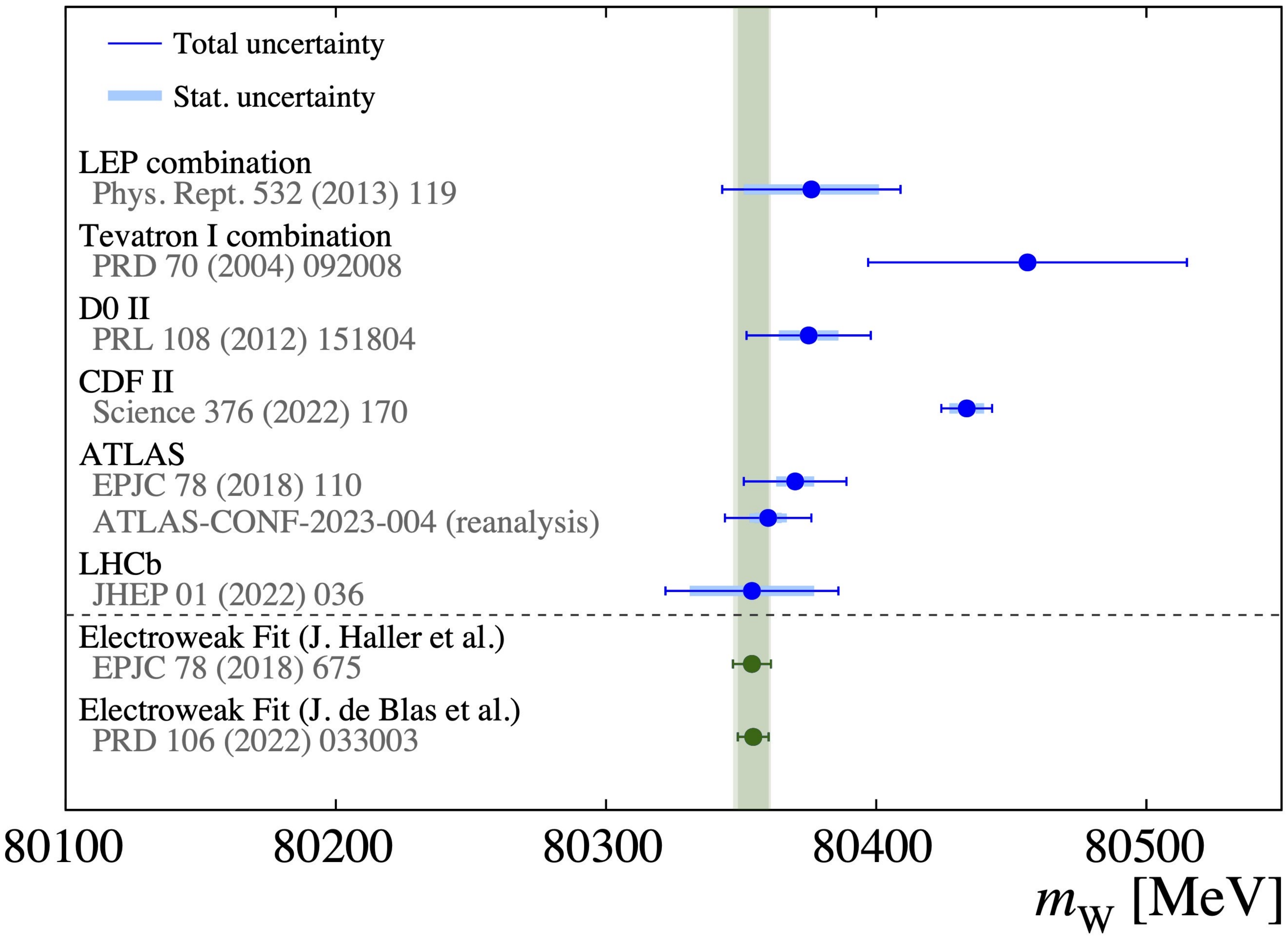
$$da_s = \left(\frac{\alpha_s(m_Z)}{0.1184} - 1\right)$$

	$124.42 \le m_H \le 125.87$ GeV	$50 \le m_H \le 450$ GeV
w_0	80.35712	80.35714
w_1	-0.06017	-0.06094
w_2	0.0	-0.00971
w_3	0.0	0.00028
w_4	0.52749	0.52655
w_5	-0.00613	-0.00646
w_6	-0.08178	-0.08199
w_7	-0.50530	-0.50259

on-shell scheme $m_W^{os} = 80.353 \pm 0.004$ GeV (Freitas, Hollik, Walter, Weiglein)
MSbar scheme. $m_W^{\overline{MS}} = 80.351 \pm 0.003$ GeV (Degrassi, Gambino, Giardino)

parametric uncertainties $\delta m_W^{par} = \pm 0.005$ GeV due to the $(\alpha, G_\mu, m_Z, m_H, m_t)$ values

Experimental determinations of the W boson mass



Are all the uncertainties, including the theoretical ones, properly included, for a determination at the O(10 MeV) level ?

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The weak mixing angle(s)

- in the classical SM lagrangian the weak mixing angle expresses the amount of mixing between $SU(2)_L$ and $U(1)_Y$ necessary to identify the electromagnetic current. $$\tan\theta_W = \frac{g'}{g}$$

The weak mixing angle(s)

- in the classical SM lagrangian the weak mixing angle expresses the amount of mixing between $SU(2)_L$ and $U(1)_Y$ necessary to identify the electromagnetic current.

$$\tan\theta_W = \frac{g'}{g}$$

- upon renormalisation, various definitions are possible, with sensitivity to different subsets of quantum corrections

  - on-shell definition:
    Sirlin, 1980

    $$\sin^2\theta_{OS} = 1 - \frac{m_W^2}{m_Z^2}$$

    definition valid to all orders

  - MSbar definition:
    Marciano, Sirlin, 1980; Degrassi, Sirlin, 1991

    $$\frac{G_\mu}{\sqrt{2}} = \frac{g_0^2}{8m_{W,0}^2} \longrightarrow \hat{s}^2\hat{c}^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu m_Z^2(1-\Delta\hat{r})} \qquad \hat{s}^2 \equiv \sin^2\hat{\theta}(\mu_R = m_Z)$$

    weak dependence on top-quark corrections

  - the effective leptonic weak mixing angle enters in the definition of the effective Z-f-fbar vertex at the Z resonance ($q^2 = m_Z^2$), when f is a lepton

$$\mathcal{M}^{eff}_{Zf\bar{f}} = \bar{u}_l\gamma_\alpha\left[\mathcal{G}_v^f(m_Z^2) - \mathcal{G}_a^f(m_Z^2)\gamma_5\right] v_l\,\varepsilon_Z^\alpha \qquad 4\,|Q_f|\sin^2\theta^f_{eff} = 1 - \frac{\mathcal{G}_v^f}{\mathcal{G}_a^f}$$

The effective leptonic weak mixing angle: theoretical prediction

- parameterization of the full two-loop EW calculation + different sets of 3- and 4-loop corrections

I.Dubovyk, A.Freitas, J.Gluza, T.Riemann, J.Usovitsch, arXiv:1906.08815

$$\sin^2\theta^f_{\rm eff} = s_0 + d_1 L_H + d_2 L_H^2 + d_3 L_H^4 + d_4\Delta_\alpha + d_5\Delta_{\rm t} + d_6\Delta_{\rm t}^2 + d_7\Delta_{\rm t} L_H + d_8\Delta_{\alpha_{\rm s}} + d_9\Delta_{\alpha_{\rm s}}\Delta_{\rm t} + d_{10}\Delta_{\rm Z}$$

$$L_{\rm H} = \log\frac{M_{\rm H}}{125.7\,{\rm GeV}}, \qquad \Delta_{\rm t} = \left(\frac{m_{\rm t}}{173.2\,{\rm GeV}}\right)^2 - 1,$$

$$\Delta_{\alpha_{\rm s}} = \frac{\alpha_{\rm s}(M_{\rm Z})}{0.1184} - 1, \qquad \Delta_\alpha = \frac{\Delta\alpha}{0.059} - 1, \qquad \Delta_{\rm Z} = \frac{M_{\rm Z}}{91.1876\,{\rm GeV}} - 1$$

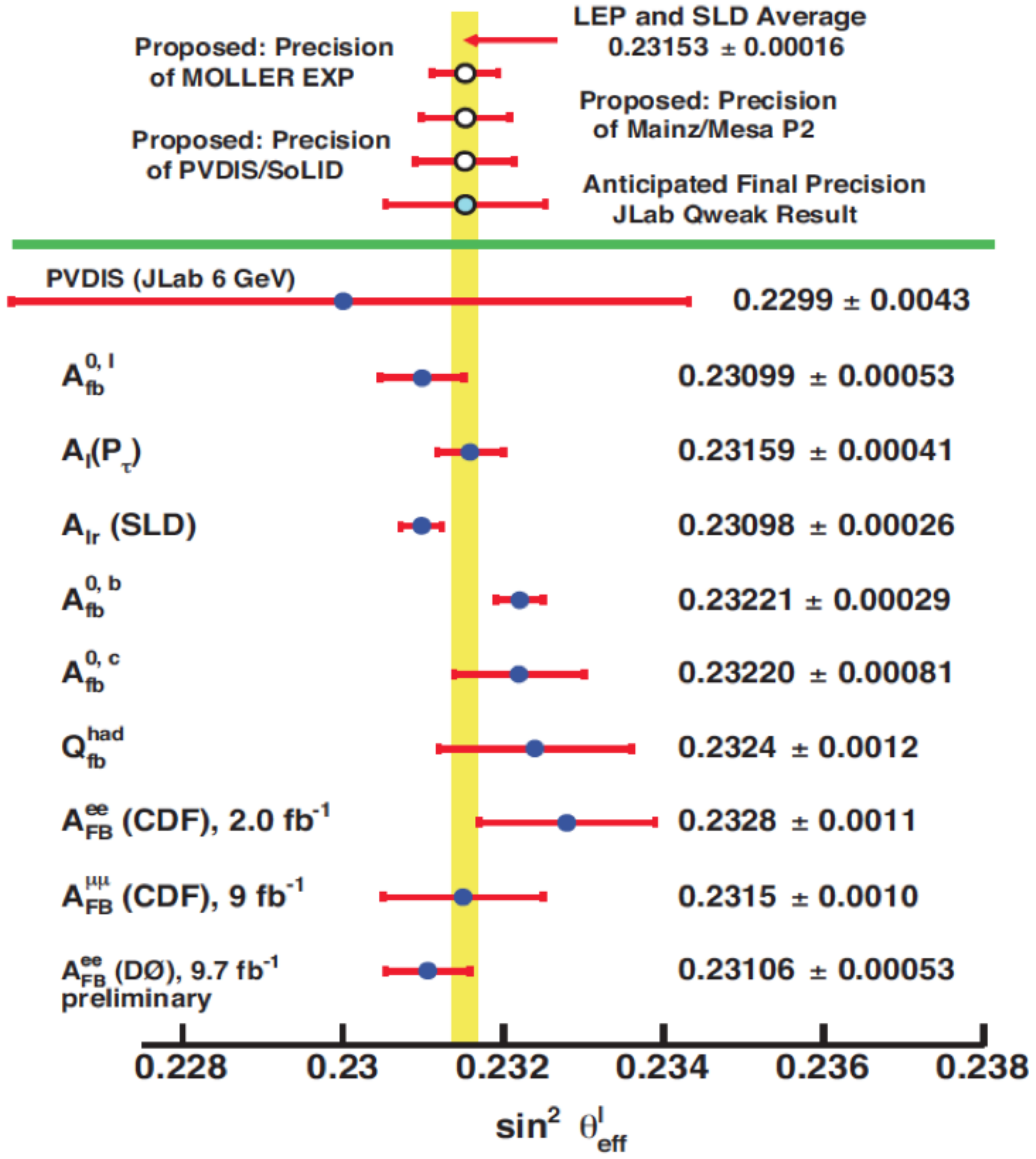
Observable	s_0	d_1	d_2	d_3	d_4	d_5
$\sin^2\theta^\ell_{\rm eff}\times 10^4$	2314.64	4.616	0.539	−0.0737	206	−25.71
$\sin^2\theta^b_{\rm eff}\times 10^4$	2327.04	4.638	0.558	−0.0700	207	−9.554

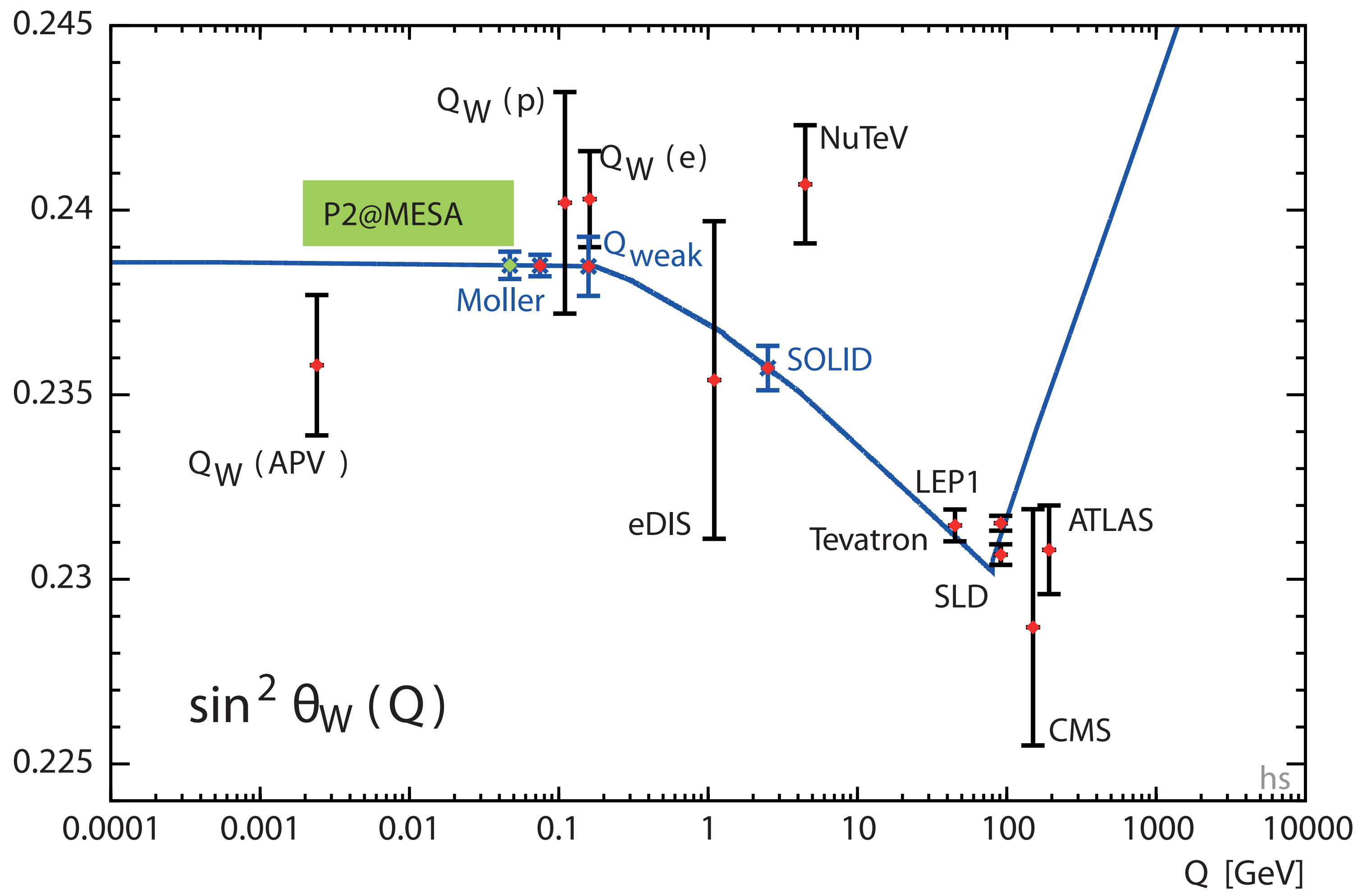
Observable	d_6	d_7	d_8	d_9	d_{10}	max. dev.
$\sin^2\theta^\ell_{\rm eff}\times 10^4$	4.00	0.288	3.88	−6.49	−6560	< 0.056
$\sin^2\theta^b_{\rm eff}\times 10^4$	3.83	0.179	2.41	−8.24	−6630	< 0.025

Comparison of different weak mixing angle determinations

The sensible comparison of different determinations of $\sin^2\theta_W$ offers a test of the SM
→ LEP/SLD longstanding discrepancies might be clarified

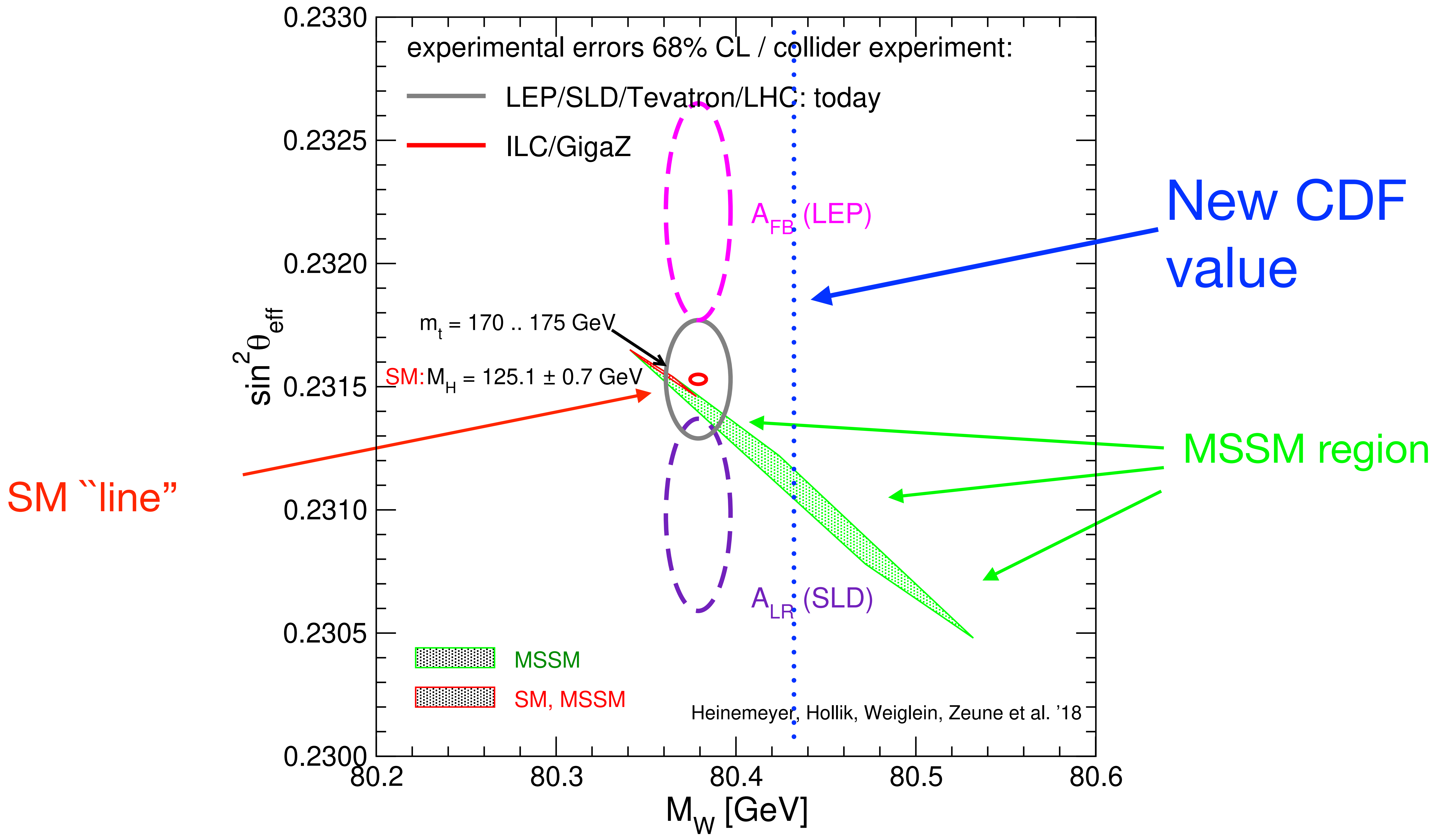
e+e- and hadron colliders determinations are based on observables with different systematics
but also use different definitions to fit the data (WARNING!)
→ For a meaningful test, it is important to compare the same weak mixing angle. (cfr. different definitions)





Relevance of a simultaneous study of m_W and of the weak mixing angle

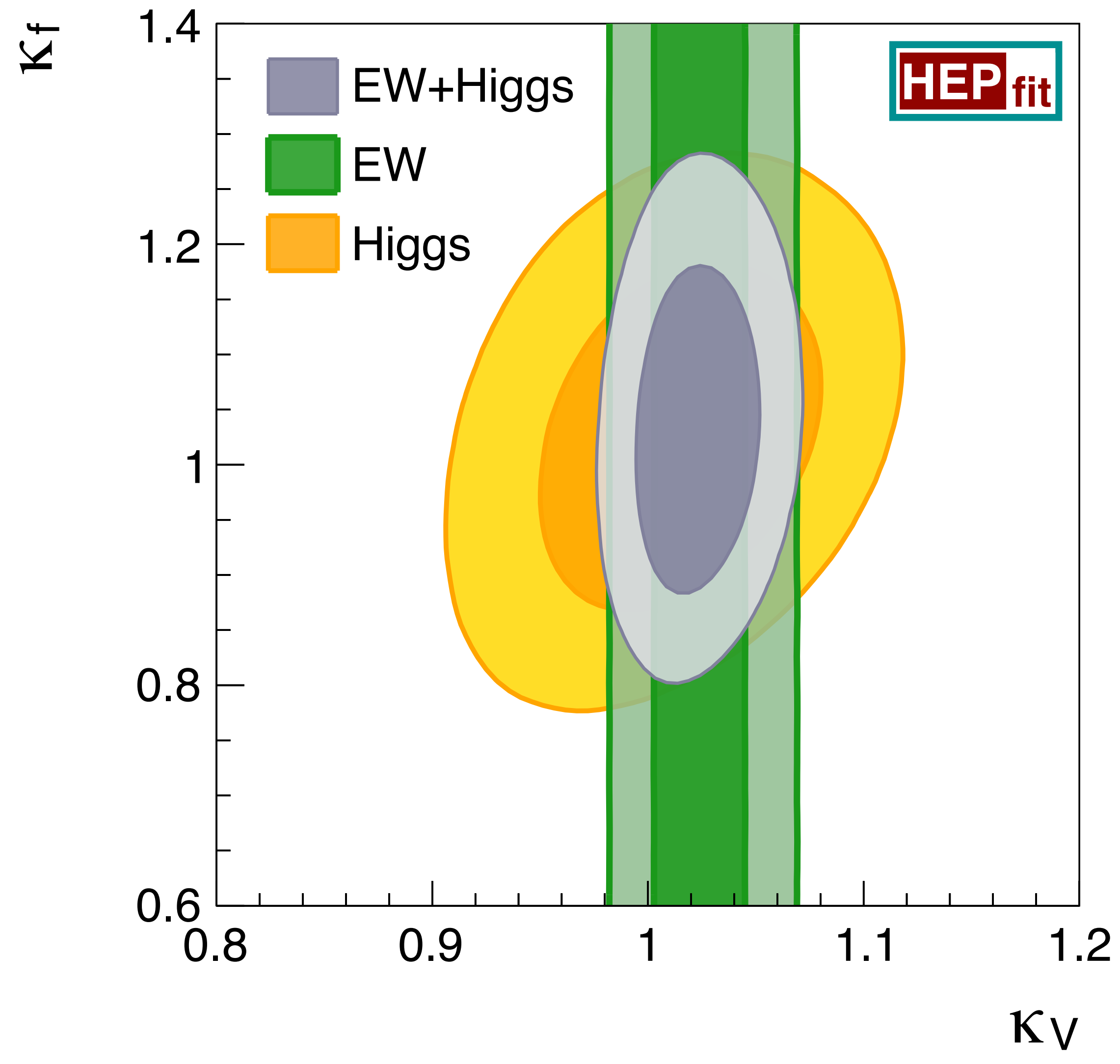
[S. Heinemeyer, W. Hollik, G. W., L. Zeune '18]



independent determination of these two parameters crucial for testing different New Physics alternatives

Relevance of new high-precision measurement of EW parameters

de Blas et al, arXiv:1608.01509



$$\mathcal{L}_{\rm Eff} = \sum_{d=4}^{\infty} \frac{1}{\Lambda^{d-4}} \mathcal{L}_d = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \cdots$$

$$\mathcal{L}_d = \sum_i C_i^d \mathcal{O}_i \qquad [\mathcal{O}_i] = d \longrightarrow \left(\frac{q}{\Lambda}\right)^{d-4}$$

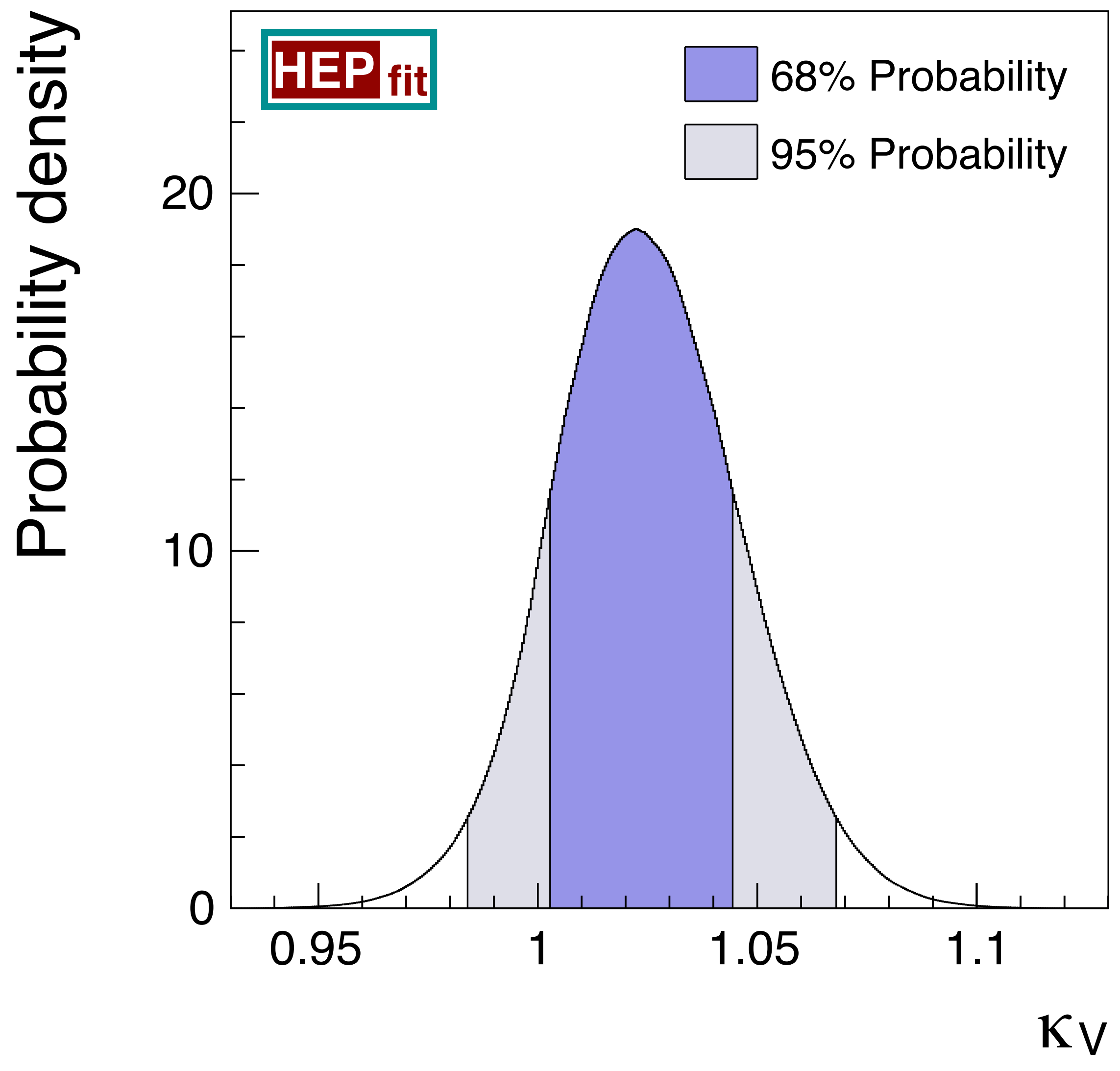
Effects suppressed by $q = v, E < \Lambda$

Λ: Cut-off of the EFT

$$\mathcal{O}_{\phi WB} = \phi^\dagger \sigma_a \phi B^{\mu\nu} W^a_{\mu\nu}$$

EWSB $\to$ $v^2 B^{\mu\nu} W^3_{\mu\nu}$ gauge boson masses

EWSB $\to$ $v h B^{\mu\nu} W^3_{\mu\nu}$ h→ZZ, γ γ



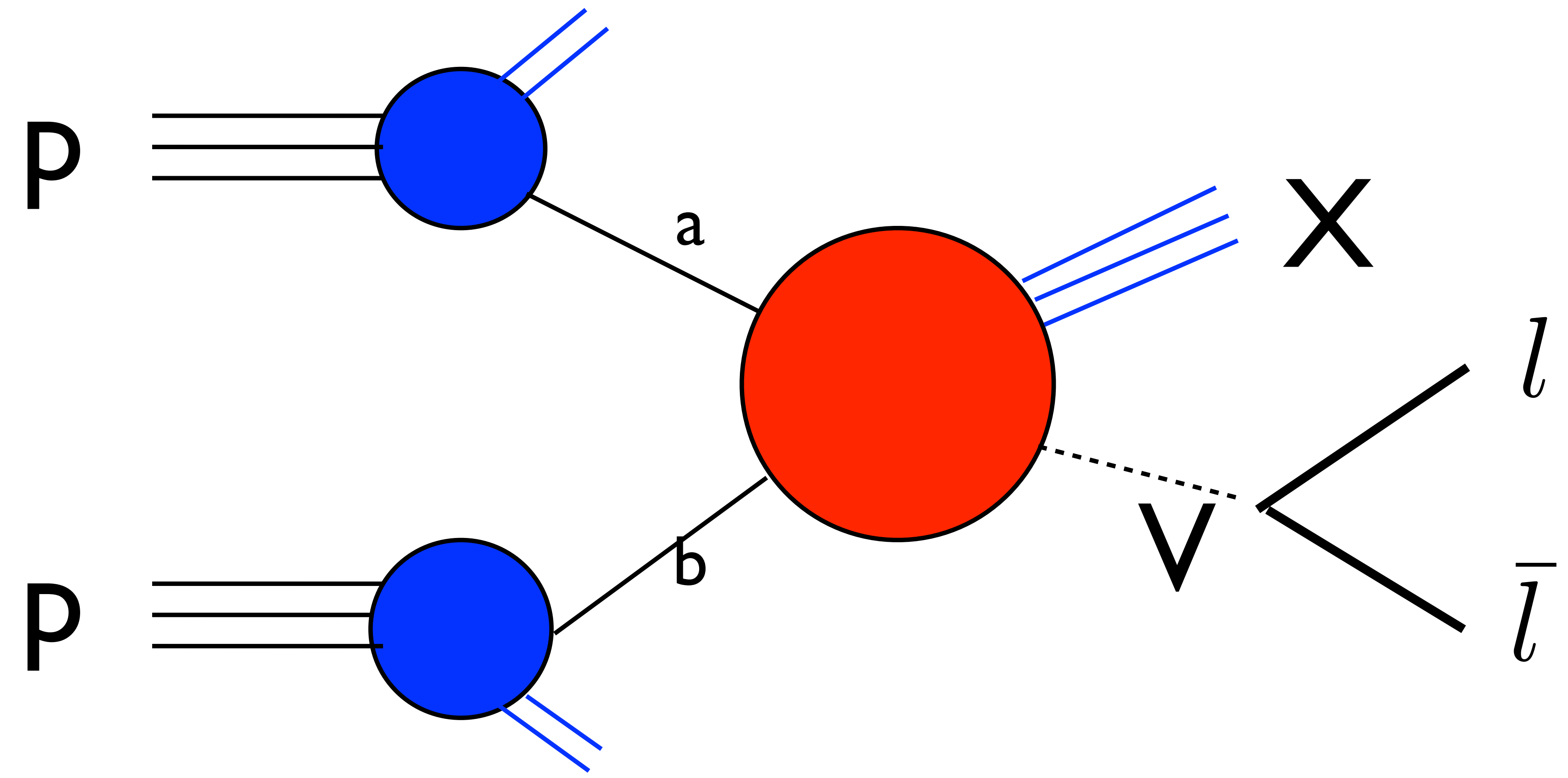
$$M_W^2 = M_Z^2 c^2 \left[1 - \frac{c^2}{c^2 - s^2}\left(\frac{1}{2}C_{\phi D} + 2\frac{s}{c}C_{\phi WB} + \frac{s^2}{c^2}\Delta_{G_\mu}\right)\frac{v^2}{\Lambda^2}\right]$$

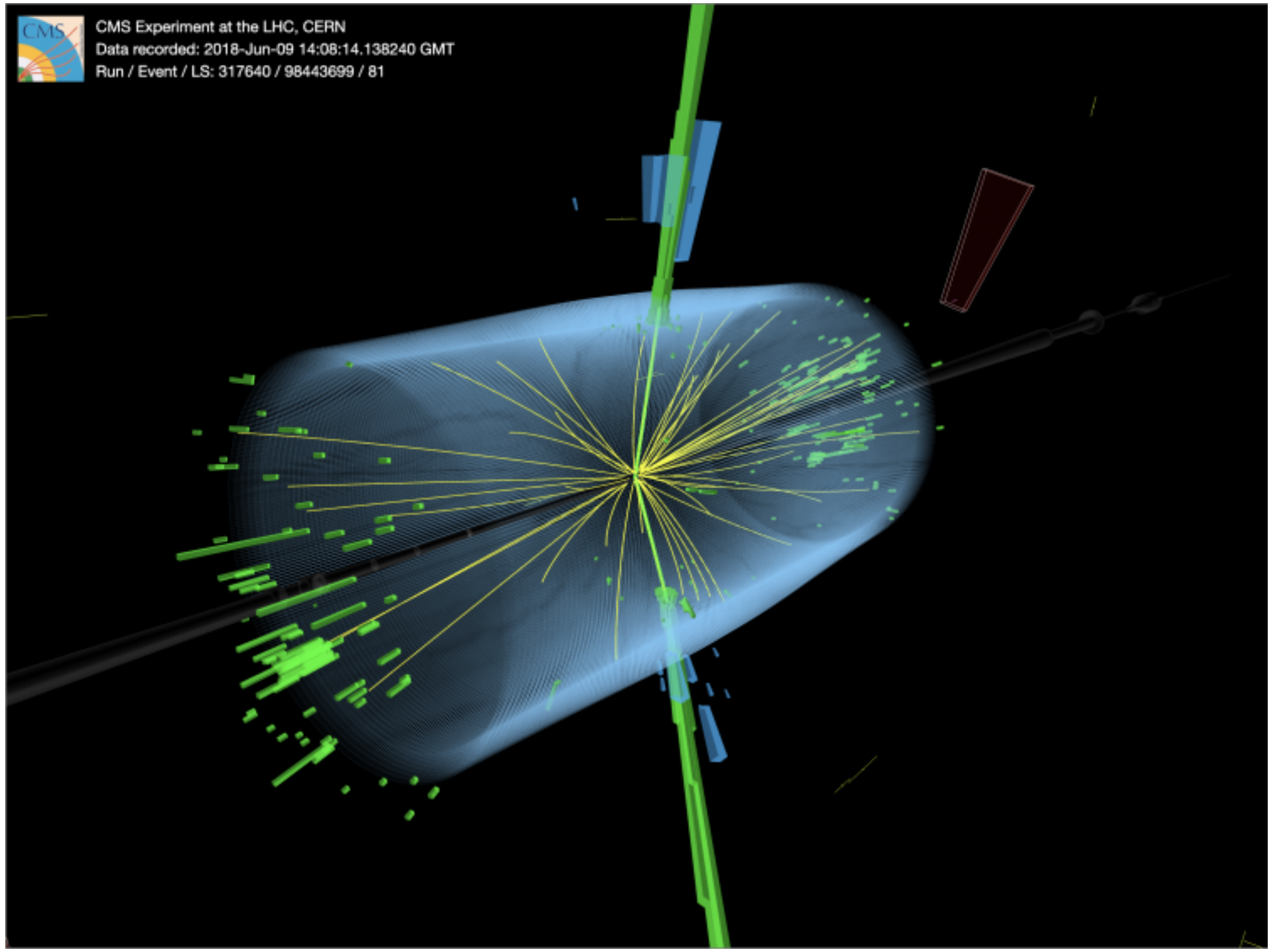
A precise measurement of m_W and $\sin^2\theta_{eff}$ constrains several dim-6 operators contributing to Higgs and gauge interaction vertices.

Today still one of the strongest constraints

Physical processes, observables and parameter determination

Lepton-pair Drell-Yan production at hadron colliders

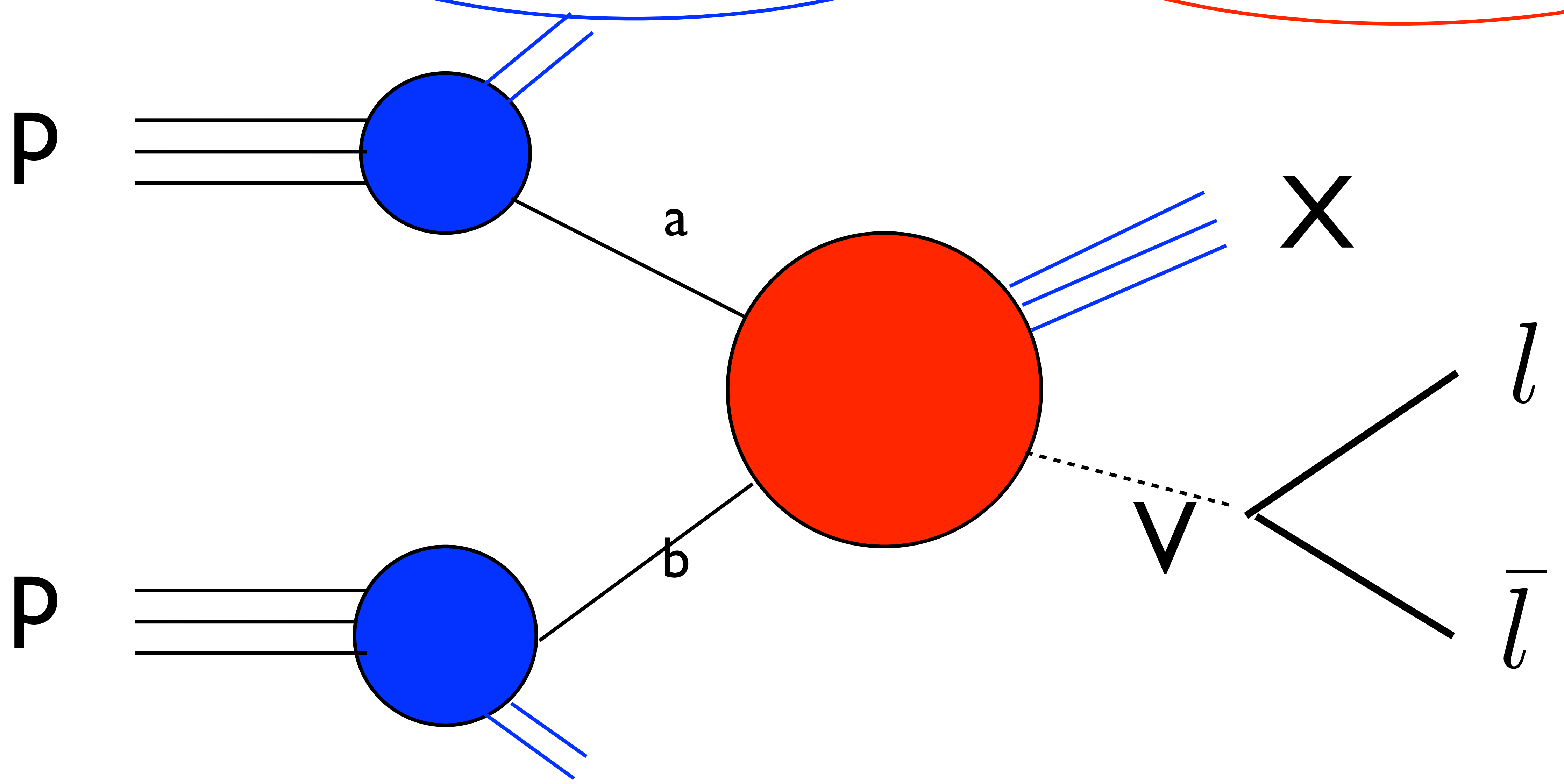




- Test of perturbative QCD
- Determination of the proton structure
- Discovery of W and Z bosons (1983)
- High-precision determination of W and Z properties
- Background to New Physics searches

Lepton-pair Drell-Yan production at hadron colliders

$$\sigma(P_1, P_2; m_V) = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_{h_1,a}(x_1, M_F) f_{h_2,b}(x_2, M_F) \, \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2, \alpha_s(\mu), M_F)$$



The factorisation theorems guarantee the validity of the above picture up to power correction effects

The interplay of QCD and EW interactions appears both in the partonic cross section and in the proton PDFs

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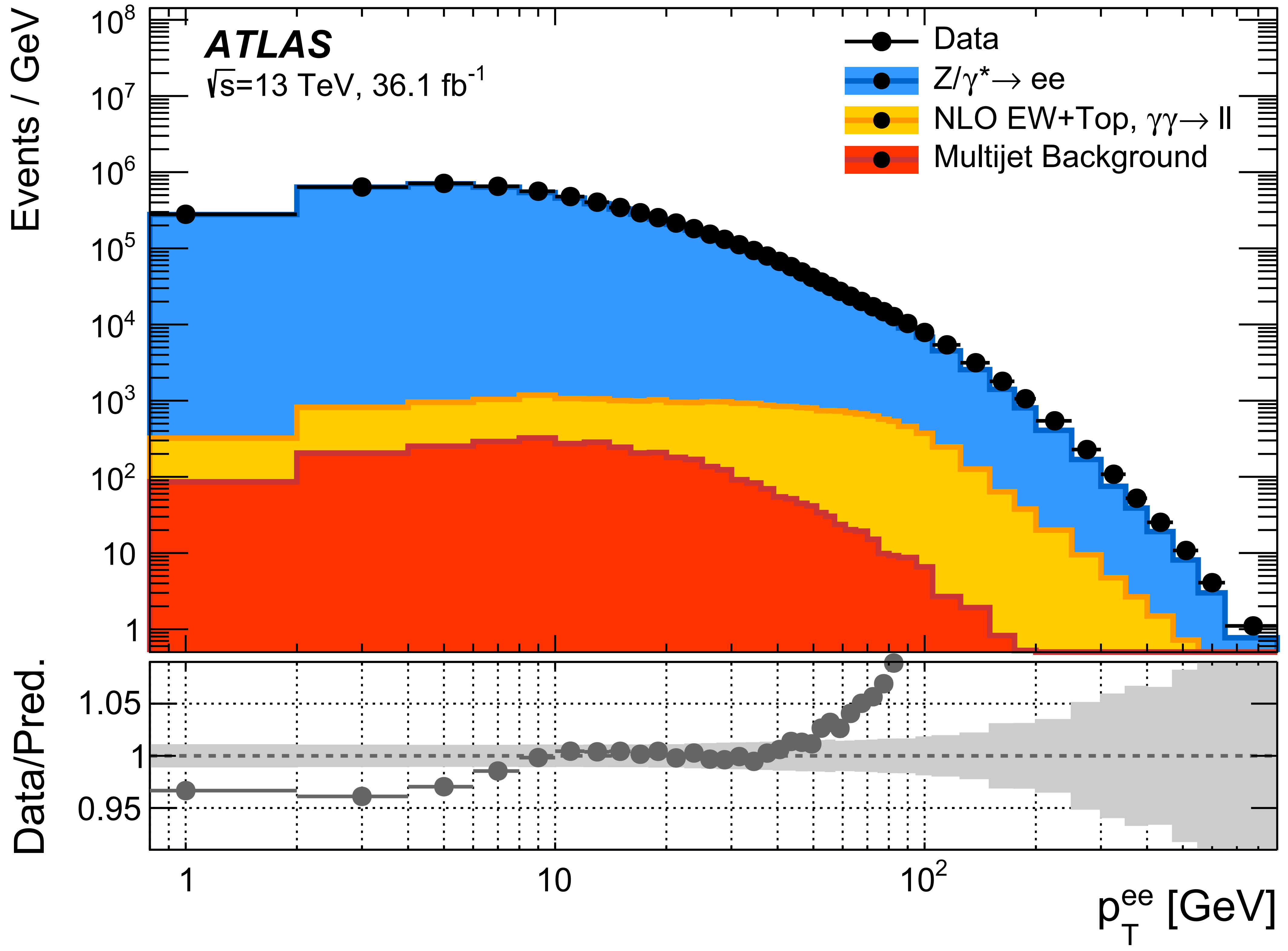
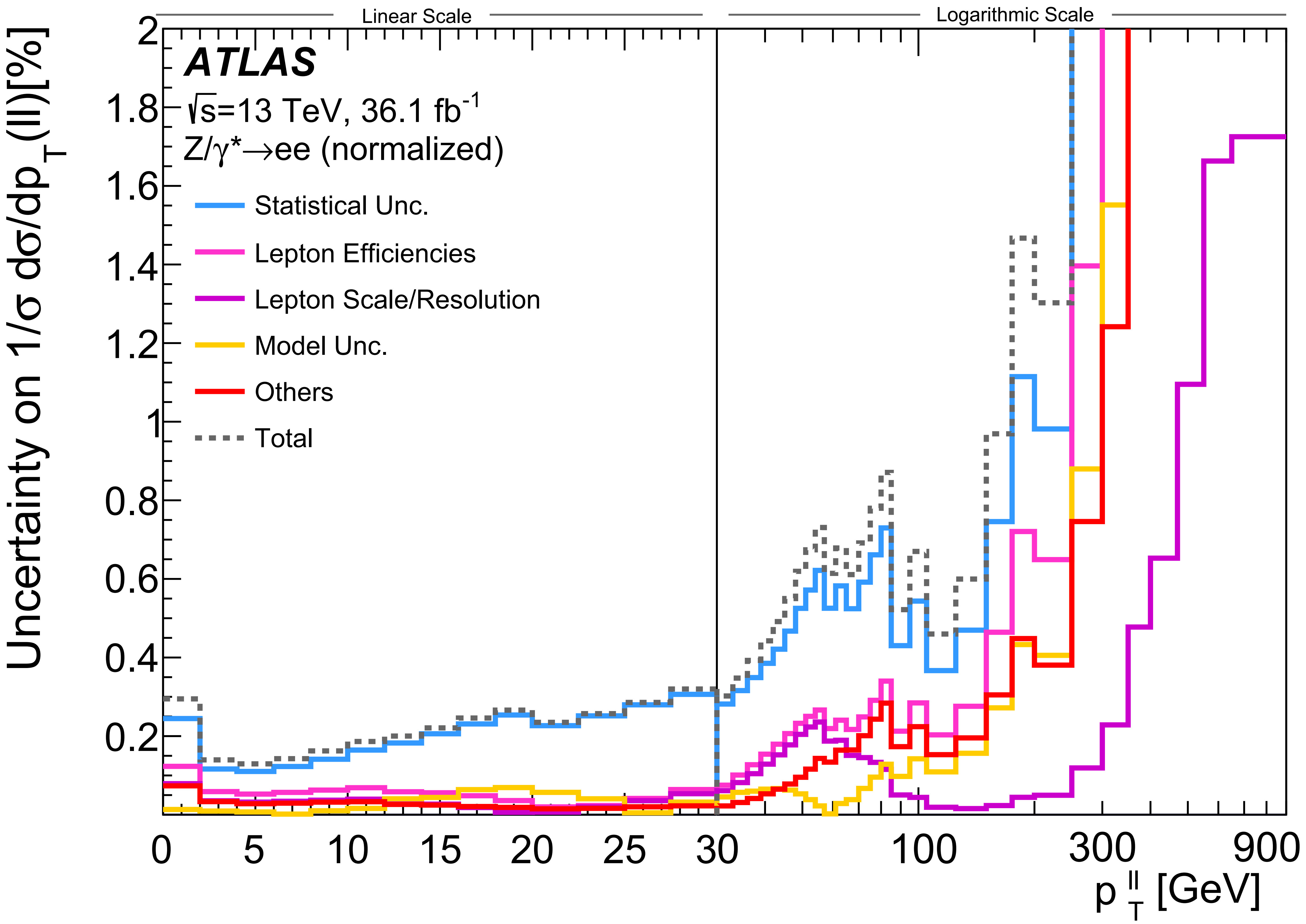
Lepton-pair transverse momentum distribution

- A crucial role in QCD tests and precision EW measurements (m_W in particular) is played by the $p_\perp^{\ell^+\ell^-}$ distribution
- The impressive experimental precision is a formidable test of the theory predictions, QCD in first place
- At per mille level higher-order QCD resummation matched with fixed order corrections

non-perturbative QCD effects and heavy quarks corrections are relevant

EW corrections

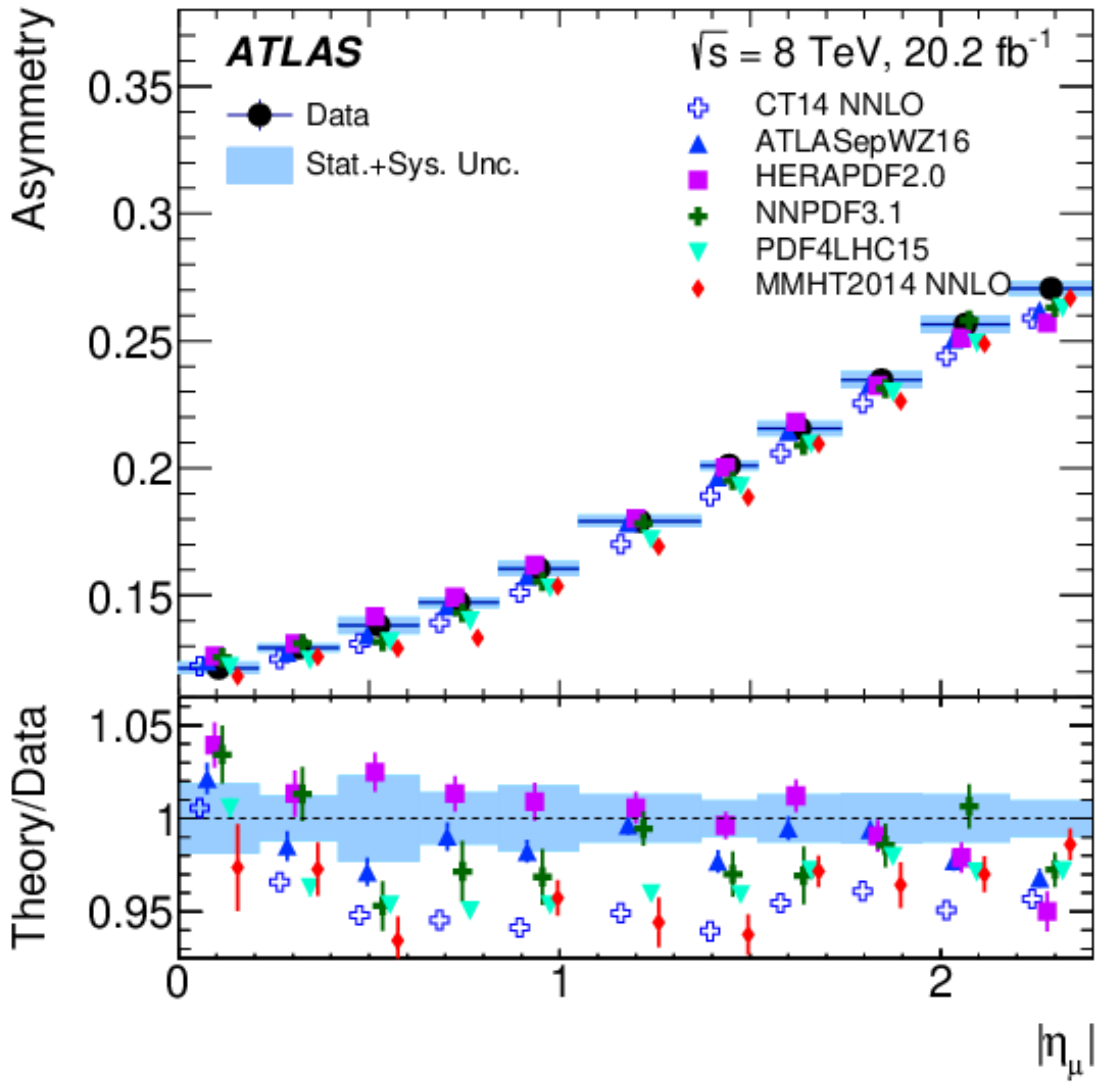


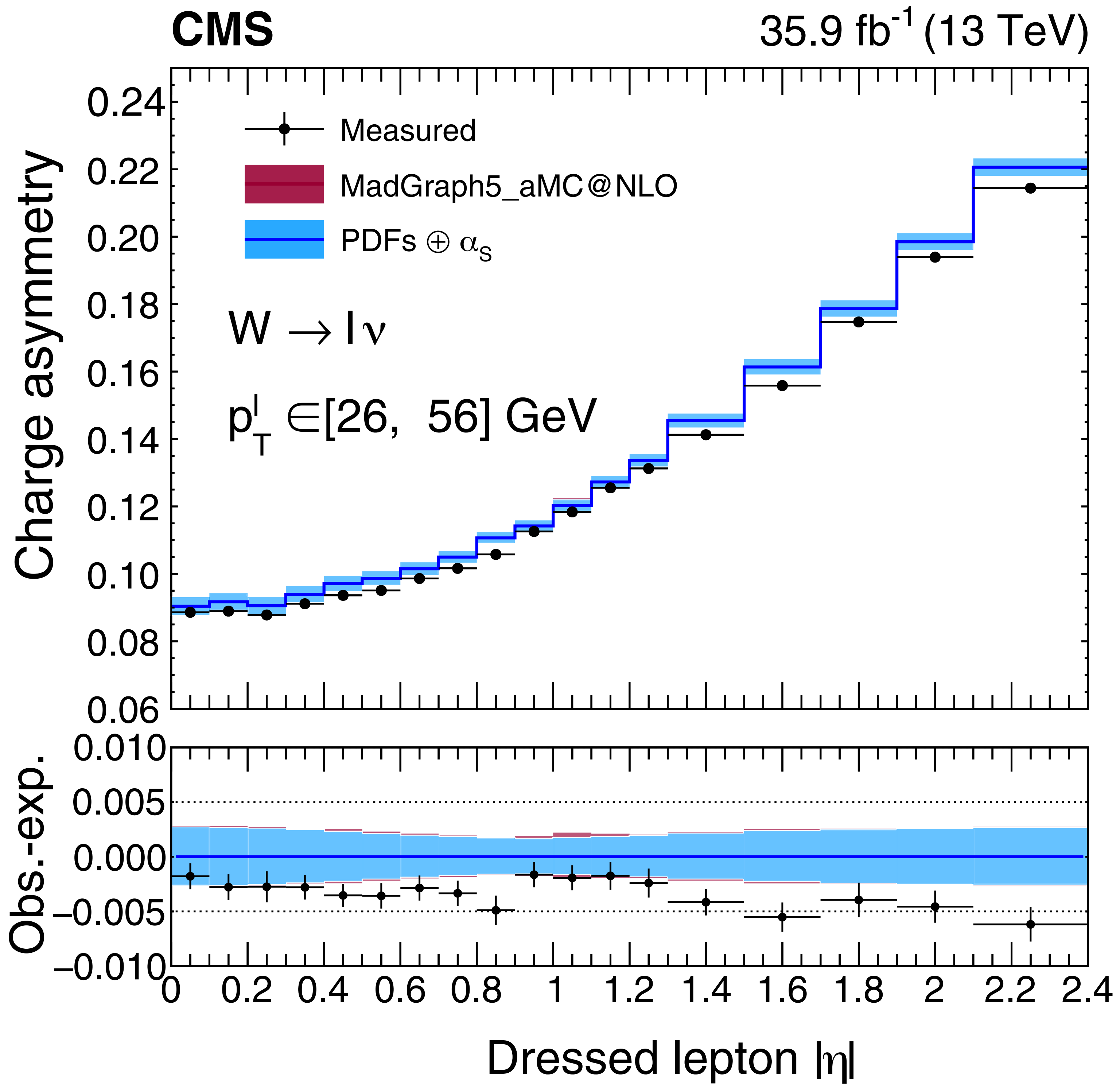


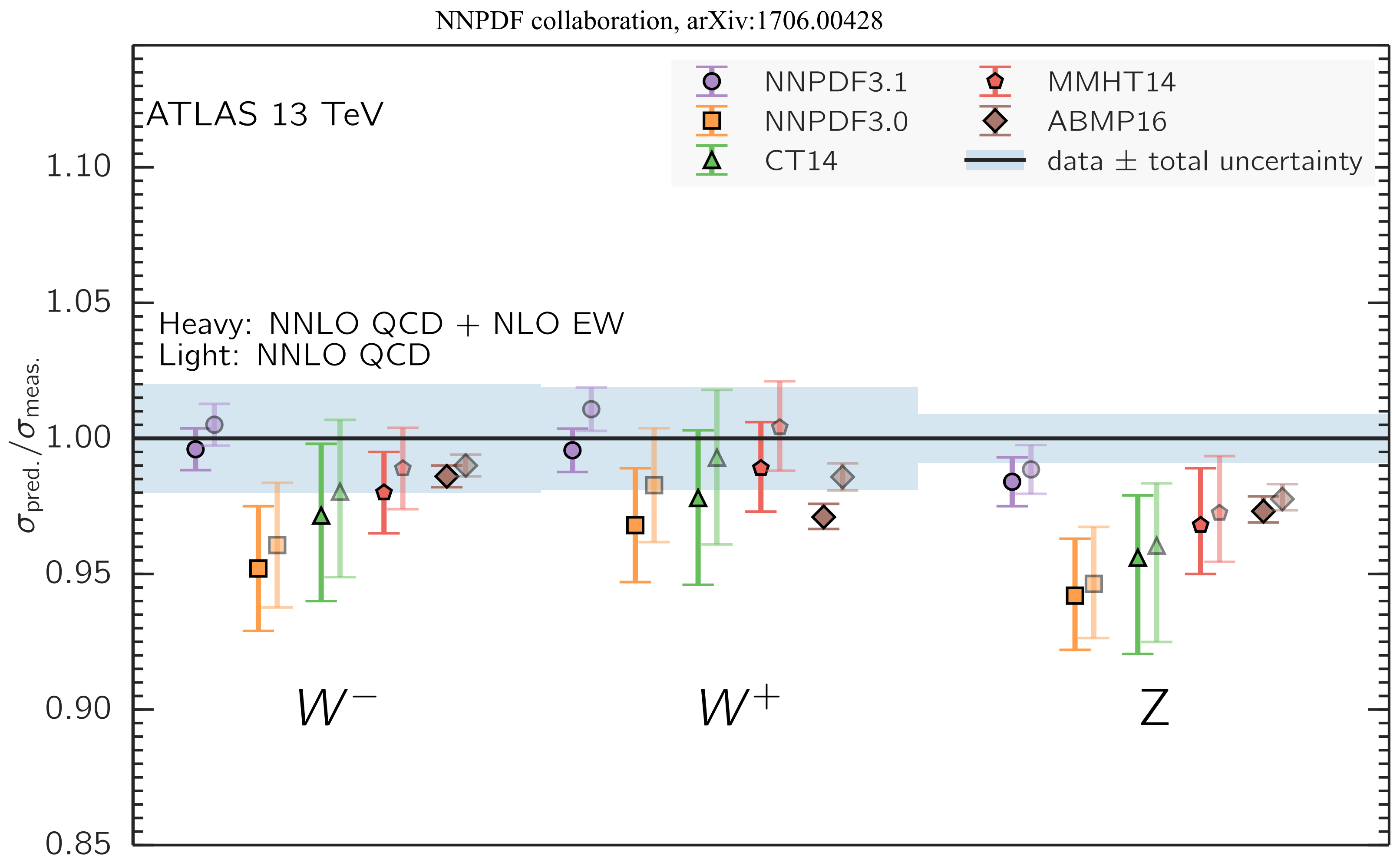
At CERN the EW WG has a subgroup scrutinising the predictions of this observable by different collaborations

Charge asymmetry in charged-current Drell-Yan

- An important role in the determination of proton structure is played by the charge-asymmetry rapidity distribution
  - ▻ needed to improve the flavour separation
  - ▻ precise results at parton level for this quantity make its contribution to the PDF fit more significant
    - → importance of NNLO and N3LO calculations
  - ▻ in a fiducial volume the rapidity and transverse momentum dependencies are connected by kinematics
    - → impact on the m_W determination

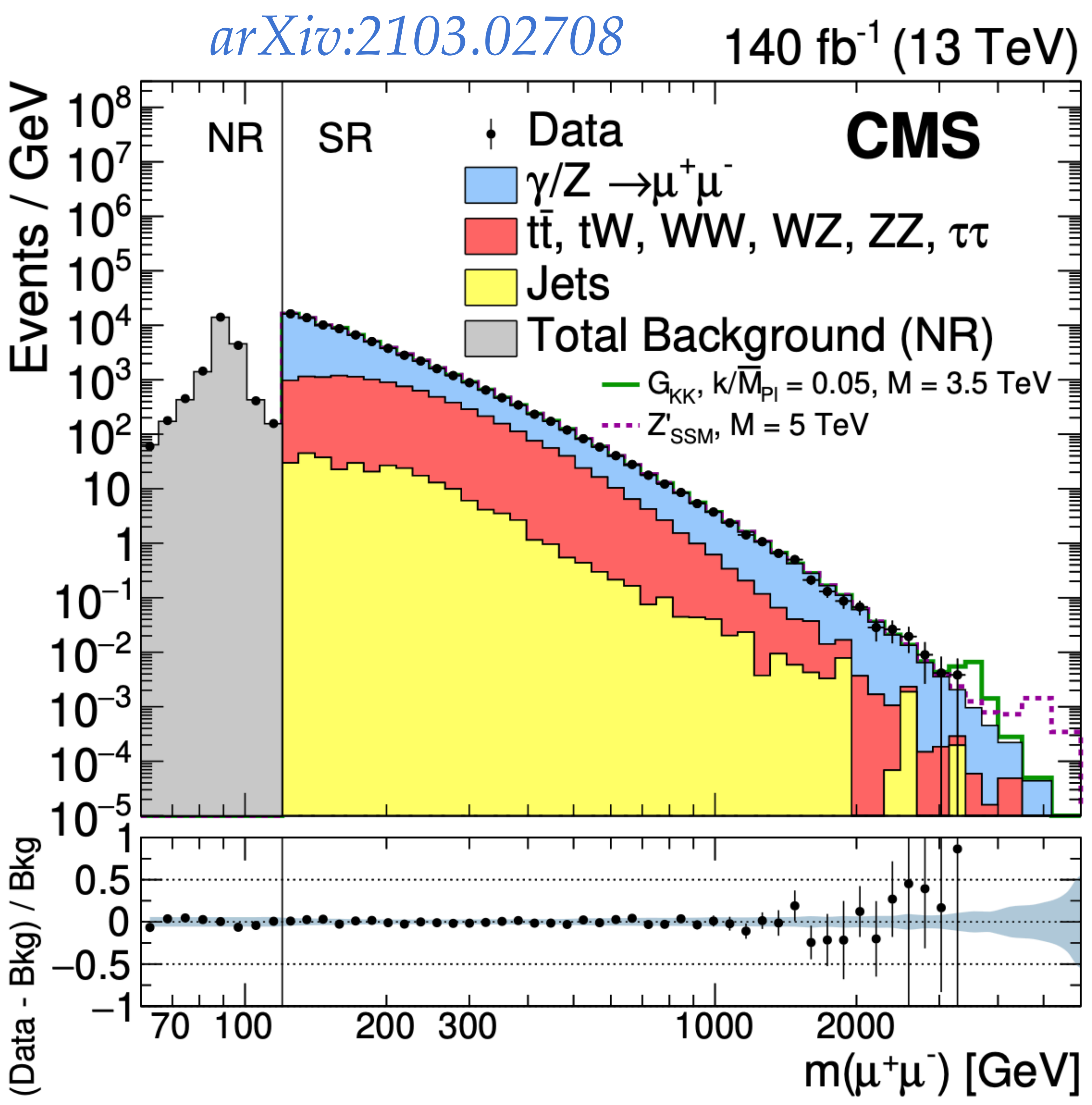






on-shell gauge boson production as a PDF benchmark

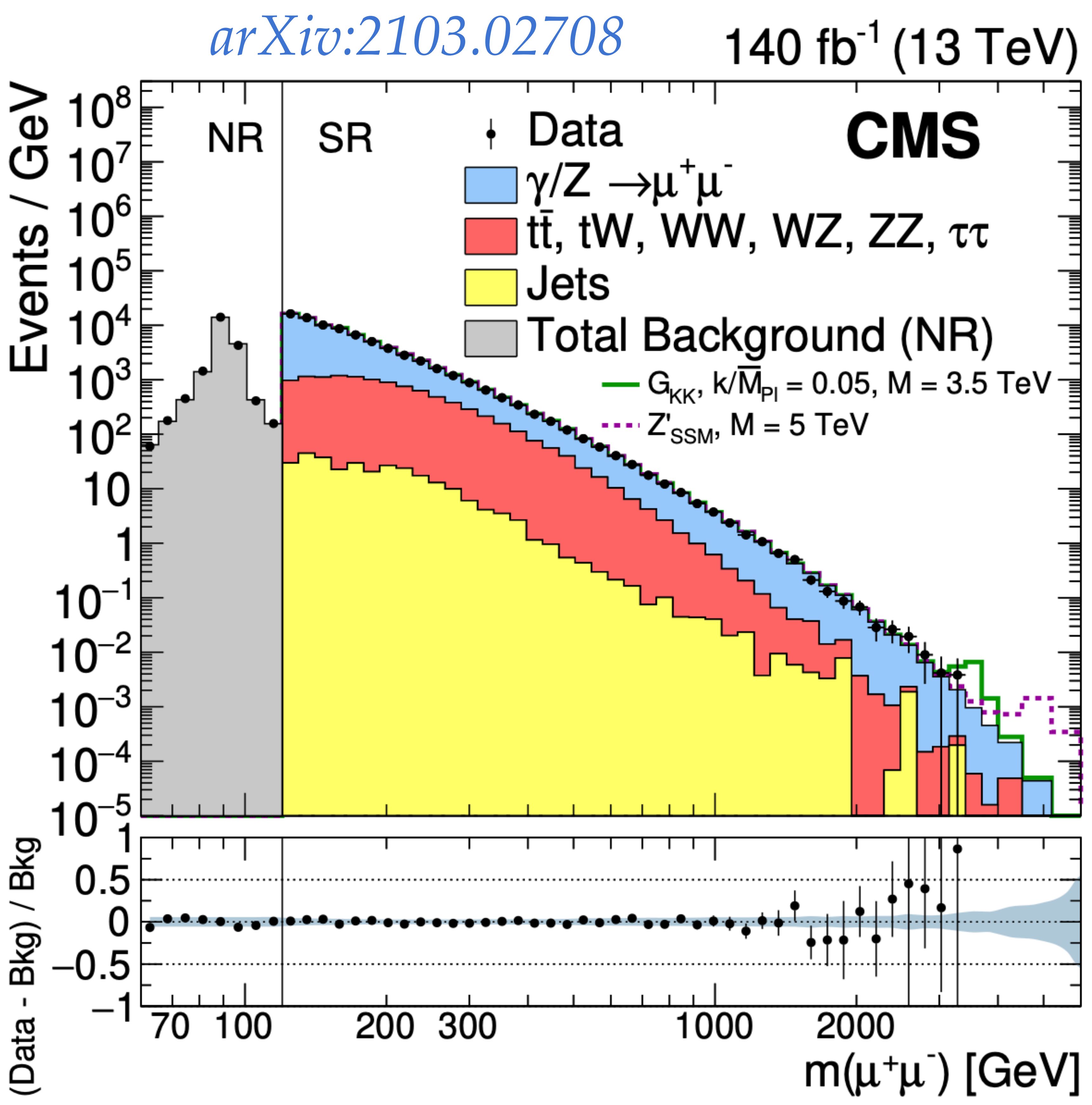
Relevance of Neutral Current Drell-Yan measurements: searches for New Physics signals



mass window [GeV]	stat. unc. 140fb^{-1}	stat. unc. 3ab^{-1}
$600<m_{\mu\mu}<900$	1.4%	0.2%
$900<m_{\mu\mu}<1300$	3.2%	0.6%

At the end of High-Luminosity LHC we will be able to test the TeV region with data at per mille level
i.e.
to test the SM at the level of its quantum corrections

Relevance of Neutral Current Drell-Yan measurements: searches for New Physics signals



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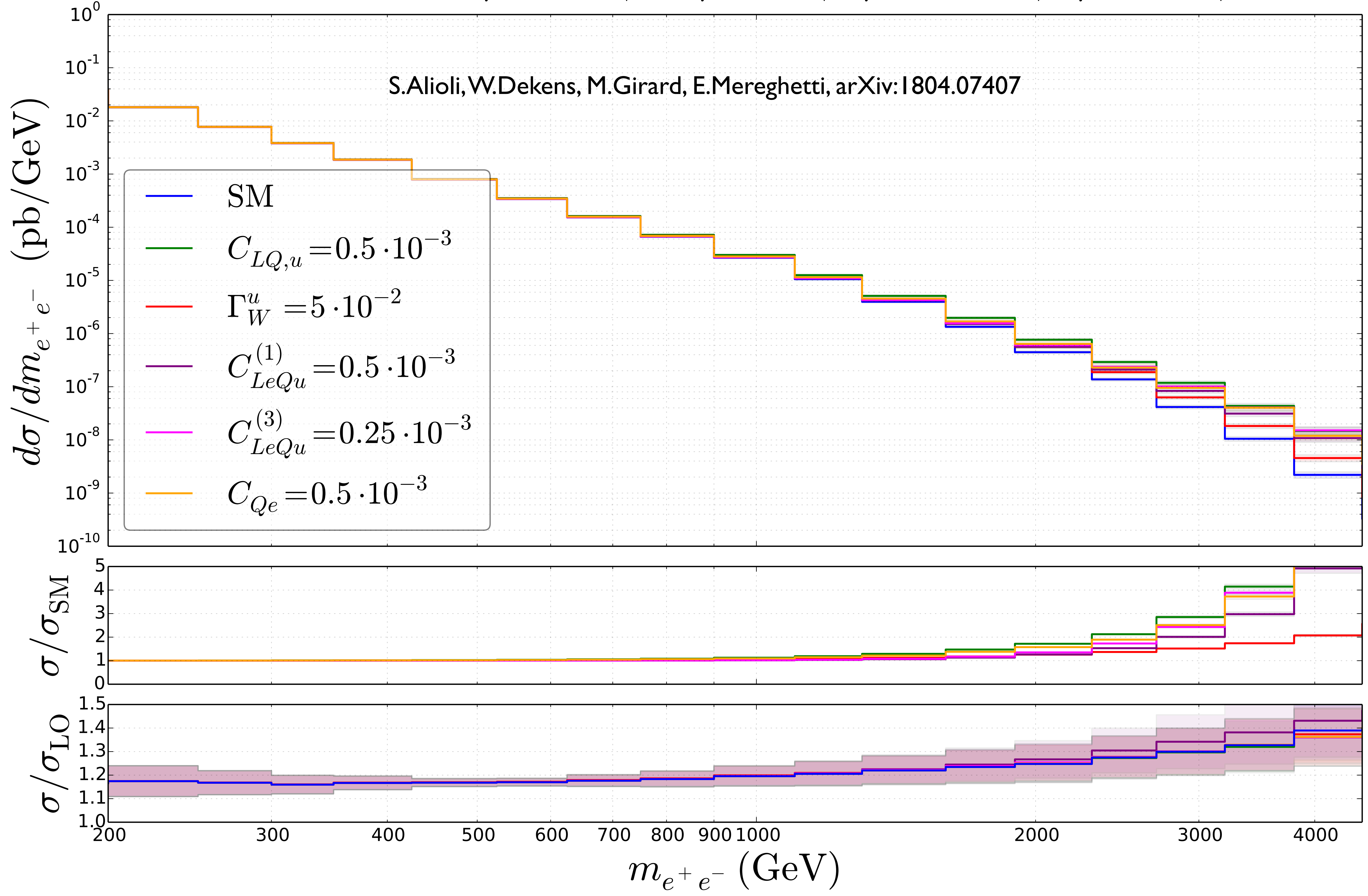
$600<m_{\mu\mu}<900$	1.4%	0.2%
$900<m_{\mu\mu}<1300$	3.2%	0.6%

$$\mathcal{L} = \mathcal{L}_{X^2\varphi^2} + \mathcal{L}_{\psi^2 X\varphi} + \mathcal{L}_{\psi^2\varphi^2 D} + \mathcal{L}_{\psi^2\varphi^3} + \mathcal{L}_{\psi^4}$$



A deviation from the SM prediction can point towards New Physics

Is the SM prediction under control at the O(0.5%) level in the TeV region of the $m_{\ell\ell}$ distribution ?

Neutral current Drell-Yan in a fixed-order expansion

Drell-Yan (1970)

Baur, Brein, Hollik, Schappacher, Wackeroth (2001)

$$\sigma(h_1 h_2 \to \ell\bar{\ell} + X) = \sigma^{(0,0)} +$$

Altarelli, Ellis, Martinelli (1979)

$$\alpha_s\, \sigma^{(1,0)} + \alpha\, \sigma^{(0,1)} +$$

Hamberg, Matsuura, van Nerveen, (1991)
Anastasiou, Dixon, Melnikov, Petriello, (2003)
Catani, Cieri, Ferrera, de Florian, Grazzini (2009)

$$\alpha_s^2\, \sigma^{(2,0)} + \alpha\, \alpha_s\, \sigma^{(1,1)} + \alpha^2\, \sigma^{(0,2)} +$$

$$\alpha_s^3\, \sigma^{(3,0)} + \ldots$$

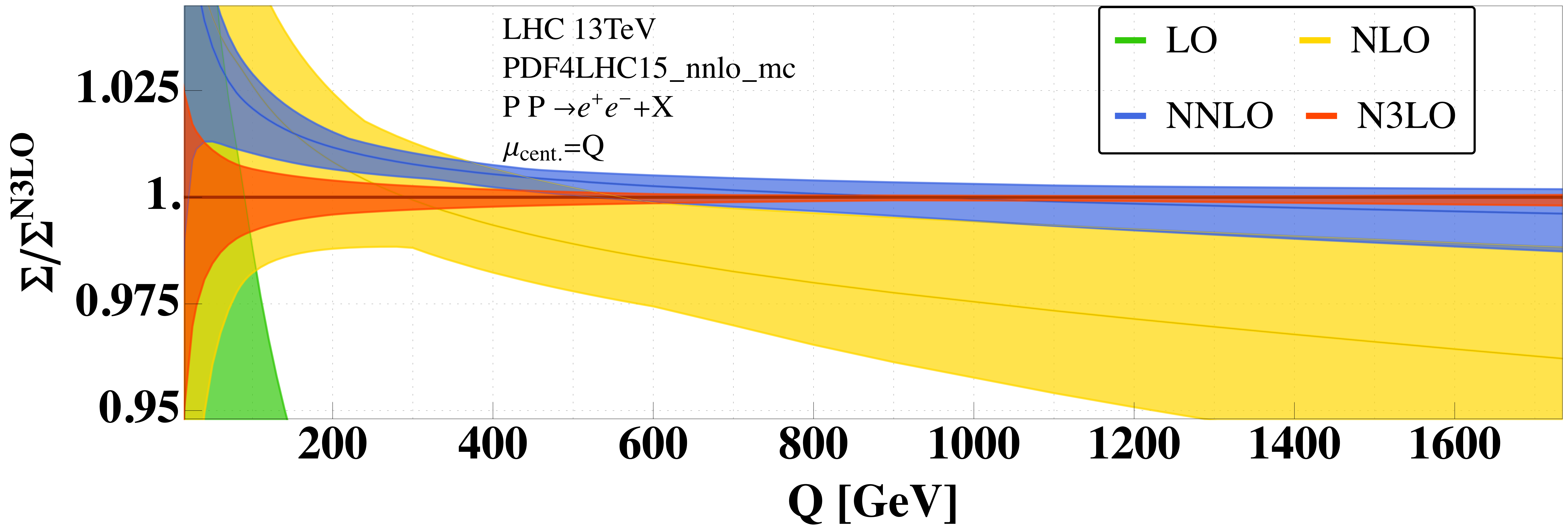
C.Duhr, B.Mistlberger, arXiv:2111.10379

still missing
Sudakov high-energy approximations

R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, (2021)
T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, (2022)
F.Buccioni, F.Caola, H.Chawdhry, F.Devoto, M.Heller, A.von Manteuffel, K.Melnikov, R.Röntsch, C.Signorile-Signorile, (2022)

Progress in the QCD calculations and simulations: lepton-pair invariant mass

C.Duhr, B.Mistlberger, arXiv:2111.10379



Thanks to the N3LO-QCD results for the Drell-Yan cross section, scale variation band at the few per mille level at any Q

The PDFs are not yet at N3LO

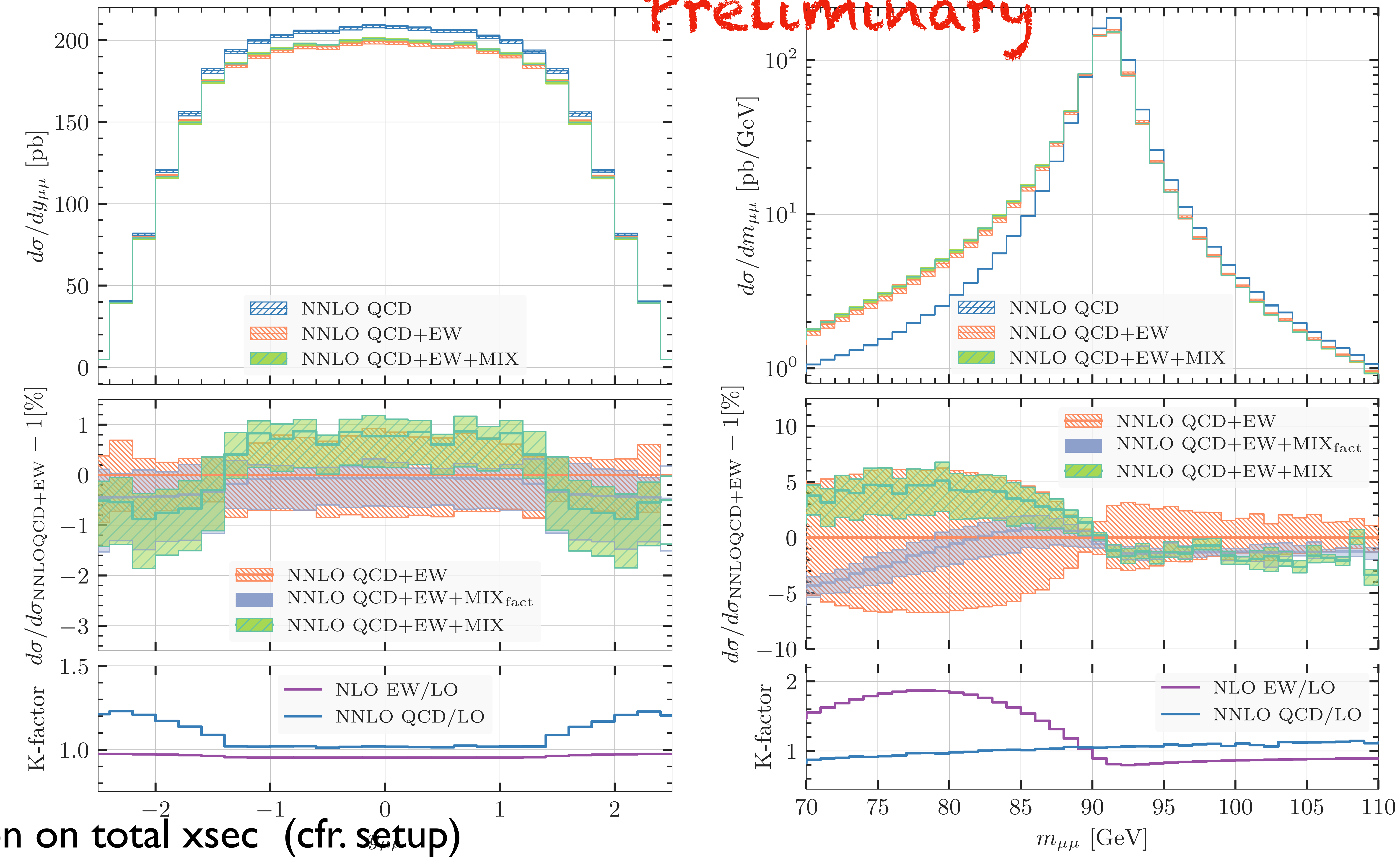
This is promising, in view of the program of searches for deviation from the SM in the TeV range

What about NNLO QCD-EW and NNLO-EW corrections ?

Phenomenology of Neutral Current Drell-Yan including exact NNLO QCD-EW corrections

R.Bonciani, L.Buonocore, S.Devoto, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, arXiv:2106.11953 , Phys.Rev.Lett. 128 (2022) 1, 012002 and work in preparation

Preliminary



Sub-percent correction on total xsec (cfr. setup)

Non-trivial distortion of the rapidity distribution (absent in the naive factorised approximation)

Large effects below the Z resonance (the factorised approximation fails) → impact on the $\sin^2\theta_{eff}$ determination

O(-1.5%) effects above the resonance → ongoing precision studies in the CERN EW WG

Phenomenology of Neutral Current Drell-Yan including exact NNLO QCD-EW corrections

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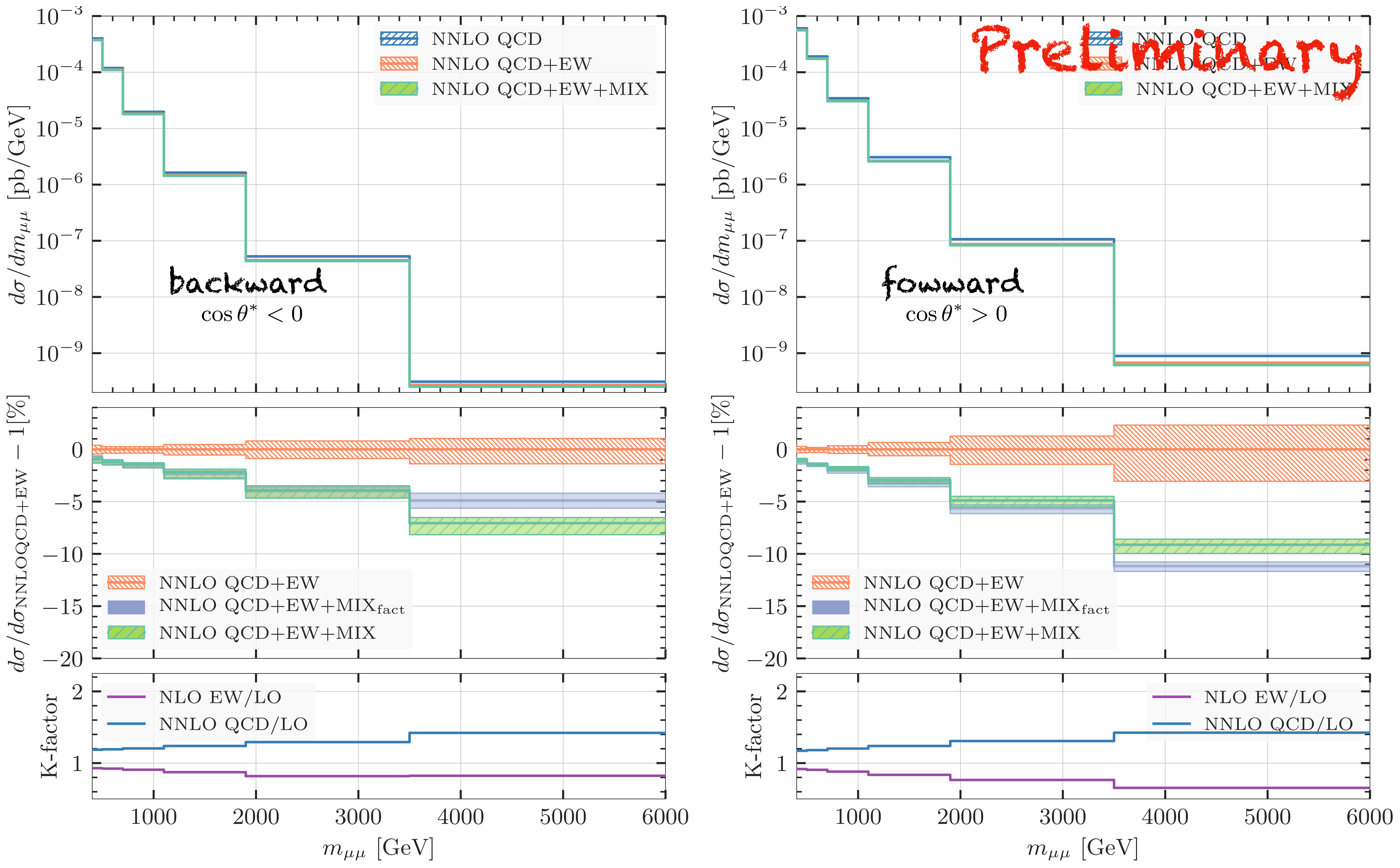
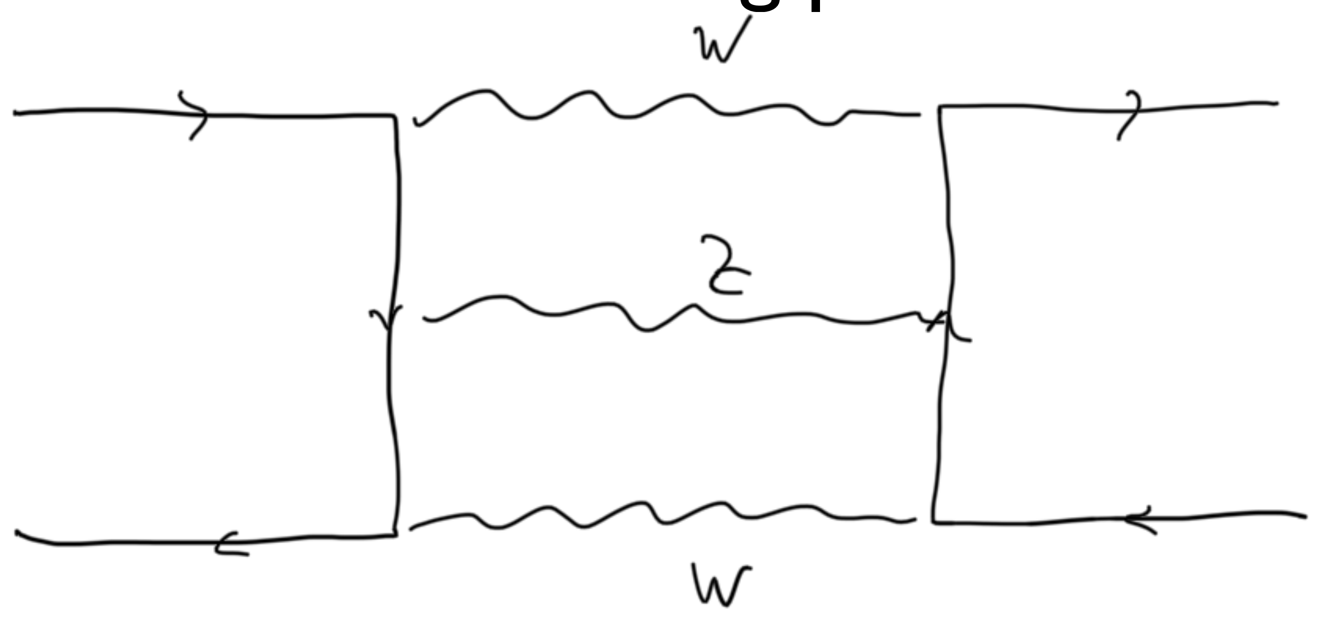


Negative mixed NNLO QCD-EW effects (-3% or more) at large invariant masses,

absent in any additive combination $\rightarrow$ impact on the searches for new physics

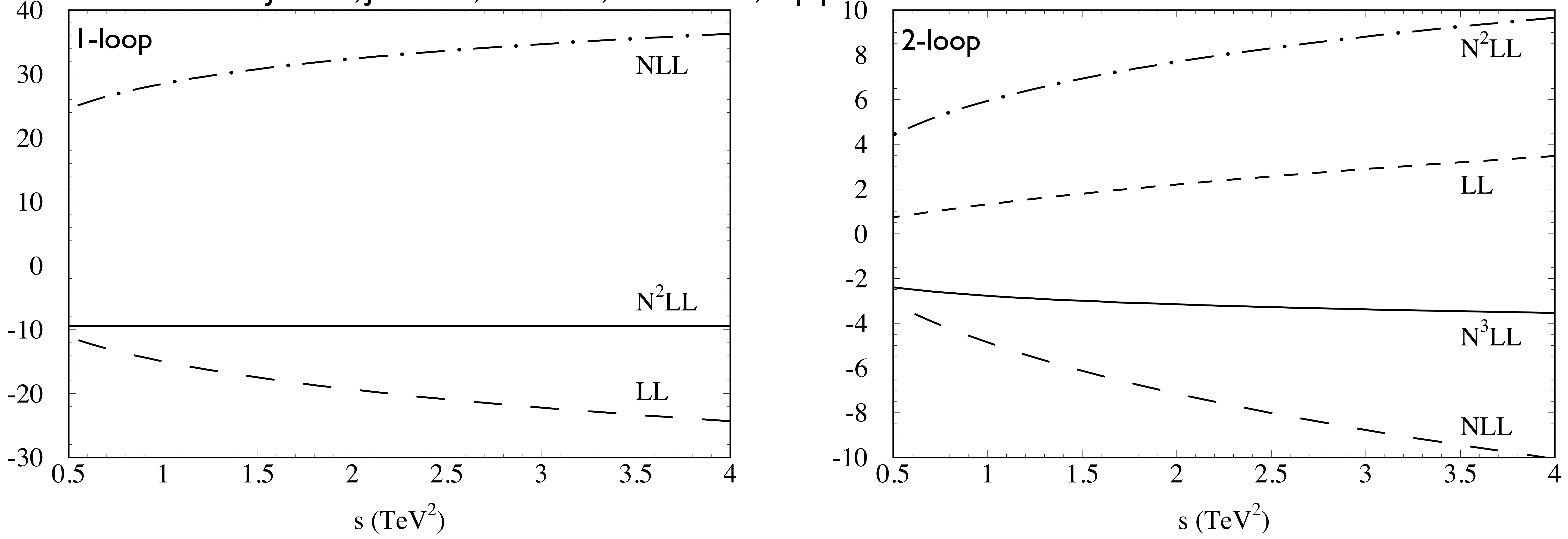
Need for a full NNLO-EW calculation to reduce the uncertainties to sub-percent level

The NNLO-EW corrections to scattering processes are still today one of the frontiers in QFT



The NNLO-EW corrections could modify in a non-trivial way the large-mass/momentum tails of the distributions
Large logarithmic corrections (EW Sudakov logs) appear in the virtual corrections
At two-loop level, we have up to the fourth power of $\log(s/m_V^2)$,

B.Jantzen, J.H.Kühn, A.A.Penin, V.A.Smirnov, hep-ph/0509157

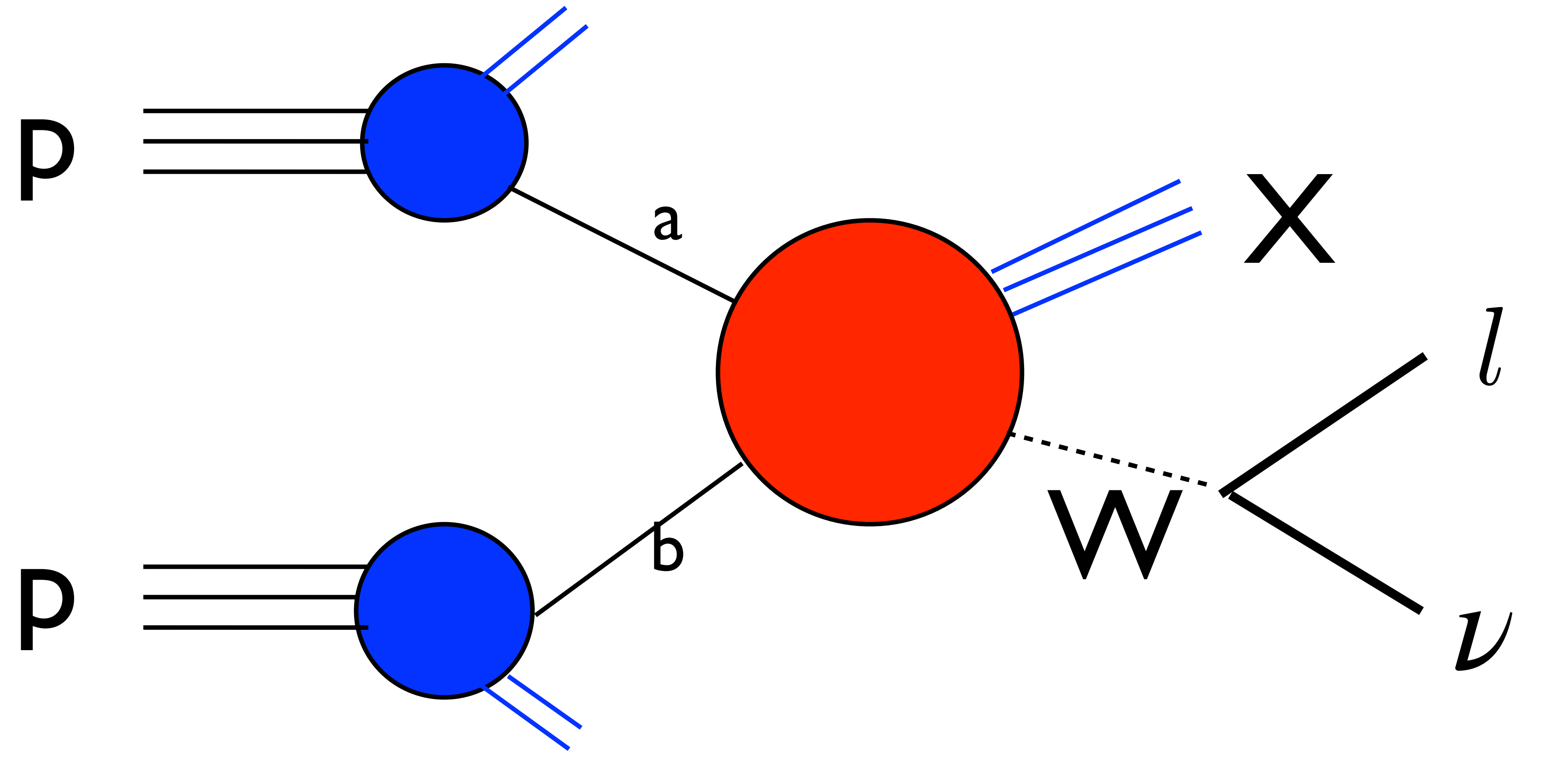


corrections to $e^+e^- \to q\bar{q}$ due to EW Sudakov logs

urgently needed to match sub-percent precision in the TeV region, but also to match FCC-ee precision

W-boson mass determination

m_W determination at hadron colliders

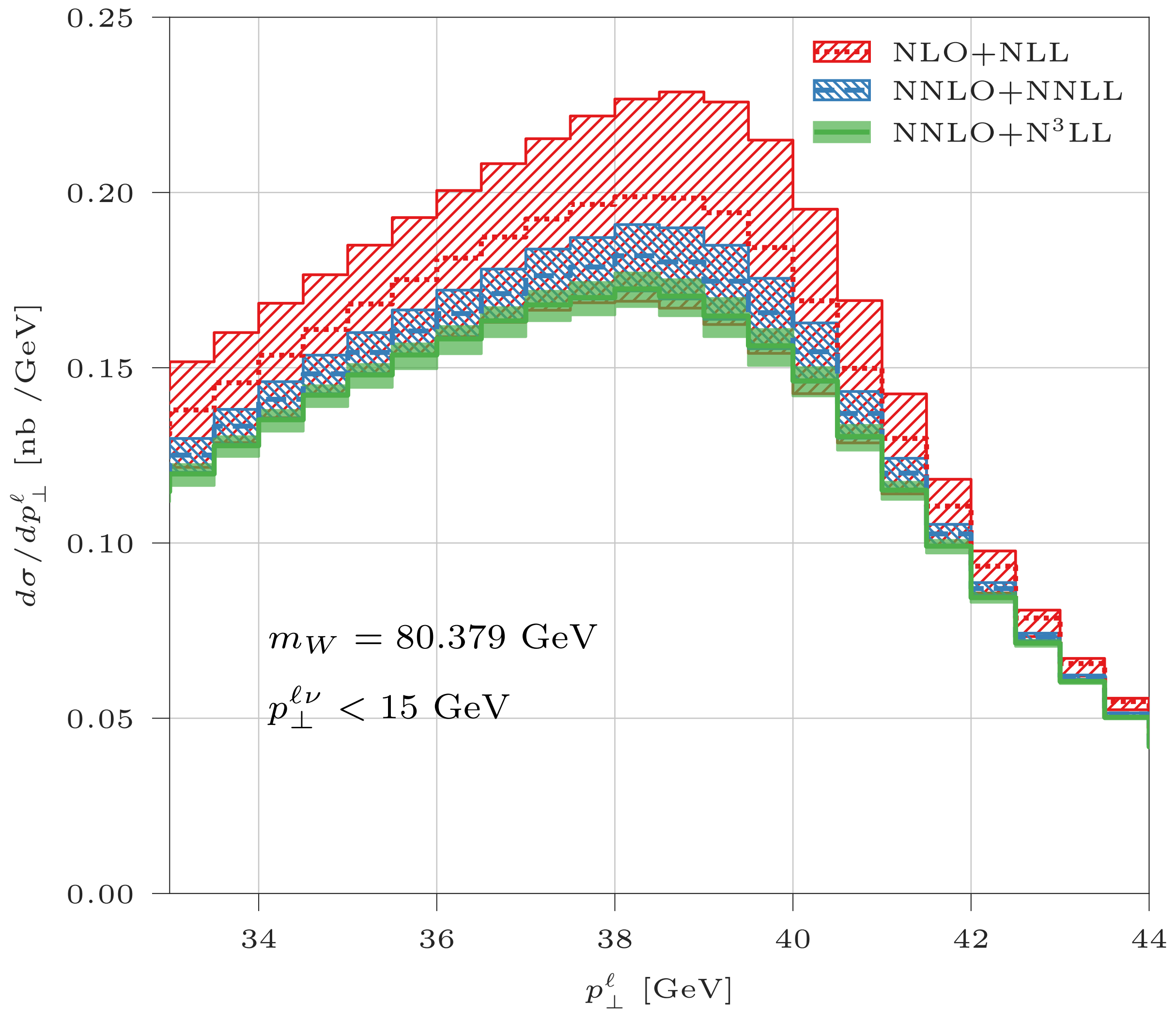


- In charged-current DY,
  it is NOT possible to reconstruct the lepton-neutrino invariant mass
  Full reconstruction is possible (but not easy) only in the transverse plane

- A generic observable has a linear response to an m_W variation
  With a goal for the relative error of 10^{-4}, the problem seems to be unsolvable

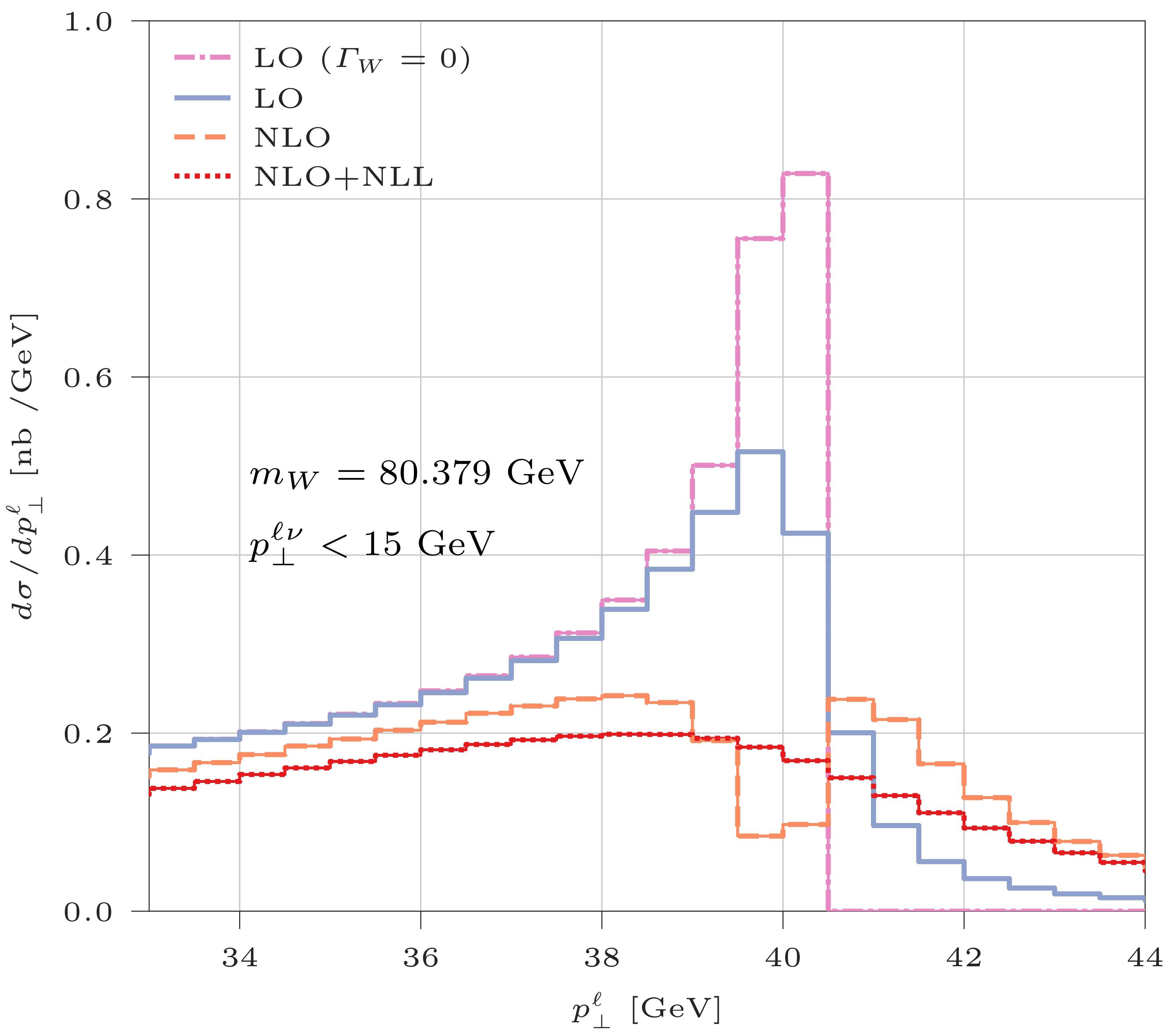
- m_W extracted from the study of the shape of the $p_\perp^l, M_\perp$ and $E_\perp^{miss}$ distributions in CC-DY
  thanks to the jacobian peak that enhances the sensitivity to m_W

$$\frac{d}{dp_\perp^2} \to \frac{2}{s}\frac{1}{\sqrt{1-4p_\perp^2/s}}\frac{d}{d\cos\theta} \sim \frac{d}{dp_\perp^2} \to \frac{2}{s}\frac{1}{\sqrt{1-4p_\perp^2/m_W^2}}\frac{d}{d\cos\theta}$$



→ enhanced sensitivity at the 10^{-3} level ($p_\perp^l$ distribution)
or even at the 10^{-2} level ($M_\perp$ distribution)

The lepton transverse momentum distribution in charged-current Drell-Yan



The lepton transverse momentum distribution has a jacobian peak induced by the factor $1/\sqrt{1-\frac{s}{4p_\perp^2}}$.

When studying the W resonance region, the peak appears at $p_\perp \sim \frac{m_W}{2}$

Kinematical end point at $\frac{m_W}{2}$ at LO

The decay width allows to populate the upper tail of the distribution

Sensitivity to soft radiation $\rightarrow$ double peak at NLO-QCD

The QCD-ISR next-to-leading-log resummation broadens the distribution and cures the sensitivity to soft radiation at the jacobian peak.

In the $p_\perp^\ell$ spectrum the sensitivity to m_W and important QCD features are closely intertwined

m_W determination at hadron colliders: template fitting

Given one experimental kinematical distribution

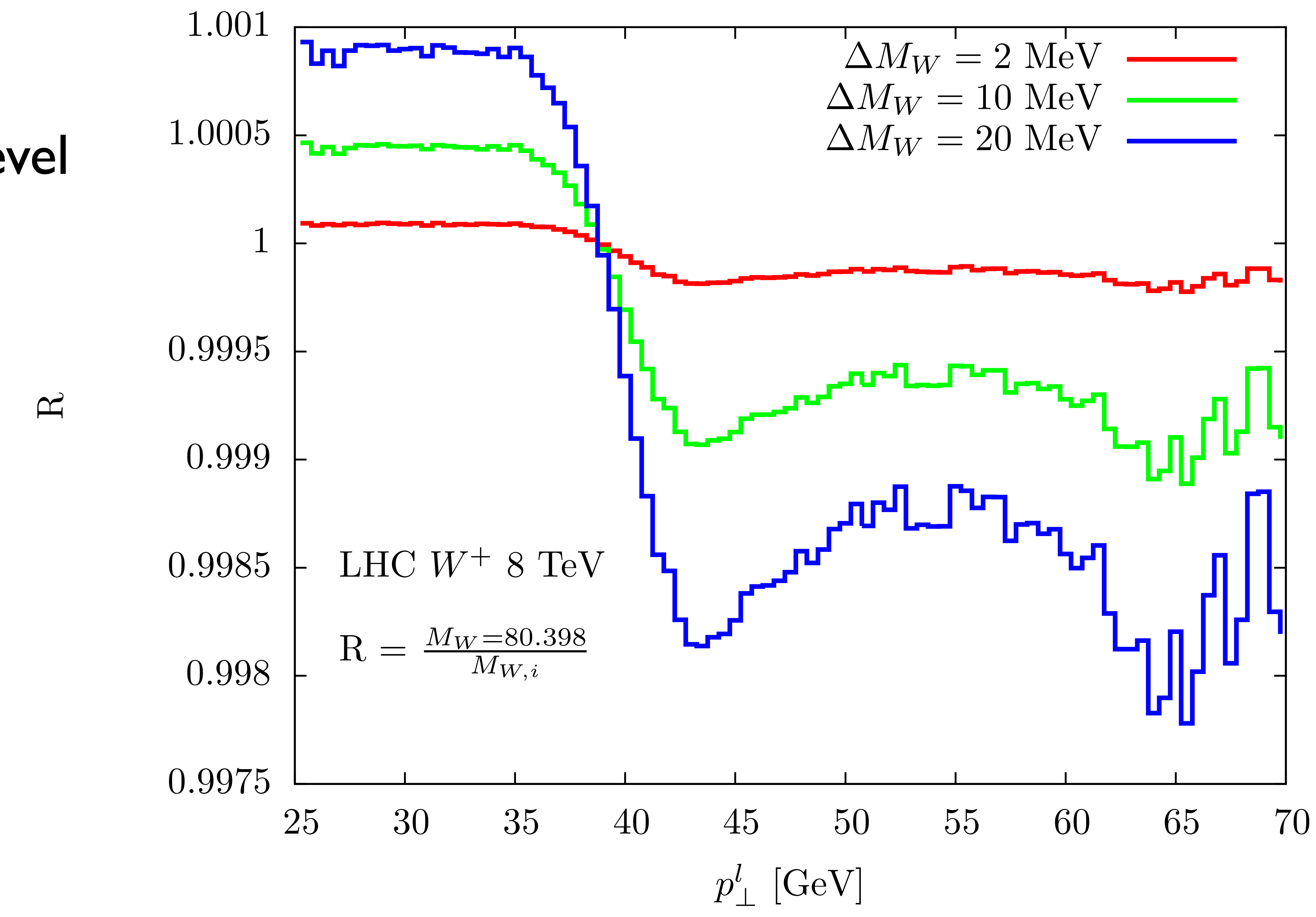
- we compute the corresponding theoretical distribution for several hypotheses of one Lagrangian input parameters (e.g. m_W)
- we compute, for each $m_W^{(k)}$ hypothesis, a χ_k^2 defined in a certain interval around the jacobian peak (fitting window)
- we look for the minimum of the χ^2 distribution

The m_W value associated to the position of the minimum of the χ^2 distribution is the experimental result

A determination at the 10^{-4} level requires
a control over the shape of the distributions at the per mille level

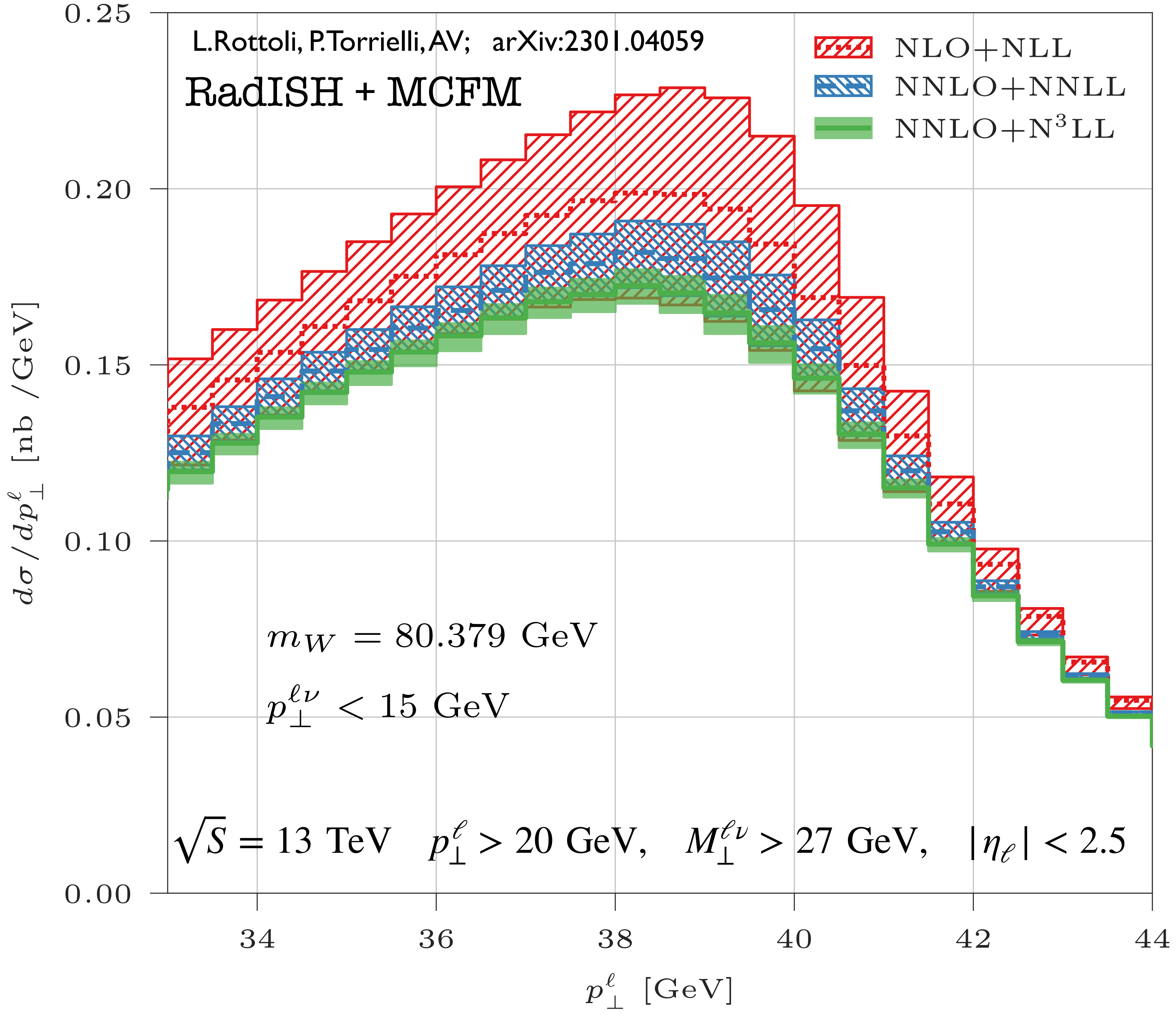
The theoretical uncertainties of the templates
contribute to the theoretical systematic error on m_W

- higher-order QCD
- non-perturbative QCD
- PDF uncertainties
- heavy quarks corrections
- EW corrections



Template fitting: description of the single lepton transverse momentum distribution

The template fitting procedure is acceptable if the data are described by the theoretical distribution with high quality



Scale variation of the NNLO+N3LL prediction for ptlep provides a set of equally good templates
but the width of the uncertainty band is at the few percent level
a factor 10 larger than the naive estimate would require !

→ data driven approach

a Monte Carlo event generator is tuned to the data in NCDY ($p_\perp^Z$)
for one QCD scale choice

↓

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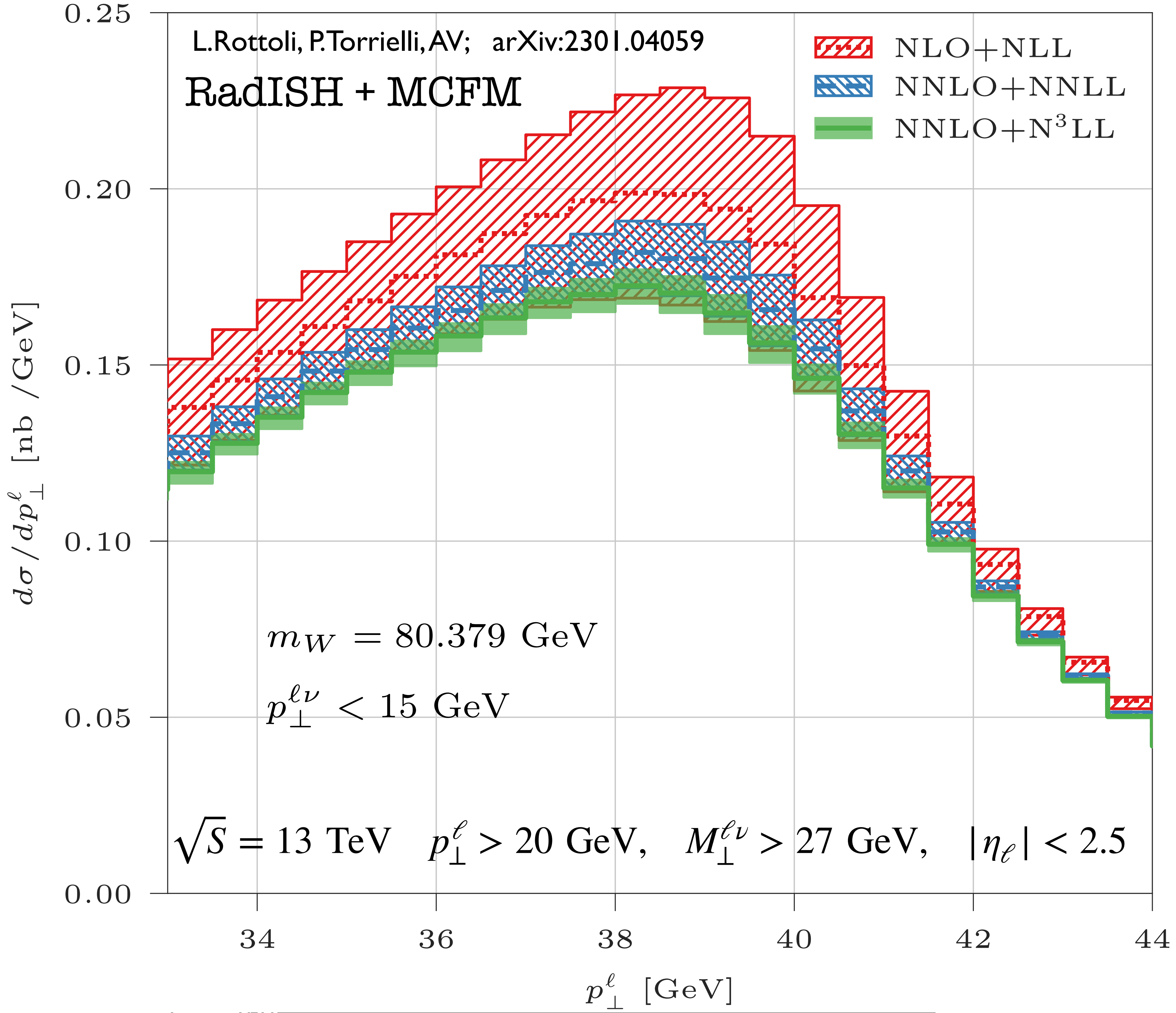

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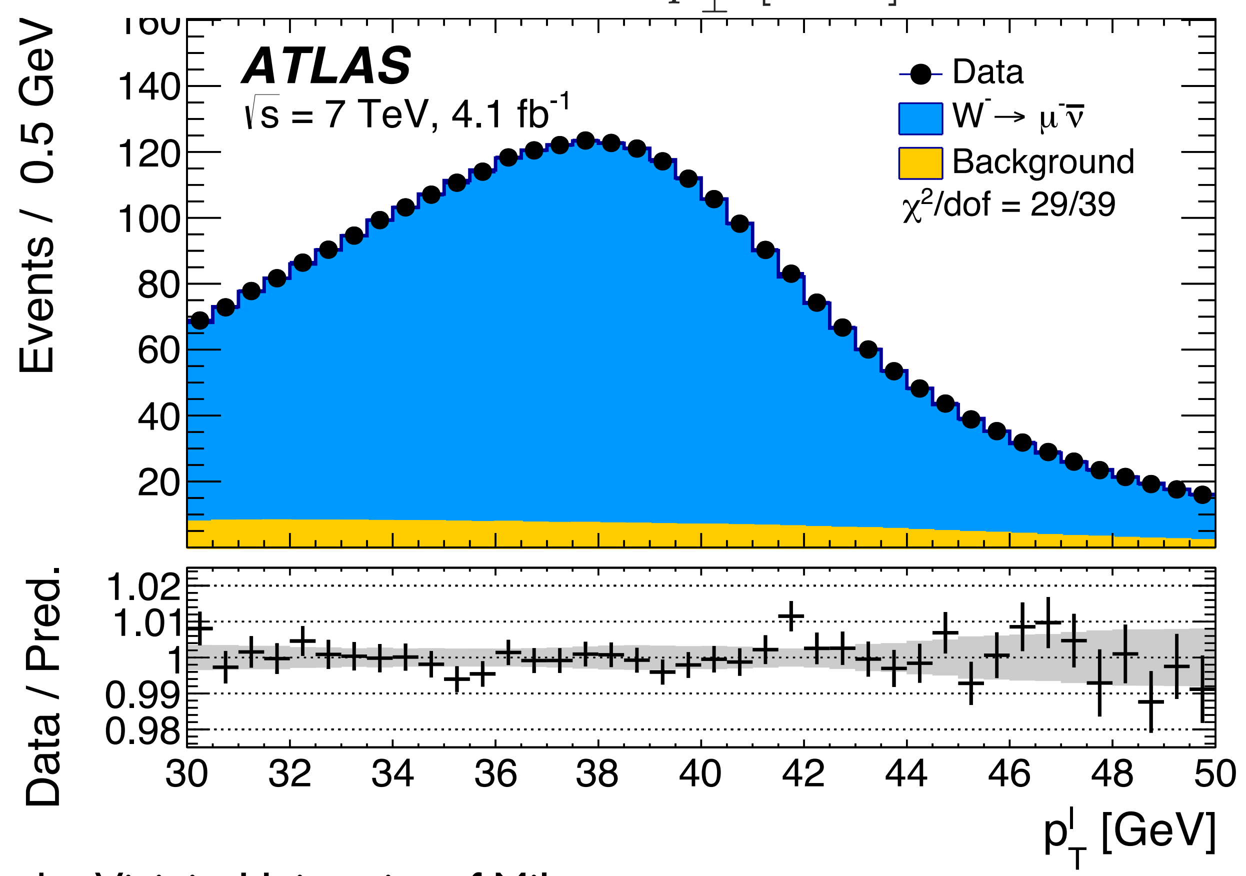
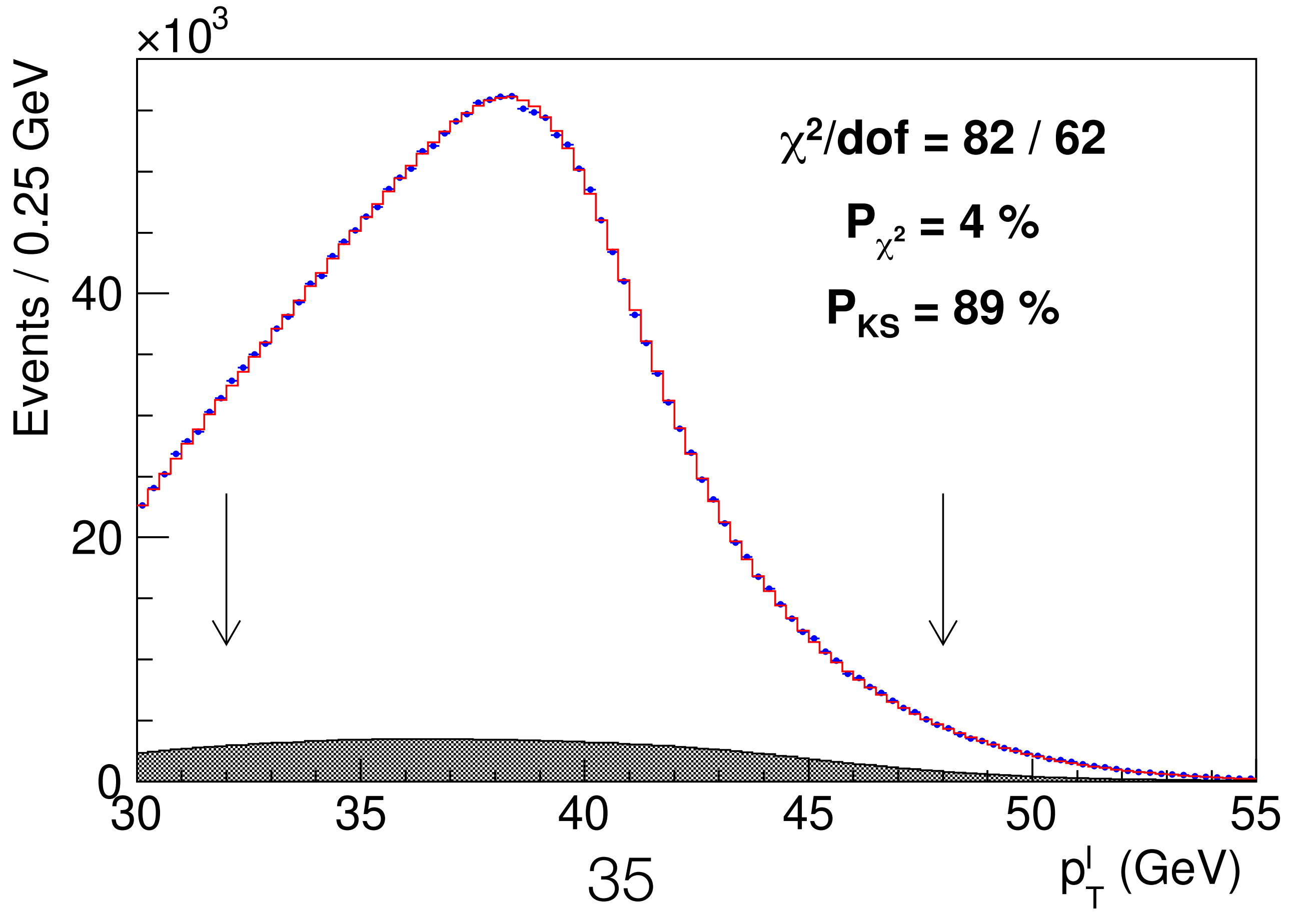
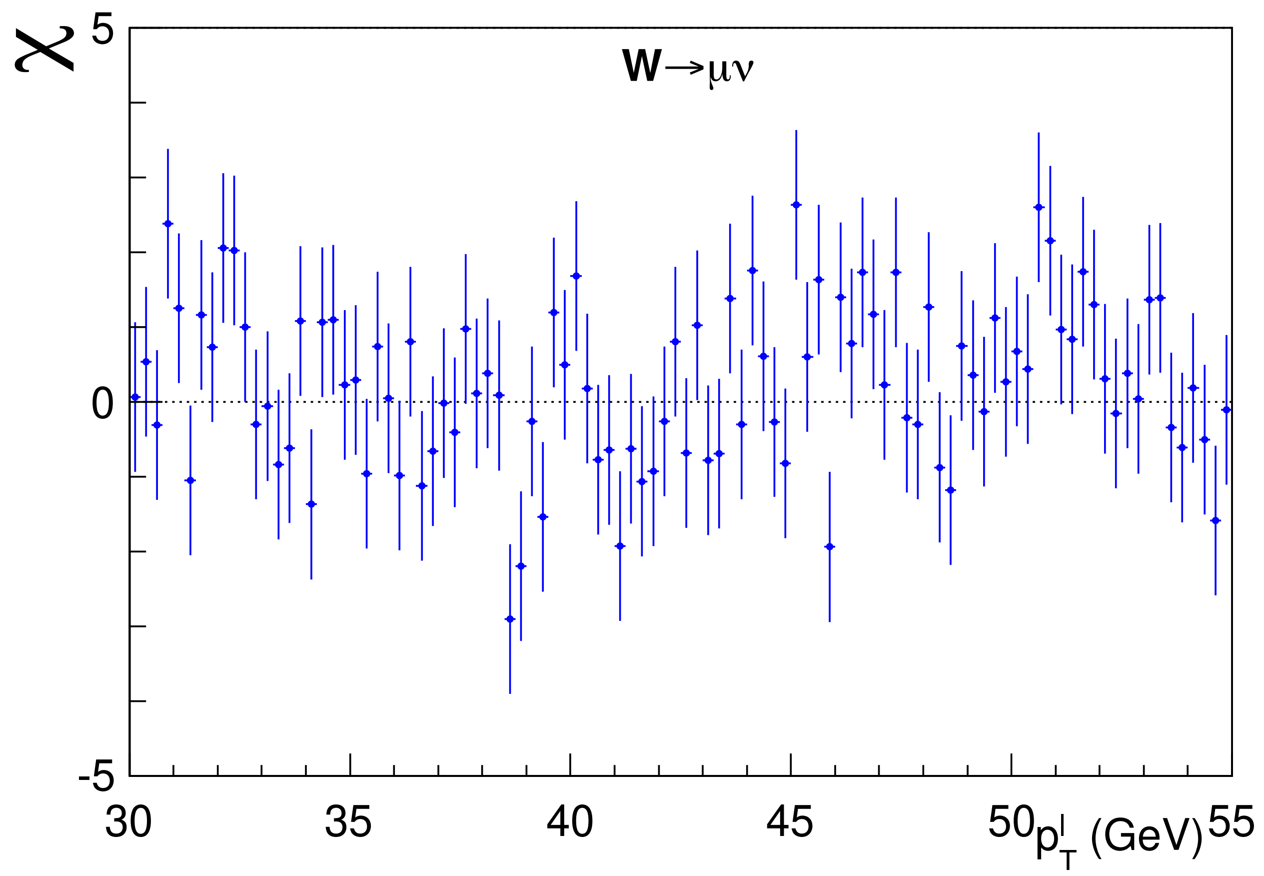


CDF collaboration, Scince 376, 170-176 (2022)





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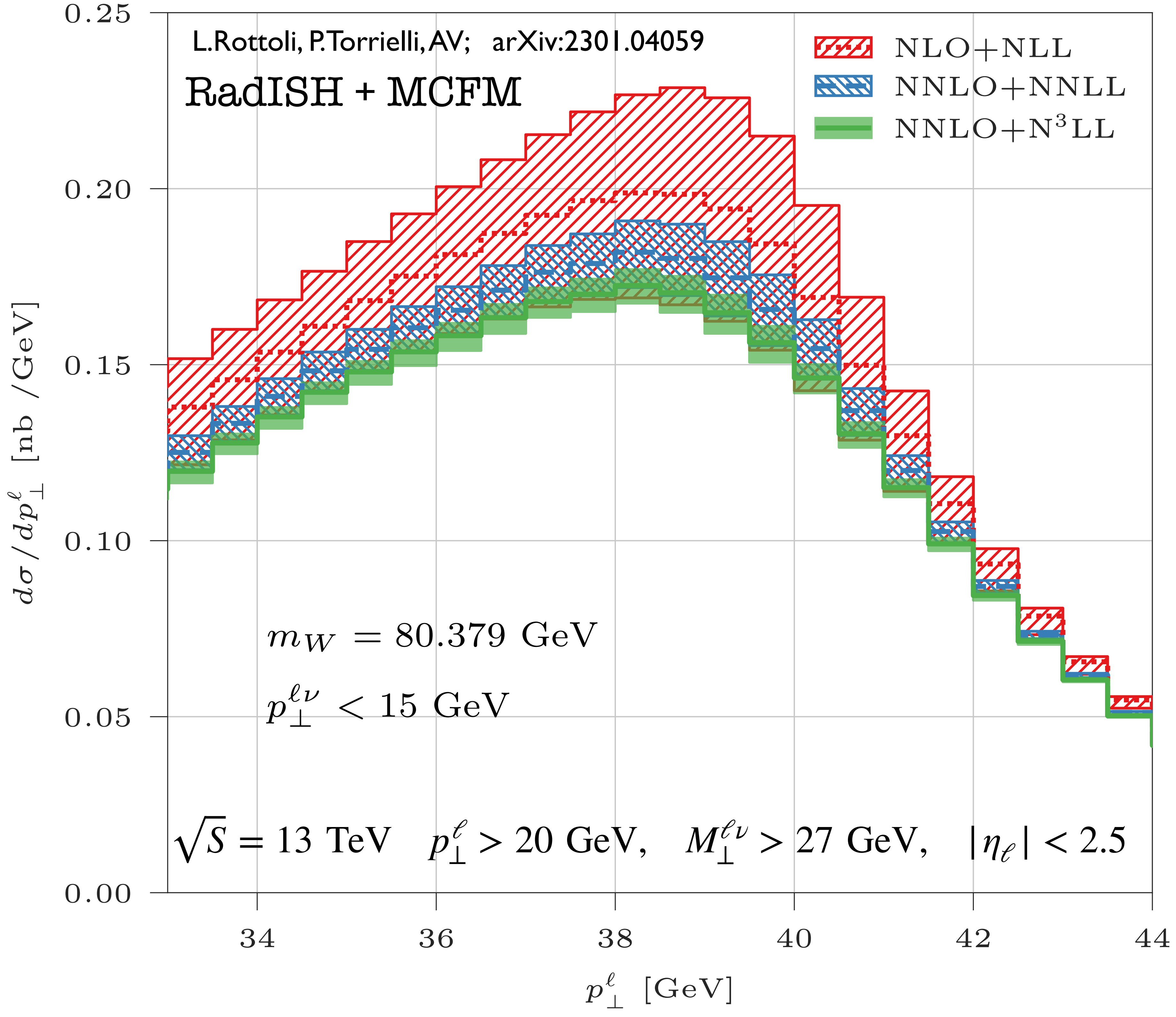

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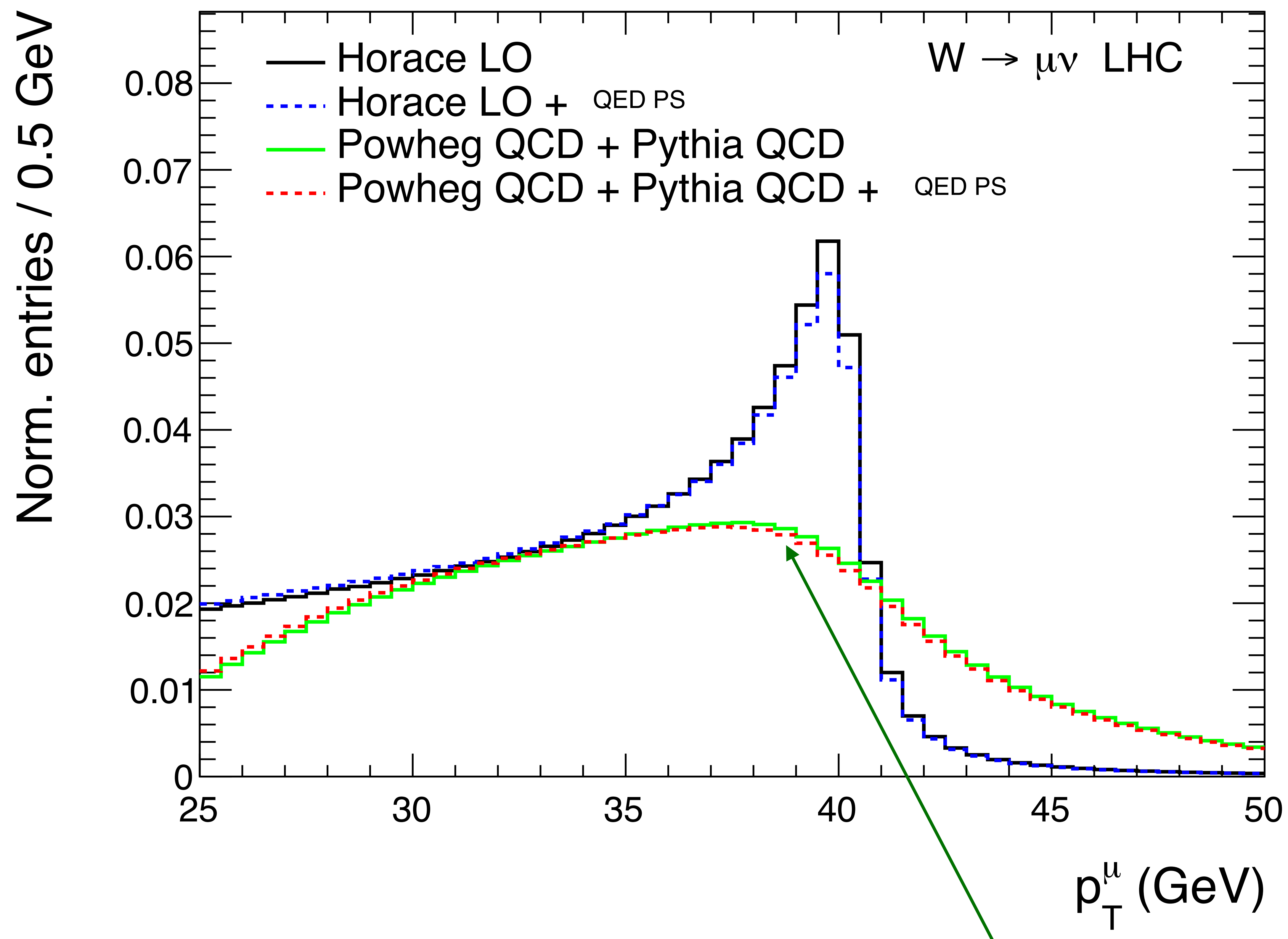
a Monte Carlo event generator is tuned to the data in NCDY ($p_\perp^Z$)
for one QCD scale choice
↓

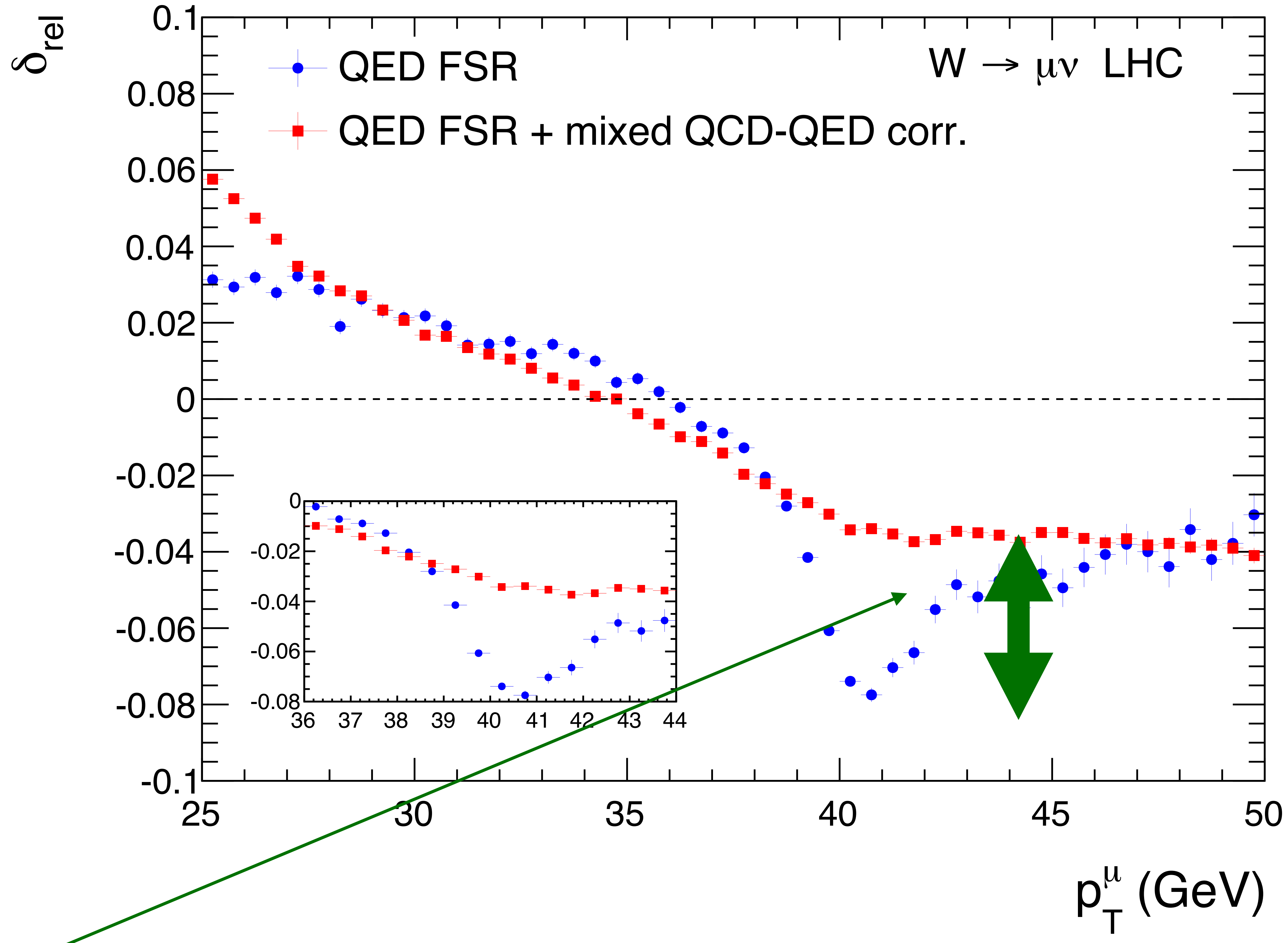
A data driven approach improves the accuracy of the model (i.e. its ability to describe the data)
does not improve the precision of the model (the intrinsic ambiguities in the model formulation)

What are the limitations of the transfer of information from NCDY to CCDY ?

Interplay of QCD and QED corrections

C.Carloni Calame, M.Chiesa, H.Martinez, G.Montagna, O.Nicrosini, F.Piccinini, AV, arXiv:1612.02841





- very large impact of initial-state QCD radiation on the ptlep distribution
- large radiative corrections due to QED final state radiation at the jacobian peak
- very large interplay of QCD and QED corrections redefining the precise shape of the jacobian peak

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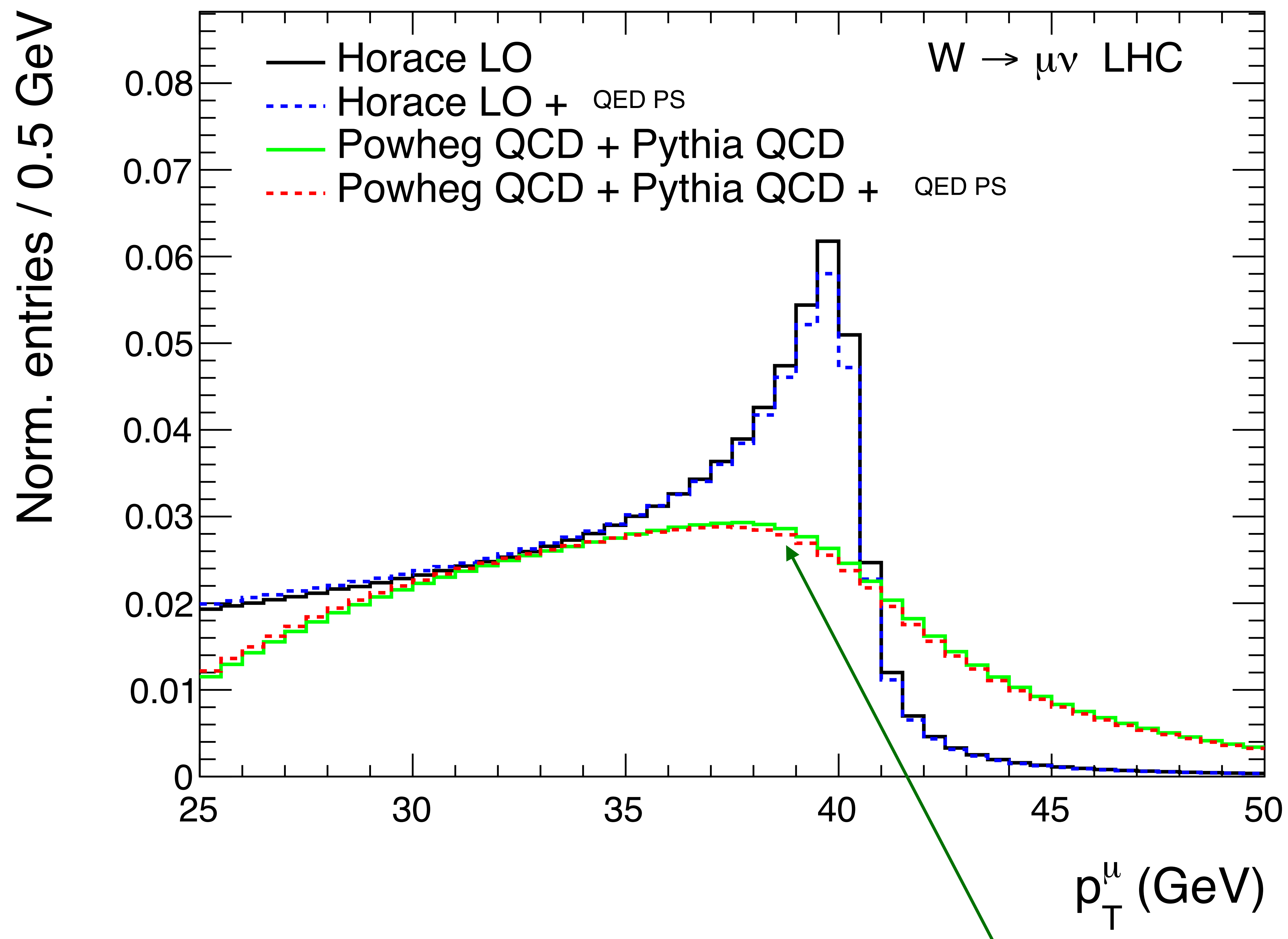

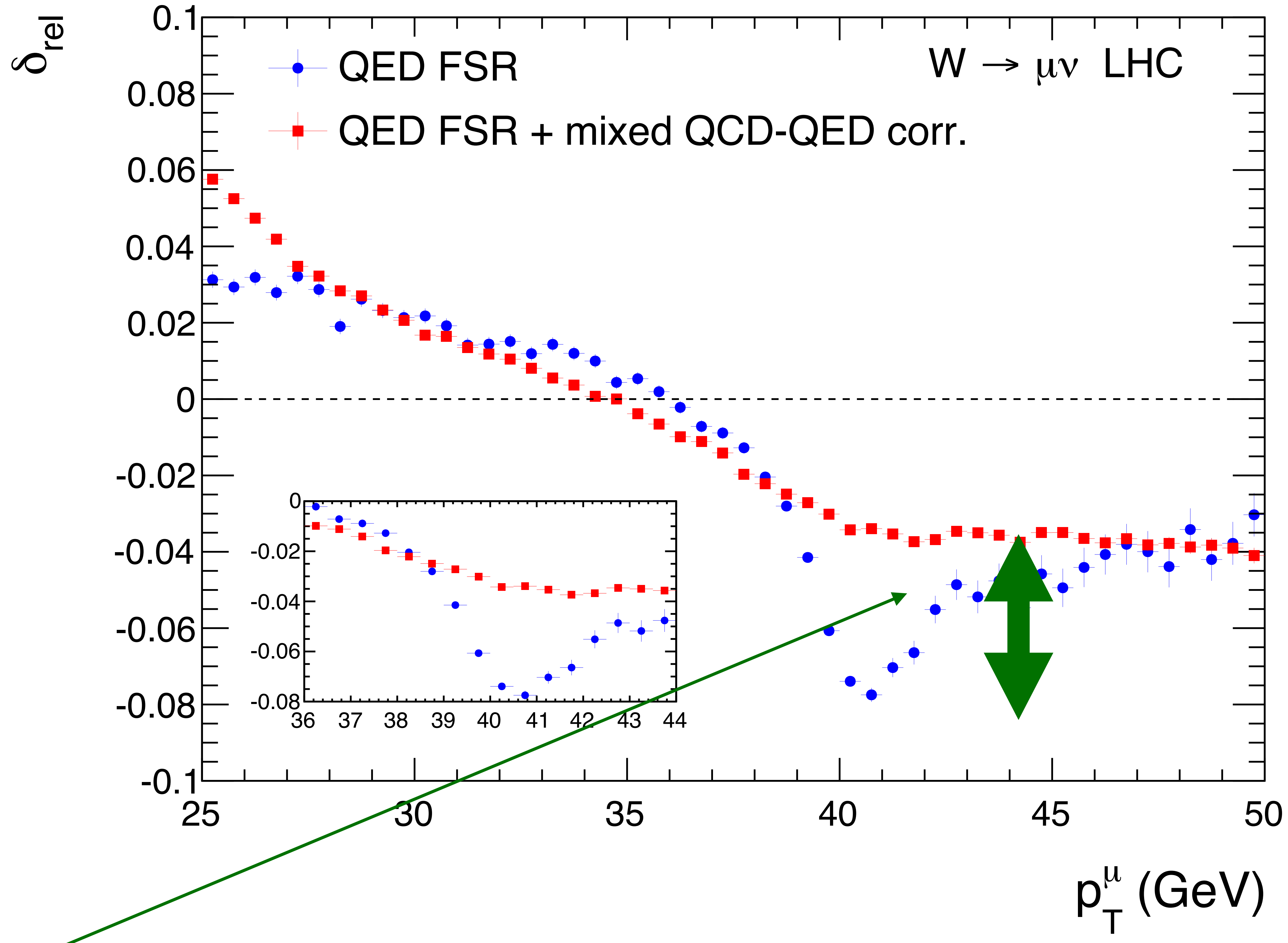

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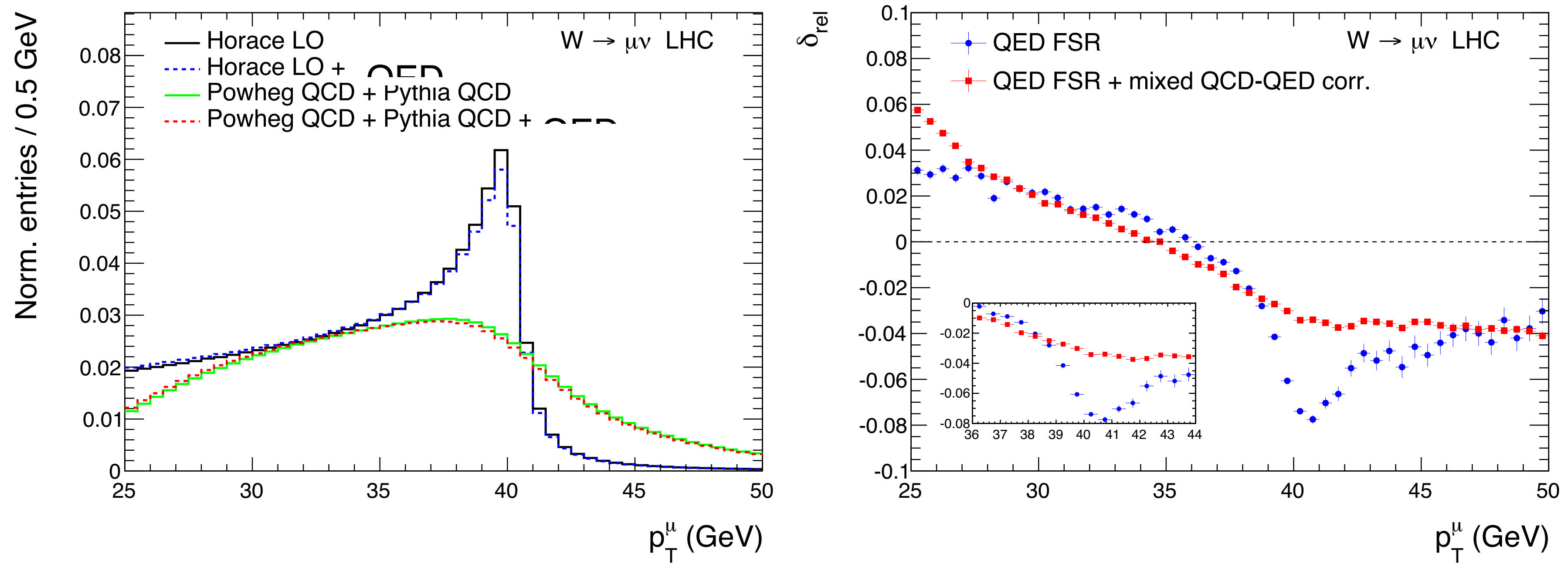
NLO-QCD + QCDPS + QEDPS is the lowest order meaningful approximation of this observable

the precise size of the mixed QCDxQED corrections (and uncertainties) depends on the choice for the QCD modelling

Impact of EW and mixed QCDxEW corrections on MW

C.Carloni Calame, M.Chiesa, H.Martinez, G.Montagna, O.Nicrosini, F.Piccinini, AV, arXiv:1612.02841

$pp \to W^+$, $\sqrt{s} = 14$ TeV	M_W shifts (MeV)			
Templates accuracy: LO	$W^+ \to \mu^+\nu$		$W^+ \to e^+\nu$	
Pseudo–data accuracy	M_T	p_T^ℓ	M_T	p_T^ℓ
1 HORACE only FSR-LL at $\mathcal{O}(\alpha)$	-94±1	-104±1	-204±1	-230±2
2 HORACE FSR-LL	-89±1	-97±1	-179±1	-195±1
3 HORACE NLO-EW with QED shower	-90±1	-94±1	-177±1	-190±2
4 HORACE FSR-LL + Pairs	-94±1	-102±1	-182±2	-199±1
5 PHOTOS FSR-LL	-92±1	-100±2	-182±1	-199±2



- QED FSR plays the major role
- subleading QED and weak induce further O(4 MeV) shifts

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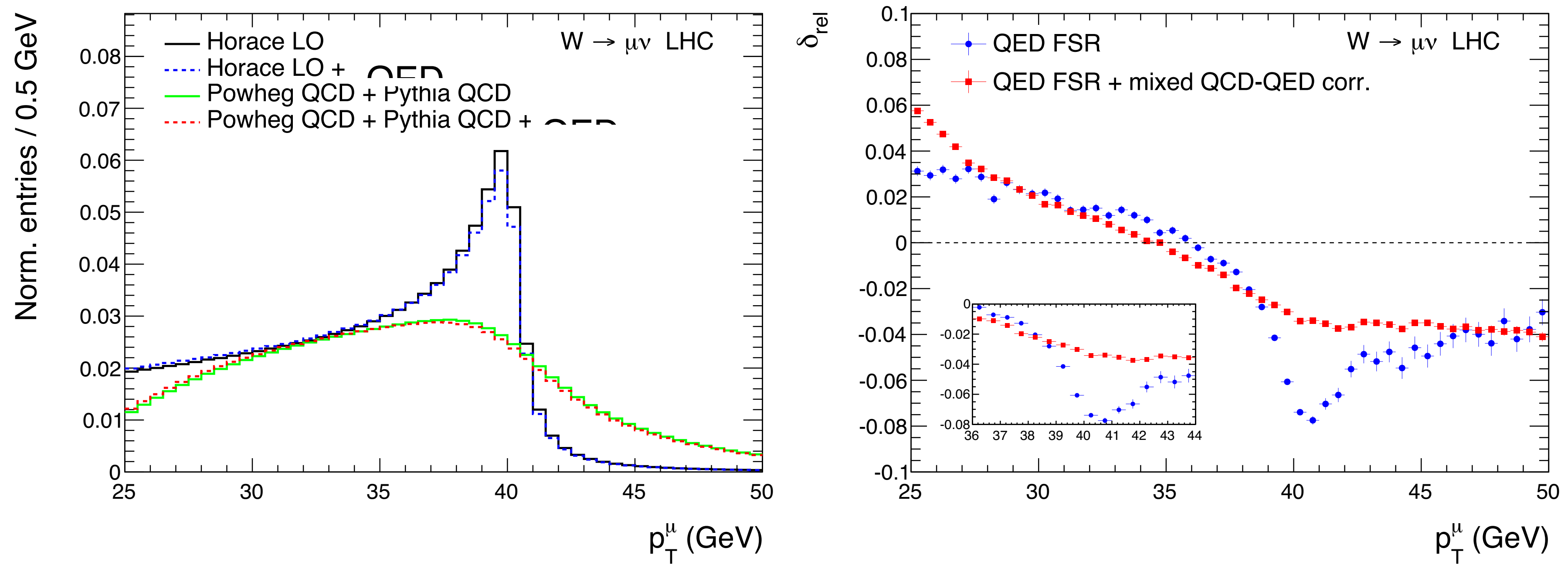

	$pp \to W^+$, $\sqrt{s} = 14$ TeV Templates accuracy: LO Pseudo–data accuracy	M_W shifts (MeV) $W^+ \to \mu^+\nu$ M_T	 p_T^ℓ	 $W^+ \to e^+\nu$ M_T	 p_T^ℓ
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- QED FSR plays the major role
- subleading QED and weak induce further O(4 MeV) shifts

the impact on MW of the mixed QCD QED-FSR corrections strongly depends on the underlying QCD shape/model

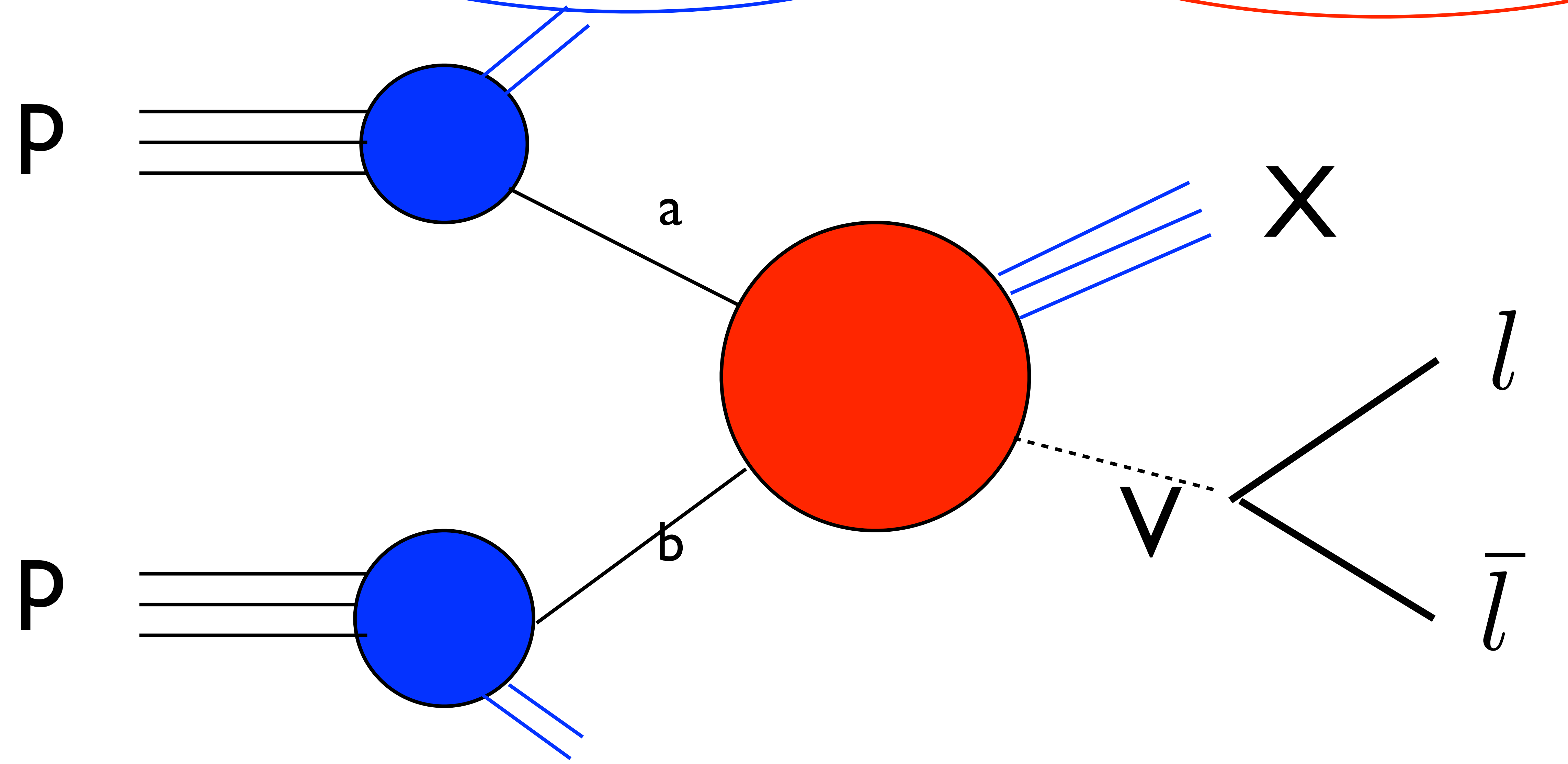
	$pp \to W^+$, $\sqrt{s} = 14$ TeV Templates accuracy: NLO-QCD+QCD$_{PS}$ Pseudodata accuracy	 QED FSR	M_W shifts (MeV) $W^+ \to \mu^+\nu$ M_T	 p_T^ℓ	 $W^+ \to e^+\nu$(dres) M_T	 p_T^ℓ
1	NLO-QCD+(QCD+QED)$_{PS}$	PYTHIA	-95.2±0.6	-400±3	-38.0±0.6	-149±2
2	NLO-QCD+(QCD+QED)$_{PS}$	PHOTOS	-88.0±0.6	-368±2	-38.4±0.6	-150±3
3	NLO-(QCD+EW)+(QCD+QED)$_{PS}$two-rad	PYTHIA	-89.0±0.6	-371±3	-38.8±0.6	-157±3
4	NLO-(QCD+EW)+(QCD+QED)$_{PS}$two-rad	PHOTOS	-88.6±0.6	-370±3	-39.2±0.6	-159±2

the bulk of the corrections is included in the analyses

- what is the associated uncertainty ?
- what happens if we change the underlying QCD model ?

PDF uncertainties and MW determination

$$\sigma(P_1, P_2; m_V) = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_{h_1,a}(x_1, M_F) f_{h_2,b}(x_2, M_F) \, \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2, \alpha_s(\mu), M_F)$$



- in a fiducial volume the rapidity and transverse momentum dependencies are connected by kinematics

$\rightarrow$ the PDF uncertainties (longitudinal d.o.f.) are "transmitted" to the transverse observables $\rightarrow$ impact on m_W

- proton PDF uncertainty parameterised with replicas $\rightarrow$ each one yields a different m_W fit result

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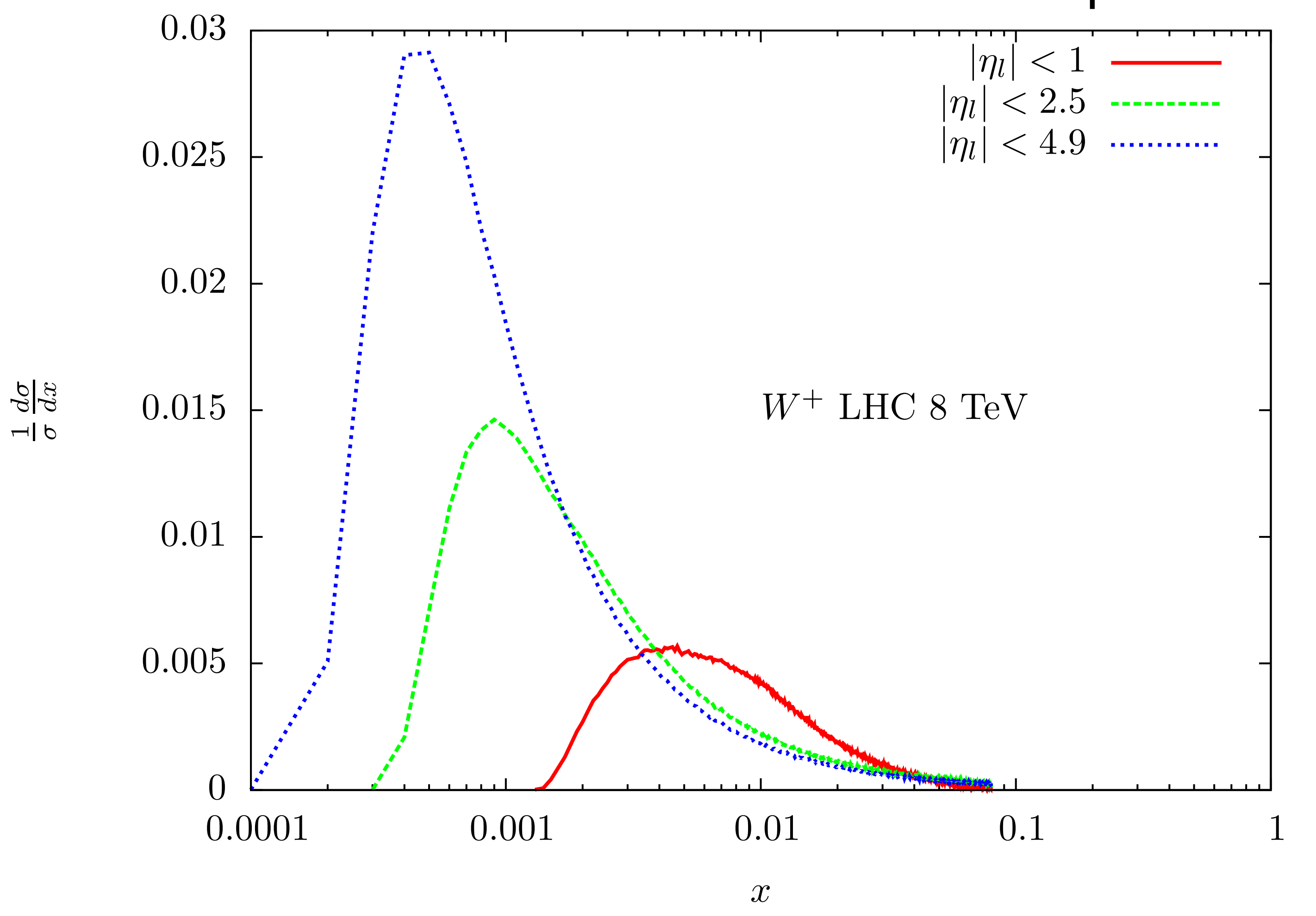
PDF uncertainties and acceptance cuts; anticorrelations

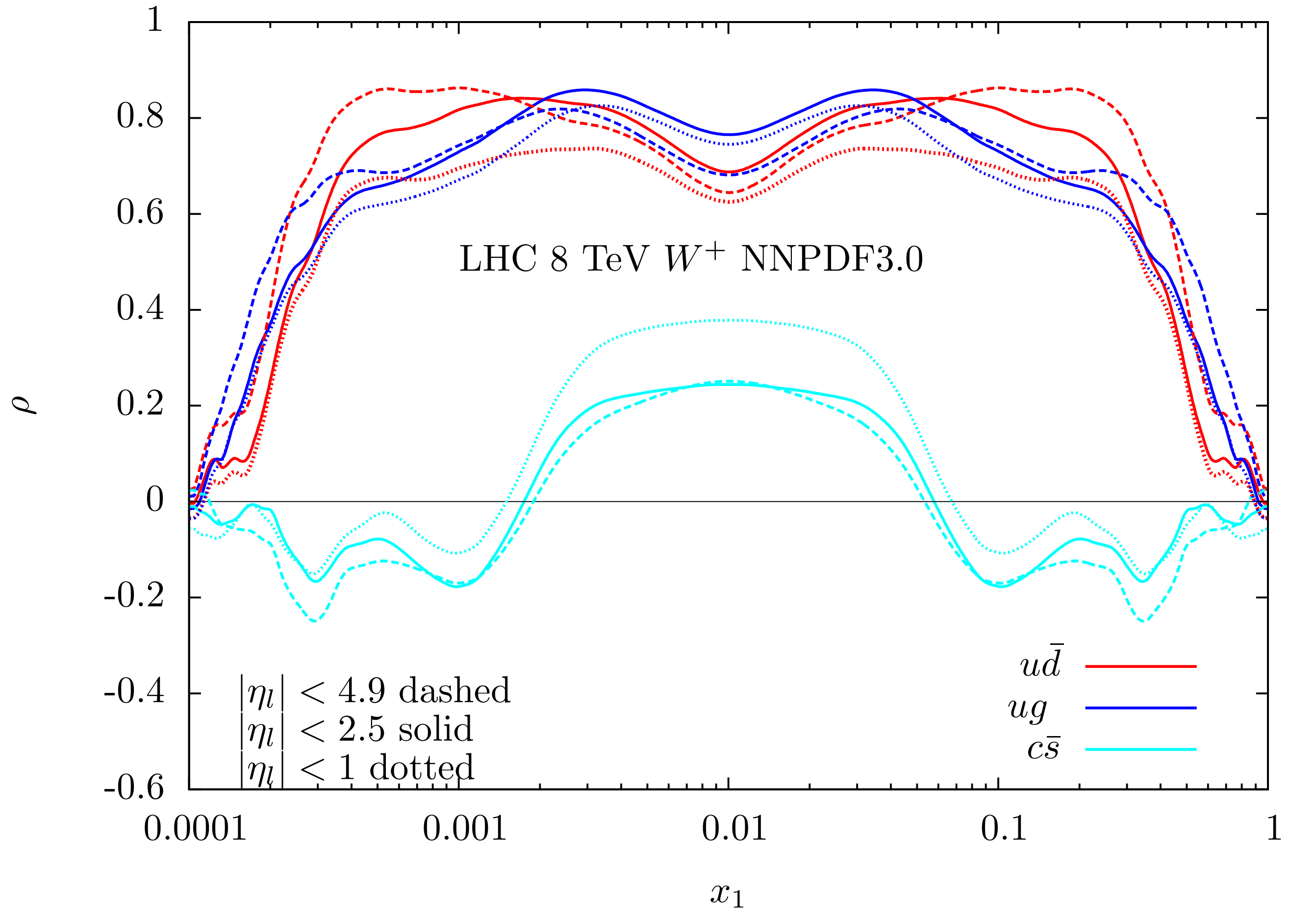
G.Bozzi, L.Citelli, AV, arXiv:1501.05587, G.Bozzi, L.Citelli, M.Vesterinen, AV, arXiv:1508.06954

normalized distributions			
cut on $p_\perp^W$	cut on $\|\eta_l\|$	CT10	NNPDF3.0
inclusive	$\|\eta_l\| < 2.5$	$80.400 + 0.032 - 0.027$	80.398 ± 0.014
$p_\perp^W < 20$ GeV	$\|\eta_l\| < 2.5$	$80.396 + 0.027 - 0.020$	80.394 ± 0.012
$p_\perp^W < 15$ GeV	$\|\eta_l\| < 2.5$	$80.396 + 0.017 - 0.018$	80.395 ± 0.009
$p_\perp^W < 10$ GeV	$\|\eta_l\| < 2.5$	$80.392 + 0.015 - 0.012$	80.394 ± 0.007
$p_\perp^W < 15$ GeV	$\|\eta_l\| < 1.0$	$80.400 + 0.032 - 0.021$	80.406 ± 0.017
$p_\perp^W < 15$ GeV	$\|\eta_l\| < 2.5$	$80.396 + 0.017 - 0.018$	80.395 ± 0.009
$p_\perp^W < 15$ GeV	$\|\eta_l\| < 4.9$	$80.400 + 0.009 - 0.004$	80.401 ± 0.003
$p_\perp^W < 15$ GeV	$1.0 < \|\eta_l\| < 2.5$	$80.392 + 0.025 - 0.018$	80.388 ± 0.012

normalized cross section differential in partonic x



correlation of parton-parton luminosities with the ptl bin 40.5 GeV



- the normalized ptlep distribution, integrated over the whole lepton-pair rapidity range, does not depend on *x* and very weakly on the PDF replica

- PDF sum rules → non trivial compensations between different rapidity intervals among different flavors → Anticorrelation

- MW measurement at LHCb significantly reduces the global PDF uncertainty
- W+ and W- determinations are anti correlated w.r.t. PDFs their combination benefits of a reduction of the PDF error

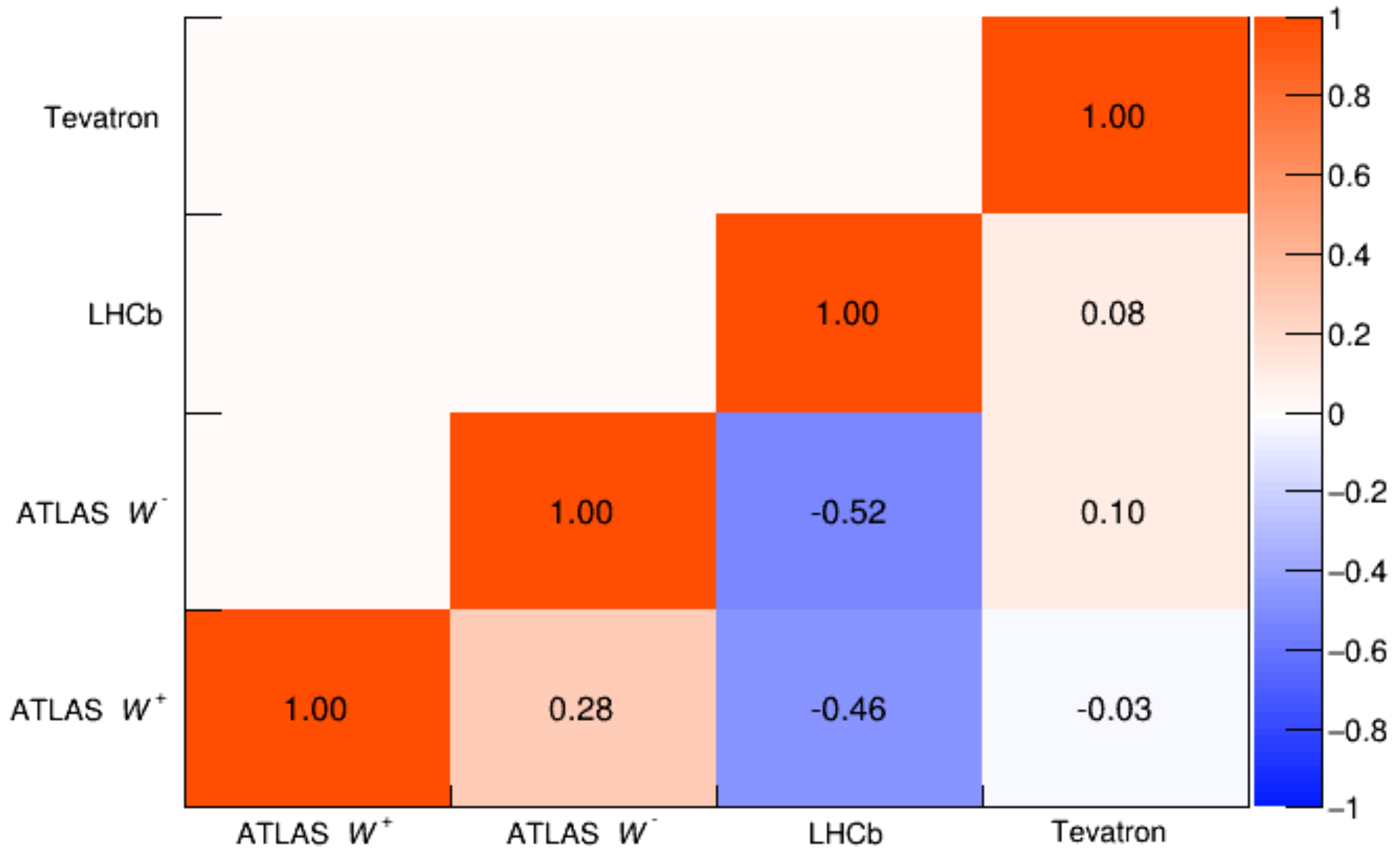
PDF correlations in the MW combination LHC-TeV MW working group, arXiv:2308.09417

Correlations needed in the combination

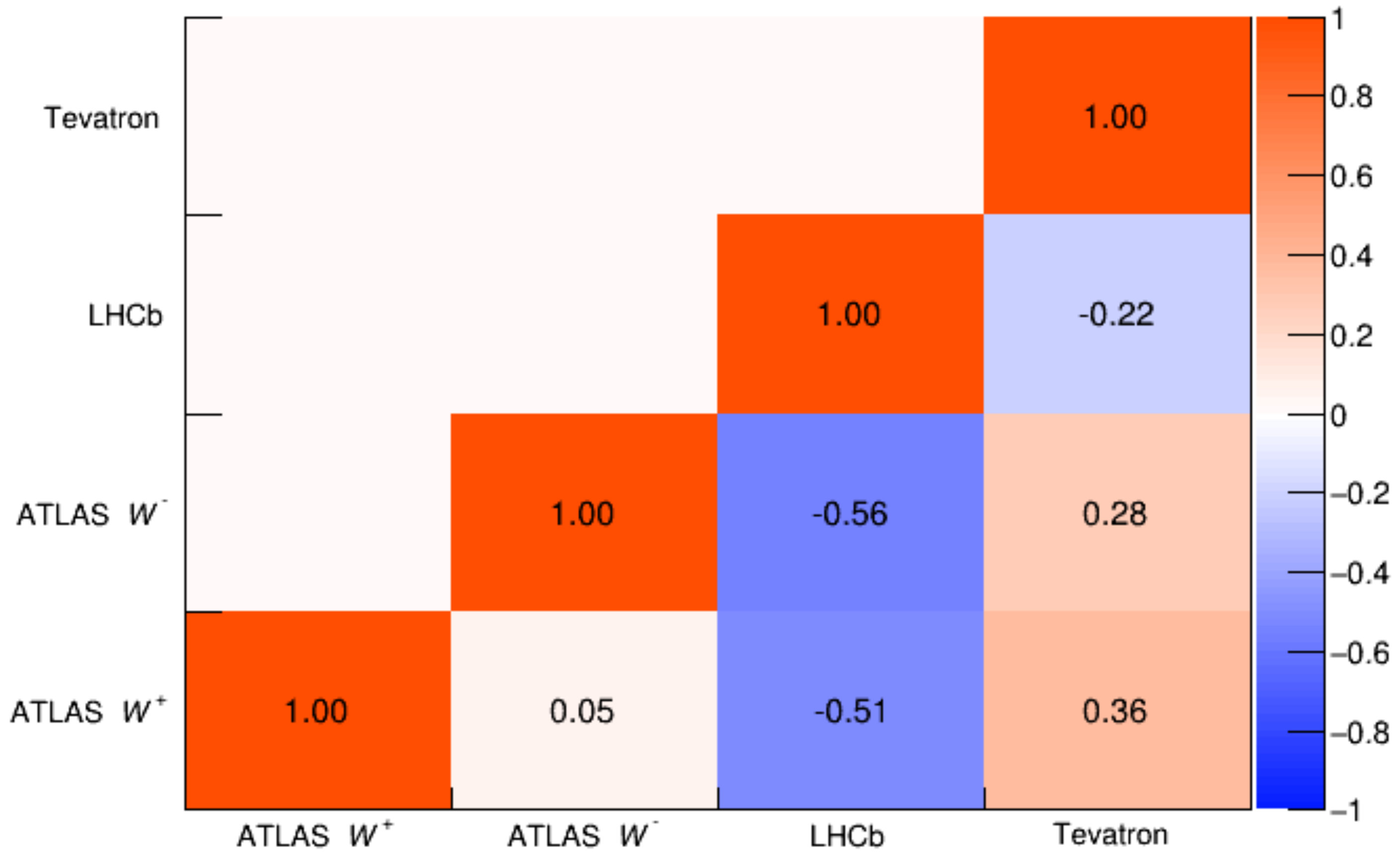
PDF anti-correlations between experiments leads to more stable results and reduced PDF dependence

Significantly different correlations between the various PDF sets

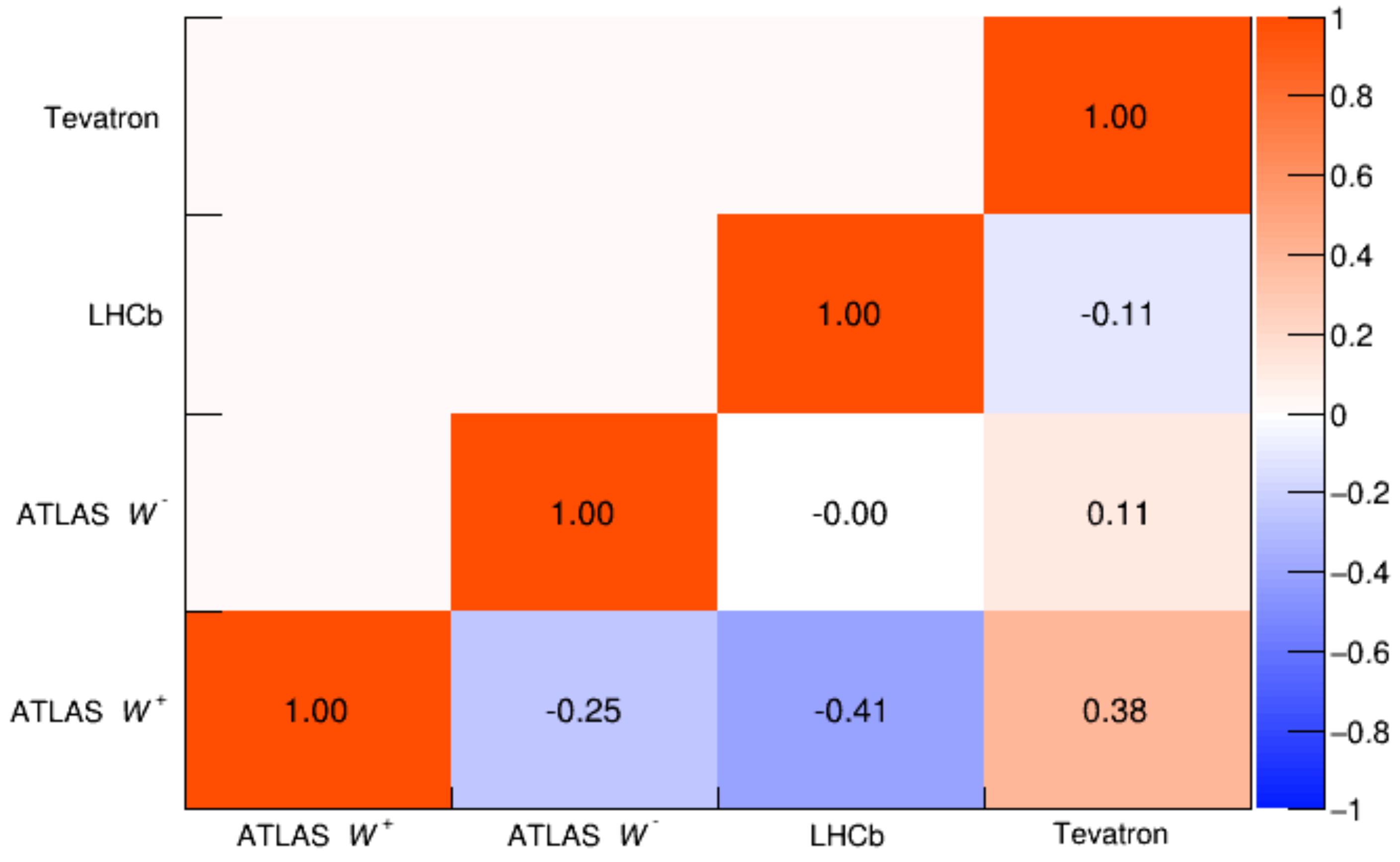
CT18



MSHT20



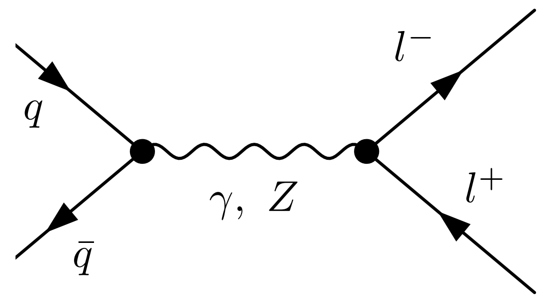
NNPDF4.0



Weak mixing angle determinations

Weak mixing angle determination at hadron colliders (I)

$$\mathcal{M}^{eff}_{Zl^+l^-} = \bar{u}_l \gamma_\alpha \left[\mathcal{G}^f_v(m_Z^2) - \mathcal{G}^f_a(m_Z^2)\gamma_5\right] v_l \varepsilon^\alpha_Z$$



invariant mass Forward-Backward asymmetry in NCDY

$$F(M_{l^+l^-}) = \int_0^1 \frac{d\sigma}{d\cos\theta^*} d\cos\theta^* \qquad B(M_{l^+l^-}) = \int_{-1}^0 \frac{d\sigma}{d\cos\theta^*} d\cos\theta^*$$

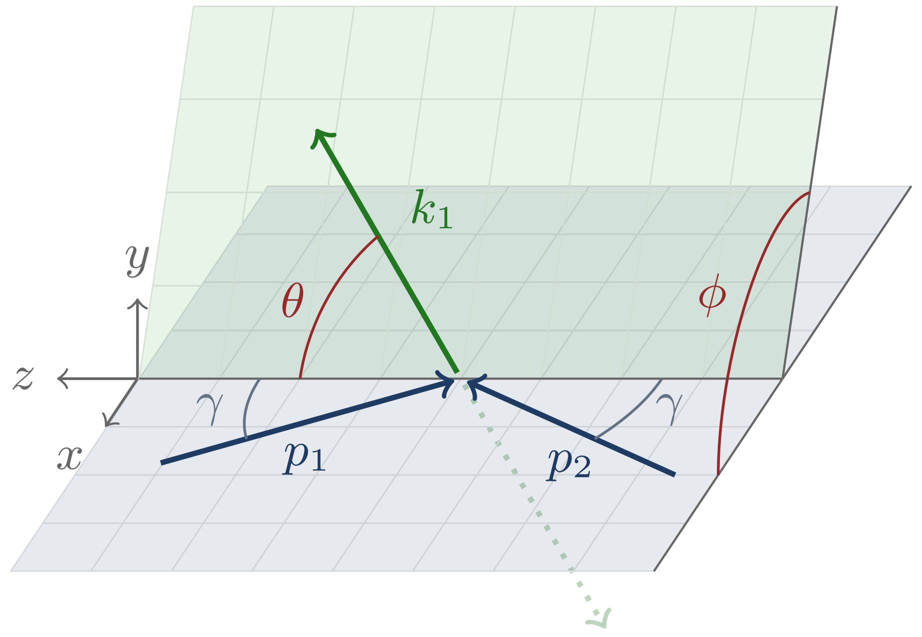
$$A_{FB}(M_{l^+l^-}) = \frac{F(M_{l^+l^-}) - B(M_{l^+l^-})}{F(M_{l^+l^-}) + B(M_{l^+l^-})}$$

Sensitive to parity violation

scattering angle defined in the Collins-Soper frame $\rightarrow$ "Forward" ("Backward")

$$\cos\theta^* = f \frac{2}{M(l^+l^-)\sqrt{M^2(l^+l^-) + p_t^2(l^+l^-)}} [p^+(l^-)p^-(l+) - p^-(l^-)p^+(l^+)]$$

$$p^\pm = \frac{1}{\sqrt{2}}(E \pm p_z) \qquad f = \frac{|p_z(l^+l^-)|}{p_z(l^+l^-)}$$



we would like to appreciate parity violation like at LEP,
observing an asymmetry with respect to the direction of the incoming particle

$\rightarrow$ it is not possible because we have both $q\bar{q}$ and $\bar{q}q$ annihilation processes

$\rightarrow$ at the LHC the symmetry of the collider (p-p) removes one possible preferred direction

but…

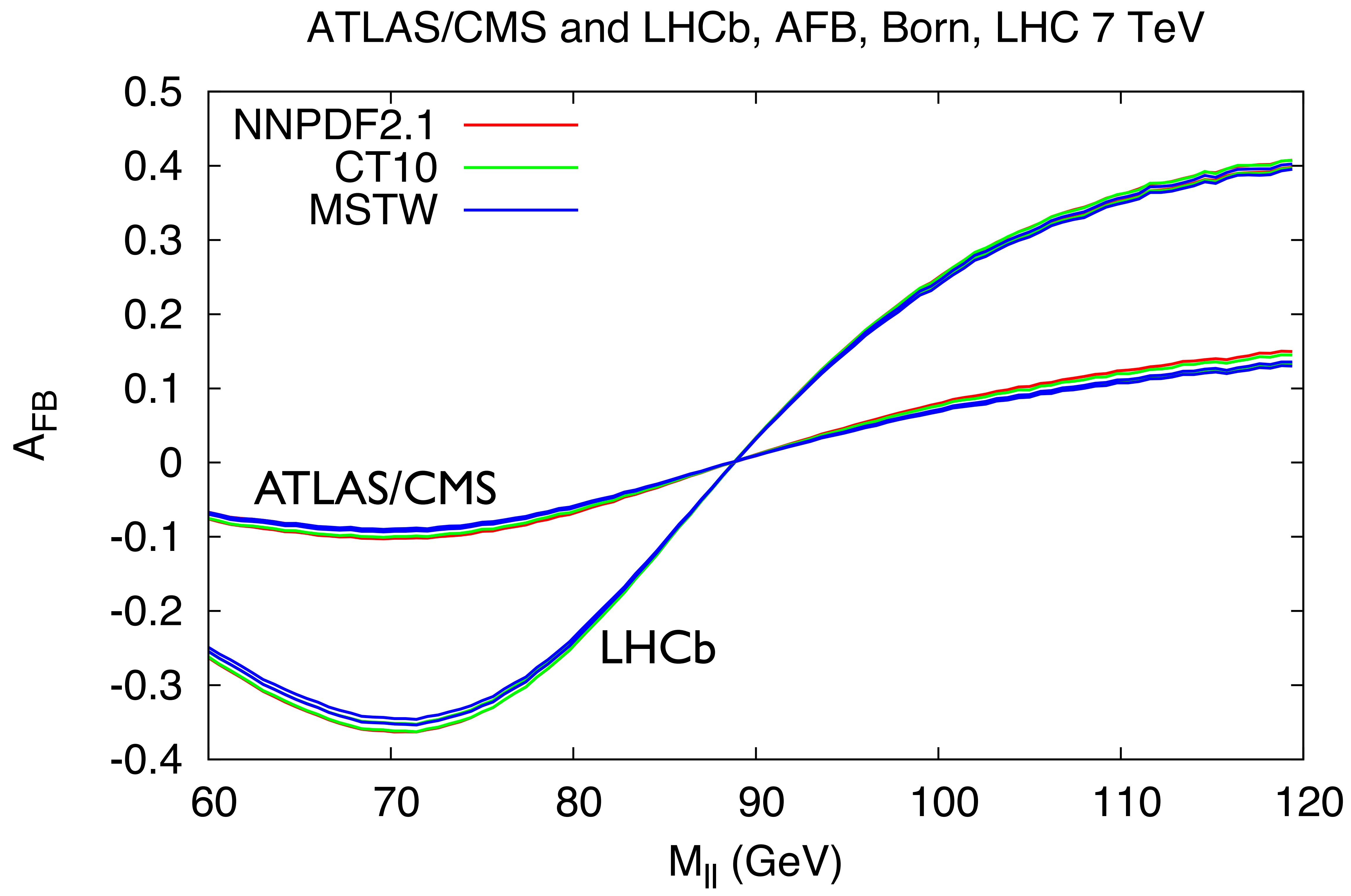
Weak mixing angle determination at hadron colliders (II)

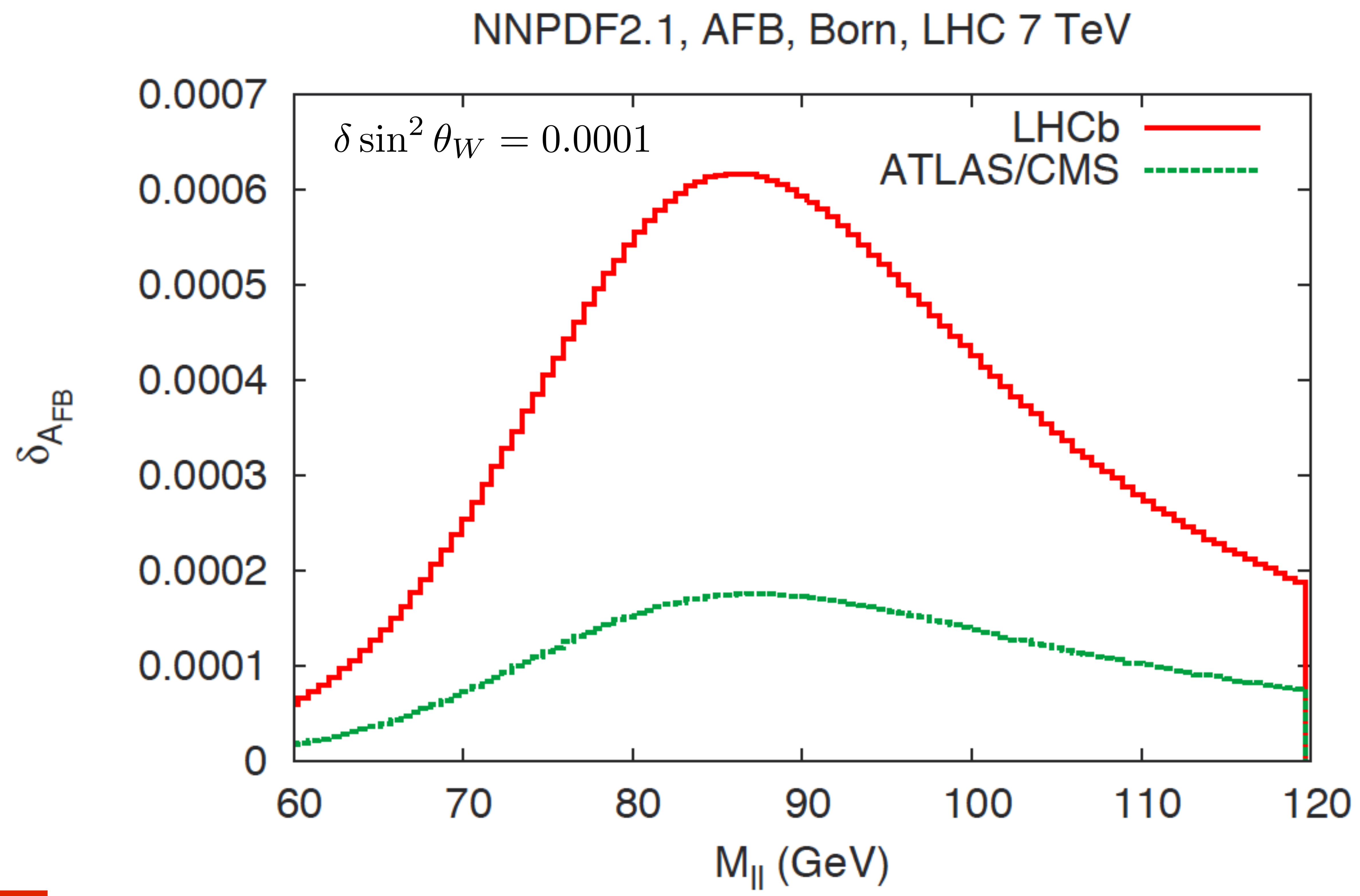
…but

at a given lepton-pair rapidity Y, $q\bar{q}$ and $\bar{q}q$ have different weight because of the PDFs $\Rightarrow$ do not cancel each other

the parton luminosity unbalance is due to the different x dependence of the valence and sea quarks

AFB is more pronounced at large Y, e.g. at LHCb

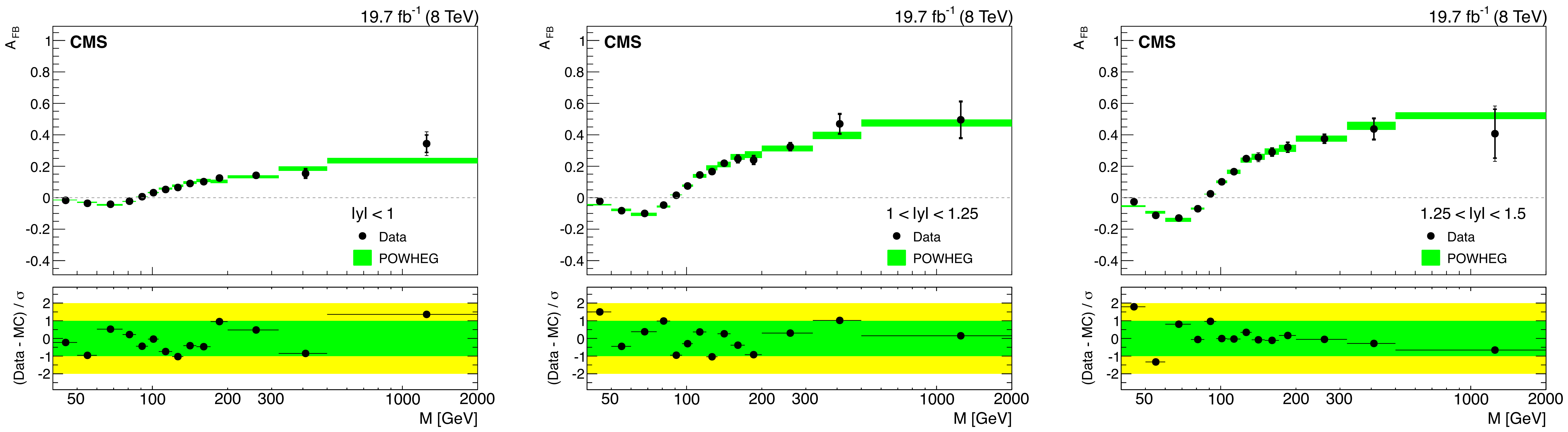


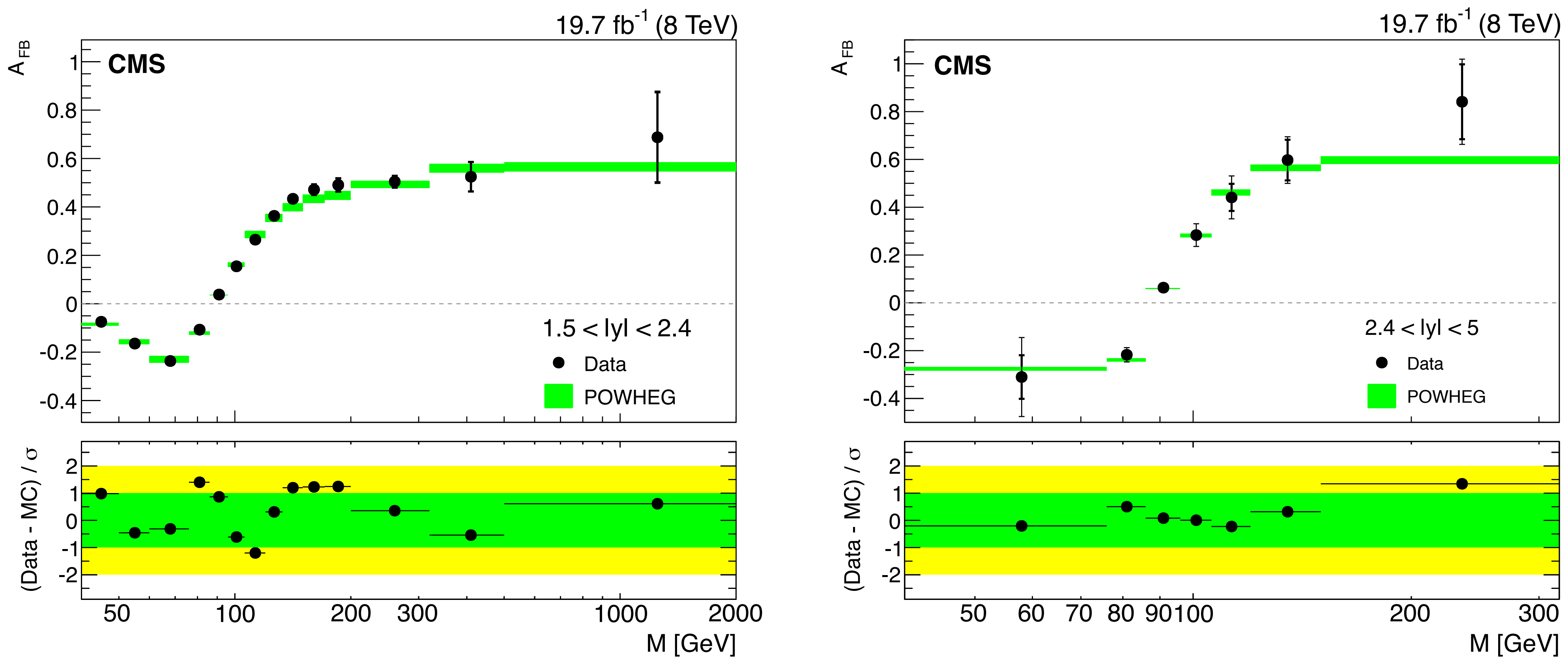


the AFB slope around m_Z has a linear dependence on $\sin^2\theta^{\ell}_{eff}$

AFB probes a PDF weighted combination of up, down and leptonic effective angles

The invariant mass Forward-Backward asymmetry in neutral-current DY and $\sin^2\theta^{\ell}_{eff}$



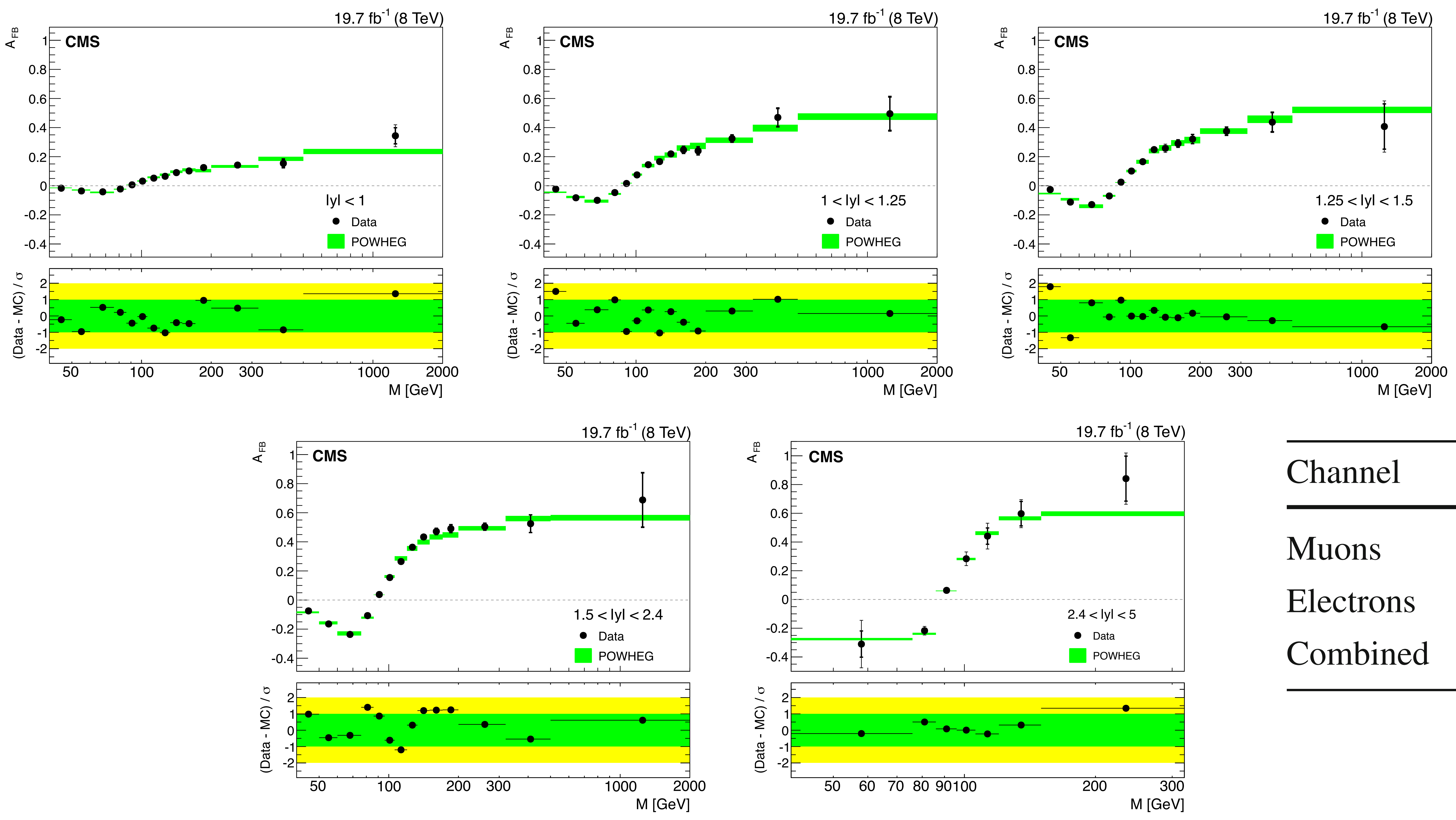


Channel	Not constraining PDFs	Constraining PDFs
Muons	0.23125 ± 0.00054	0.23125 ± 0.00032
Electrons	0.23054 ± 0.00064	0.23056 ± 0.00045
Combined	0.23102 ± 0.00057	0.23101 ± 0.00030

A determination of $\sin^2\theta^{lep}_{eff}$ competitive with the LEP results (0.23152(16)) is becoming possible.

It requires the most advanced fixed- and all-order QCD and EW corrections

The invariant mass Forward-Backward asymmetry in neutral-current DY and $\sin^2\theta^{\ell}_{eff}$



Channel	Not constraining PDFs	Constraining PDFs
Muons	0.23125 ± 0.00054	0.23125 ± 0.00032
Electrons	0.23054 ± 0.00064	0.23056 ± 0.00045
Combined	0.23102 ± 0.00057	0.23101 ± 0.00030

A determination of $\sin^2\theta^{lep}_{eff}$ competitive with the LEP results (0.23152(16)) is becoming possible.

It requires the most advanced fixed- and all-order QCD and EW corrections

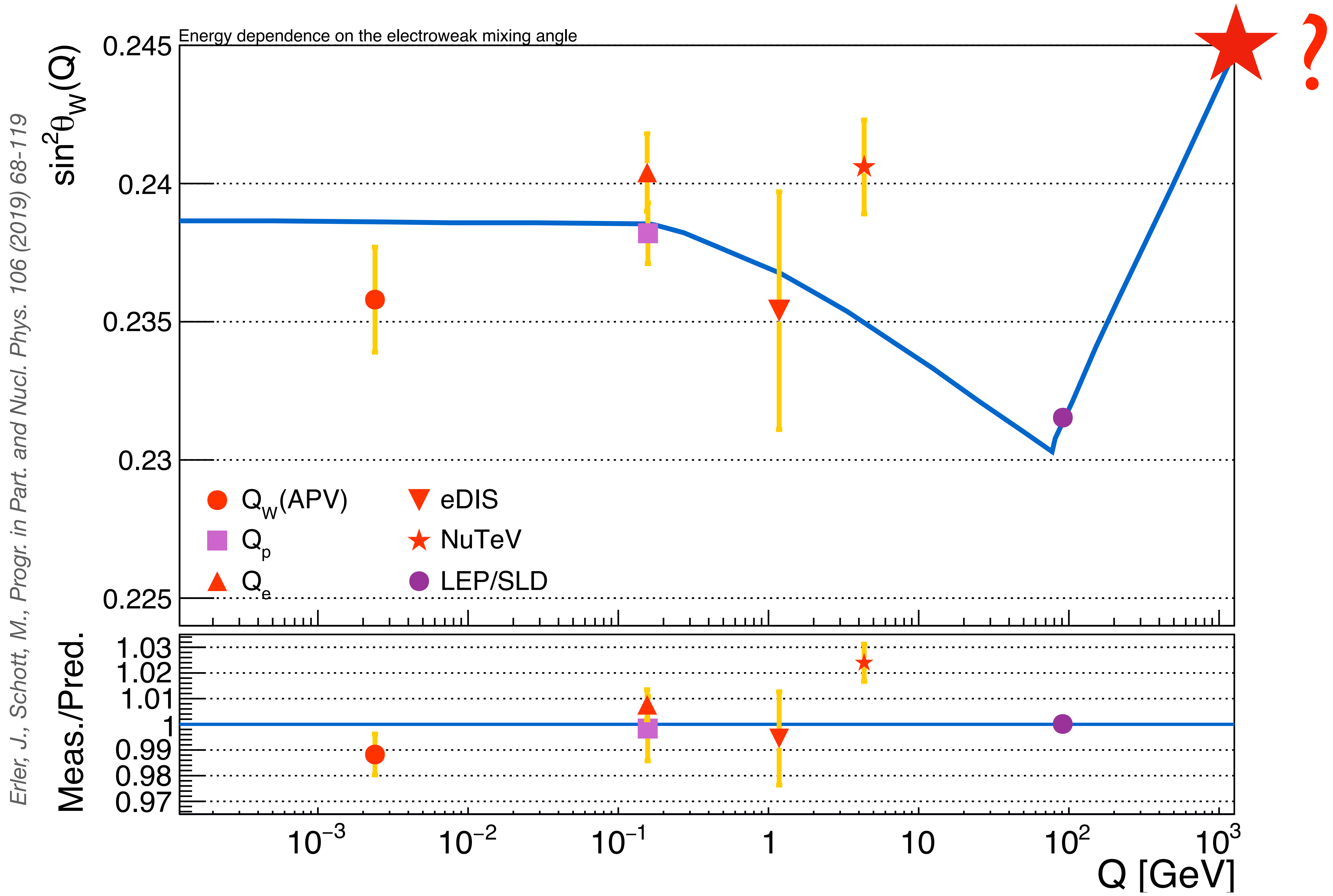
$\sin^2\theta^{lep}_{eff}$ is defined exactly at $q^2 = m_Z^2$ $\rightarrow$ it is a test of the SM at this energy scale

$\rightarrow$ a test on an extended energy range possible by studying $\sin^2\hat{\theta}_{\overline{MS}}(\mu_R^2)$

The MSbar weak mixing angle $\sin^2\hat{\theta}(\mu_R^2)$

In QFT couplings and masses are defined at a given energy scale μ_R

$\longrightarrow$ observables characterized by different energy scales allow to extract the“running” couplings testing QFT predictions



The RGE evolution depends on the number of active flavours contributing to the β-function
Above $\mu = m_W$ there is an change of sign which features a positive slope.

Can we test this prediction of the SM, i.e. 1) the running and 2) the value of the slope ?

Fitting the MSbar weak mixing angle $\sin^2\hat\theta(\mu_R^2)$

How can we fit $\sin^2\hat\theta(\mu_R^2)$?

- take the experimental lepton-pair invariant mass distribution in a given bin of mass $m_{\ell\ell}$
- set $\mu_R = m_{\ell\ell}$
- take the theoretical expression of the invariant mass distribution and fit $\sin^2\hat\theta(m_{\ell\ell}^2)$ to the data

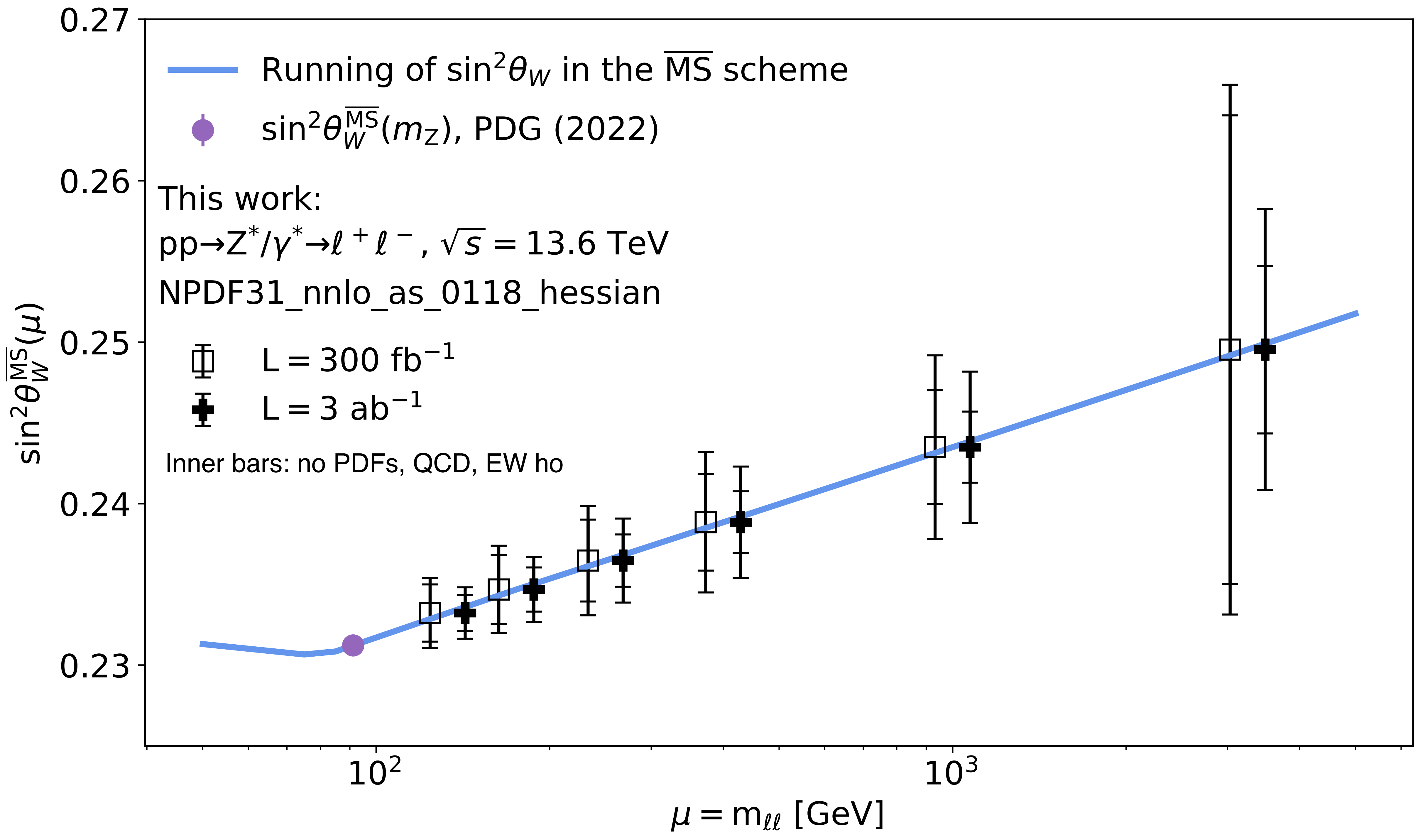
Which theoretical expression should I use ?

- if we take the LO cross section $\rightarrow$ we bias the result (faking a BSM effect)
  because we reabsorb quantum corrections not related to the coupling def
  in the fit parameter (e.g. QCD corrections)

- if we take the LO + NLO + NNLO + … cross section
  $\rightarrow$ we remove the source of bias thanks to an explicit description

$\sin^2\hat{\theta}(\mu_R)$ determination at hadron colliders at large invariant masses

S.Amoroso, M.Chiesa, C.L Del Pio, E.Lipka, F.Piccinini, F.Vazzoler, AV, arXiv:2302.10782



The running of the MSbar angle can be established at LHC in Run III and at HL-LHC with percent precision.

For the actual measurement the best theoretical predictions will be needed, to avoid interpretation mismatches: full NNLO (QCD, EW and mixed QCDxEW) and leading higher orders

Parity violation: what can be learned from precision e- p measurements at very low energies?

The P2 experiment in Mainz studies the scattering of intense polarized electron beams on protons

It offers alternative SM tests and probes of BSM physics

The asymmetry $A_{PV} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = \frac{-G_F Q^2}{4\sqrt{2}\pi\alpha_{em}} \left(Q_W - F(E_i, Q^2)\right)$ is obtained polarising the electron beam

$A_{PV}(P2) \sim -40 \cdot 10^{-9}$

- A_{PV} is proportional to the weak charge of the proton, accidentally suppressed in the SM: $Q_W(p) = 1 - 4\sin^2\theta_W \sim 0.09$

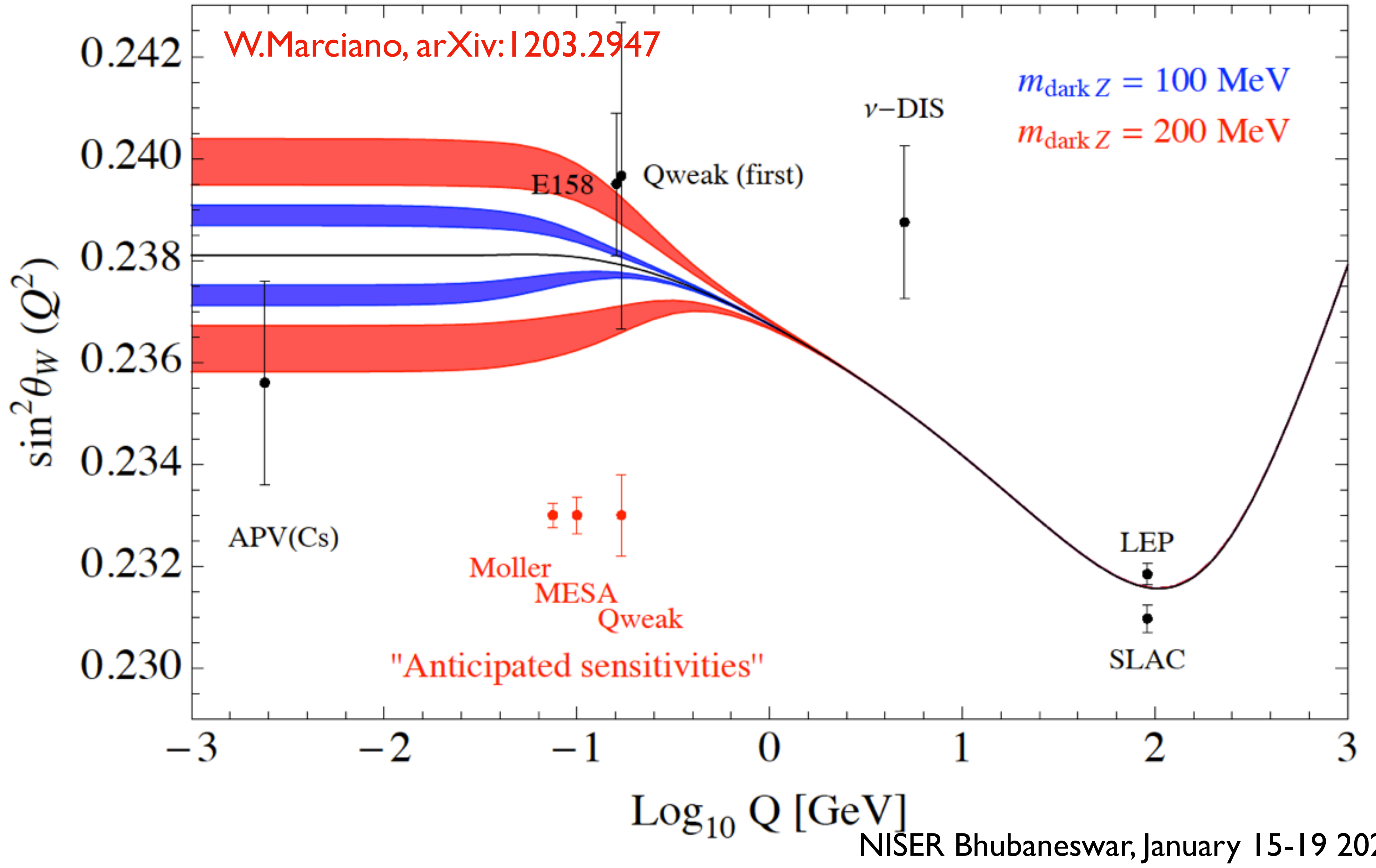
→ a measurement at the 1.4% level of $A_{PV}(P2)$ allows a determination of $\sin^2\theta_W$ with an error $\Delta\sin^2\theta_W \sim 33 \cdot 10^{-5}$

(cfr. LEP error $\Delta\sin^2\theta_W \sim 16 \cdot 10^{-5}$)

→ BSM effects might emerge with good significance

→ complementary tests at very low and very high energies for the same running parameter

→ the RGE solution depends on boundary and matching conditions then the running is a testable prediction



Concluding remarks

Conclusions

- The tests of the SM can be performed in two different ways:

  1) comparing the predictions for cross sections and asymmetries with the experimental values

     the HL-LHC precision can be matched by including at least N3LO-QCD and NNLO-EW corrections
     by using combined QCD and QED resummation of enhanced contributions

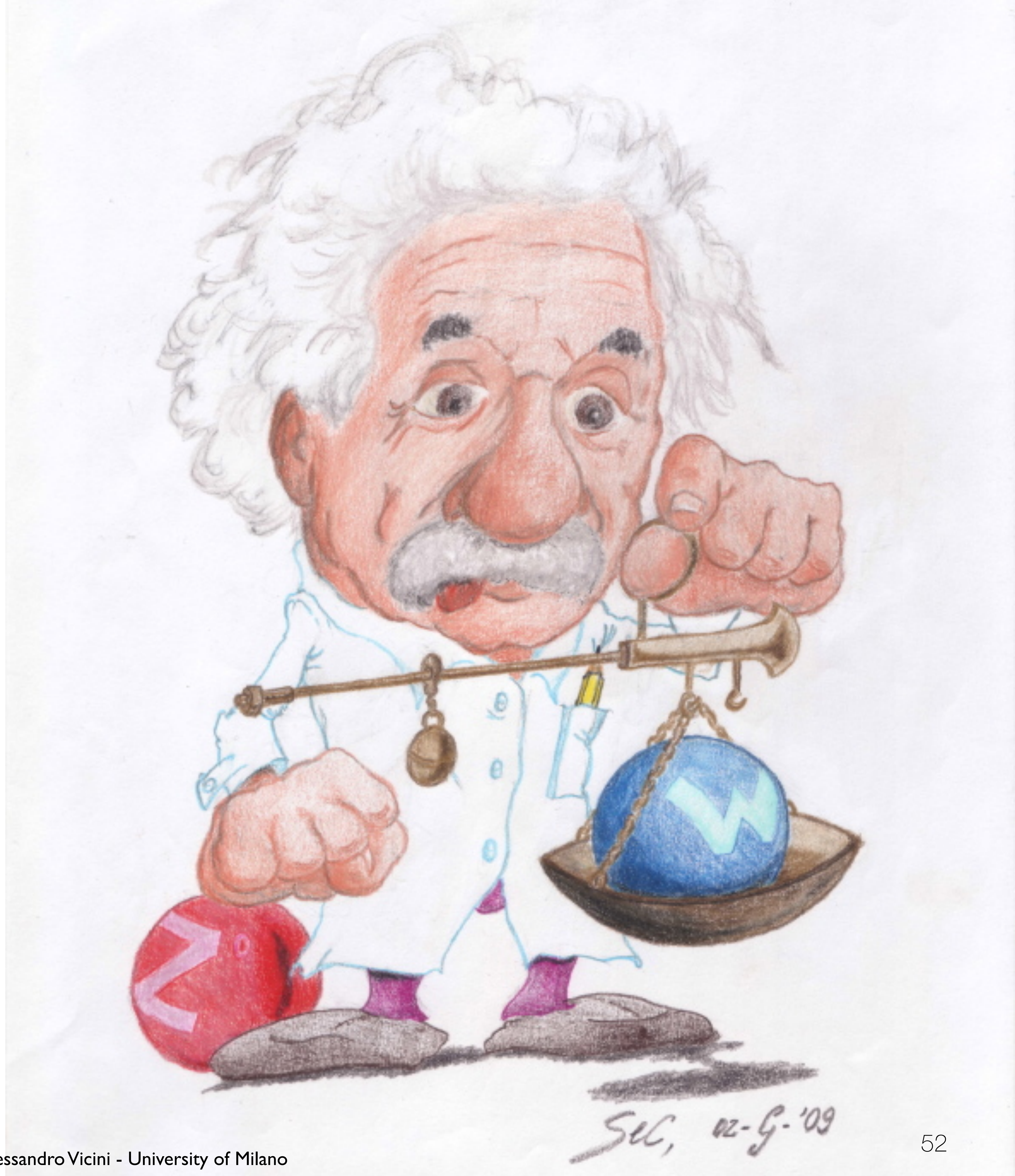
  2) comparing the predictions for the parameters of the SM with the corresponding experimental determinations

     extracting a parameter from the data requires the usage of a fitting model with that parameter in input
     $\rightarrow$ improved calculations are needed to minimise the theoretical systematic error on the parameter determination

- Testing the energy dependence of the predictions is a powerful tool to exploit the large amount of high precision data

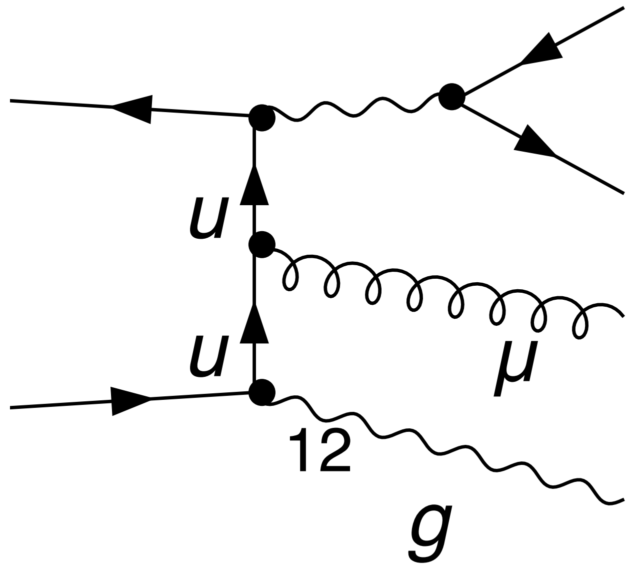
  the MSbar weak mixing angle offers the possibility to test the SM from the eV to the TeV energy range

  any BSM study (e.g. the SMEFT coefficients) must be done on top of the best SM results to avoid fake conclusions



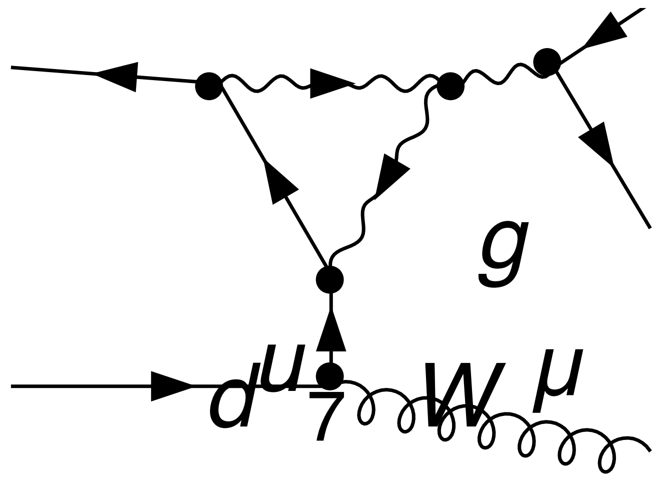
Thank you

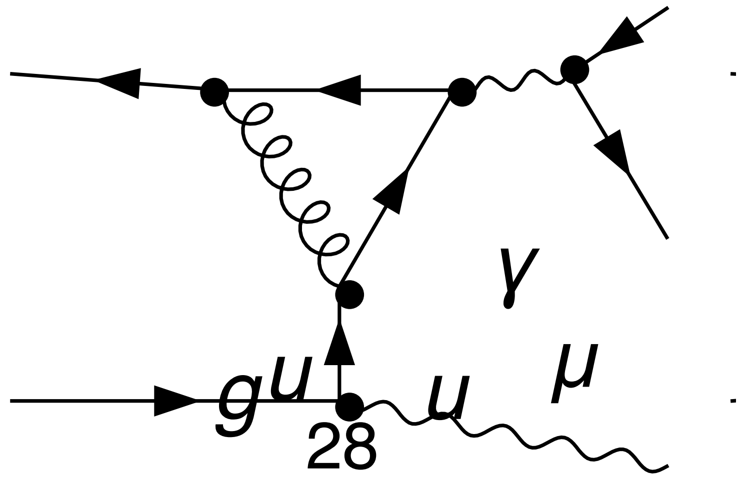
Different kinds of contributions at $\mathcal{O}(\alpha\alpha_s)$ and corresponding problems



double-real contributions

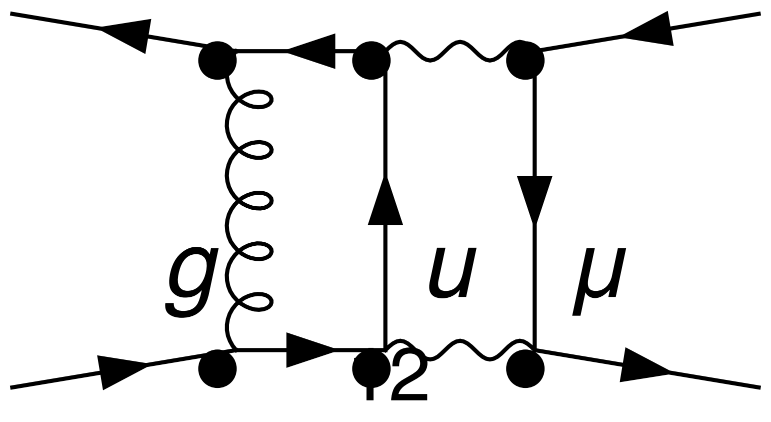
amplitudes are easily generated with OpenLoops
IR subtraction
care about the numerical convergence when aiming at 0.1% precision

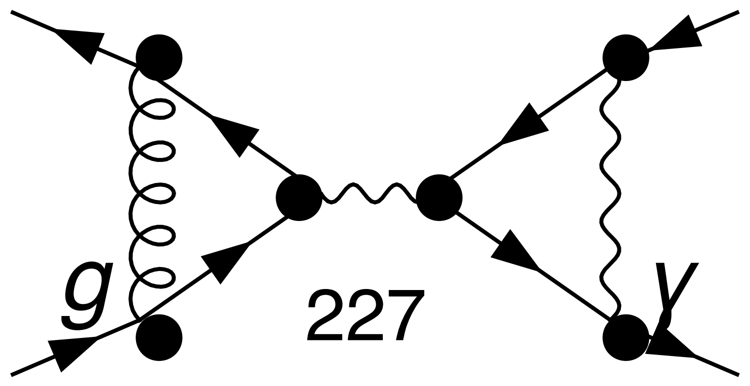




real-virtual contributions

amplitudes are easily generated with OpenLoops or Recola
1-loop UV renormalisation and IR subtraction
care about the numerical convergence when aiming at 0.1% precision





double-virtual contributions

generation of the amplitudes
γ_5 treatment
2-loop UV renormalization
subtraction of the IR divergences
solution and evaluation of the Master Integrals
numerical evaluation of the squared matrix element

The double-real and real-virtual corrections already known from studies of the large transverse momentum lepton pair final state
A.Denner, S.Dittmaier, T.Kasprzik, A.Muck, arXiv:1103.0914, A.Denner, S.Dittmaier, M.Hecht, C.Pasold, arXiv:1510.08742 J.Lindert et al., arXiv:1705.04664
Now we can consider the inclusive spectrum, also in the $q_T \to 0$ limit

General structure of the inclusive cross section and the q_T-subtraction formalism in Matrix

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathcal{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

IR structure associated to the QCD-QED part derived from NNLO-QCD results via abelianisation
(de Florian, Rodrigo, Sborlini, 2016, de Florian, Der , Fabre, 2018)

the q_T-subtraction formalism has been extended to the case of massive final-state emitters (heavy quarks in QCD, leptons in EW)
(Catani, Torre, Grazzini, 2014, Buonocore, Grazzini, Tramontano 2019.)

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$$\int d\sigma_R^{(1,1)} \sim \sum_{i=1}^{4} c_i \ln^i r_{cut} + c_0 + \mathcal{O}(r_{cut}^m) \qquad \rightarrow \qquad \int \left(d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right) \sim c_0 + \mathcal{O}(r_{cut}^m)$$

The counterterm removes the IR sensitivity to the cutoff variable
→ we need small values of the cutoff and explicit numerical tests to quantify the bias induced by the cutoff choice
we can fit the r_{cut} dependence and extrapolate in the $r_{cut} \rightarrow 0$ limit
(cfr. Buonocore, Kallweit, Rottoli, Wiesemann, arXiv:2111.13661, Camarda, Cieri, Ferrera, arXiv:2111.14509)

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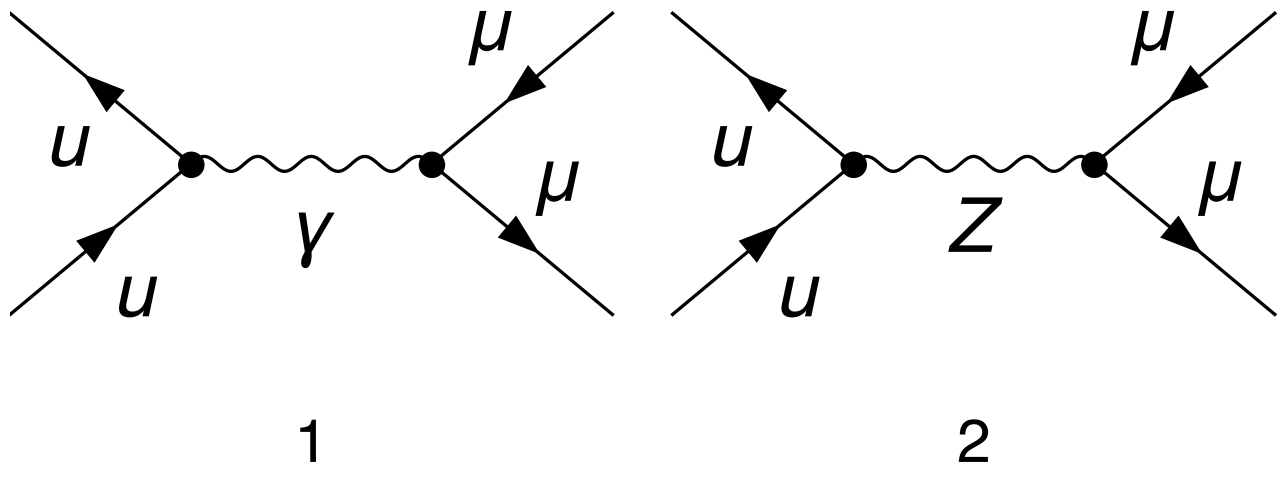
$$\mathscr{H}^{(1,1)} = H^{(1,1)} C_1 C_2 \qquad 2\mathrm{Re}\langle \mathscr{M}^{(0,0)} | \mathscr{M}^{(1,1)} \rangle = \sum_{k=-4}^{0} \varepsilon^k f_i(s,t,m) \qquad |\mathscr{M}_{fin}\rangle \equiv (1-I)|\mathscr{M}\rangle \qquad H \propto \langle \mathscr{M}_0 | \mathscr{M}_{fin} \rangle$$

The IR poles are removed from the full 2-loop amplitude by means of a subtraction procedure (matching the real radiation one)

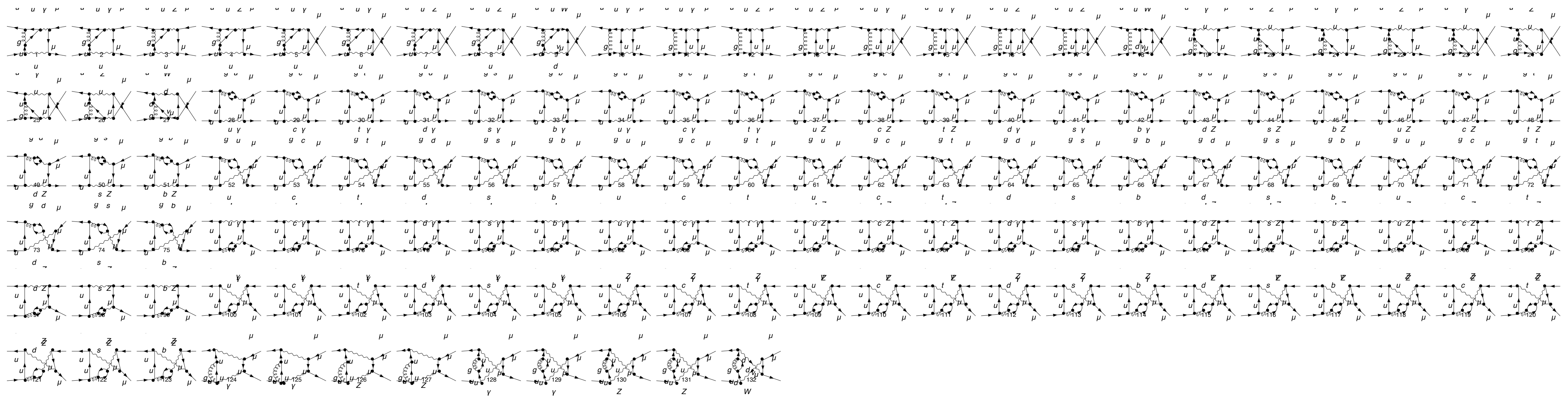
The double virtual amplitude: generation of the amplitude

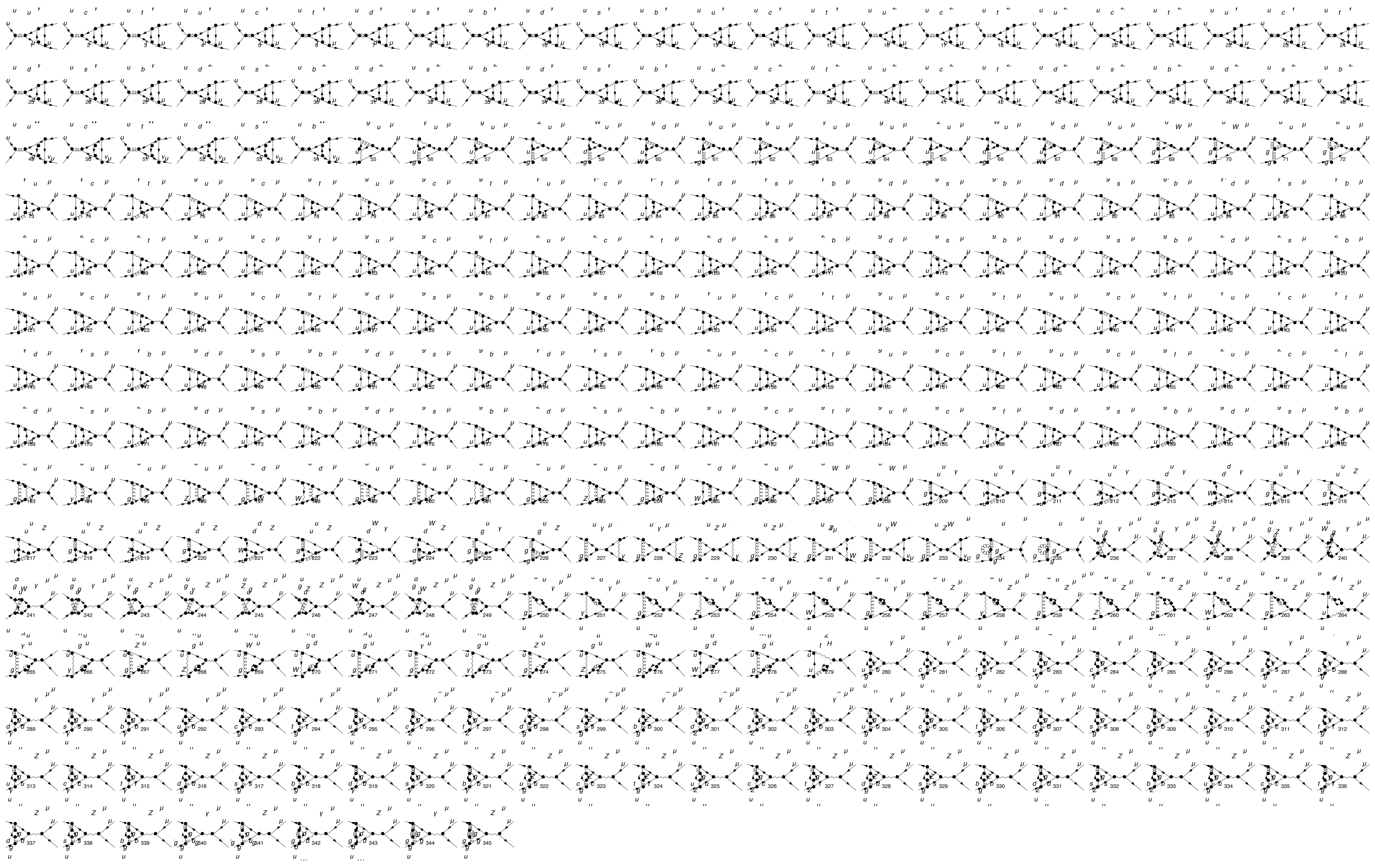
$\mathscr{M}^{(0,0)}(q\bar{q} \to l\bar{l}) =$



$\mathscr{M}^{(1,1)}(q\bar{q} \to l\bar{l}) =$ O(1000) self-energies + O(300) vertex corrections +O(130) box corrections + 1loop x 1loop

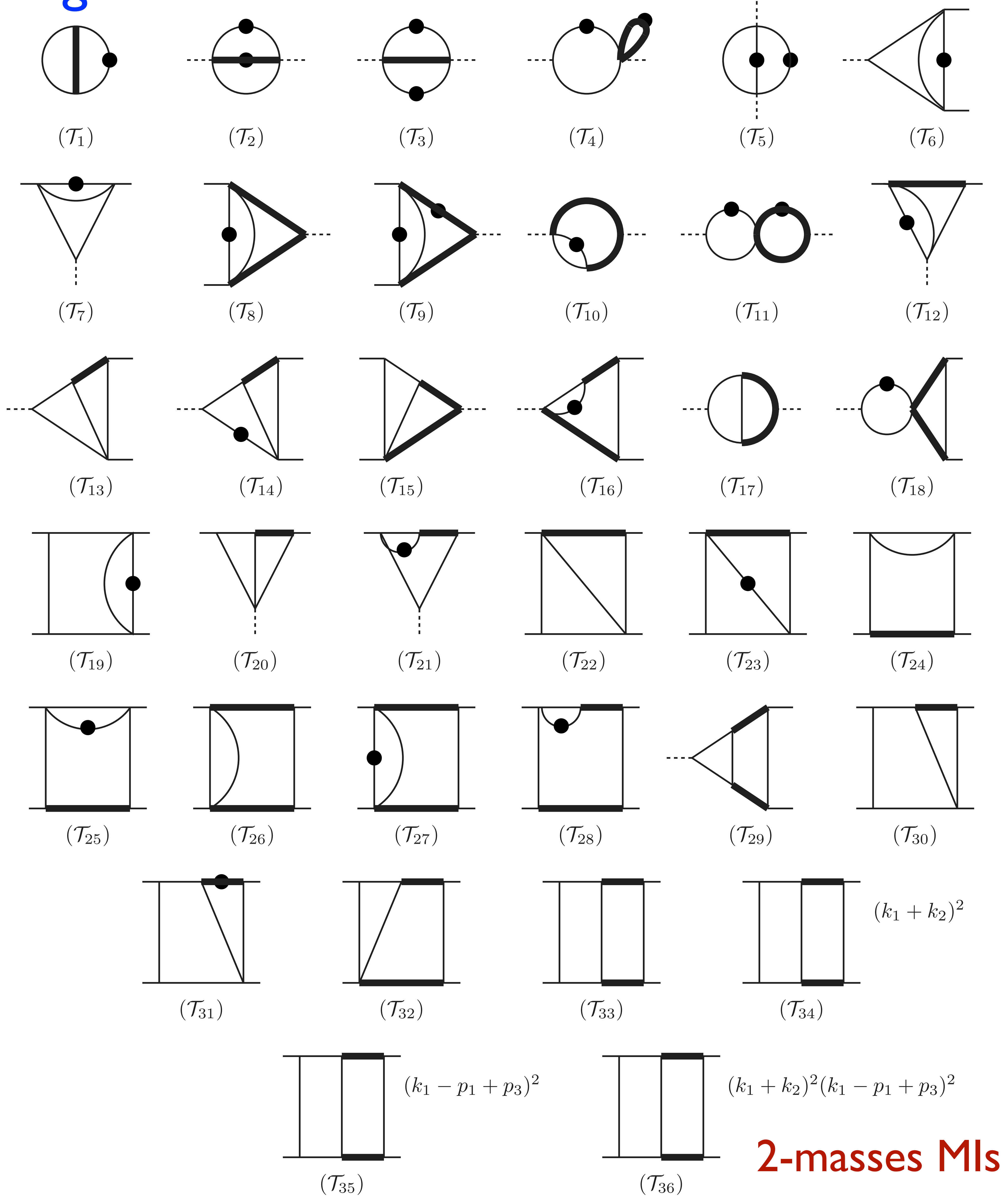
(before discarding all those vanishing for colour conservation, e.g. no fermonic triangles)





The double virtual amplitude: reduction to Master Integrals

$$2\mathrm{Re}\left(\mathcal{M}^{(1,1)}(\mathcal{M}^{(0,0)})^\dagger\right) = \sum_{i=1}^{N_{MI}} c_i(s,t,m;\varepsilon)\ \mathcal{T}_i(s,t,m;\varepsilon)$$



The double virtual amplitude: reduction to Master Integrals

$$2\mathrm{Re}\left(\mathcal{M}^{(1,1)}(\mathcal{M}^{(0,0)})^\dagger\right) = \sum_{i=1}^{N_{MI}} c_i(s,t,m;\varepsilon)\ \mathcal{T}_i(s,t,m;\varepsilon)$$

The coefficients c_i are rational functions of the invariants, masses and of ε
The size of the total expression can rapidly "explode"
→ careful work to identify the patterns of recurring subexpressions keeping the total size in the O(1-10 MB) range

The complexity of the MIs depends on the number of energy scales
MIs relevant for the QCD-QED corrections, with massive final state

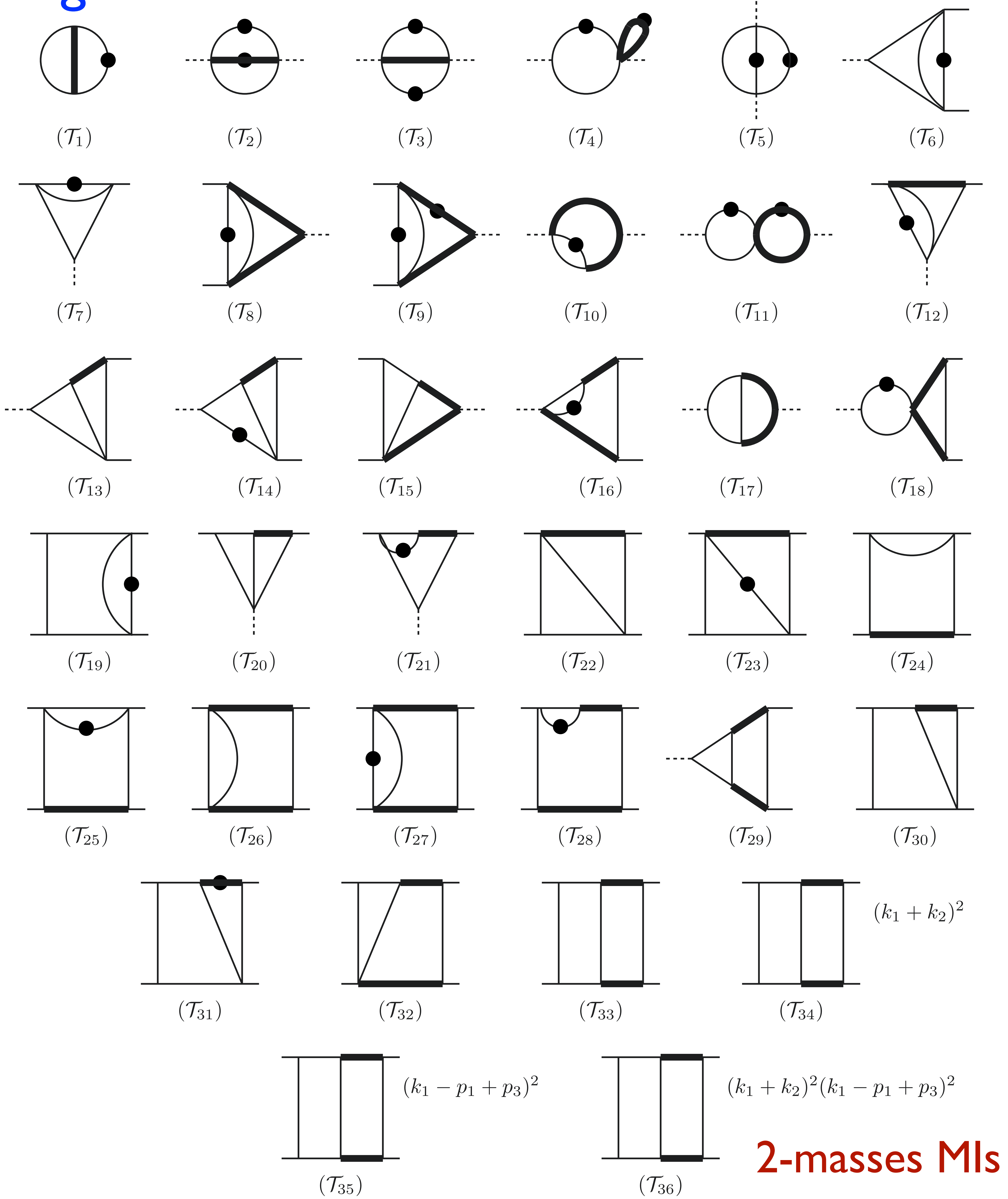
Bonciani, Ferroglia, Gehrmann, Maitre, Studerus., arXiv:0806.2301, 0906.3671

MIs with 1or 2 internal mass relevant for the EW form factor

Aglietti, Bonciani, hep-ph/0304028, hep-ph/0401193

31 MIs with 1 mass and 36 MIs with 2 masses including boxes, relevant for the QCD-weak corrections to the full Drell-Yan

Bonciani, Di Vita, Mastrolia, Schubert., arXiv:1604.08581



In the 2-mass case, 5 box integrals in Chen-Goncharov representation
→ problematic numerical evaluation→ need an alternative strategy

cfr. also Heller, von Manteuffel, Schabinger, arXiv:1907.00491 for a representation of the MIs in terms of GPLs
arXiv:2012.05918 for a description of the 2-loop virtual amplitude

Evaluation of the Master Integrals by series expansions

T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, 2205.03345

The Master Integrals satisfy a system of differential equations.

The MIs are replaced by formal series with unknown coefficients $\rightarrow$ eqs for the unknown coefficients of the series.

The package `DiffExp` by M.Hidding, arXiv:2006.05510 implements this idea, for real valued masses, with real kinematical vars.

But we need complex-valued masses of W and Z bosons (unstable particles) $\rightarrow$ we wrote a new package (`SeaSyde`)

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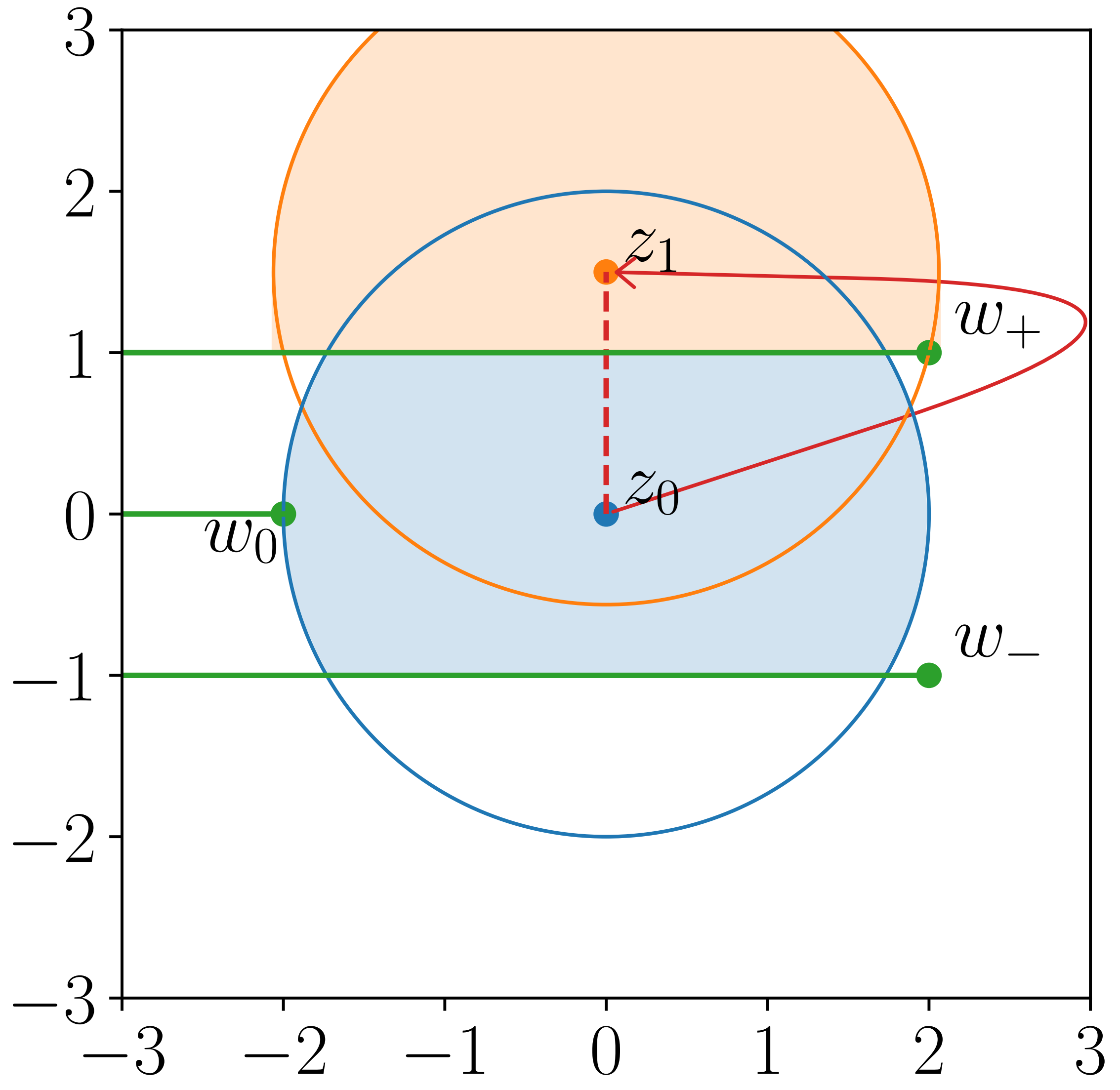

Complete knowledge about the singular structure of the MI can be read directly from the differential equation matrix

The solution can be computed with an arbitrary number of significant digits, but not in closed form $\rightarrow$ semi-analytical

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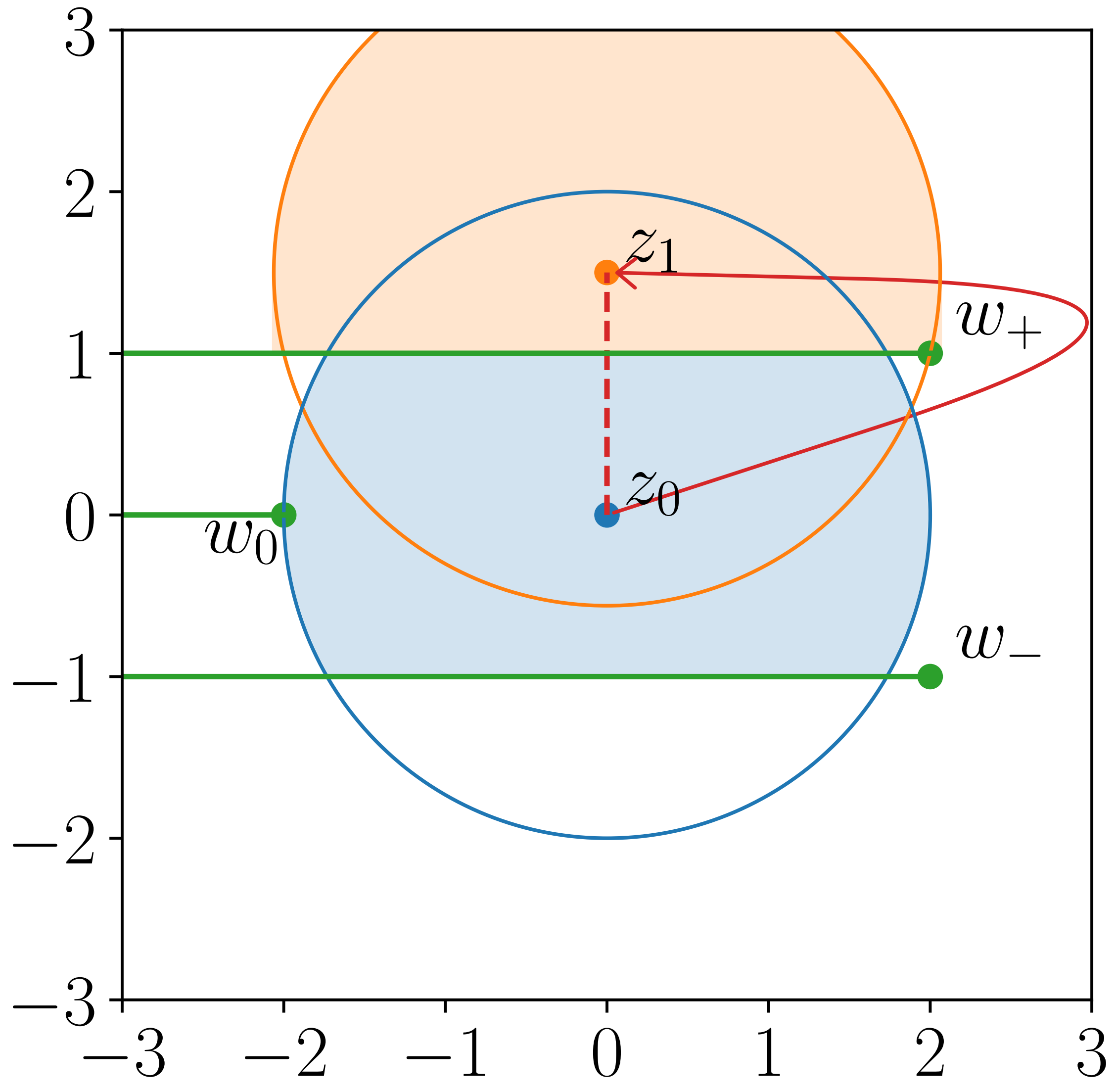

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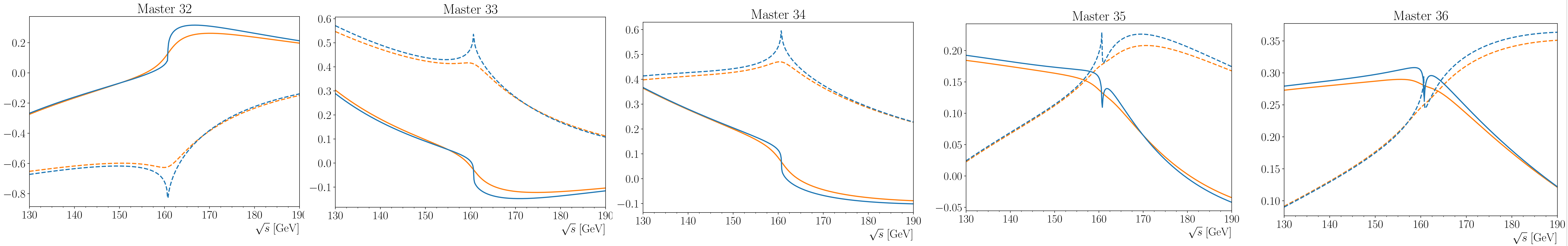


Numerical evaluation of the hard coefficient function

The interference term $2\text{Re}\langle \mathscr{M}^{(1,1),fin} | \mathscr{M}^{(0,0)} \rangle$ contributes to the hard function $H^{(1,1)}$

After the subtraction of all the universal IR divergences, it is a finite correction

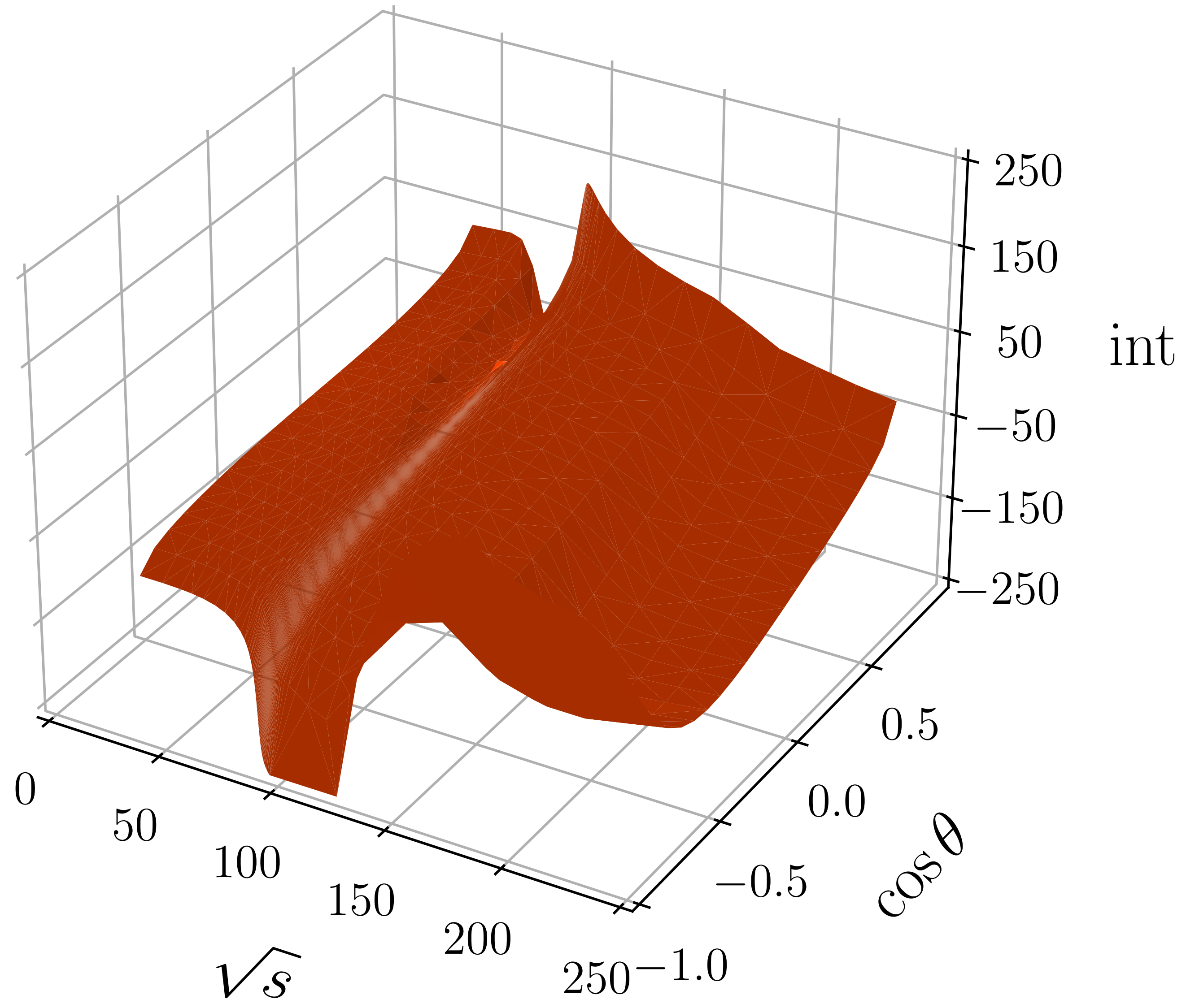
It has been published in arXiv:2201.01754 and is available as a `Mathematica` notebook

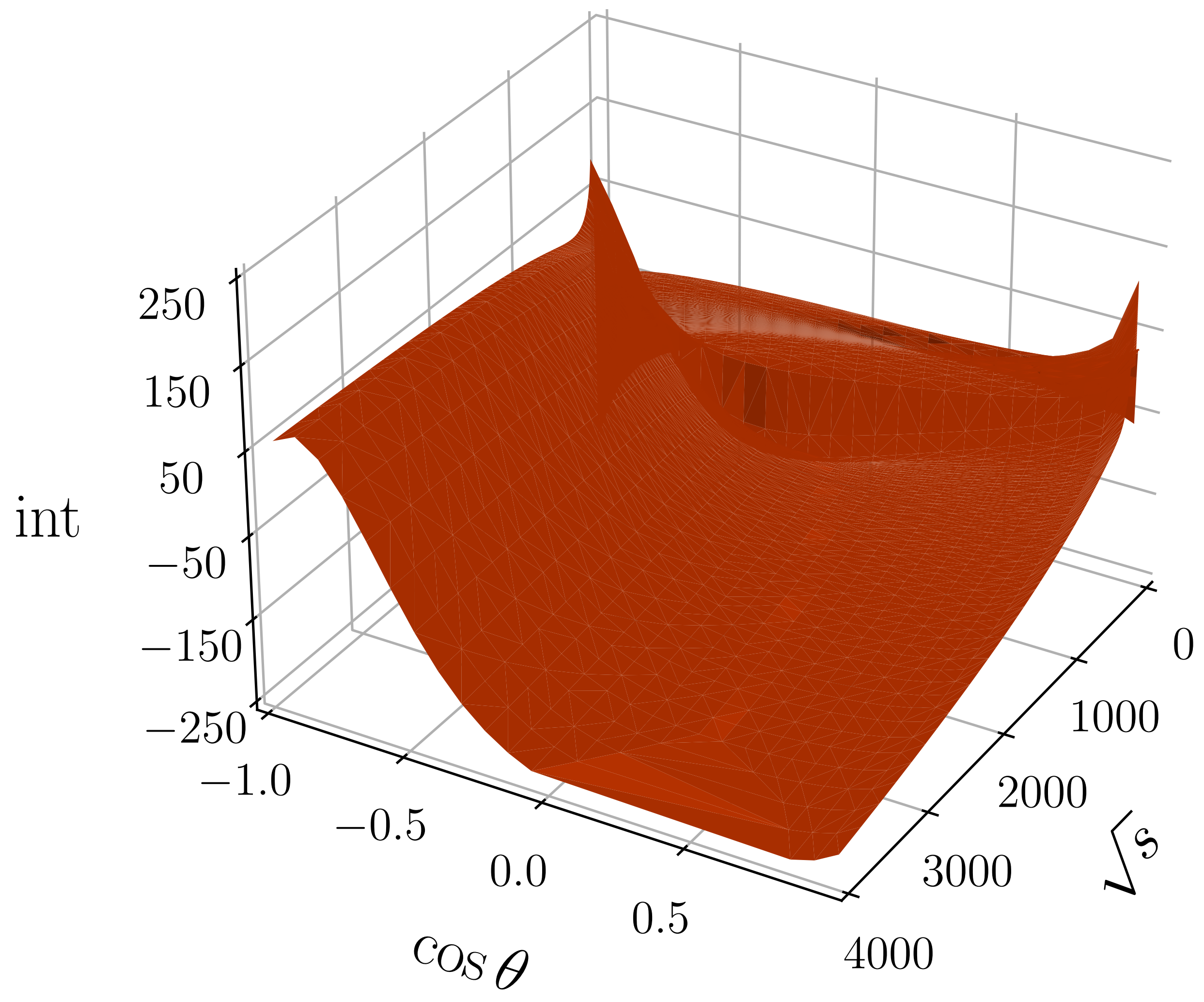
Several checks of the MIs performed with `Fiesta`, `PySecDec` and `AMFlow`

A numerical grid has been prepared for all the 36 MIs, with `GiNaC` and `SeaSyde` (T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, 2205.03345),
covering the whole $2 \to 2$ phase space in (s,t),
in O(12 h) on one 32-cores machine

$\rightarrow$ a numerical grid for $2\text{Re}\langle \mathscr{M}^{(1,1),fin} | \mathscr{M}^{(0,0)} \rangle$ has been prepared
values at arbitrary phase space points with excellent accuracy via interpolation, with negligible evaluation time

in units $\frac{\alpha}{2\pi}\frac{\alpha_s}{2\pi}\sigma_0$





Compatibility and combination of world W-boson mass determinations

LHC-TeV MW working group, arXiv:2308.09417

CDF, Science 376 (2022) 170; D0, PRL 103 (2009) 141801 and PRD 89 (2014) 012005; ATLAS, EPJC 78 (2018) 110; LHCb, JHEP 01 (2022) 036; LEP, Phys Rept 532 (2013) 119

Input Measurements for combination

- CDF – $p\bar{p}$ collisions @ $\sqrt{s}$ = 1.96 TeV; fit variables are p_T^l , p_T^ν and m_T.
- D0 – two separate measurements using $p\bar{p}$ collisions @ $\sqrt{s}$ = 1.96 TeV; fit variables are p_T^e, m_T and p_T^ν.
- ATLAS – pp collisions @ $\sqrt{s}$ = 7 TeV; central region at LHC; fit variables are p_T^l and m_T. [*Original analysis used following agreement to use published results*]
- LHCb – pp collisions @ $\sqrt{s}$ = 13 TeV; forward region at LHC; fit variable is q/p_T^μ.
- LEP – legacy combination from LEP experiments.

Experiment	Event requirements	Fit ranges
CDF	$30 < p_T^\ell < 55$ GeV $\lvert\eta_\ell\rvert < 1$ $30 < E_T^{miss} < 55$ GeV $65 < m_T < 90$ GeV $u_T < 15$ GeV	$32 < p_T^\ell < 48$ GeV $32 < E_T^{miss} < 48$ GeV $60 < m_T < 100$ GeV
D0	$p_T^e > 25$ GeV $\lvert\eta_\ell\rvert < 1.05$ $E_T^{miss} > 25$ GeV $m_T > 50$ GeV $u_T < 15$ GeV	$32 < p_T^e < 48$ GeV $65 < m_T < 90$ GeV
ATLAS	$p_T^\ell > 30$ GeV $\lvert\eta_\ell\rvert < 2.4$ $E_T^{miss} > 30$ GeV $m_T > 60$ GeV $u_T < 30$ GeV	$32 < p_T^\ell < 45$ GeV $66 < m_T < 99$ GeV
LHCb	$p_T^\mu > 24$ GeV $2.2 < \eta_\mu < 4.4$	$28 < p_T^\mu < 52$ GeV

QCD challenges

The measurements span two decades → remarkable theoretical progress

The analyses are based on different PDF sets and event generators, with different theoretical content

- D0: RESBOS CP (N2LO, N2LL) with CTEQ66 PDFs (NLO)
- CDF: RESBOS C (NLO, N2LL) with CTEQ6M PDFs (NLO) [CDF publication applied a correction to reproduce Resbos2 + NNPDF3.1]
- ATLAS: POWHEG + Pythia8 (NLO+PS) with DYTurbo for Angular Distribution (N2LO) with CT10 PDFs (NNLO)
- LHCb: POWHEG + Pythia8 (NLO+PS) with DYTurbo for Angular Distribution (N2LO) with averaged result from MSHT20, NNPDF31 and CT18 PDFs (NLO)

The combination study seeks to "update" the measurements to a common QCD framework before their compatibility is assessed and, eventually, the results are combined

$$m_W^{update} = m_W^{ref} + \delta m_W^{PDF} + \delta m_W^{pol} + \delta m_W^{other}$$

- m_W^{ref}: Published value
- δm_W^{PDF}: Update to common PDF
- δm_W^{pol}: Common W polarisation
- δm_W^{other}: Additional (small) updates

The LHCb measurement has been "repeated", using the same code framework but different PDF sets
Effect of updates on other measurements estimated with two simulated samples from two models

Fitting pseudodata

The impact on m_W is estimated by fitting reference and updated distribution using the same fitting model

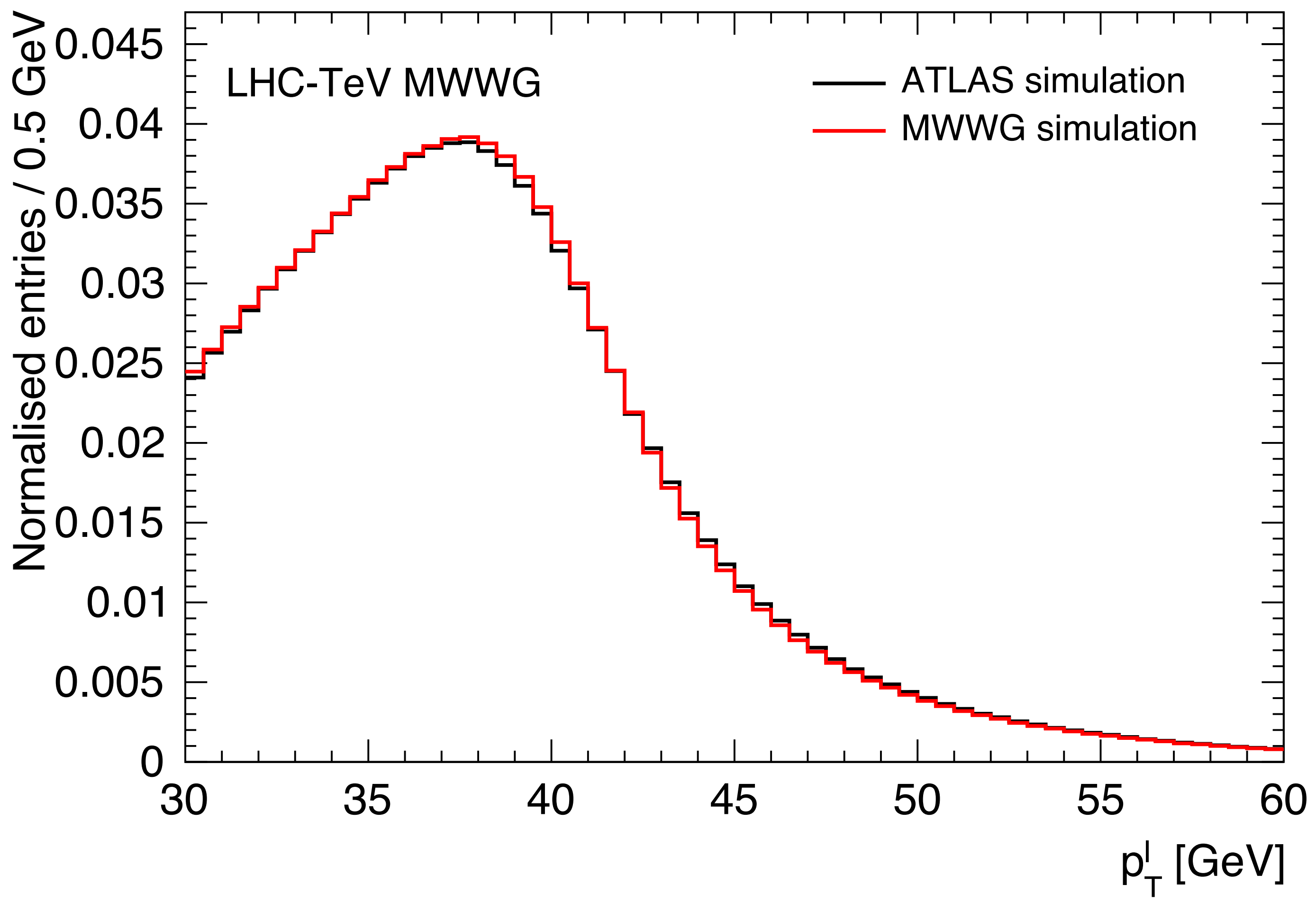
The comparison of PDF effects has been performed using the Wj-MINNLO event generator

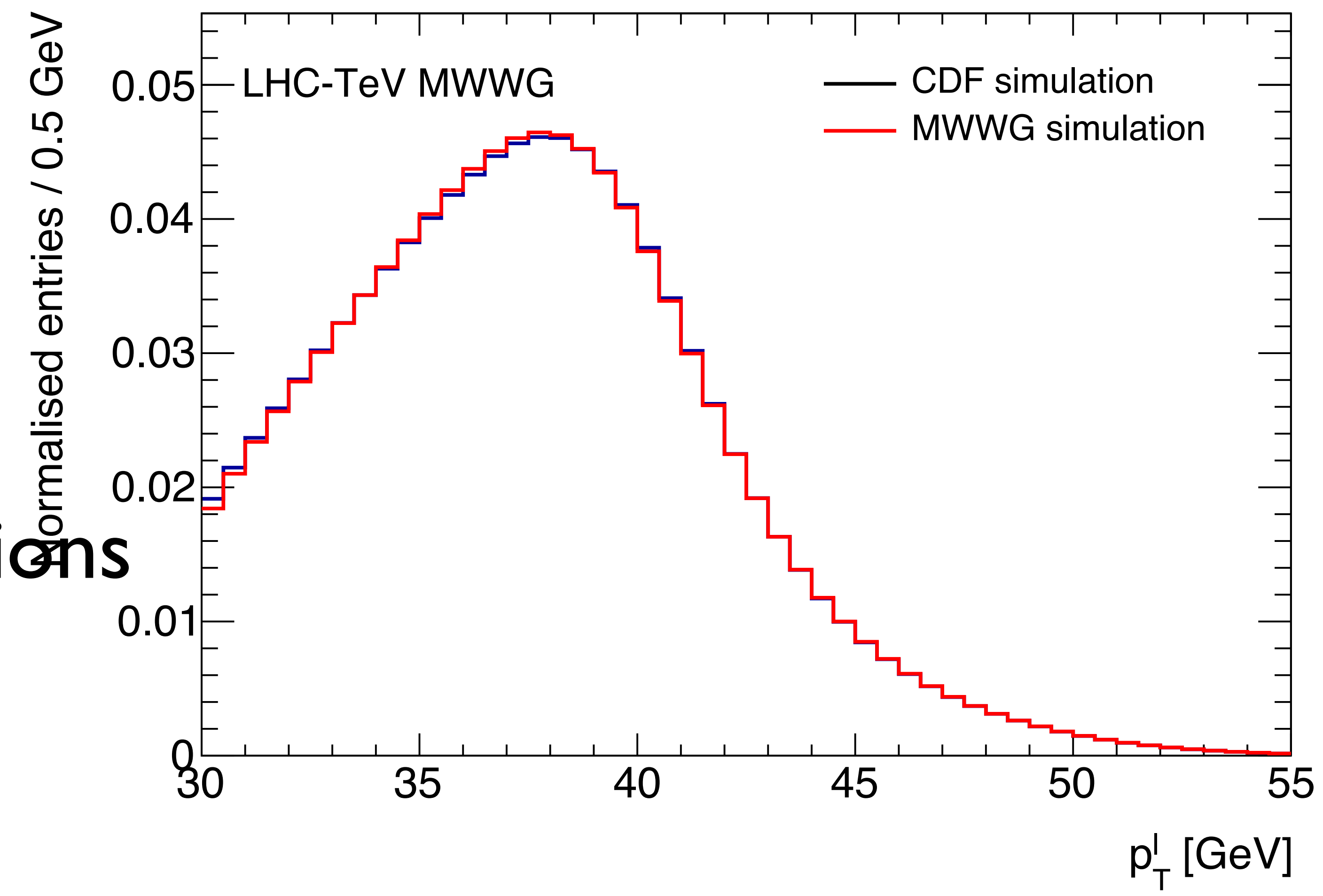
The reference generators for the study of pQCD corrections are ResBos (CDF,D0) and DYTurbo (ATLAS, LHCb)

Detector emulation

The ATLAS, CDF and D0 detectors have been emulated

- η- and $p_\perp$-dependent smearing of leptons
- Recoil modelling includes lepton removal and event activity effects
- Agreement typically at the percent level between the full simulation and the LHC-TeV MWWG emulation
- Small imperfections in the emulation lead to MeV-level uncertainties on δm_W





The $p_\perp^Z$ ($p_\perp^W$) constraint

After all the updates, the distributions are reweighed to reproduce the exp. $p_\perp^Z$ distributions
The constraints by $p_\perp^W$ are also included, when available.

Compatibility of PDF sets with Drell-Yan data

Measurement	NNPDF3.1	NNPDF4.0	MMHT14	MSHT20	CT14	CT18	ABMP16
CDF y_Z	24 / 28	28 / 28	30 / 28	32 / 28	29 / 28	27 / 28	31 / 28
CDF A_W	11 / 13	14 / 13	12 / 13	28 / 13	12 / 13	11 / 13	21 / 13
D0 y_Z	22 / 28	23 / 28	23 / 28	24 / 28	22 / 28	22 / 28	22 / 28
D0 $W \to e\nu$ A_ℓ	22 / 13	23 / 13	52 / 13	42 / 13	21 / 13	19 / 13	26 / 13
D0 $W \to \mu\nu$ A_ℓ	12 / 10	12 / 10	11 / 10	11 / 10	11 / 10	12 / 10	11 / 10
ATLAS peak CC y_Z	13 / 12	13 / 12	58 / 12	17 / 12	12 / 12	11 / 12	18 / 12
ATLAS W^- y_ℓ	12 / 11	12 / 11	33 / 11	16 / 11	13 / 11	10 / 11	14 / 11
ATLAS W^+ y_ℓ	9 / 11	9 / 11	15 / 11	12 / 11	9 / 11	9 / 11	10 / 11
Correlated χ^2	75	62	210	88	81	41	83
Total χ^2 / d.o.f.	200 / 126	196 / 126	444 / 126	270 / 126	210 / 126	162 / 126	236 / 126
p(χ^2, n)	0.003%	0.007%	$< 10^{-10}$	$< 10^{-10}$	0.0004%	1.5%	10^{-8}

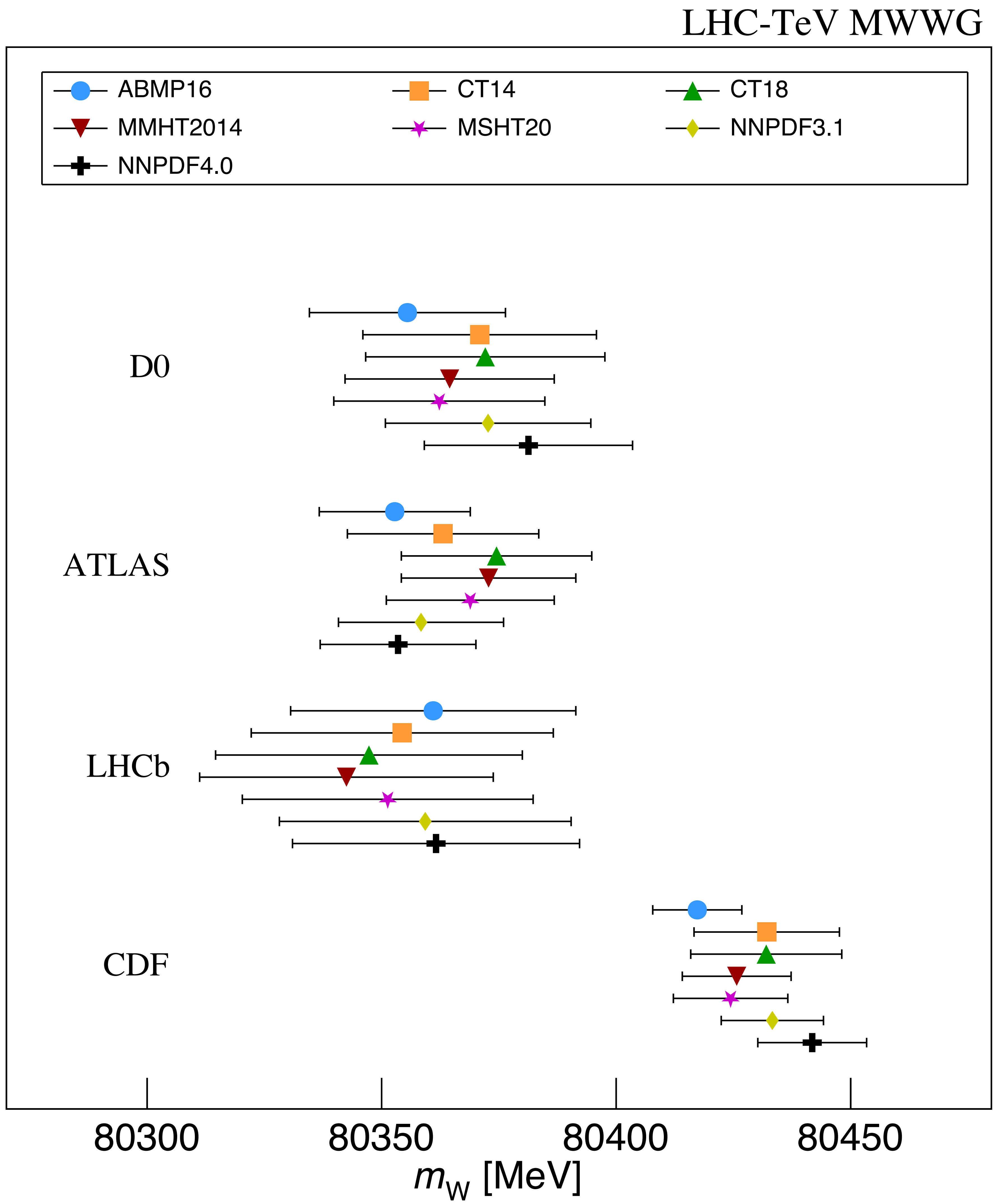
No PDF set provides a good description of the full Tevatron+LHC dataset

Best description given by CT18 (which has larger uncertainties)

CT18 therefore taken as the default PDF set

Combination

Input measurements with updates applied

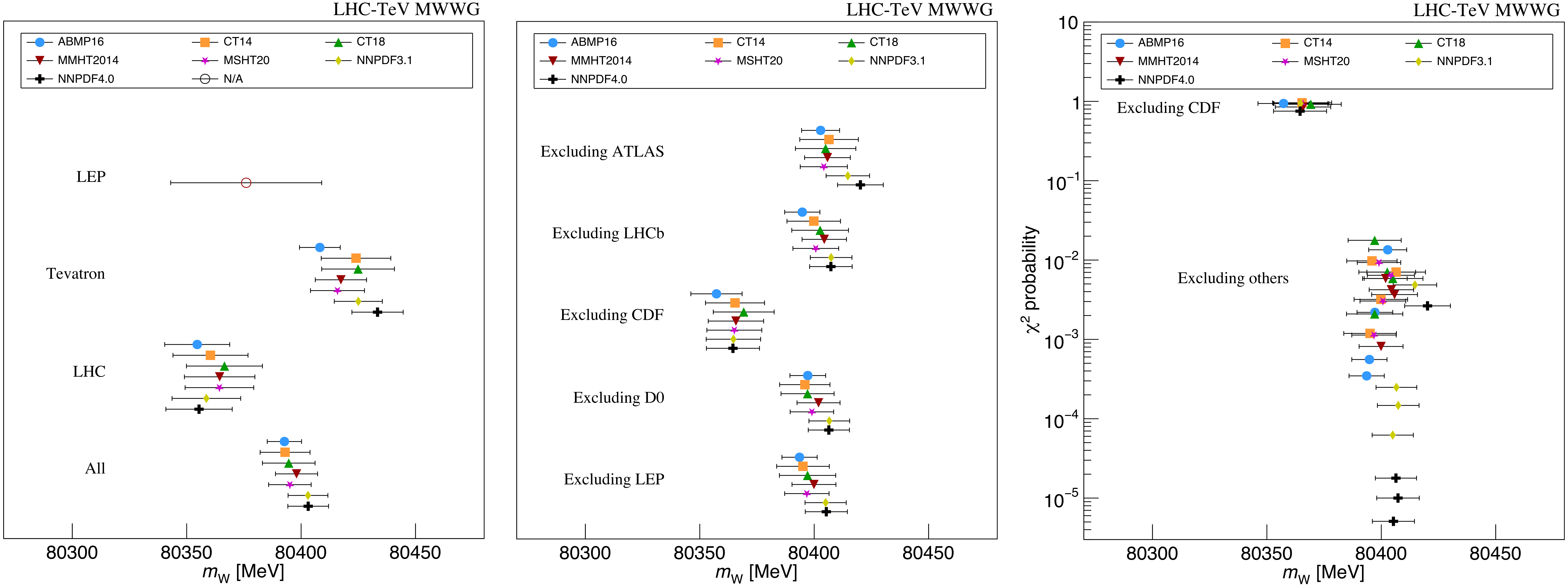


All experiments (4 d.o.f.)				
PDF set	m_W	σ_{PDF}	χ^2	$p(\chi^2, n)$
ABMP16	80392.7 ± 7.5	3.2	29	0.0008%
CT14	80393.0 ± 10.9	7.1	16	0.3%
CT18	80394.6 ± 11.5	7.7	15	0.5%
MMHT2014	80398.0 ± 9.2	5.8	17	0.2%
MSHT20	80395.1 ± 9.3	5.8	16	0.3%
NNPDF3.1	80403.0 ± 8.7	5.3	23	0.1%
NNPDF4.0	80403.1 ± 8.9	5.3	28	0.001%

No combination of all measurements provides a good χ^2 probability

the full combination, including CDF, is disfavoured

MW combinations (cfr arXiv:2308.09417 for all the preparatory steps of the combination)



Combinations with CDF excluded have good compatibility: $m_W = 80369.2 \pm 13.3$ MeV (CT18)

the χ^2 probability is 91%

relative weights: 42% (ATLAS), 23% (D0), 18% (LHCb), 16% (LEP)

The inclusion of CDF brings the χ^2 probability below 0.5%

PDF effects from the study of the $p_\perp^\ell$ or $p_\perp^\nu$ distributions

δm_W^{PDF}

PDF set	D0 p_T^ℓ	D0 p_T^ν	CDF p_T^ℓ	CDF p_T^ν	ATLAS W^+	ATLAS W^-	LHCb
CTEQ6	−17.0	−17.7	0.0	0.0	–	–	–
CTEQ6.6	0.0	0.0	15.0	17.0	–	–	–
CT10	0.4	−1.3	16.0	16.3	0.0	0.0	–
CT14	−9.7	−10.6	5.8	6.8	-1.2	−5.8	1.1
CT18	−8.2	−9.3	7.2	7.7	12.1	−2.3	−6.0
ABMP16	−19.6	−21.5	−1.4	−2.4	−22.5	−3.1	7.7
MMHT2014	−10.4	−12.7	6.1	5.5	−2.6	9.9	−10.8
MSHT20	−13.7	−15.4	3.6	4.1	−20.9	4.5	−2.0
NNPDF3.1	−1.0	−1.2	14.0	15.1	−14.1	-1.8	6.0
NNPDF4.0	6.7	8.1	20.8	24.1	−22.4	6.9	8.3

$\sigma_{PDF}(m_W)$

PDF set	D0	CDF	ATLAS	LHCb
CTEQ6	–	14.1	–	–
CTEQ6.6	15.1	–	–	–
CT10	–	–	9.2	–
CT14	13.8	12.4	11.4	10.8
CT18	14.9	13.4	10.0	12.2
ABMP16	4.5	3.9	4.0	3.0
MMHT2014	8.8	7.7	8.8	8.0
MSHT20	9.4	8.5	7.8	6.8
NNPDF3.1	7.7	6.6	7.4	7.0
NNPDF4.0	8.6	7.7	5.3	4.1

The Tevatron combination did not consider

$$\delta m_W^{PDF}(\text{CTEQ6,CTEQ6.6}) \sim 17 \text{ MeV}$$

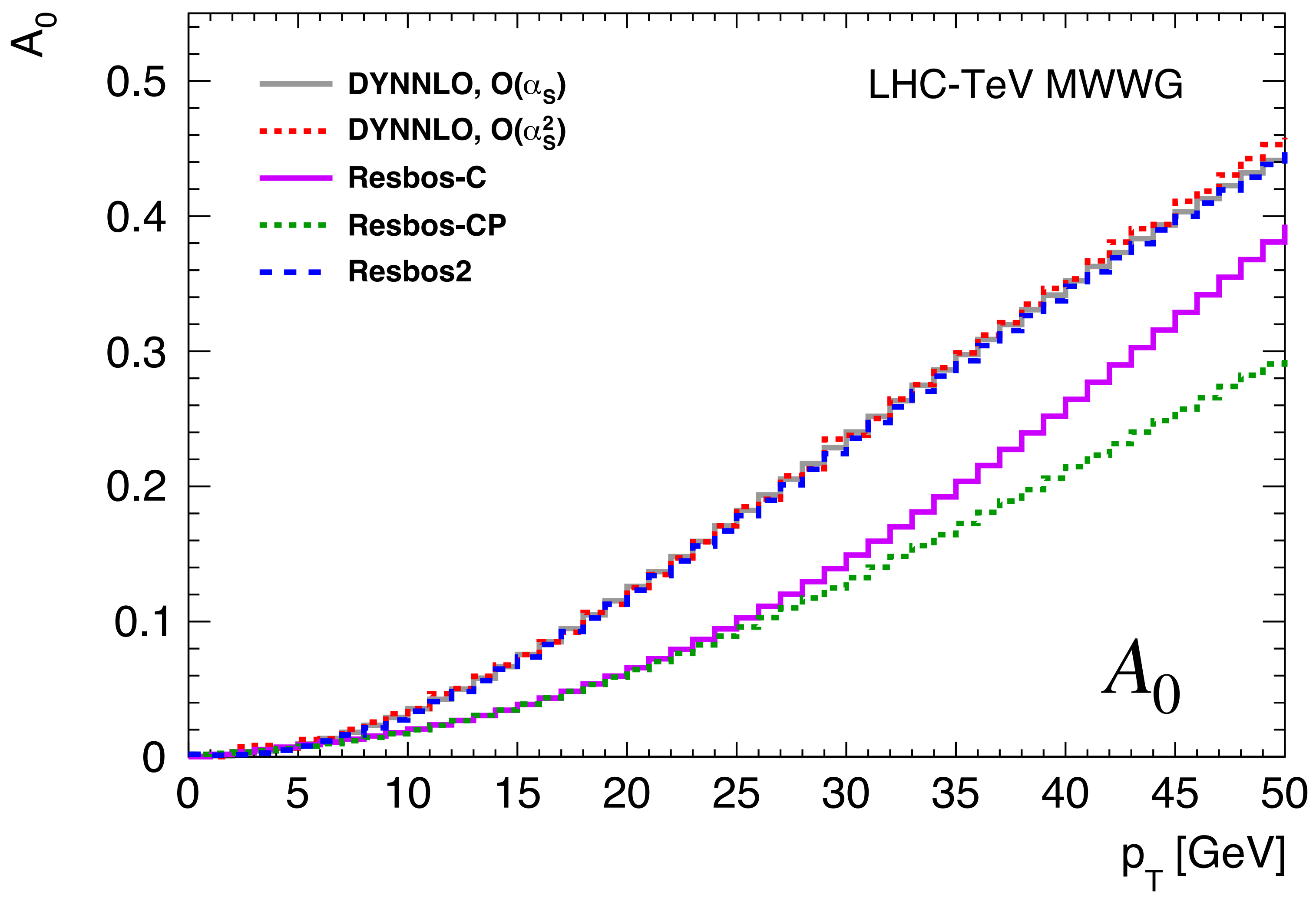
Uncertainties here in some cases larger than in original publications
e.g.for CDF the NNPDF3.1 uncertainty from 3.9 to 6.6 MeV

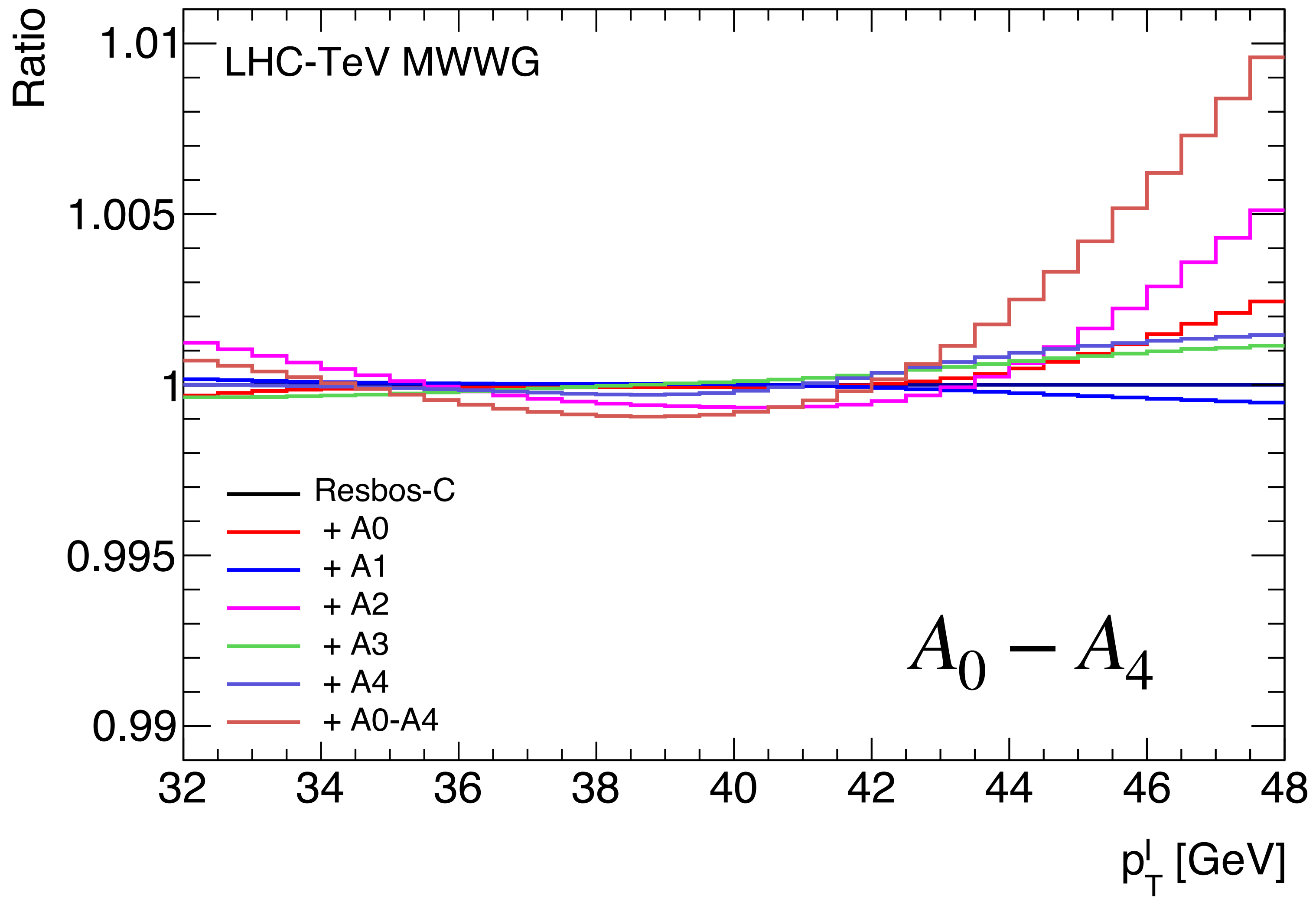
Leptonic angular distributions and QCD corrections

$$\frac{d\sigma}{dp_\perp^W dy_W dm_W d\Omega} = \frac{d\sigma}{dp_\perp^W dy_W dm_W}\Big\{1+\cos^2\theta + \frac{1}{2}A_0(1-3\cos^2\theta) + A_1\sin 2\theta\cos\phi + \frac{1}{2}A_2\sin^2\theta\cos 2\phi + A_3\sin\theta\cos\phi + A_4\cos\theta + A_5\sin^2\theta\sin 2\phi + A_6\sin 2\theta\sin\phi + A_7\sin\theta\sin\phi\Big\}$$

ATLAS and LHCb use DYTurbo and quote an uncertainty on the A_i $\rightarrow$ no additional corrections

Fits to data using ResBos-C (CDF) or ResBos-CP (D0) ported so that the $A_0 - A_4$ combinations matches the ResBos2 prediction at $\mathcal{O}(\alpha_s)$





CDF δm_W^{pol}

Coefficient	m_T	p_T^ℓ	p_T^ν
A_0	-6.3	-2.6	-9.1
A_1	1.1	1.3	0.3
A_2	-0.7	0.4	-3.2
A_3	-2.1	-4.2	1.0
A_4	-1.4	-3.3	-1.6
$A_0 - A_4$	-9.5	-8.4	-12.5
ResBos2	-10.2±1.1	-7.6±1.2	-11.8±1.4
Difference	-0.7±1.1	0.8±1.2	0.7±1.4

D0 δm_W^{pol}

Coefficient	m_T	p_T^ℓ	p_T^ν
A_0	-9.8	-7.3	-15.6
A_1	1.9	2.4	1.8
A_2	3.0	3.3	-2.7
A_3	-1.6	-2.9	0.4
A_4	0.2	-2.3	0.5
$A_0 - A_4$	-6.4	-6.9	-15.8
ResBos2	-7.8±1.0	-6.6±1.1	-16.5±1.2
Difference	-1.4±1.0	0.3±1.1	-0.7±1.2

Combination of the different m_W determinations

Results combined using BLUE

Validation by reproducing internal experimental combinations

The CDF measurement contains an *a posteriori* shift $\delta m_W \sim 3$ MeV
accounting for (CTEQ6M→NNPDF3.1, mass modelling, polarisation effects) removed before the combination

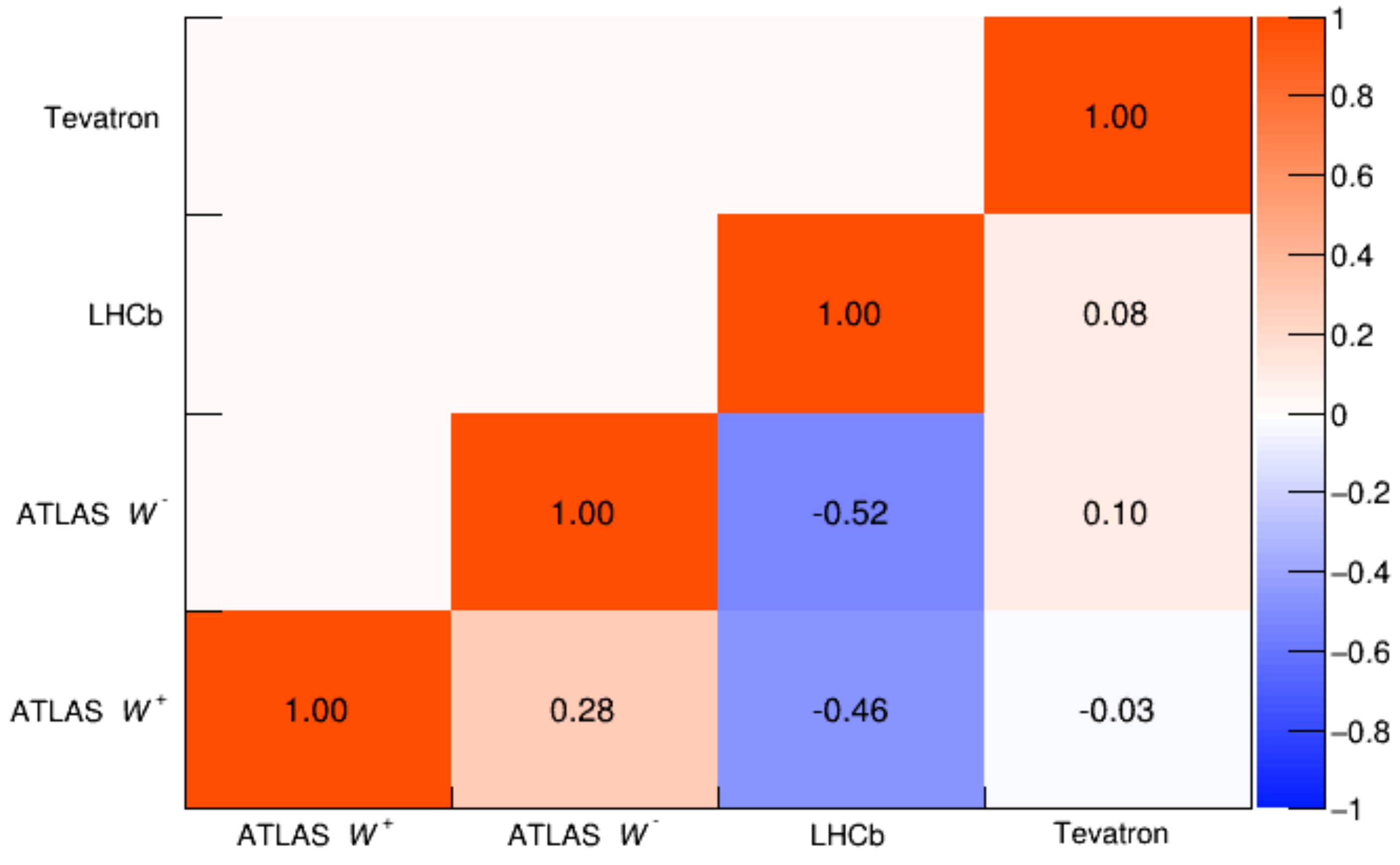
PDF correlations in the combination

Correlations needed in the combination

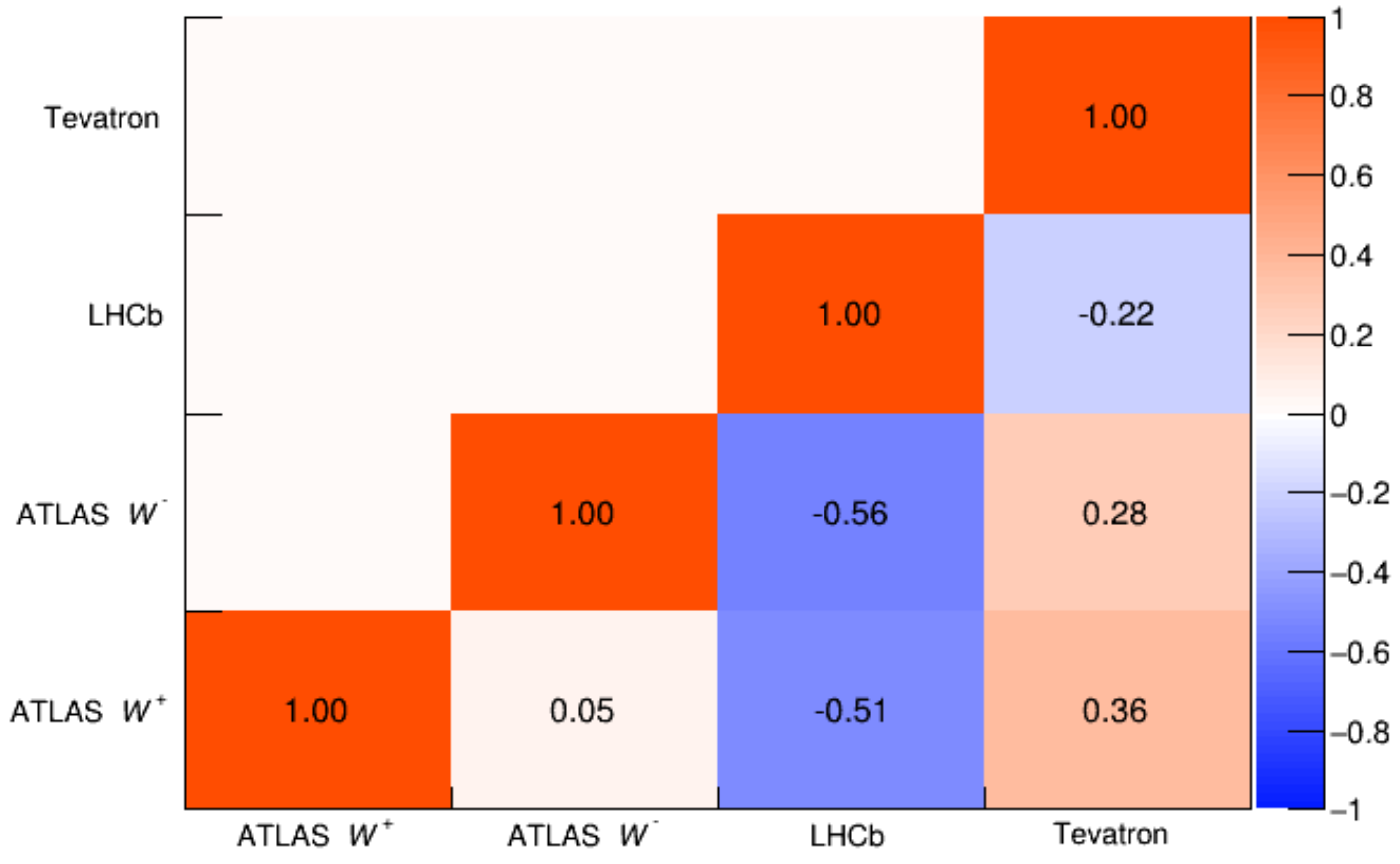
Significantly different correlations between the various PDF sets

PDF anti-correlations between experiments leads to more stable results and reduced PDF dependence
cfr. G.Bozzi, L.Citelli, AV, M.Vesterinen, arXiv:1501.05587, arXiv:1508.06954

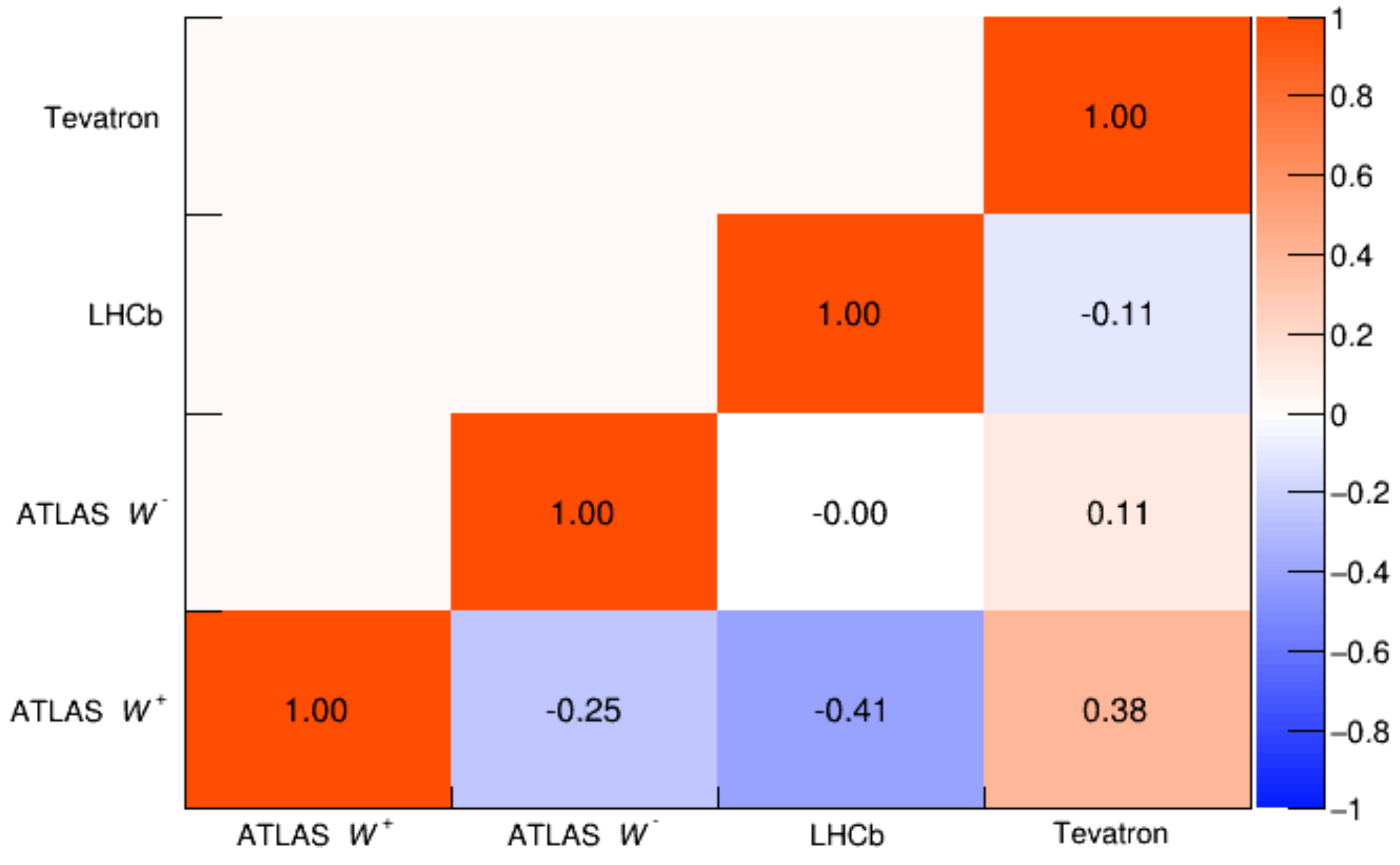
CT18



MSHT20



NNPDF4.0



Conclusions about the m_W combination effort

Extensive effort to provide a common treatment of PDF and pQCD modelling for the m_W determination at hadron colliders

The updated treatment is unable to solve the tension between the existing measurements

The full combination $m_W = 80394.6 \pm 11.5$ MeV (CT18) is disfavoured due to low χ^2 probability (0.5%)

The combination with CDF excluded $m_W = 80369.2 \pm 13.3$ MeV (CT18) has good χ^2 probability (91%)

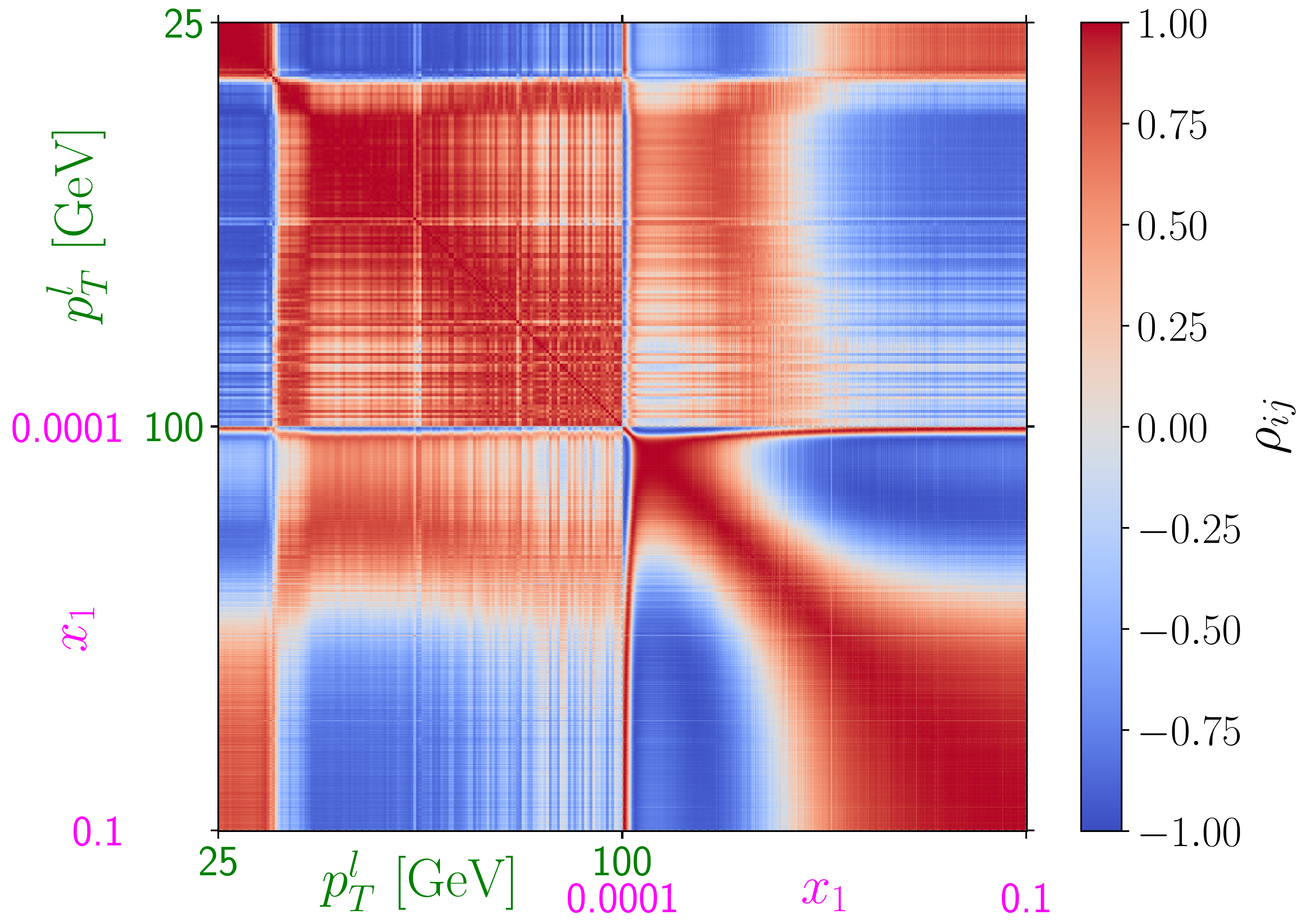
PDF uncertainty on MW: exploiting the theoretical constraints

E.Bagnaschi, AV, Phys.Rev.Lett.126 (2021) 4, 041801

all PDF replicas are correlated because the parton densities are developed in the same QCD framework
1) obey sum rules, 2) satisfy DGLAP equations, 3) are based on the same data set

the "unitarity constraint" of each parton density affects the parton-parton luminosities, which, convoluted with the partonic xsec, in turn affect the hadron-level xsec

$$\rho_{ij} = \frac{\langle \left(\mathcal{O}_i - \langle \mathcal{O}_i \rangle_{PDF}\right) \left(\mathcal{O}_j - \langle \mathcal{O}_j \rangle_{PDF}\right) \rangle_{PDF}}{\sigma_i \sigma_j}$$



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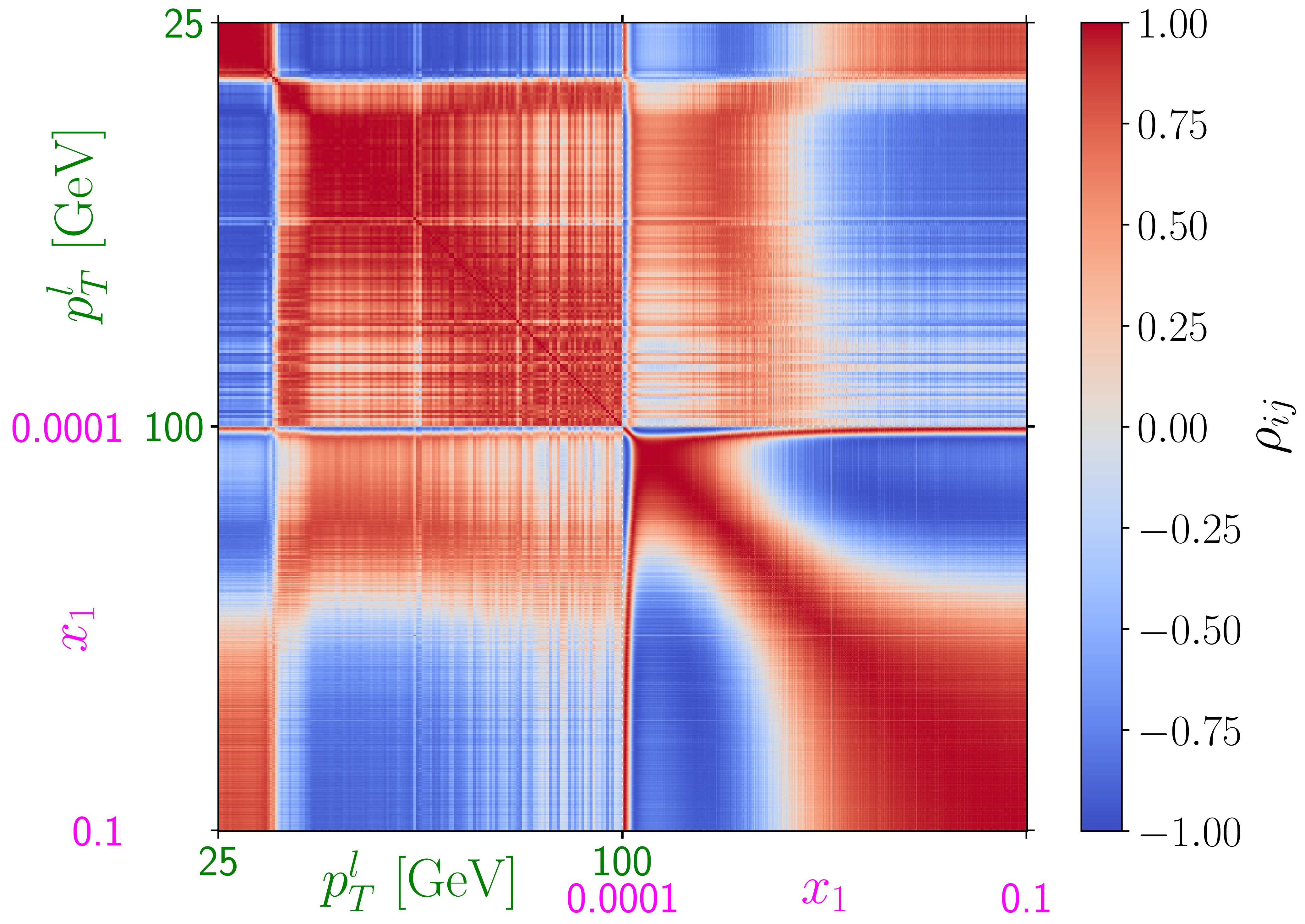


$$\chi^2_{k,min} = \sum_{r,s \in bins} \left(\mathcal{T}_{0,k} - \mathcal{D}^{exp}\right)_r C^{-1}_{rs} \left(\mathcal{T}_{0,k} - \mathcal{D}^{exp}\right)_s$$

$$C = \Sigma_{PDF} + \Sigma_{stat} + \Sigma_{MC} + \Sigma_{exp\,syst}$$ total covariance

Inserting the information about PDFs in the covariance matrix leads to a profiling action "in situ", given by the data themselves

the PDF uncertainty can be reduced to the few MeV level
thanks to the strong anti correlated behaviour of the two tails of $p_\perp^\ell$

ATLAS has used this idea to profile PDFs and reduce their impact

Comments on the data driven approach to fit the W-boson mass

- The Monte Carlo event generators typically have (N)LO+(N)LL QCD perturbative accuracy
  → to match the data they might require a reweighing factor larger than a code N3LO+N3LL

- The tuning to the data should be done in association to QCD scale variations
  → starting from different pQCD scale choices, we can achieve by construction the same description of NCDY
  with different reweighing functions
  but
  we should check how the different alternatives behave when propagated to CCDY

- The tuning assumes that the reweighing factor derived from $p_\perp^Z$
  applies equally well to the $p_\perp^W$ and to the lepton transverse momentum in CCDY

- The tuning assumes that the missing factor taken from the data is universal, i.e. identical for NCDY and CCDY
  but
  several elements of difference:
  - masses and phase-space factors, acceptances
  - different electric charges (QED corrections)
  - different initial states (→ PDFs, heavy quarks effects)

- It is possible that BSM physics is reabsorbed in the tuning

- The interpretation of the fitted value is not necessarily the SM lagrangian parameter

The W boson mass: theoretical prediction

on-shell scheme: dominant contributions to Δr

$$\Delta r = \Delta\alpha - \frac{c_w^2}{s_w^2}\Delta\rho + \Delta r_{\rm rem}$$

$$\Delta\alpha = \Pi^{\gamma}_{\rm ferm}(M_Z^2) - \Pi^{\gamma}_{\rm ferm}(0) \quad \to \quad \alpha(M_Z) = \frac{\alpha}{1-\Delta\alpha}$$

$$\Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2} = 3\frac{G_F m_t^2}{8\pi^2\sqrt{2}} \quad \text{[one-loop]} \quad \sim \frac{m_t^2}{v^2} \sim \alpha_t$$

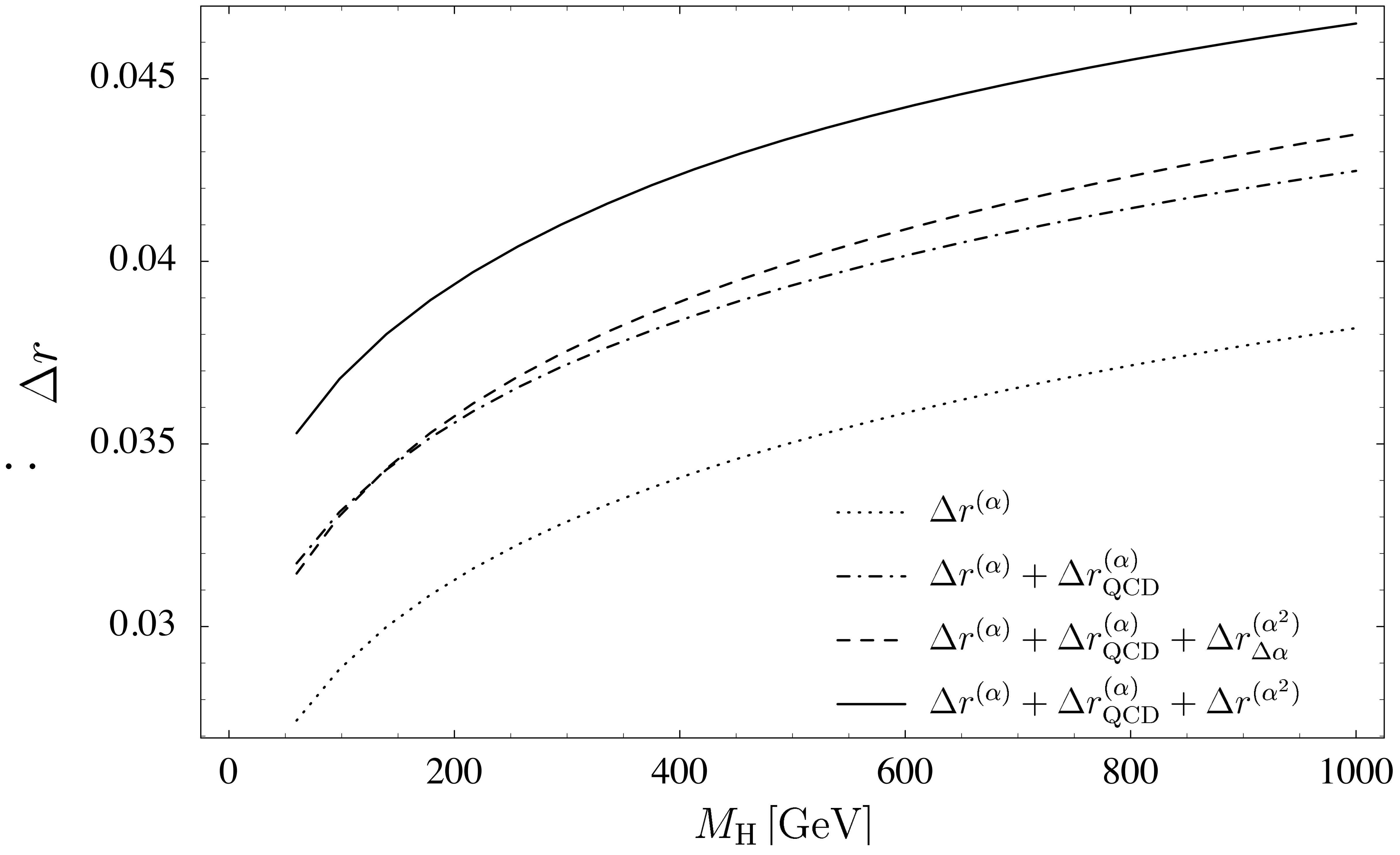
beyond one-loop order: $\sim \alpha^2,\ \alpha\alpha_t,\ \alpha_t^2,\ \alpha^2\alpha_t,\ \alpha\alpha_t^2,\ \alpha_t^3, \ldots$

reducible higher order terms from $\Delta\alpha$ and $\Delta\rho$ via

$$1+\Delta r \to \frac{1}{(1-\Delta\alpha)\left(1+\frac{c_w^2}{s_w^2}\Delta\rho\right)+\cdots}$$

$$\rho = 1+\Delta\rho \to \frac{1}{1-\Delta\rho}$$

effects of higher-order terms on Δr



(Consoli, Hollik, Jegerlehner)

The dilepton invariant mass distribution in NC-DY at high mass and $\sin^2\hat\theta(\mu_R^2)$

The triple-differential cross section at LO

$$\frac{d^3\sigma}{dm_{\ell\ell}dy_{\ell\ell}d\cos\theta_{CS}} = \frac{\pi\alpha^2}{3m_{\ell\ell}s}\Bigg((1+\cos^2\theta_{CS})\sum_q S_q[f_q(x_1,Q^2)f_{\bar q}(x_2,Q^2)+f_q(x_2,Q^2)f_{\bar q}(x_1,Q^2)] + \cos\theta_{CS}\sum_q A_q \mathrm{sign}(y_{\ell\ell})\cdot[f_q(x_1,Q^2)f_{\bar q}(x_2,Q^2)-f_q(x_2,Q^2)f_{\bar q}(x_1,Q^2)]\Bigg)$$

$$S_q = e_\ell^2 e_q^2 + P_{\gamma Z}\cdot e_\ell v_\ell e_q v_q + P_{ZZ}\cdot(v_\ell^2+a_\ell^2)(v_q^2+a_q^2)$$
$$A_q = P_{\gamma Z}\cdot 2e_\ell a_\ell e_q a_q + P_{ZZ}\cdot 8v_\ell a_\ell v_q a_q,$$

$$P_{\gamma Z}(m_{\ell\ell}) = \frac{2m_{\ell\ell}^2(m_{\ell\ell}^2-m_Z^2)}{\sin^2\theta_W\cos^2\theta_W[(m_{\ell\ell}^2-m_Z^2)^2+\Gamma_Z^2 m_Z^2]}$$
$$P_{ZZ}(m_{\ell\ell}) = \frac{m_{\ell\ell}^4}{\sin^4\theta_W\cos^4\theta_W[(m_{\ell\ell}^2-m_Z^2)^2+\Gamma_Z^2 m_Z^2]}$$

The 3D differential xsec exhibits a dependence on the specific $\sin^2\theta_W$ value, modulated by the different combinations of γ and Z propagators.

At the Z resonance, specific sensitivity to $\sin^2\theta_W$, via the ratio of vector/axial-vector couplings, assessed from the study of A_{FB} and A_{LR} asymmetries

Also at large invariant masses the xsec features a sensitivity to $\sin^2\theta_W$, stemming from both normalisation and angular-dependent factors!

→ at NLO-EW we can study $\sin^2\hat\theta(\mu_R)$, the MSbar renormalised mixing angle and exploit the large mass range to test the running of this quantity

Determination of $\sin^2\theta_{eff}^{lep}$ in the LHC framework

A few differences compared to the LEP measurement and analysis framework

- the initial state is a mixture, weighted by PDFs, of different quark flavours

  → PDF uncertainty + problems to disentangle individual Z decay widths

- the precision on the Z peak cross section is lower than the one at LEP for e+e-→hadrons

  → σ_{had} was at LEP an important constraint of the pseudo-observable fit

- the experimental analysis involves an invariant mass window (instead of only $q^2=M_Z^2$)

  → non-factorisable contributions spoil the factorisation (initial)x(final) form factors

→ it is not possible to pursue the LEP approach in terms of pseudo-observables at LHC

$$A_{FB}^{exp}(m_Z^2) - \mathcal{A}_{nonfact} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f$$

→ a template fit approach in the full SM is needed to analyse the AFB data and offers a well defined procedure

- to extract $\sin^2\theta_{eff}^{lep}$
- to assign the associated theoretical uncertainties

→ we need to be able to prepare templates of $A_{FB}(m_{\ell\ell}^2)$ for different values of $\sin^2\theta_{eff}^{lep}$

An electroweak scheme with $(G_\mu, m_Z, \sin^2\theta^\ell_{eff})$ as inputs

M.Chiesa, F.Piccinini, AV, arXiv:1906.11569

The weak mixing angle is related to the left- and right-handed (vector and axial-vector) couplings of the Z boson to fermions

$$\sin^2\theta^l_{eff} = \frac{I_3^l}{2Q_l}\left(1 - \frac{g_V^l}{g_A^l}\right) = \frac{I_3^l}{Q_l}\left(\frac{-g_R^l}{g_L^l - g_R^l}\right)$$

The request that the tree-level relation holds to all orders fixes the counterterm for $\sin^2\theta^{lep}_{eff}$ on-shell definition

$$\delta\sin^2\theta^\ell_{eff} = -\frac{1}{2}\frac{g_L^\ell g_R^\ell}{(g_L^\ell - g_R^\ell)^2}\,\mathrm{Re}\left(\frac{\delta g_L^\ell}{g_L^\ell} - \frac{\delta g_R^\ell}{g_R^\ell}\right)$$

The renormalised angle is identified with the LEP leptonic effective weak mixing angle

The Z mass is defined in the complex mass scheme.

Δr is evaluated with $\sin^2\theta^{lep}_{eff}$ as input and differs from the usual (α, m_W, m_Z) expression

See also D.C.Kennedy, B.W.Lynn,Nucl.Phys.B322, 1; F.M.Renard, C.Verzegnassi, Phys.Rev.D52,1369;
A.Ferroglia, G.Ossola, A.Sirlin,Phys.Lett.B507,147; A.Ferroglia, G.Ossola, M.Passera, A.Sirlin,Phys.Rev.D65 (2002) 113002

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This scheme allows to express any observable as $\mathcal{O} = \mathcal{O}(G_\mu, m_Z, \sin^2\theta^{lep}_{eff})$

so that templates as a function of $\sin^2\theta^{lep}_{eff}$ can be easily generated

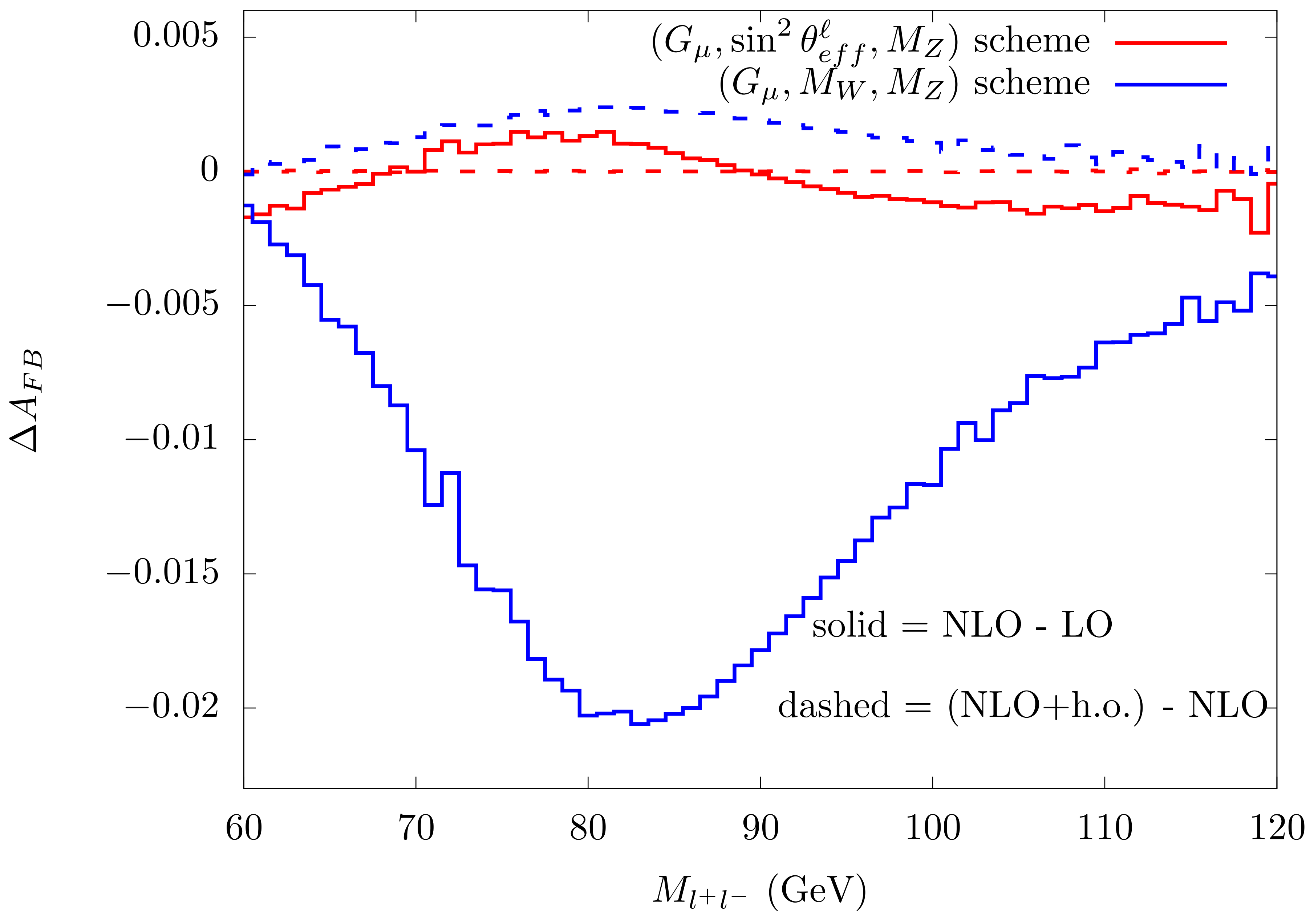
→ direct relation between the data and the parameter of interest

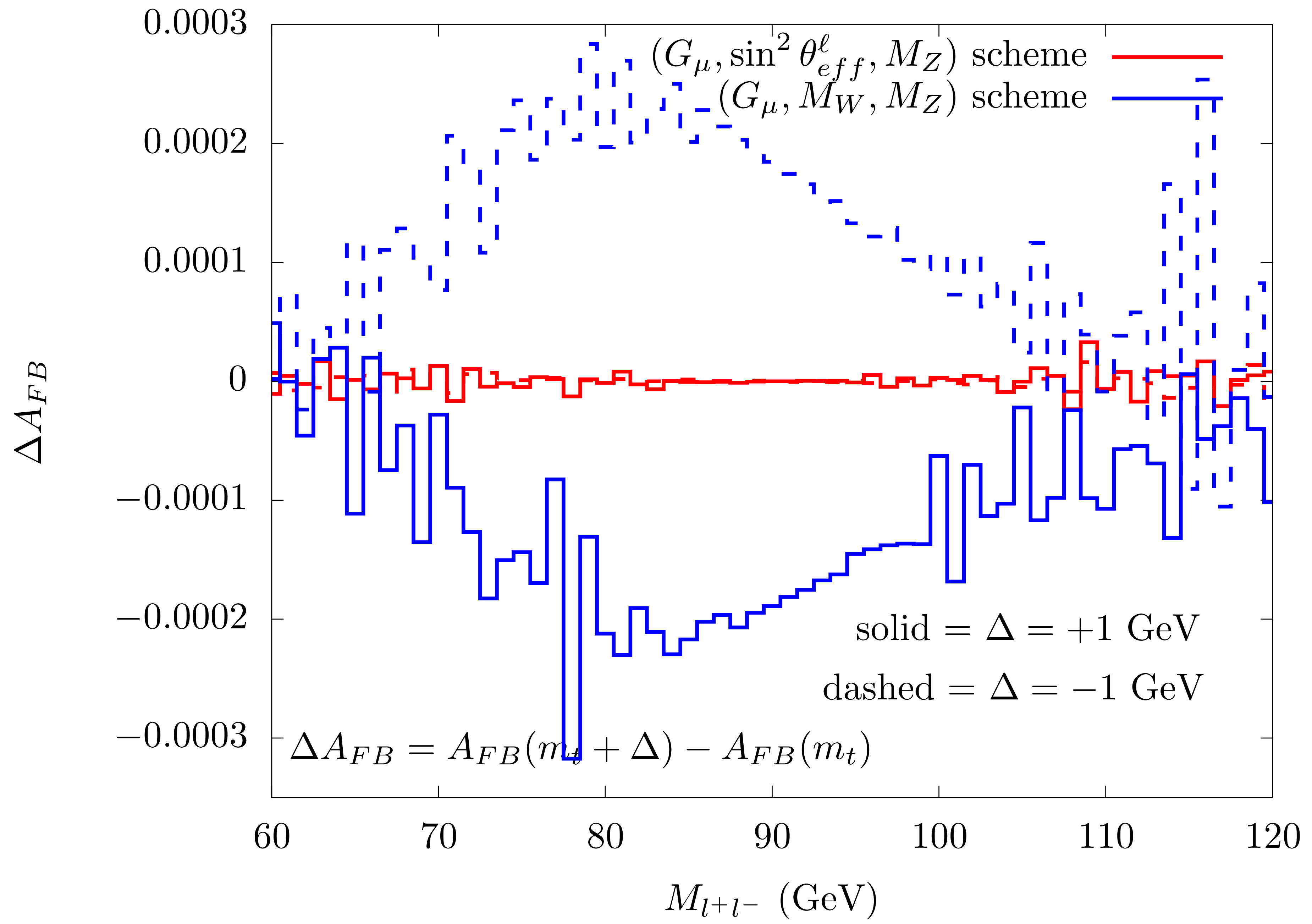
→ simple estimate of all the systematic effects, theoretical and experimental

The result of the fit in this scheme can be directly combined with LEP results

A_{FB} m_t parametric uncertainty and perturbative convergence

M.Chiesa, F.Piccinini, AV, arXiv:1906.11569





prediction for A_{FB} at the LHC in the $(G_\mu, m_Z, \sin^2\theta^\ell_{eff})$ input scheme (red), comparison with (G_μ, m_W, m_Z) (blue)

faster perturbative convergence → good control over the systematic uncertainties of the templates used to fit the data

very weak parametric m_t dependence

$(G_\mu, m_Z, \sin^2\theta^\ell_{eff})$ offer a very effective parameterisation of the Z resonance in terms of normalisation, position, shape

$\sin^2\hat{\theta}(\mu_R)$ determination at hadron colliders at large invariant masses

S.Amoroso, M.Chiesa, C.L Del Pio, E.Lipka, F.Piccinini, F.Vazzoler, AV, arXiv:2302.10782

The study has to be performed at least at NLO-EW.

The amplitude has at NLO-EW different groups of corrections: QED, weak.
Only a specific subset of such corrections contributes to the redefinition of the renormalised parameter, while the rest (e.g. boxes and part of the vertices) is a genuine process dependent correction.

In order to claim that we are sensitive to the precise $\sin^2\hat{\theta}(\mu_R)$ value,
$\sin^2\hat{\theta}(\mu_R)$ must be among the input parameters of the renormalised lagrangian.
A new version of the POWHEG NC DY QCD+EW has been prepared,
which admits as input parameters ($\hat{\alpha}(\mu_R), \sin\hat{\theta}(\mu_R), m_Z$) , renormalised at NLO-EW .

Thanks to this choice, $\sin^2\hat{\theta}(\mu_R)$ can be left as a free fit parameter, and extracted from the data.
The explicit presence of the other corrections, insensitive to $\sin^2\hat{\theta}(\mu_R)$, allows to correctly estimate the dependence on this parameter, at each mass scale.

We need to estimate the change of the xsec, for a given $\sin^2\hat{\theta}(\mu_R)$ variation. In the sensitivity study we identify the minimal variation which can be appreciated in the fit to the data, for given experimental errors.

The weak mixing angle at low energy scales

Goal: testing the parity-violating structure of the weak interactions at different energy scales

Problems: a) define an observable quantity, analogous to $\sin^2\theta_{eff}^{lep}$ at $q^2 = m_Z^2$,

now e.g. at $q^2 = 0$ for the t-channel processes like e-p or e-e- scattering

b) given the large size of the NLO corrections at $q^2 = 0$, the fixed-order result is not sufficient

we have to resum to all orders large classes of radiative corrections in the definition of a running parameter

Solution 1: introduction of $\sin^2\theta_{eff}^{e^-e^-}$ at $q^2 = 0$ to describe Møller scattering   Ferroglia, Ossola, Sirlin, hep-ph/0307200

it absorbs the effect of the EW corrections to the Møller amplitude

in a new effective parameter $\sin^2\theta_{eff}^{e^-e^-}$, via a gauge-invariant form factor $\kappa(q^2 = 0)$,

in a tree-level-like structure

this parameter is a physical observable which can be i) predicted and ii) measured → comparison with $\sin^2\theta_{eff}^{lep}$

Solution 2: the definition of $\sin^2\hat{\theta}(\mu_R)$ in the MSbar scheme is strictly bound to the presence of a renormalisation scale μ_R

$\sin^2\hat{\theta}(\mu_R)$ satisfies the RGE (→ it needs a boundary condition computed at one given scale q^2)

this quantity can be predicted in the SM using $(\alpha(0), G_\mu, m_Z)$ as basic input parameters

the scale μ_R allows to probe the size of resummed radiative correction to the couplings at different scales

The running of $\sin^2\hat\theta(\mu_R)$ and the prediction of $\sin^2\hat\theta(0)$ Erler,Ramsey-Musolf, hep-ph/0409169

given $\sin^2\hat\theta(m_Z^2)$, we want to study a process with $Q^2 \ll m_Z^2$ $\rightarrow$ the radiative corrections contain large $\log(Q^2/m_Z^2)$ factors

in the MSbar scheme, the RGE allows to compute the coupling at an arbitrary scale μ^2, once the value at a given Q^2 is known

$\sin^2\hat\theta(Q^2) = \hat\kappa(Q^2,\mu^2)\ \sin^2\hat\theta(\mu^2)$ setting $\mu^2 = Q^2$ resums the large $\log(Q^2/\mu^2)$ in $\sin^2\theta(\mu^2)$

the behaviour at the physical thresholds is fixed via matching conditions

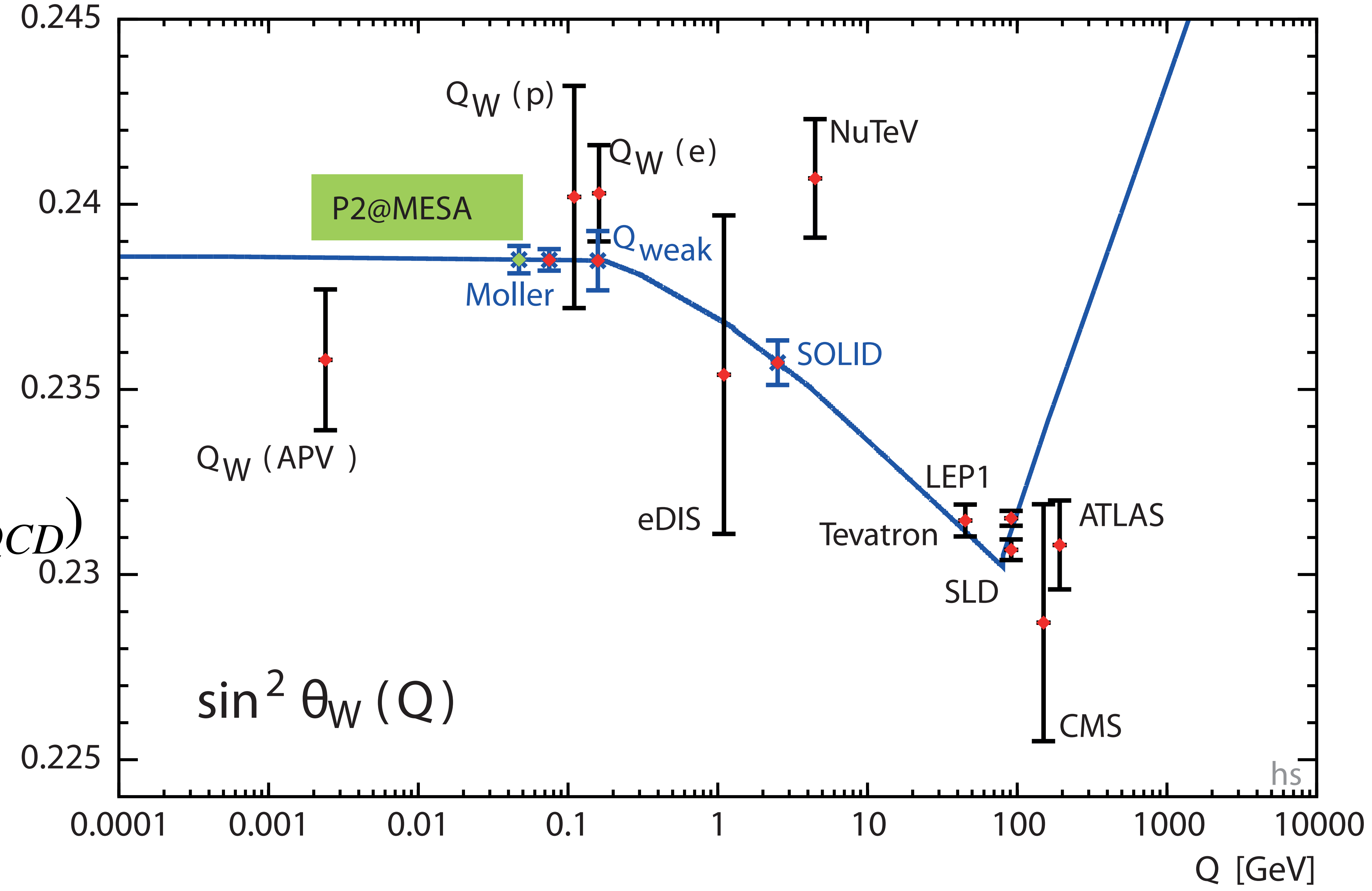
$$\sin^2\theta_W(\mu)_{\overline{MS}} = \frac{\alpha(\mu)_{\overline{MS}}}{\alpha(\mu_0)_{\overline{MS}}}\sin^2\theta_W(\mu_0)_{\overline{MS}} + \lambda_1\left[1-\frac{\alpha(\mu)}{\alpha(\mu_0)}\right] + \frac{\alpha(\mu)}{\pi}\left[\frac{\lambda_2}{3}\ln\frac{\mu^2}{\mu_0^2}+\frac{3\lambda_3}{4}\ln\frac{\alpha(\mu)_{\overline{MS}}}{\alpha(\mu_0)_{\overline{MS}}}+\tilde\sigma(\mu_0)-\tilde\sigma(\mu)\right].$$

we predict $\sin^2\hat\theta(0) = \hat\kappa(0)\ \sin^2\hat\theta(m_Z^2)$
resumming large perturbative corrections in $\hat\kappa(0)$

in ep scattering non-perturbative contributions enter via $\Sigma_{\gamma Z}(\mu\sim\Lambda_{QCD})$
and are treated along with the e.m. coupling

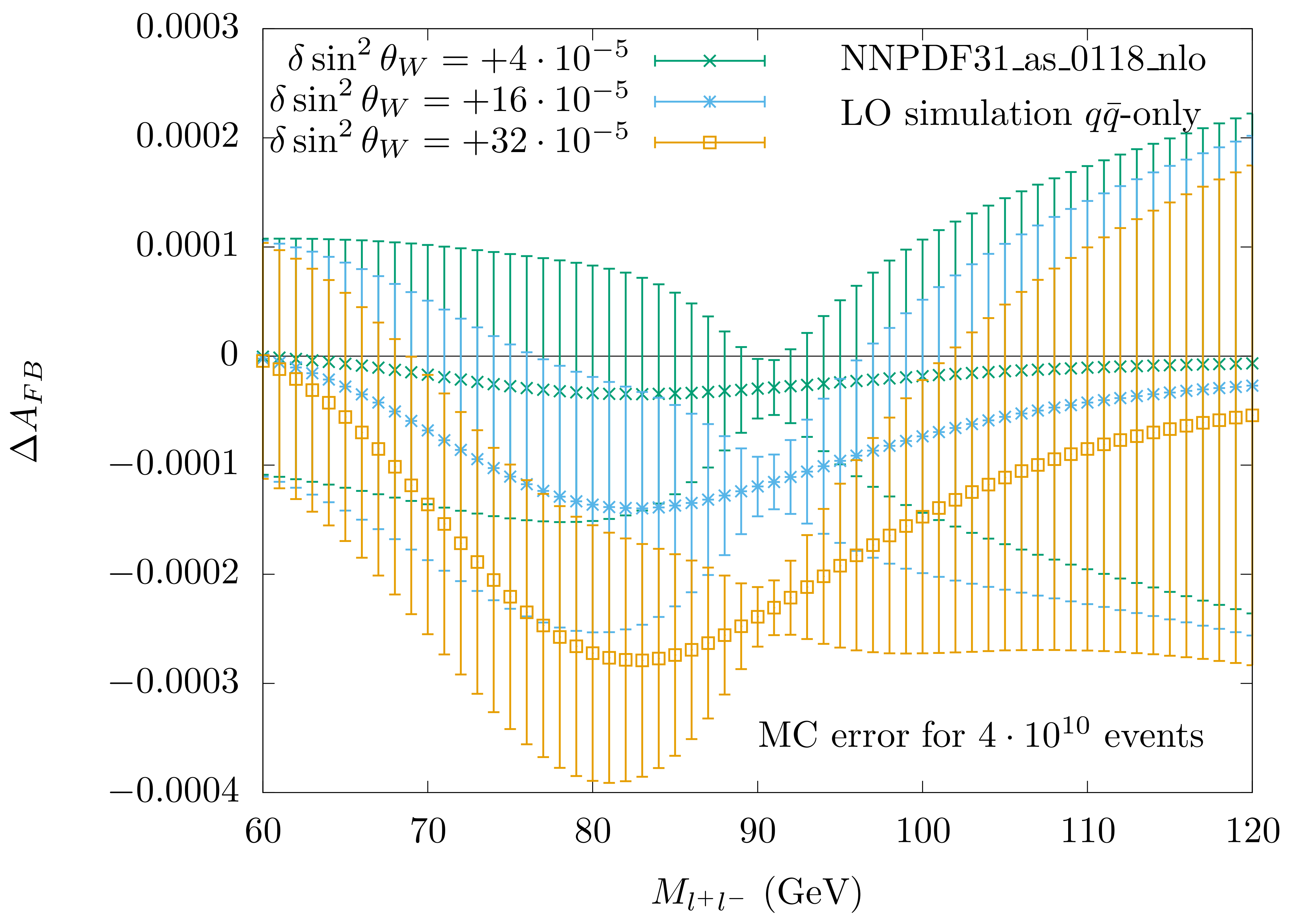
gauge invariance is respected in the MSbar $\hat\kappa$ factor

$\hat\kappa(0) = 1.03232 \pm 0.00029$
$\sin^2\hat\theta(m_Z^2) = 0.23124(6) \rightarrow \sin^2\hat\theta(0) = 0.23871(9)$



Kumar, Mantry, Marciano, Soudry, arXiv:1302.6263

Estimate of $\sin^2\theta_{eff}^{lep}$: template fit approach



$$\chi_i^2 = \sum_{j=1}^{N_{bins}} \frac{(t_j^{(i)} - d_j)^2}{(\sigma_j^{templ})^2 + (\sigma_j^{data})^2} \qquad i = 1,\ldots,N_{templ}$$

t^(i) are templates of the AFB distribution computed at LO, with NNPDF3.1 QCD-only, for different values of $\sin^2\theta_{eff}^{lep}$ labelled by i

d are (pseudo)data

Plotting χ^2_i as a function of *i* yields a parabola, whose minimum selects the preferred $\sin^2\theta_{eff}^{lep}$ value

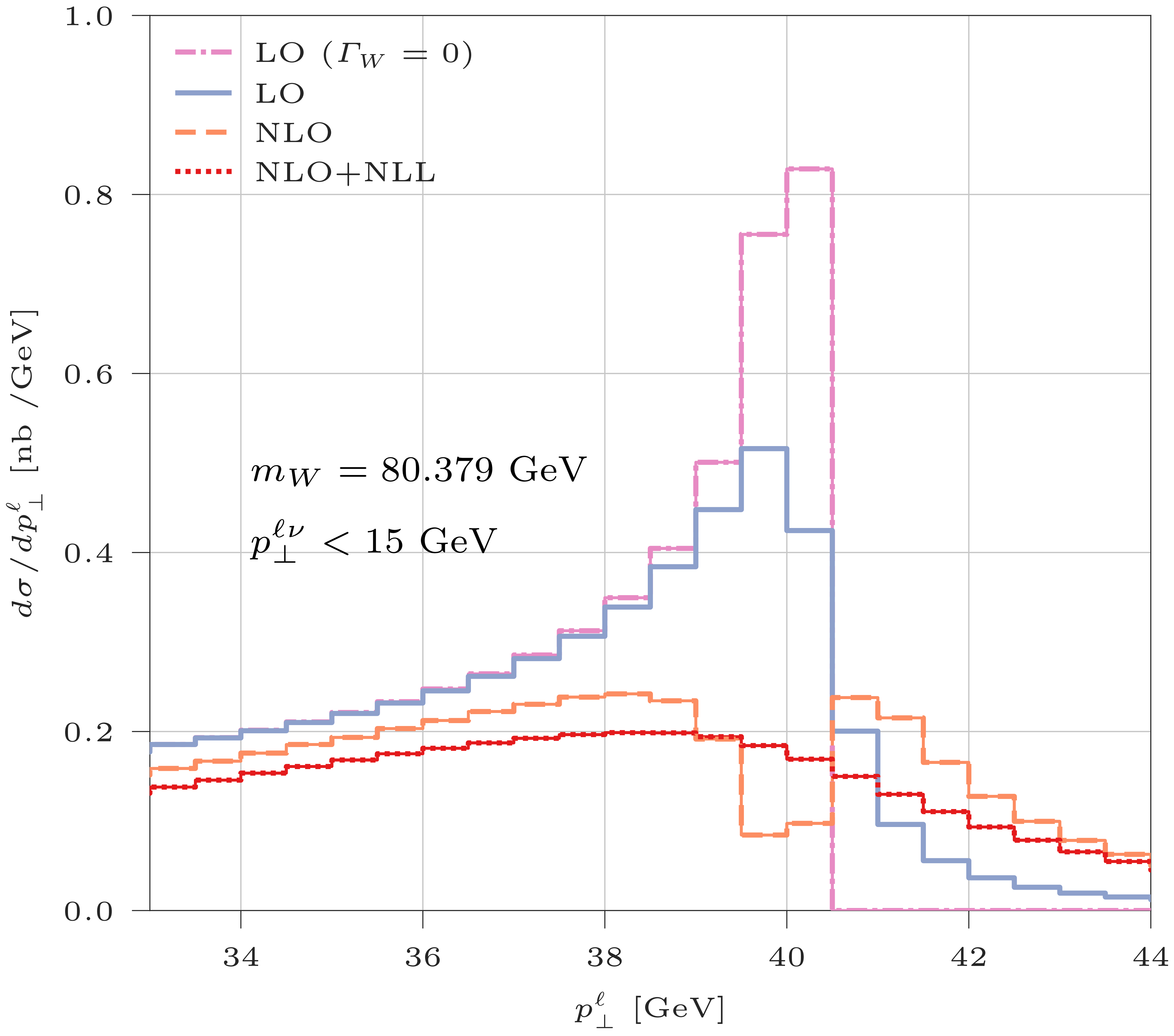
The fit is barely sensitive to $\delta\sin^2\theta_{eff}^{lep} = 4\ 10^{-5}$

A MC statistics 4 times larger would be needed
to have clear sensitivity over the whole fitting range [80,100]

m_W from a jacobian asymmetry

L.Rottoli, P.Torrielli, AV, arXiv:2301.04059

The lepton transverse momentum distribution in charged-current Drell-Yan



The lepton transverse momentum distribution has a jacobian peak induced by the factor $1/\sqrt{1-\frac{s}{4p_\perp^2}}$.

When studying the W resonance region, the peak appears at $p_\perp \sim \frac{m_W}{2}$

Kinematical end point at $\frac{m_W}{2}$ at LO

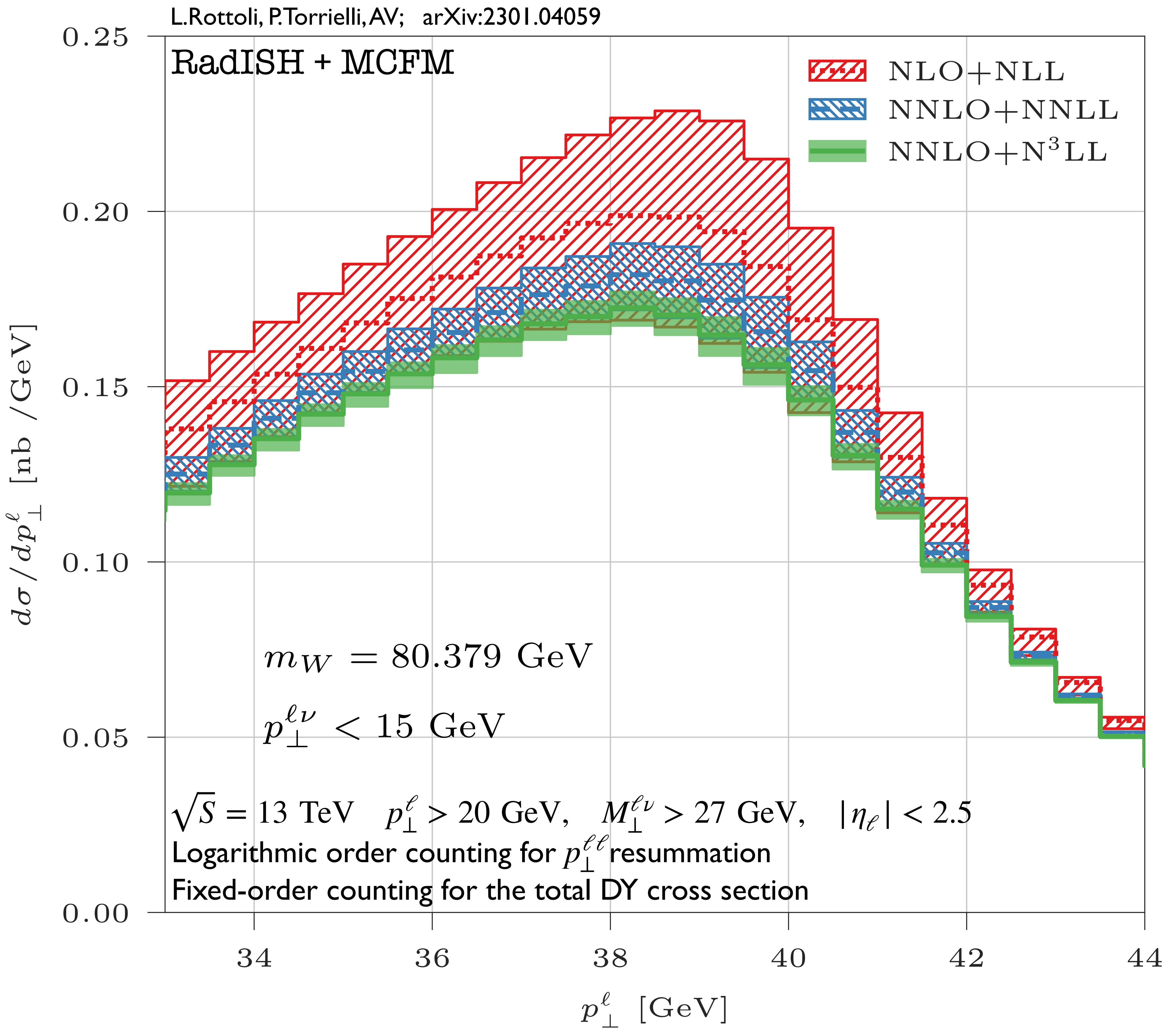
The decay width allows to populate the upper tail of the distribution

Sensitivity to soft radiation $\rightarrow$ double peak at NLO-QCD

The QCD-ISR next-to-leading-log resummation broadens the distribution and cures the sensitivity to soft radiation at the jacobian peak.

In the $p_\perp^\ell$ spectrum the sensitivity to m_W and important QCD features are closely intertwined

The lepton transverse momentum distribution in charged-current Drell-Yan



Impressive progress in QCD calculations

X.Chen, T.Gehrmann, N.Glover, A.Huss, P.Monni, E.Re, L.Rottoli, P.Torrielli, arXiv:2203.01565
X.Chen, T.Gehrmann, N.Glover, A.Huss, T.yang, H.Zhu, arXiv: 2205.11426
J.Campbell, T.Neumann, arXiv:2207.07056
S.Camarda, L.Cieri, G.Ferrera, arXiv:2303.12781

Uncertainty band based on canonical scale variations

$$\mu_{R,F} = \xi_{R,F}\sqrt{(M^{\ell\nu})^2 + (p_\perp^{\ell\nu})^2}\,, \quad \mu_Q = \xi_Q M^{\ell\nu}$$

$\xi_{R,F} \in (1/2,1,2)$ excluding ratios=4 (7 variations)

$(\xi_R, \xi_F) = (1,1)$ and $\xi_Q = (1/4,1)$ (2 variations)

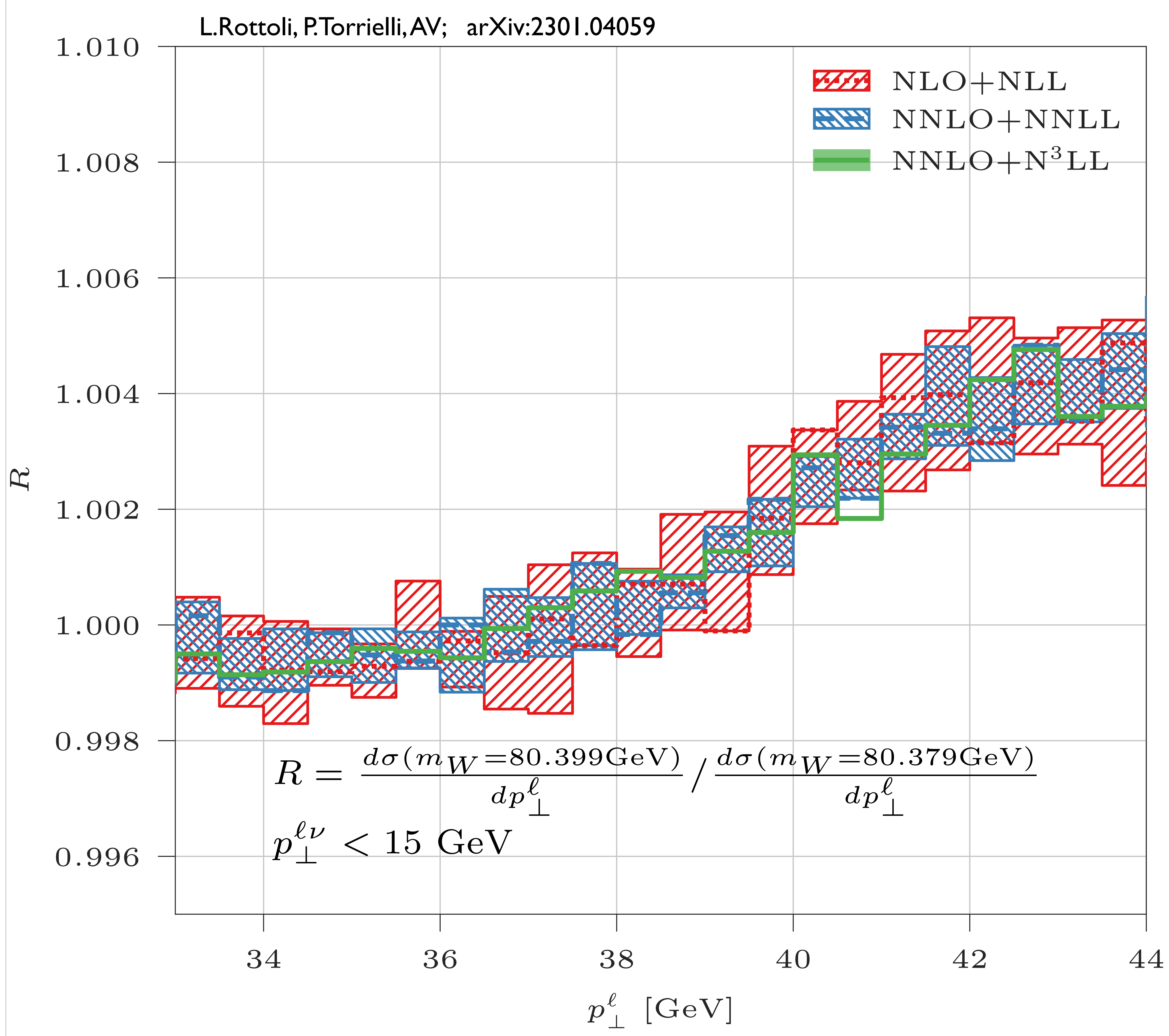
At NNLO+N3LL, residual ±2% uncertainty

The peak of the distribution is located at $p_\perp \sim 38.5$ GeV

The point of maximal sensitivity to m_W is shifted by :

- $\Gamma_W/2$ compared to the nominal value $m_W/2$
- the effect of resummed QCD radiation

Sensitivity to the W boson mass: independence from QCD approximation



The determination of m_W requires the possibility to appreciate the distortion of the distribution induced by 2 different mass hypotheses

A shift by $\Delta m_W = 20$ MeV distorts the distribution at few per mille level

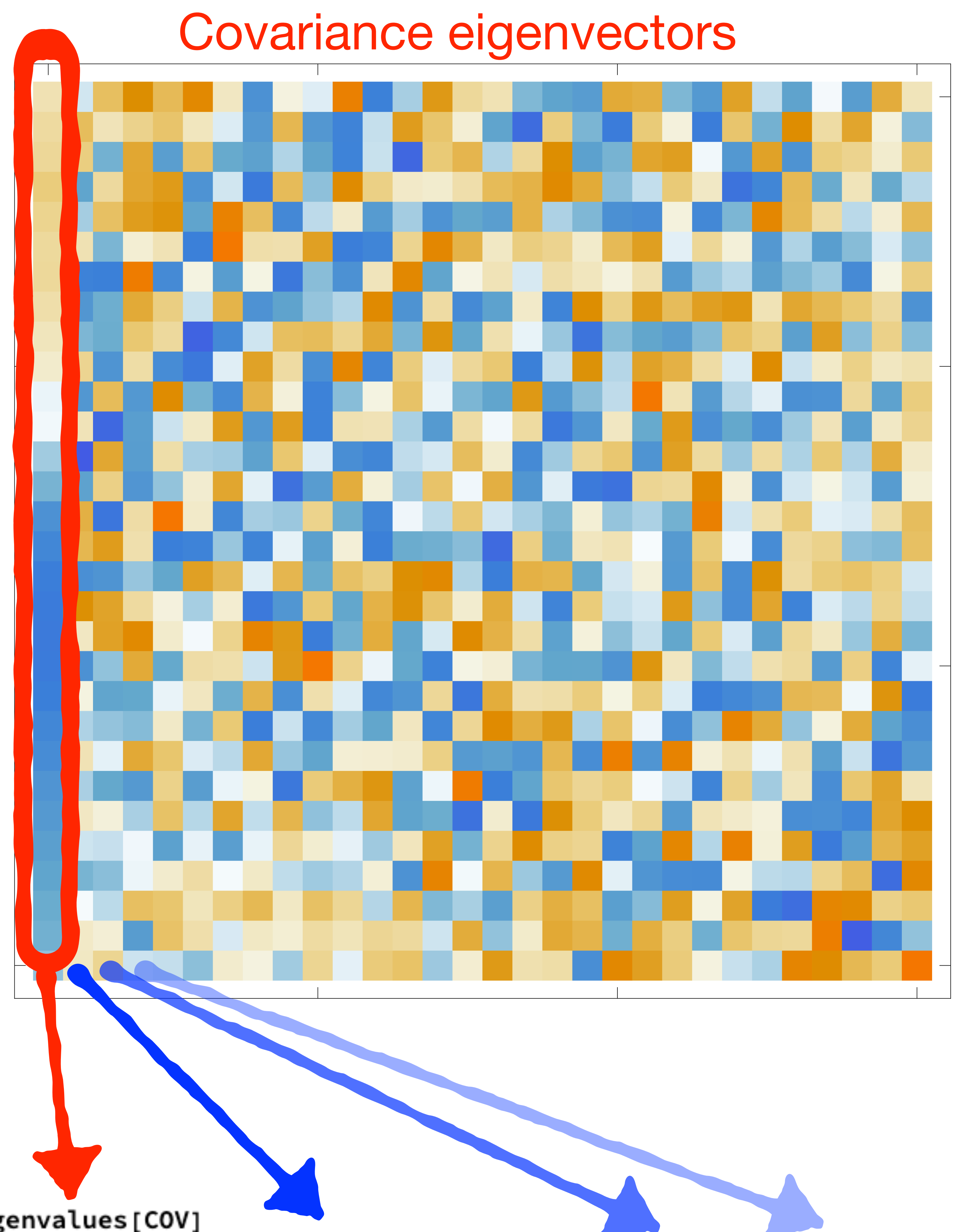
In pure QCD,
the distortion is independent of the QCD approximation or scale choice

The process can be factorized in production (with QCD effects)
times propagation and decay of the W boson.
The sensitivity to m_W stems from the propagation and decay part

The sensitivity to m_W is independent of the QCD approximation
The central value and the uncertainty on m_W instead do depend
on the QCD approximation

Where is the sensitivity to m_W ? Which bins are the most relevant?
The study of the covariance matrix for m_W variations shows that one specific combination of bins carries the bulk of the sensitivity to m_W $\rightarrow$ following this indication, we design a new observable

Sensitivity to the W boson mass: covariance with respect to m_W variations

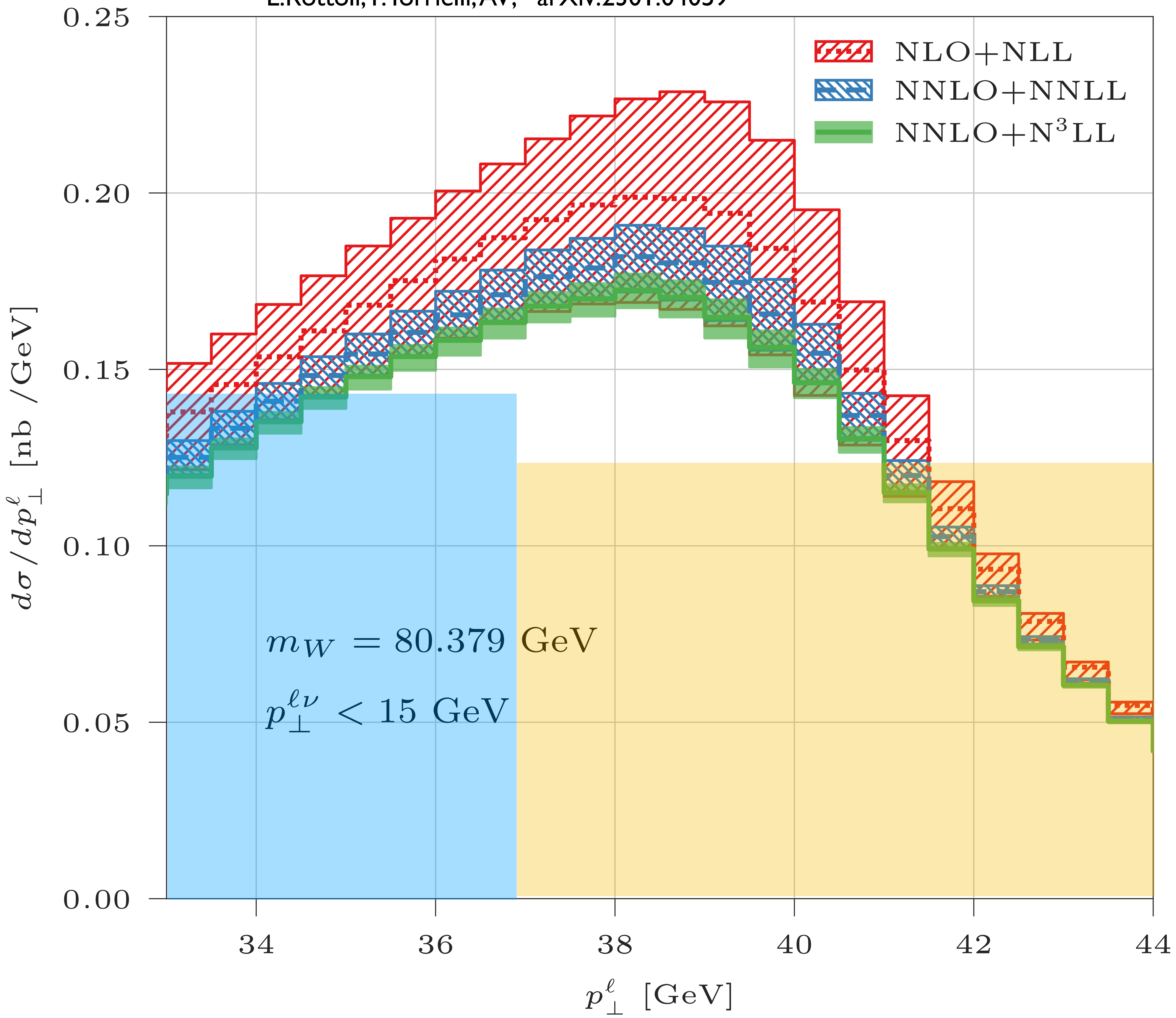



```
Eigenvalues[COV]
2.287303503 × 10^-6, 4.048282895 × 10^-9, 3.950574116 × 10^-9, 3.425317453 × 1(
2.048617953 × 10^-9, 1.474329242 × 10^-9, 1.356322185 × 10^-9, 1.148439835 × 1(
6.887799557 × 10^-10, 6.470166478 × 10^-10, 3.826204032 × 10^-10, 2.778581721 ›
8.617960844 × 10^-11, 4.87985866 × 10^-11, 2.177142745 × 10^-17, -1.057060661 ›
-5.501244758 × 10^-18, -3.029585825 × 10^-18, 1.818871791 × 10^-18, 1.21252664
```

- The $p_\perp^\ell$ spectrum includes N bins.
- After the rotation which diagonalises the m_W covariance, we have N linear combinations of the primary bins.
- The combination associated to the (by far) largest eigenvalue exhibits a very clear and simple pattern
- The point where the coefficients change sign is very stable at different orders in QCD and with different bin ranges and it is found at $p_\perp^\ell \sim 37$ GeV

The jacobian asymmetry $\mathscr{A}_{p_\perp^\ell}$

L.Rottoli, P.Torrielli, AV; arXiv:2301.04059



$$L_{p_\perp^\ell} \equiv \int_{p_\perp^{\ell,\mathrm{min}}}^{p_\perp^{\ell,\mathrm{mid}}} dp_\perp^\ell \frac{d\sigma}{dp_\perp^\ell}, \qquad U_{p_\perp^\ell} \equiv \int_{p_\perp^{\ell,\mathrm{mid}}}^{p_\perp^{\ell,\mathrm{max}}} dp_\perp^\ell \frac{d\sigma}{dp_\perp^\ell}$$

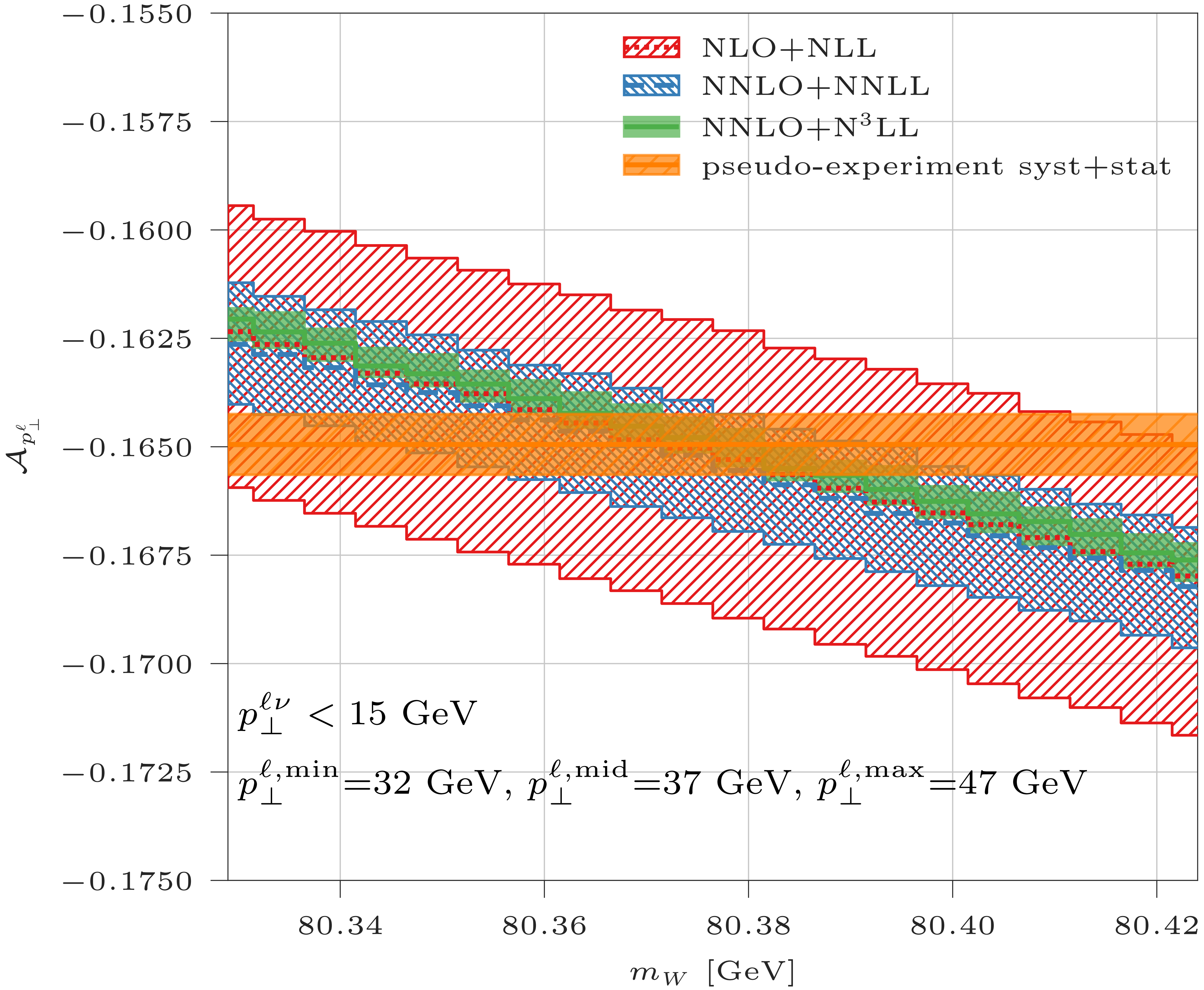
$$\mathcal{A}_{p_\perp^\ell}(p_\perp^{\ell,\mathrm{min}}, p_\perp^{\ell,\mathrm{mid}}, p_\perp^{\ell,\mathrm{max}}) \equiv \frac{L_{p_\perp^\ell} - U_{p_\perp^\ell}}{L_{p_\perp^\ell} + U_{p_\perp^\ell}}$$

The asymmetry is an observable (i.e. it is measurable via counting): its value is one single scalar number
It depends only on the edges of the two defining bins

Increasing m_W shifts the position of the peak to the right $\rightarrow$ Events migrate from the blue to the orange bin
$\rightarrow$ The asymmetry decreases

The jacobian asymmetry $\mathscr{A}_{p_\perp^\ell}$ as a function of m_W

L.Rottoli, P.Torrielli, AV; arXiv:2301.04059



The asymmetry $\mathscr{A}_{p_\perp}$ has a linear dependence on m_W, stemming from the linear dependence on the end-point position

The slope of the asymmetry expresses the sensitivity to m_W, in a given setup $(p_\perp^{\ell,min}, p_\perp^{\ell,mid}, p_\perp^{\ell,max})$

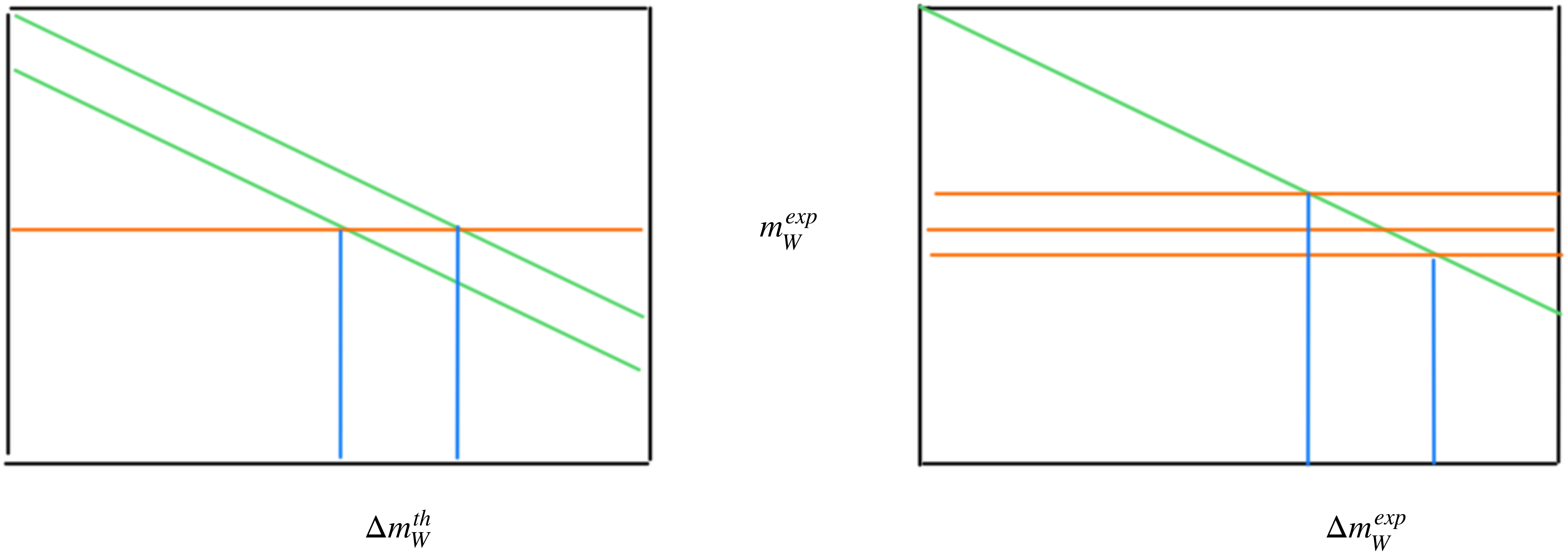
The slope is the same with every QCD approximation (factorization of QCD effects, perturbative and non-perturbative)

The "large" size of the two bins $\mathcal{O}(5-10)$ GeV leads to
- small statistical errors
- excellent stability of the QCD results (inclusive quantity)
- ease to unfold the data to particle level (m_W combination)

The experimental value and the theoretical predictions can be directly compared (m_W from the intersection of two lines)

The main systematics on the two fiducial cross sections is related to the lepton momentum scale resolution

Reading the uncertainties on m_W

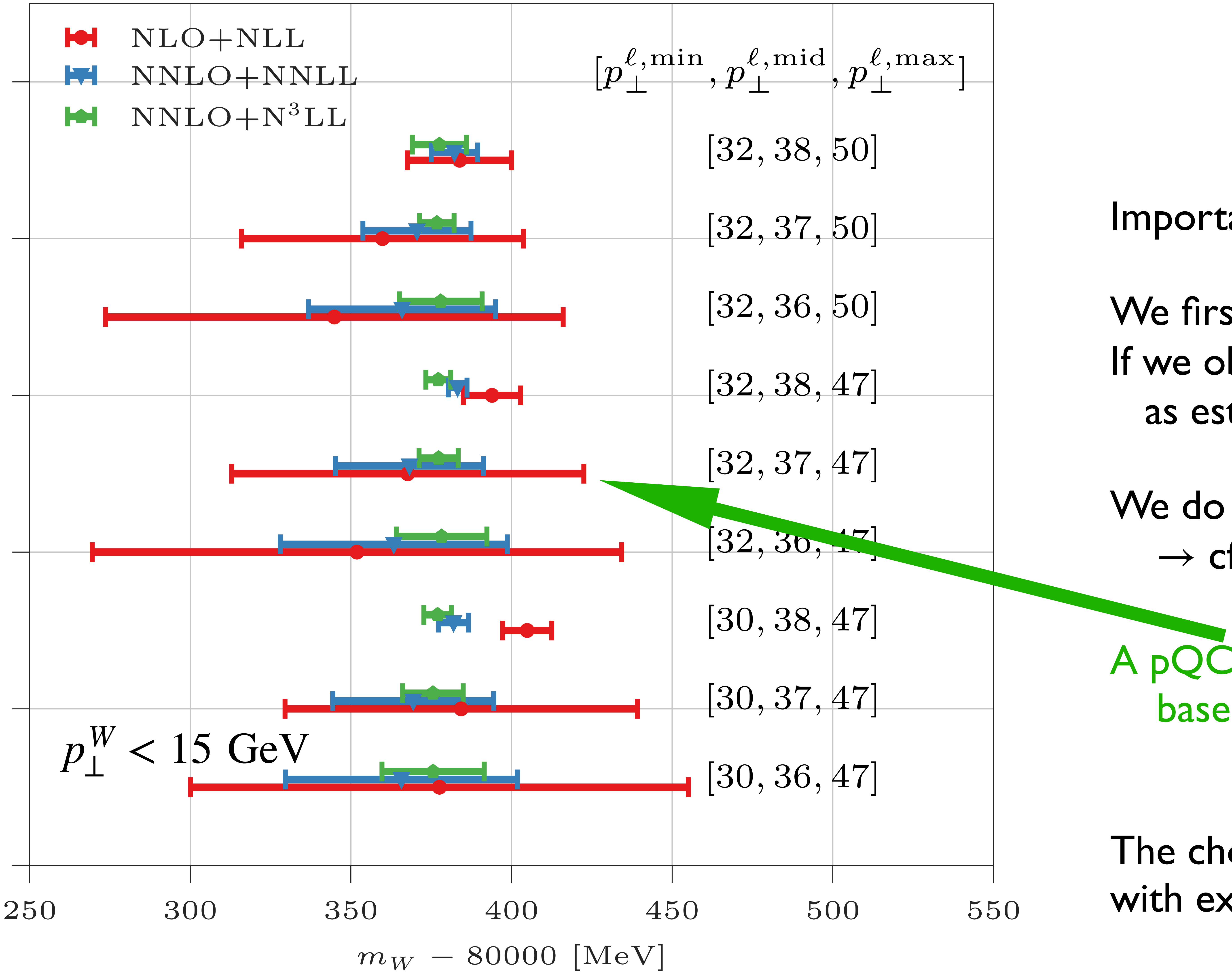


90

m_W determination at the LHC as a function of the $\mathscr{A}_{p_\perp^\ell}$ parameters (low pile-up setup)

as pseudo-experimental value we choose the NNLO+N3LL result with $m_W = 80.379$

L.Rottoli, P.Torrielli, AV; arXiv:2301.04059



Important role of the N3LL corrections

We first check the convergence order-by-order.
If we observe it, then we take the size of the m_W interval as estimator of the residual pQCD uncertainty

We do not trust the scale variations alone
→ cfr the choice with $p_\perp^{\ell,mid} = 38$ GeV

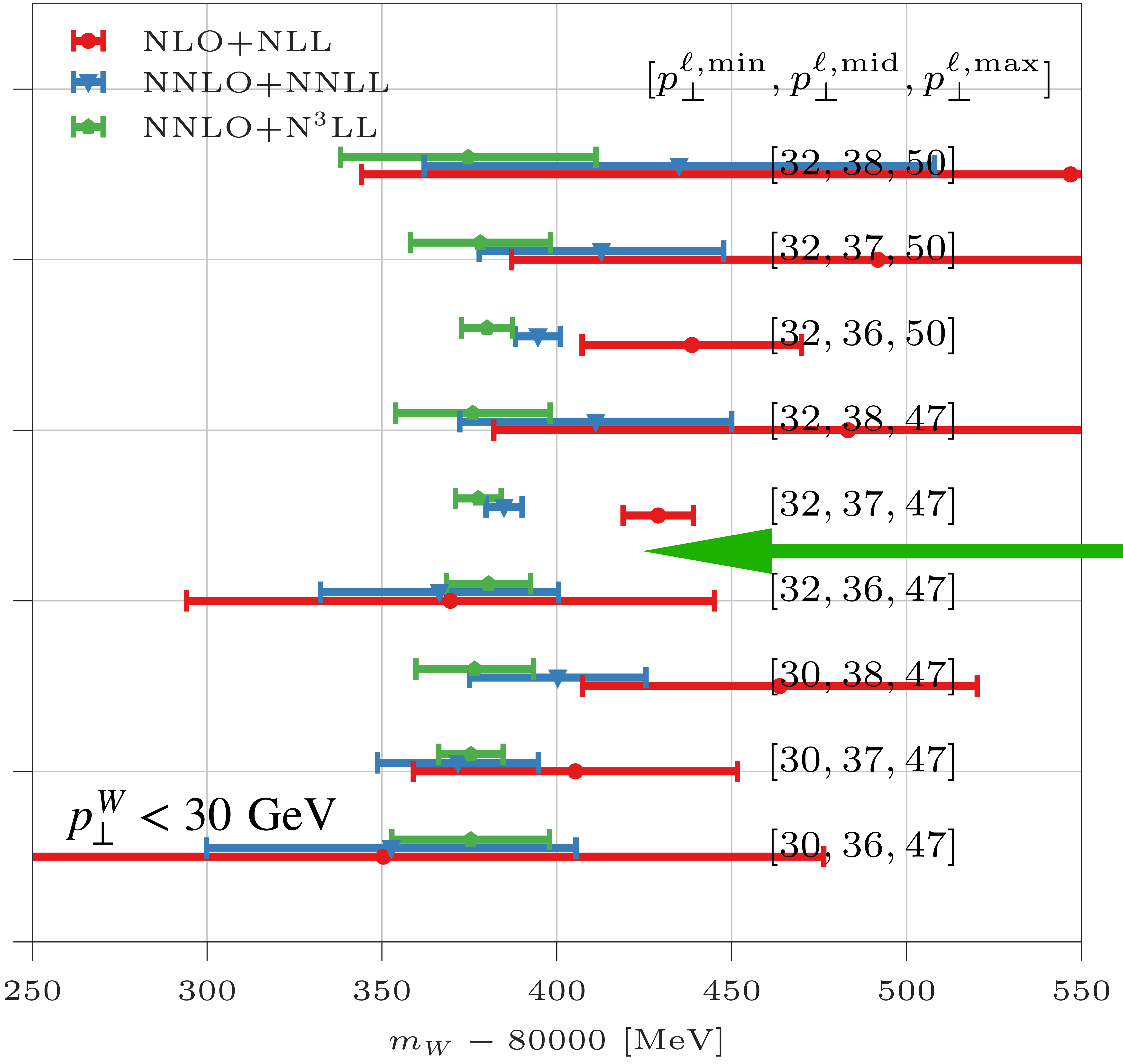
A pQCD uncertainty at the ± 5 MeV level is achievable based on CCDY data alone

The choice of the midpoint is important to identify two regions with excellent QCD convergence

m_W determination at the LHC as a function of the $\mathscr{A}_{p_\perp^\ell}$ parameters (high pile-up setup)

as pseudo-experimental value we choose the NNLO+N3LL result with $m_W = 80.379$

L.Rottoli, P.Torrielli, AV; arXiv:2301.04059



Clear impact of the acceptance cut on $p_\perp^W$

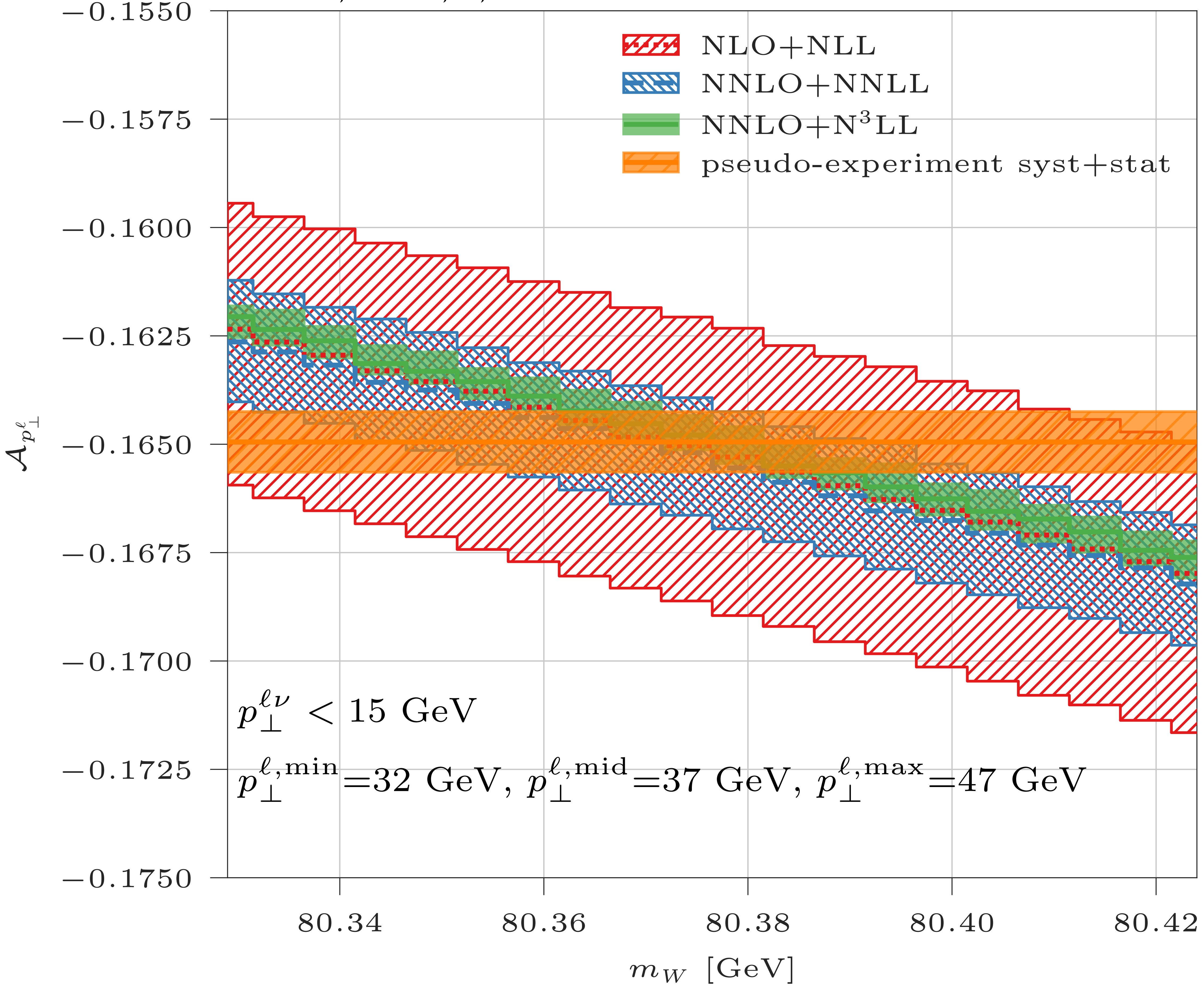
Important role of the N3LL corrections

A pQCD uncertainty below ± 10 MeV level is achievable based on CCDY data alone

The choice of the midpoint is important to identify two regions with excellent QCD convergence

What's missing?

L.Rottoli, P.Torrielli, AV;   arXiv:2301.04059



The excellent convergence in pQCD of the asymmetry $\mathscr{A}_{p_\perp}$ is the best possible starting point to discuss

- the impact on the central m_W value of
  - missing perturbative corrections (QED, QCDxEW)
  - non-perturbative effects

  → each effect yields a vertical offset of $\mathscr{A}_{p_\perp^\ell} \to m_W$ shift
  QED corrections might also change the slope
  (preliminary studies show mild QED effects)

  → the non-perturbative effects are a refinement of the study
  - impact on top of NNLO+N3LL is expected moderate
  - not a crucial element (as in the template fit case)

- the propagation of the uncertainties

  → the linearity of the dependence on m_W allows
  an easy propagation of each uncertainty source

The asymmetry in pure pQCD is just one component of the $p_\perp^\ell$ spectrum
→ additional measurements are needed, to achieve an accurate description of the data