Tree-level Triple-collinear Splittings with Massive Quarks

Prasanna Kumar Dhani

based on

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in collaboration with Germán Rodrigo and German Sborlini

Advanced School & Workshop on Multiloop Scattering Amplitudes, NISER-Bhubaneswar

15-19 January 2024

Marie Skłodowska-Curie Actions

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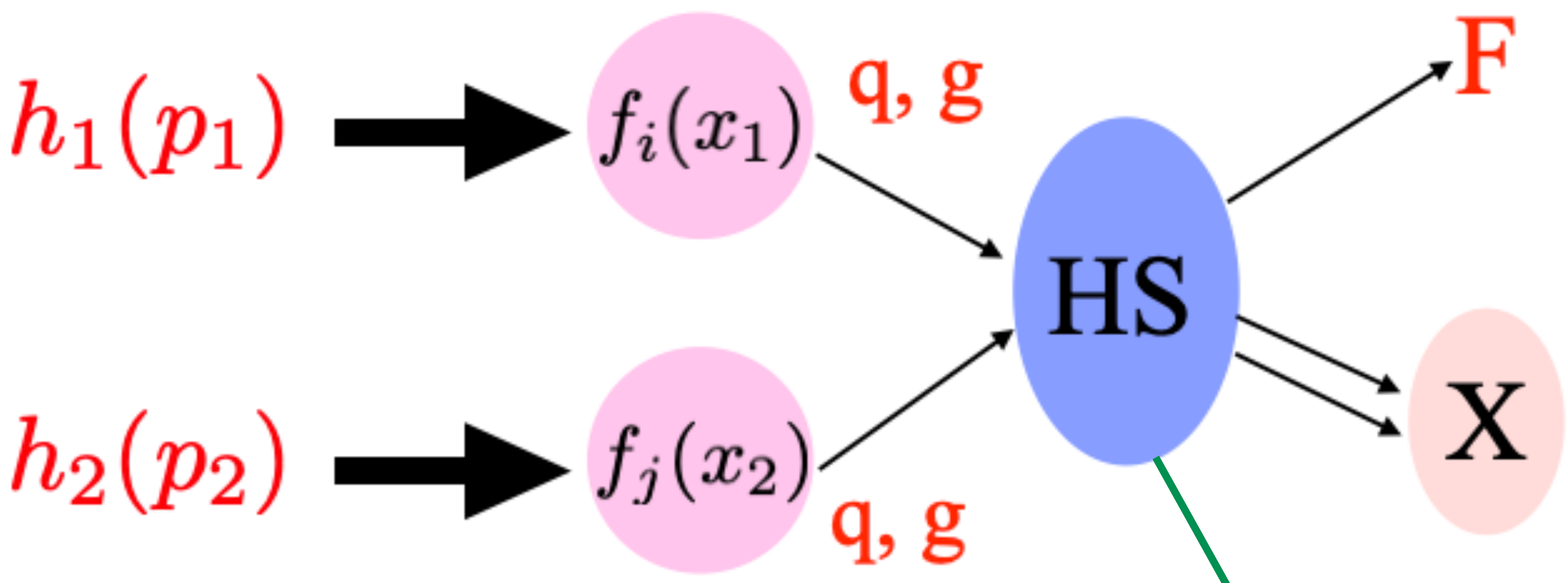
Parton Model: Short Range Factorizes from the Long Range Physics

- For a generic hadro-production process



$$\hat{\sigma}_{ij} = \hat{\sigma}_{ij}^{(0)} + \alpha_S \hat{\sigma}_{ij}^{(1)} + \ldots$$

Long distance

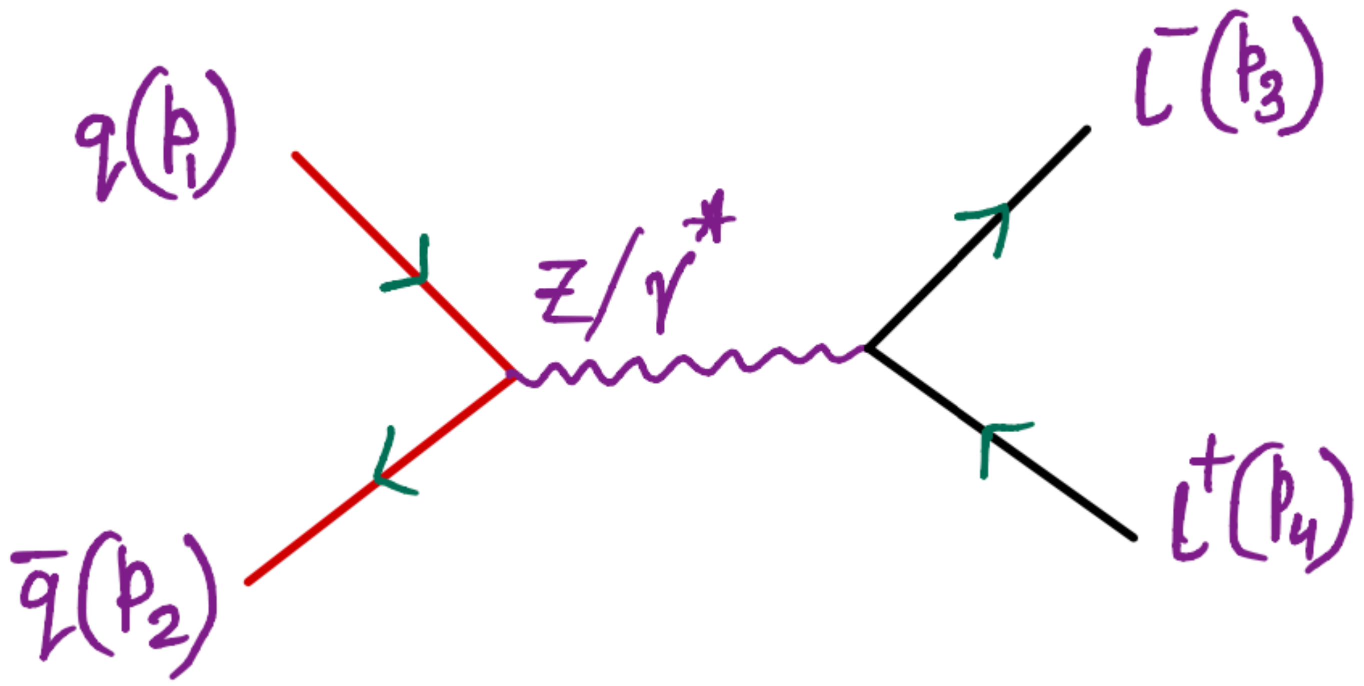
$$\sigma_{h_1,h_2} = \sum_{i,j} \int dx_1 dx_2 f_i(x_1) f_j(x_2) \hat{\sigma}_{ij} + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{Q}\right)$$

Short distance

FUNDAMENTAL !

Partonic Cross Section: Drell-Yan Process

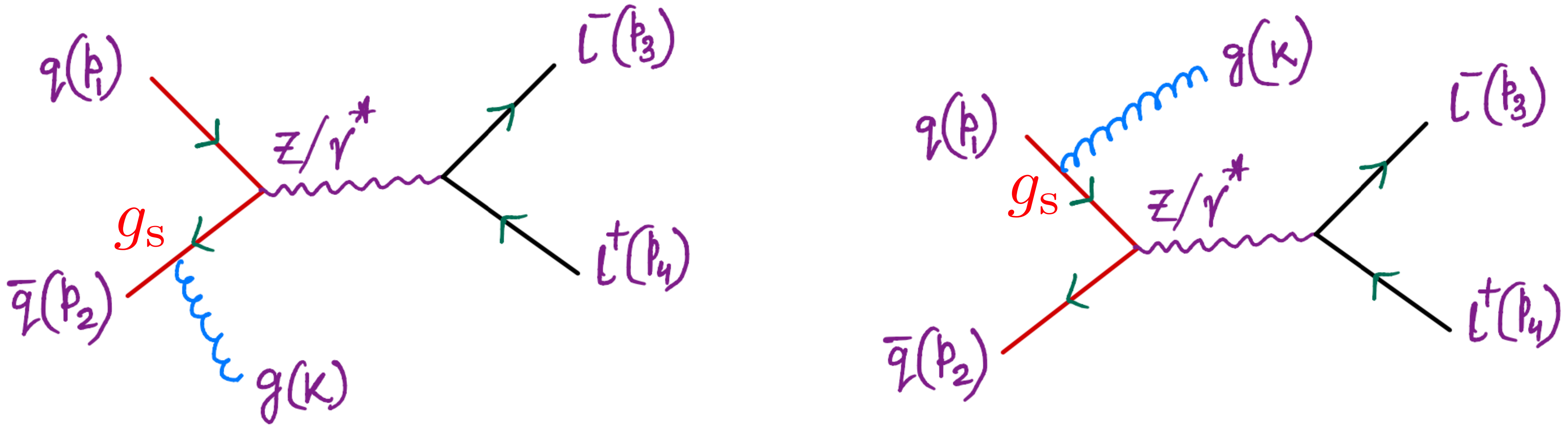
Leading order



$$\hat{\sigma} = \hat{\sigma}_0 \quad + \quad \alpha_s \hat{\sigma}_1 \quad + \quad \alpha_s^2 \hat{\sigma}_2 \quad + \quad \mathcal{O}(\alpha_s^3)$$

Next-to-leading order

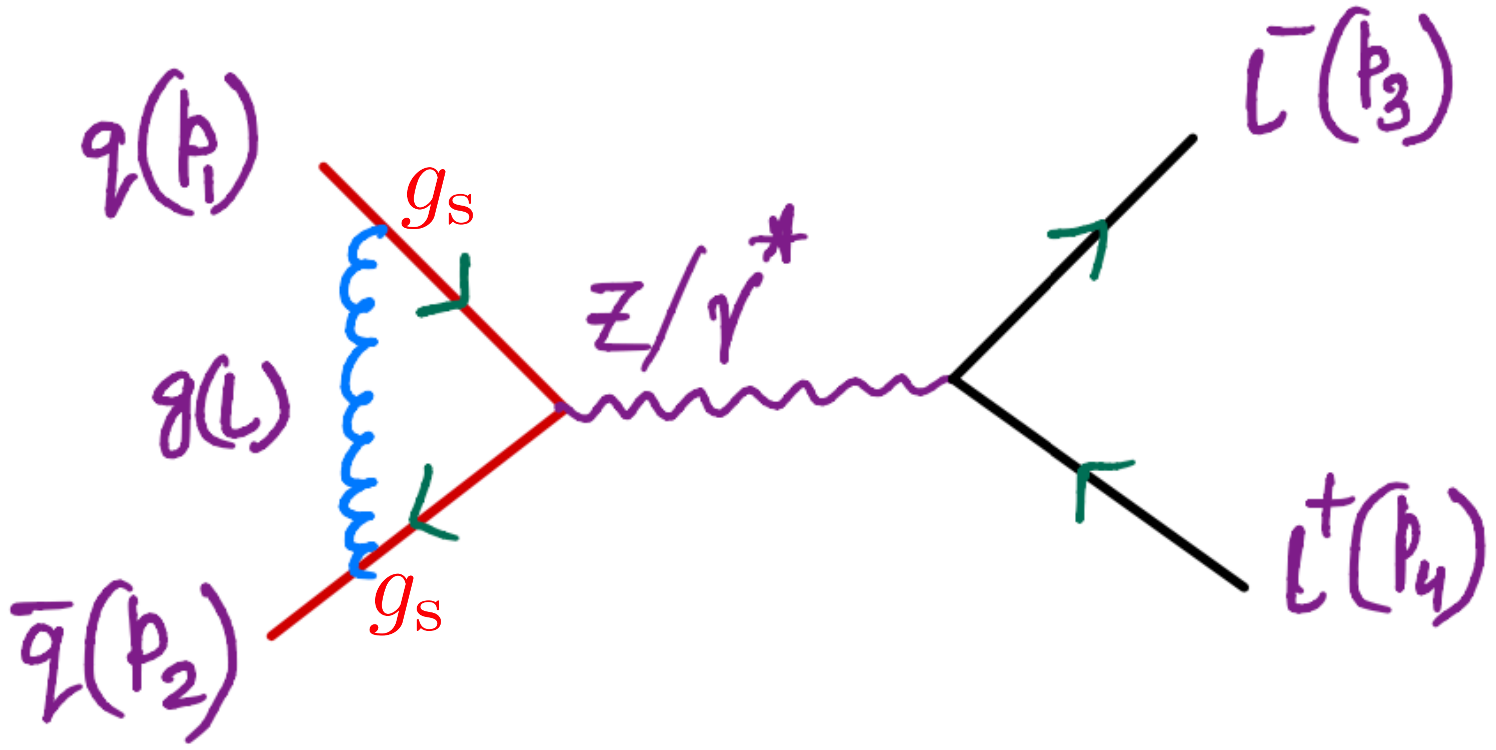
Real radiation Feynman diagrams



$k \to 0$ ⟶ Soft divergence

$k \parallel p_1$ or $k \parallel p_2$ ⟶ Collinear divergence

Virtual loop Feynman diagrams



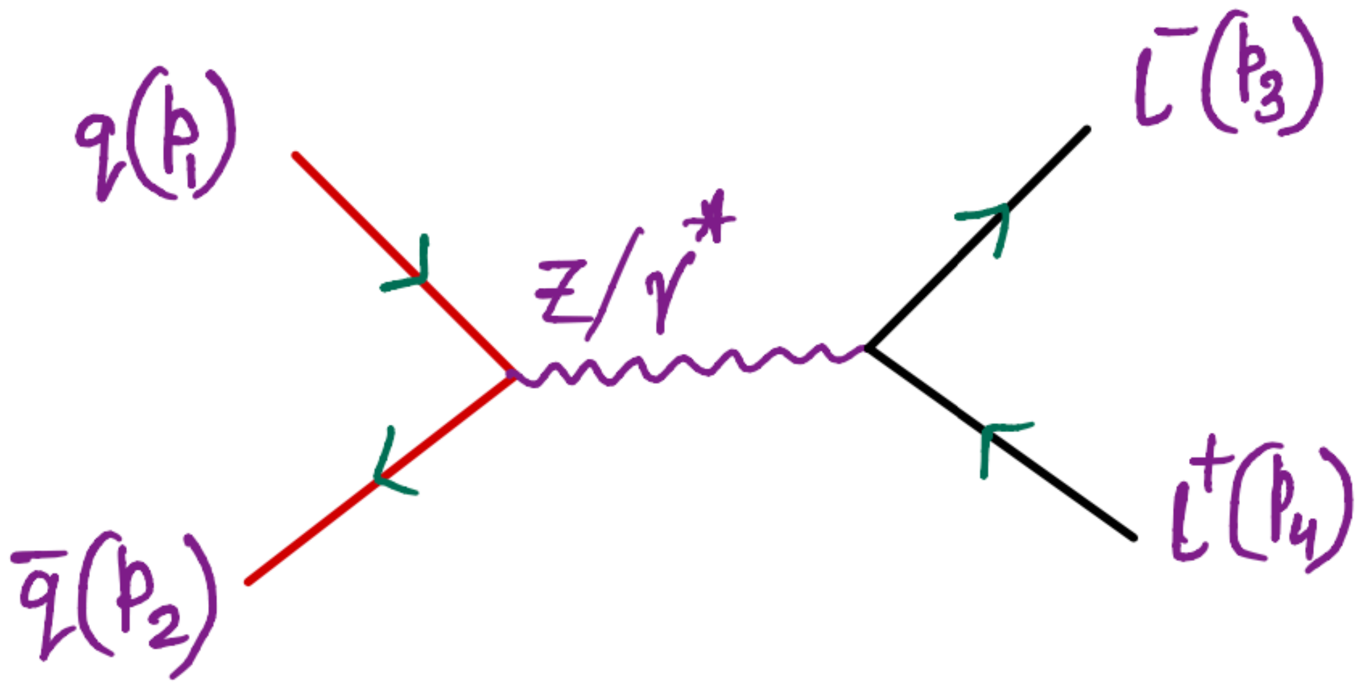
$l \to 0$ ⟶ Soft divergence

$l \parallel p_1$ or $l \parallel p_2$ ⟶ Collinear divergence

$l \to \infty$ ⟶ Ultraviolet divergence

Partonic Cross Section: Drell-Yan Process

Leading order



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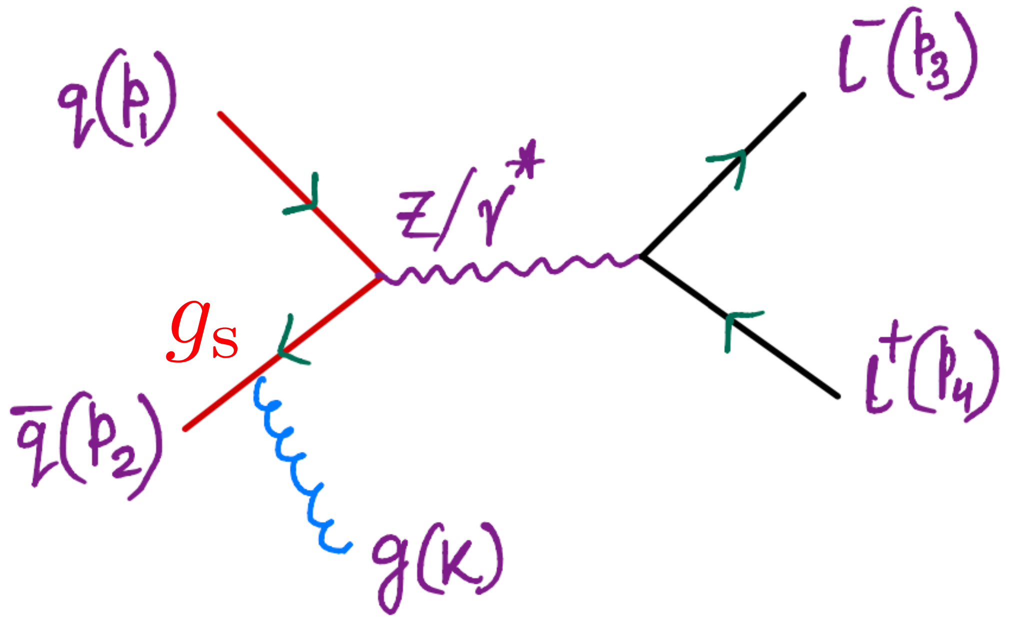
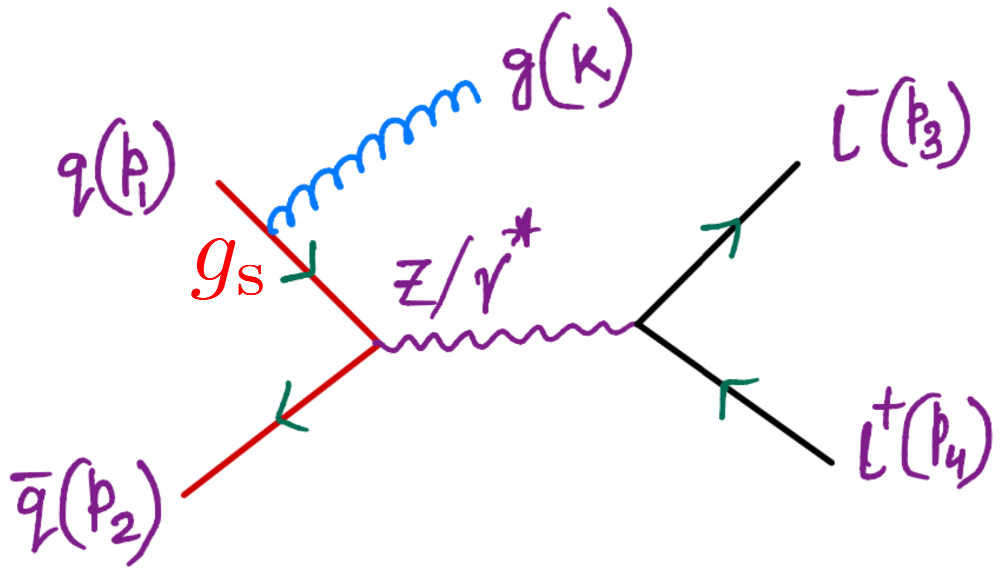
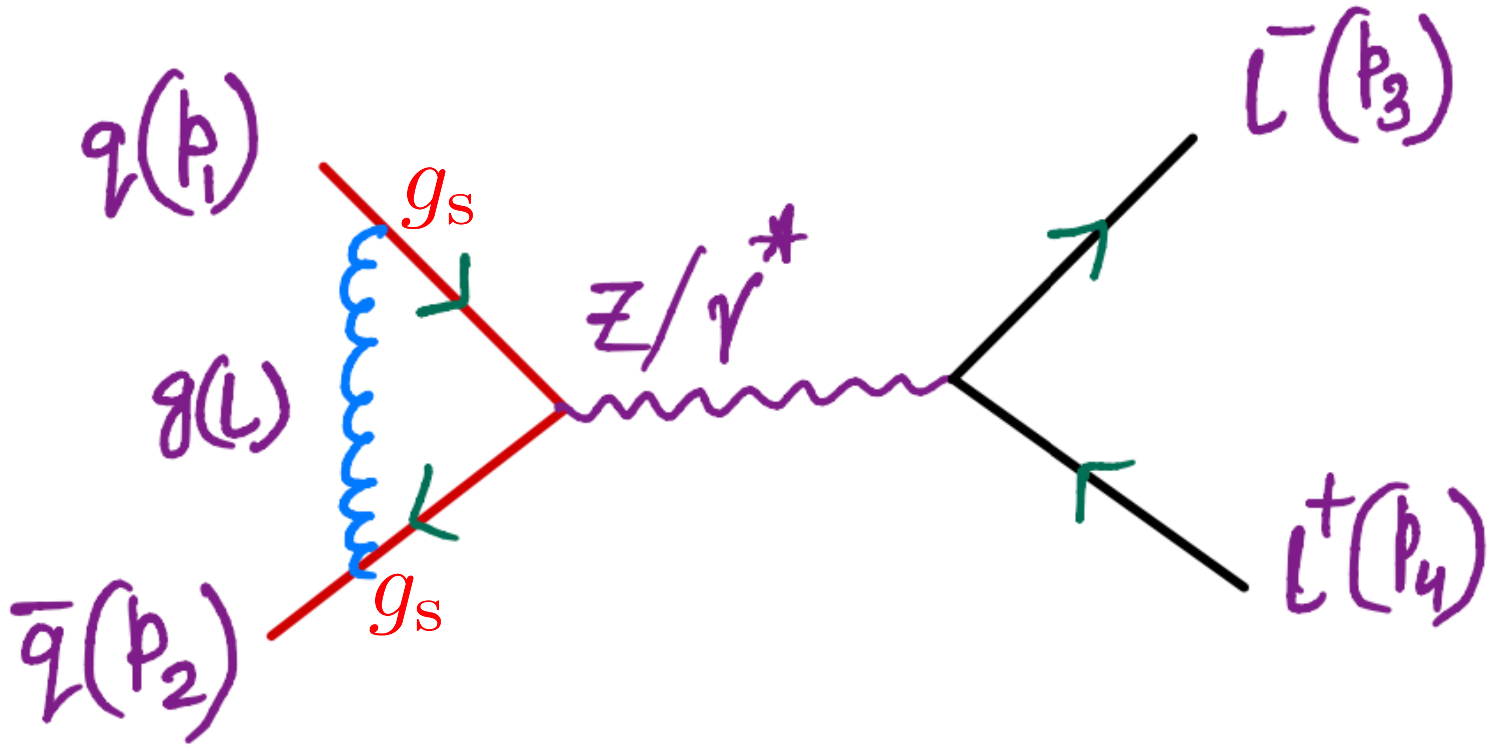




Virtual loop Feynman diagrams



$k \to 0$ → Soft divergence

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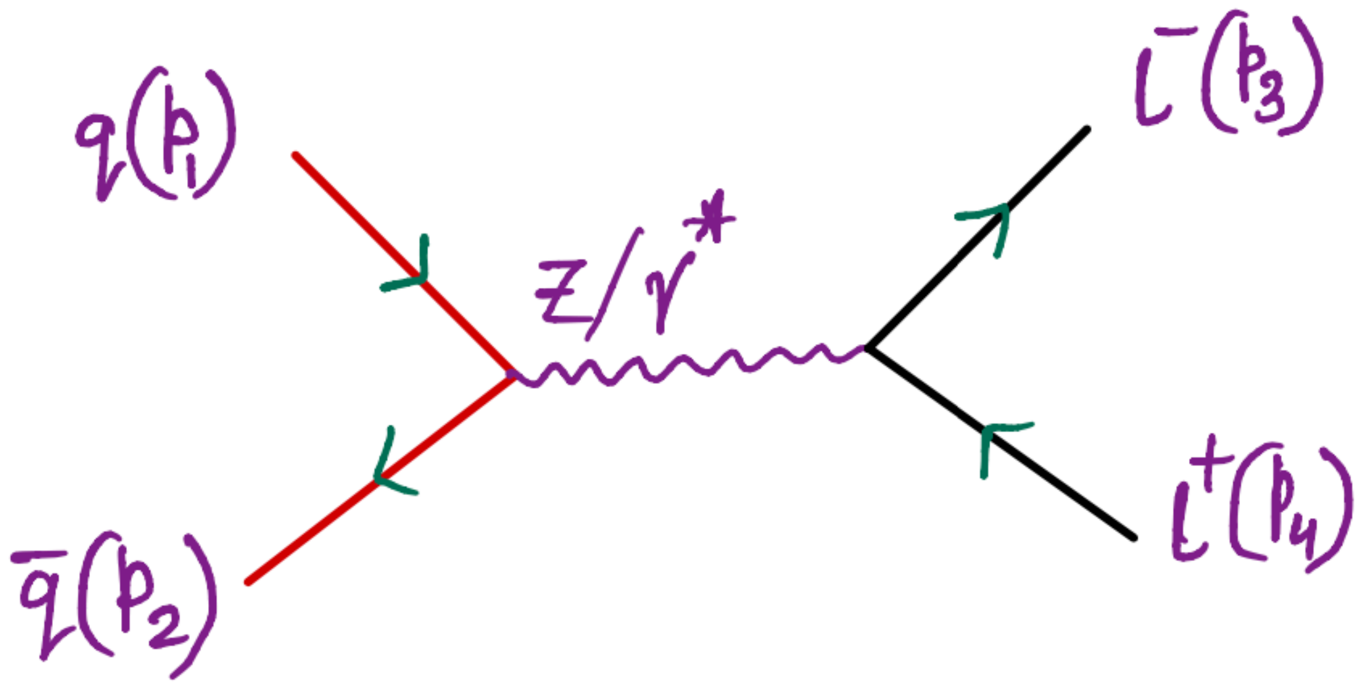
Divergent Terms Cancel!!!

Partonic Cross Section: Drell-Yan Process

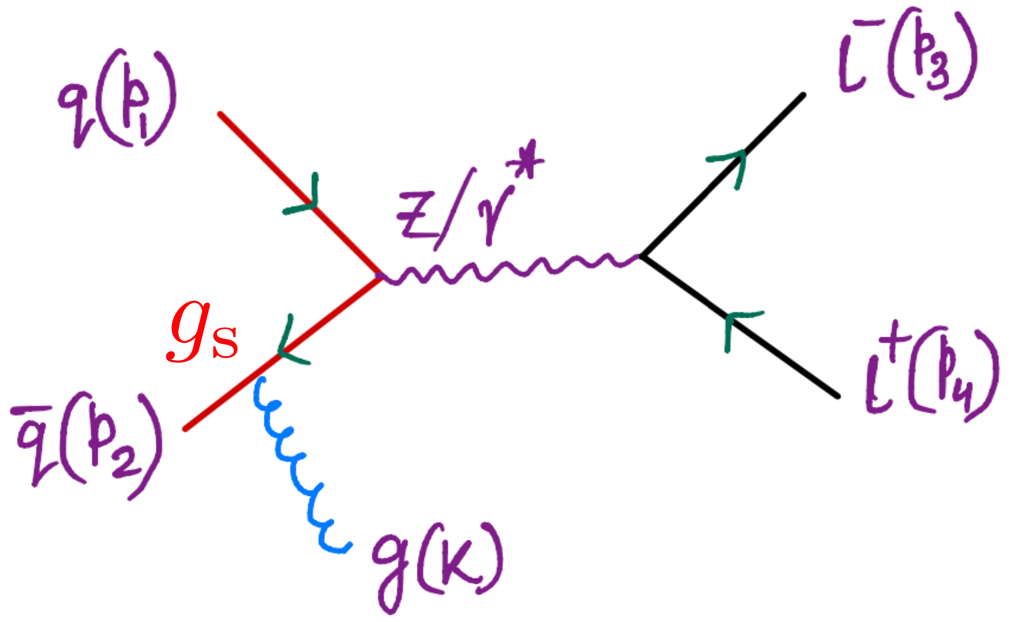
Leading order

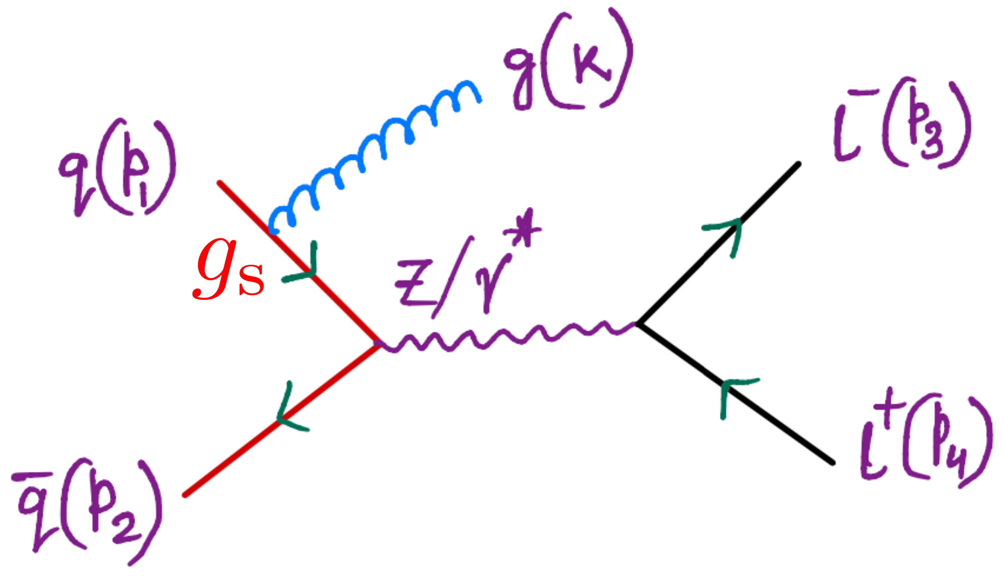


$$\hat{\sigma} = \hat{\sigma}_0 \quad + \quad \alpha_s \hat{\sigma}_1 \quad + \quad \alpha_s^2 \hat{\sigma}_2 \quad + \quad \mathcal{O}(\alpha_s^3)$$

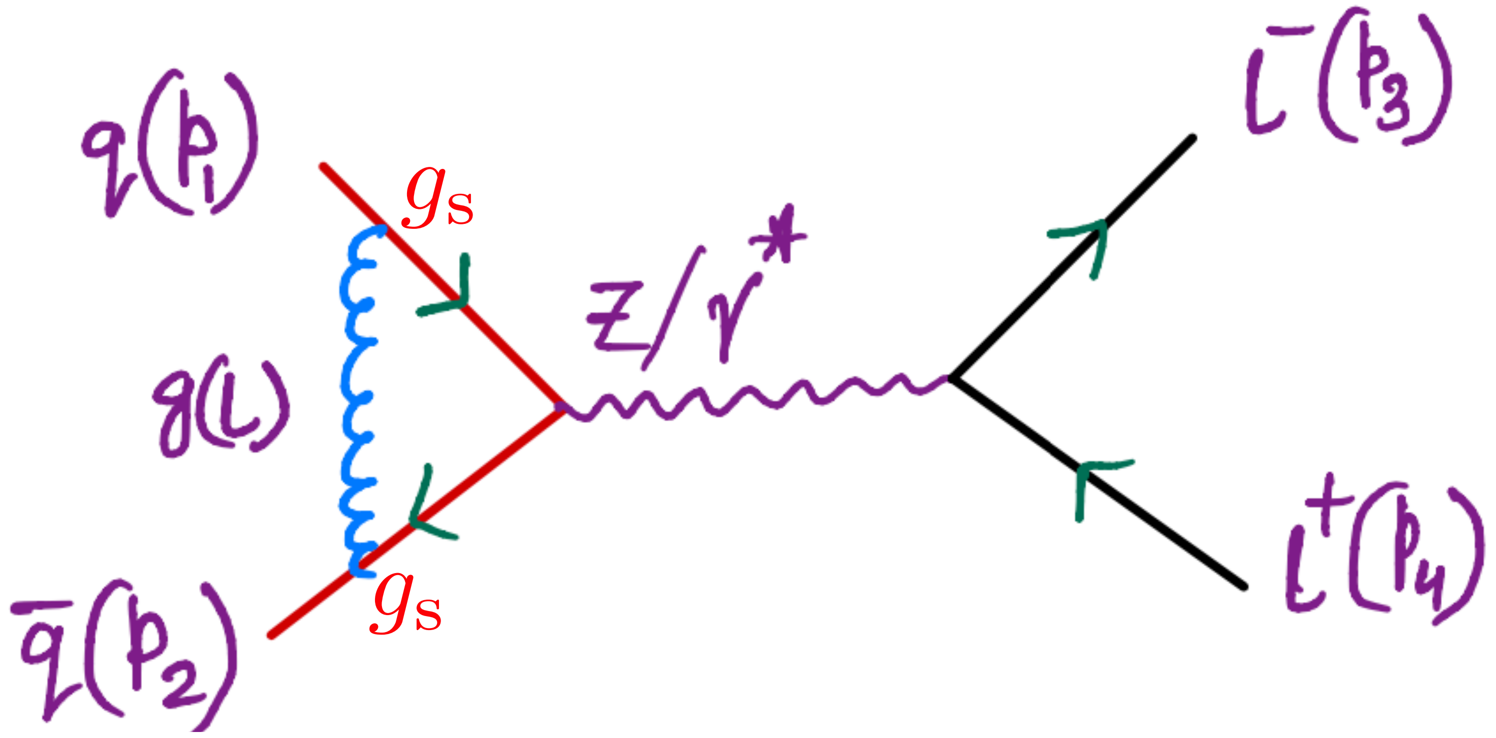
Next-to-leading order

Real radiation Feynman diagrams





Virtual loop Feynman diagrams



Identified partons → Collinear divergence remains! → Renormalise bare PDFs in the initial state and FFs in the final state

↘ Process independent

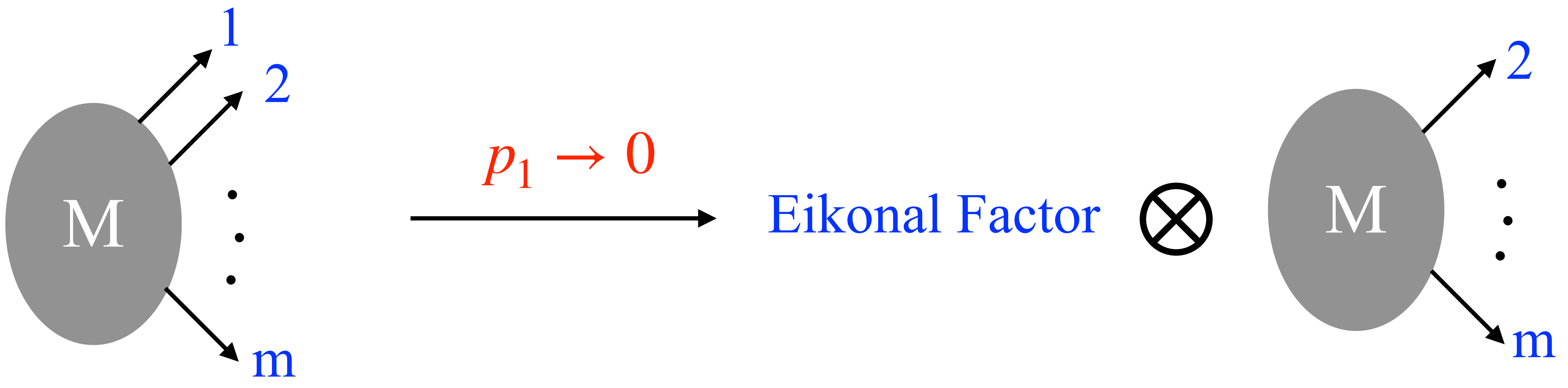
Universal Factorization

* Scattering amplitudes exhibit interesting properties in the soft and collinear limits

[Collins, Soper, Sterman (1989)]

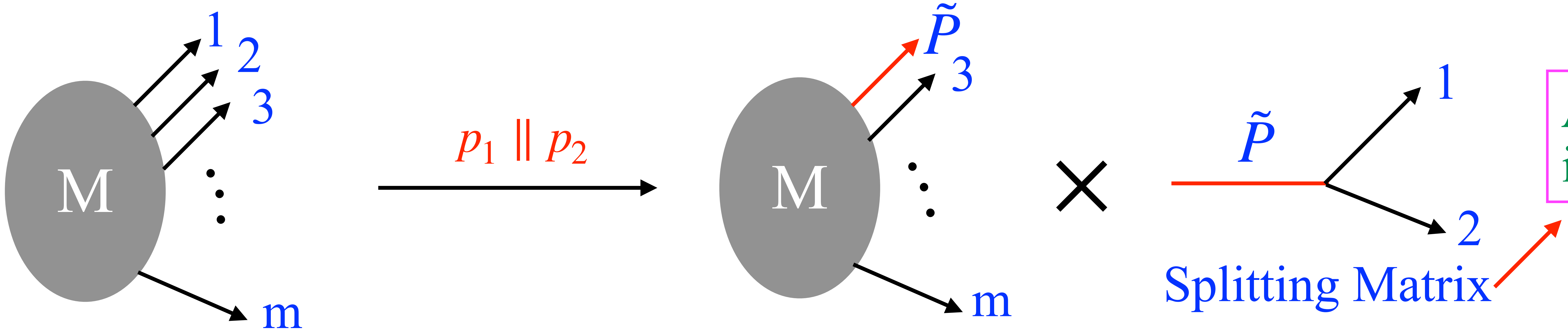
Reduction of kinematic variables (simplification), factorization, recursion properties, universal divergent behaviour, etc.

(Massless) Particles emitted with zero energy → Soft limit

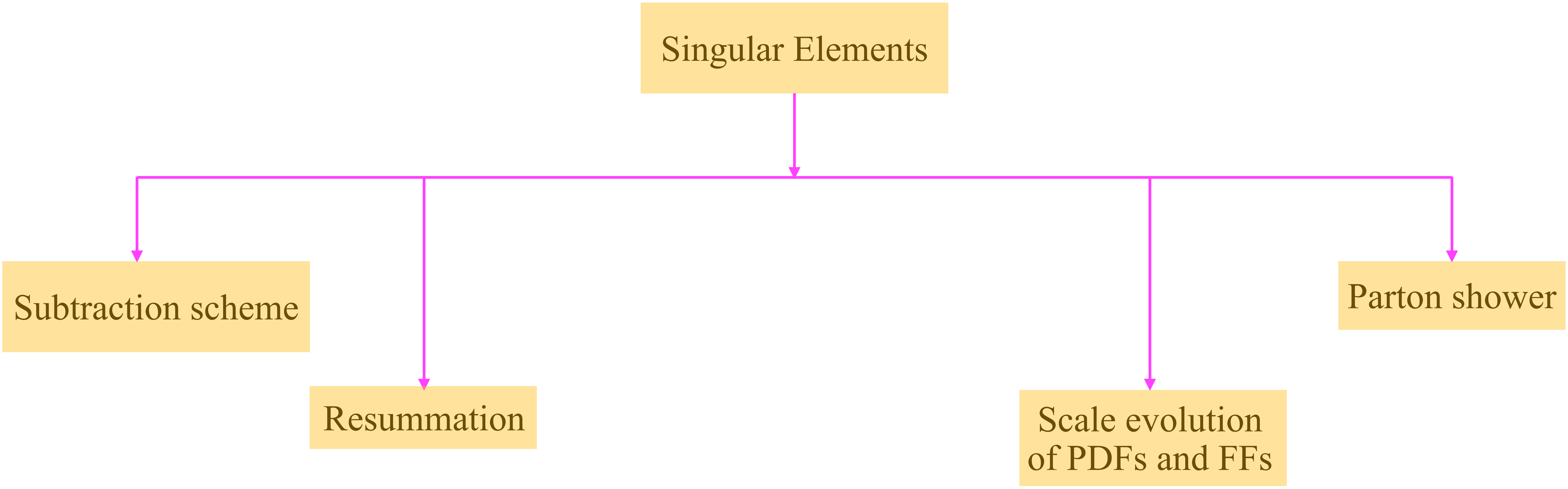


Collinear factorization is more universal…

(Massless) Particles emitted in the same direction → Collinear limit



At the squared amplitude level, it is called Splitting Kernel



- For massless case, both Soft Currents and Collinear Splitting Kernels are known to N3LO
- For massive case, Soft Currents are known to NNLO in perturbation theory
- In the case of splitting processes involving massive quarks, only Double Collinear splittings at tree-level are known [Keller '98, Catani '00]
- To control dominant mass effects such as $\ln^n\left(Q^2/m^2\right)$, it is important to calculate triple-collinear splittings at tree-level and double-collinear splittings at 1-loop level.

Outline

* Massive Splitting Processes
    - Quasi-collinear Limit
    - Double Collinear Splittings
* Triple Collinear Splitting Processes at Tree-level
    - Splitting Matrices
    - Splitting Kernels
* Summary & Future Outlook

Notations

[Catani '99, Catani '02, Catani '11, Czakon '22]

Define an on-shell momentum

$$\widetilde{P}^\mu = p^\mu_{1\ldots m} - \frac{p^2_{1\ldots m} - m^2_{1\ldots m}}{2\, n\cdot p_{1\ldots m}} n^\mu$$ such that $\widetilde{P}^2 = m^2_{1\ldots m}$

Sudakov momenta parametrisation

$$p_i^\mu = x_i \widetilde{P}^\mu + k^\mu_{\perp i} - \frac{k^2_{\perp i} + x_i^2 m^2_{1\ldots m} - m_i^2}{x_i} \frac{n^\mu}{2\, n\cdot \widetilde{P}}, \qquad i \in C \equiv \{1, \ldots, m\}$$

Boost invariant variables

$$z_i = \frac{x_i}{\sum_{j=1}^m x_j} \quad \text{with} \quad \sum_{i=1}^m z_i = 1$$

$$\tilde{k}_i^\mu = k^\mu_{\perp i} - z_i \sum_{j=1}^m k^\mu_{\perp j} \quad \text{with} \quad \sum_{i=1}^m \tilde{k}_i^\mu = 0$$

Quasi-collinear Limit

Lorenz invariants

$$\tilde{s}_{ij} \equiv 2p_i \cdot p_j = z_i z_j \left[-\left(\frac{\tilde{k}_j}{z_j} - \frac{\tilde{k}_i}{z_i} \right)^2 + \frac{m_i^2}{z_i^2} + \frac{m_j^2}{z_j^2} \right]$$

$$s_{ij} \equiv (p_i + p_j)^2 = -z_i z_j \left(\frac{\tilde{k}_j}{z_j} - \frac{\tilde{k}_i}{z_i} \right)^2 + (z_i + z_j) \left(\frac{m_i^2}{z_i} + \frac{m_j^2}{z_j} \right)$$

Quasi-collinear limit is defined by

$$m_i \to \rho\, m_i\,, \qquad k_{\perp i} \to \rho\, k_{\perp i}\,, \qquad m_{1\ldots m} \to \rho\, m_{1\ldots m} \quad \text{with} \quad \rho \to 0$$

[Keller ’98, Catani ’00]

Quasi-collinear Limit

[Keller ’98, Catani ’00]

Amplitude level factorisation

Splitting matrices → Reduced ME

$$|\mathcal{M}^{(0)}(p_1,\dots,p_m,\dots,p_n)\rangle \simeq \boldsymbol{Sp}^{(0)}_{a_1\dots a_m}(p_1,\dots,p_m;\widetilde{P})\,|\overline{\mathcal{M}}^{(0)}(\widetilde{P},p_{m+1},\dots,p_n)\rangle$$

$$Sp^{(0)(c_1,\dots,c_m;c)}_{a_1\dots a_m}\left(p_1,\dots,p_m;\widetilde{P}\right) \equiv \langle c_1,\dots,c_m|\boldsymbol{Sp}^{(0)}_{a_1\dots a_m}\left(p_1,\dots,p_m;\widetilde{P}\right)|c\rangle$$

Splitting kernel

$$\hat{P}^{(0),\,\mu\nu}_{a_1\dots a_m} \equiv \left(\frac{s_{1\dots m}-m^2_{1\dots m}}{8\pi\mu_0^{2\epsilon}\alpha_S}\right)^{m-1}\left[\boldsymbol{Sp}^{(0),\nu}_{a_1\dots a_m}(p_1,\dots,p_m;\widetilde{P})\right]^\dagger \boldsymbol{Sp}^{(0),\mu}_{a_1\dots a_m}(p_1,\dots,p_m;\widetilde{P})$$

Averaged over spin/polarisations of the parent parton

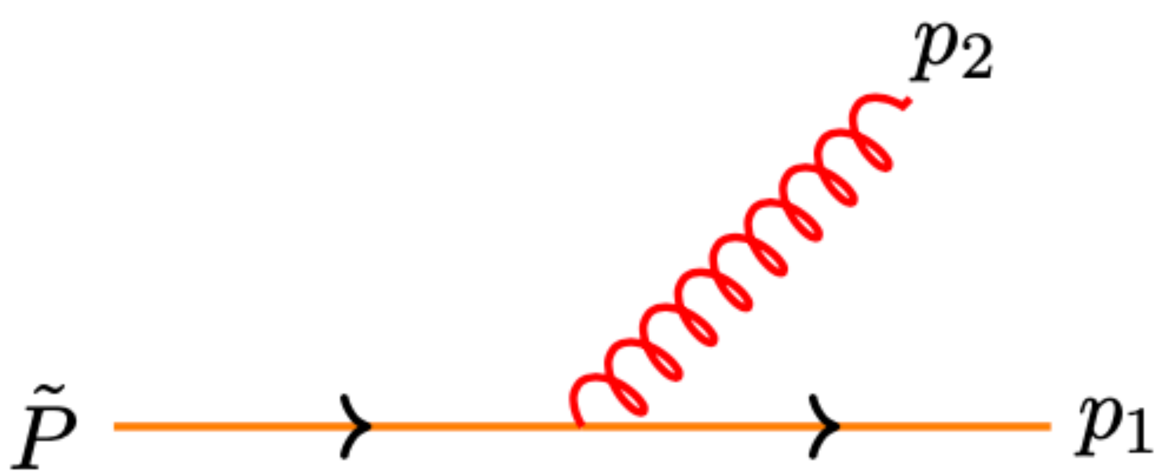
$$\langle\hat{P}^{(0)}_{a_1\cdots a_m}\rangle = \left(\frac{s_{1\dots m}-m^2_{1\dots m}}{8\pi\mu_0^{2\epsilon}\alpha_S}\right)^{m-1}\overline{|\boldsymbol{Sp}^{(0)}_{a_1\dots a_m}|^2}$$

$$d_{\mu\nu}(k,n) = -g_{\mu\nu} + \frac{k_\mu n_\nu + n_\mu k_\nu}{n\cdot k}$$

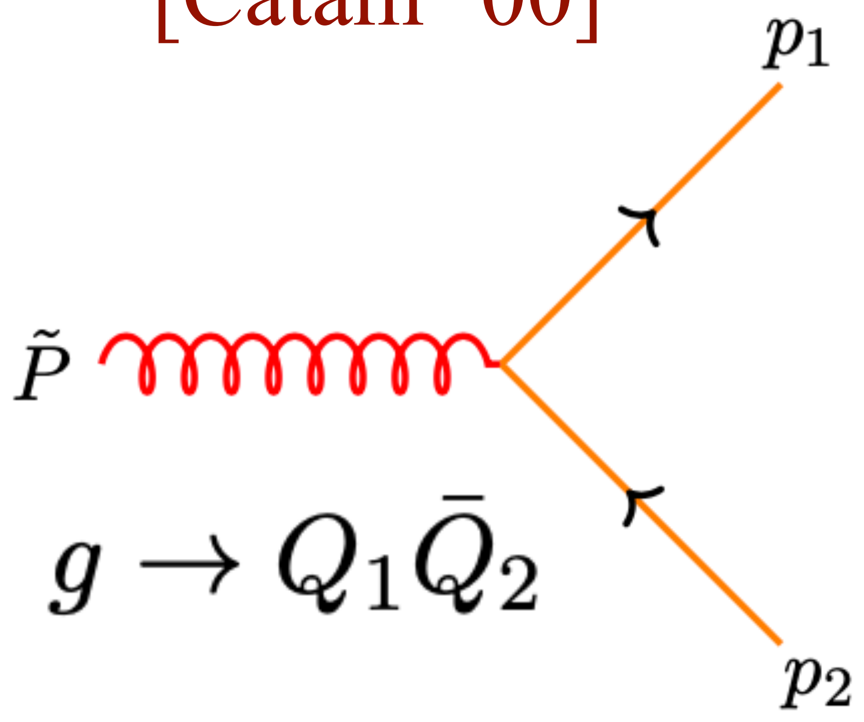
Quark: $\langle\hat{P}^{(0)}_{a_1\cdots a_m}\rangle = \frac{1}{2}\delta^{ss'}\,\hat{P}^{(0),\,ss'}_{a_1\dots a_m}$ Gluon: $\langle\hat{P}^{(0)}_{a_1\dots a_m}\rangle = \frac{d_{\mu\nu}(\widetilde{P};n)}{d-2}\hat{P}^{(0),\mu\nu}_{a_1\dots a_m}$

Tree-level Double Collinear Splittings

[Catani '00]



$Q \to Q_1 g_2$



$g \to Q_1 \bar{Q}_2$

$$\boldsymbol{Sp}^{(0)}_{Q_1 g_2} = g_{\rm s}\mu_0^{\epsilon}\boldsymbol{t}^{c_2}_{c_1 c}\frac{1}{\tilde{s}_{12}}\overline{u}(p_1)\not{\epsilon}(p_2)u(\widetilde{P})$$

$$\boldsymbol{Sp}^{(0)}_{Q_1 \bar{Q}_2} = g_{\rm s}\mu_0^{\epsilon}\boldsymbol{t}^{c}_{c_1 c_2}\frac{1}{s_{12}}\overline{u}(p_1)\not{\epsilon}^*(\widetilde{P})v(p_2)$$

$$\hat{P}^{(0),ss'}_{Q_1 g_2} = \delta^{ss'}C_F\left[\frac{1+z^2}{1-z} - \epsilon(1-z) - 2\frac{m_Q^2}{\tilde{s}_{12}}\right]$$

$$\hat{P}^{(0),\mu\nu}_{Q_1 \bar{Q}_2} = T_R\left[-g^{\mu\nu} - 4\frac{k_\perp^\mu k_\perp^\nu}{s_{12}}\right]$$

$$\langle \hat{P}^{(0)}_{Q_1 g_2}\rangle = C_F\left[\frac{1+z^2}{1-z} - \epsilon(1-z) - 2\frac{m_Q^2}{\tilde{s}_{12}}\right]$$

$m_Q \to 0$

Massless case

$$\langle \hat{P}^{(0)}_{Q_1 \bar{Q}_2}\rangle = T_R\left[1 - \frac{2}{1-\epsilon}\left(z(1-z) - \frac{m_Q^2}{s_{12}}\right)\right]$$

$m_Q \to 0$

Massless case

Splitting Matrices

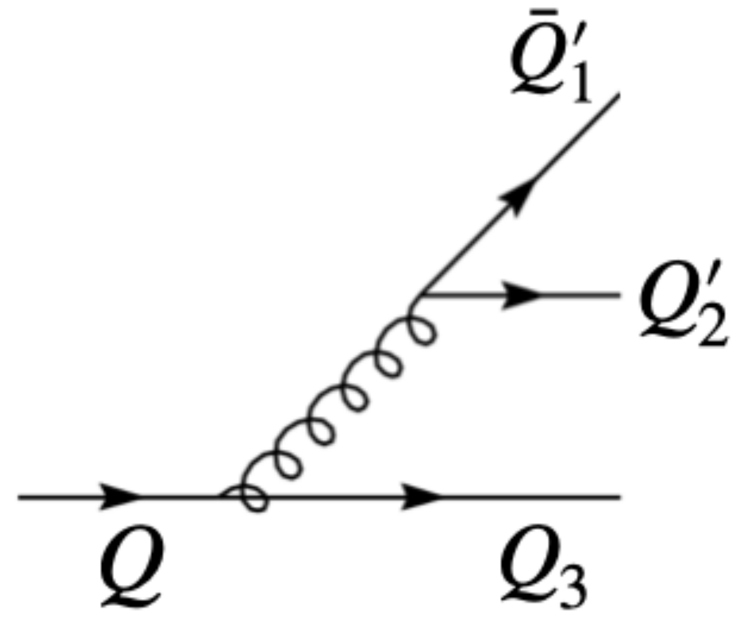
Splitting processes: $Q \to \bar{Q}'_1 Q'_2 Q_3 \quad Q \to \bar{Q}_1 Q_2 Q_3 \quad Q \to g_1 g_2 Q_3 \quad g \to g_1 Q_2 \bar{Q}_3$

$Q \to \bar{Q}_1 Q_2 Q_3$

Color structure: $\mathbf{C}_1 : T_R \left(\delta_{c_1 c_3} \delta_{c c_2} - \frac{1}{N_c} \delta_{c c_3} \delta_{c_1 c_2} \right) \qquad \mathbf{C}_2 : T_R \left(\delta_{c_1 c_2} \delta_{c c_3} - \frac{1}{N_c} \delta_{c c_2} \delta_{c_1 c_3} \right)$

Spin structure:

$$\mathbf{S}_1 : \bar{u}(p_3)\gamma^\mu u(\widetilde{P})\bar{u}(p_2)\gamma_\mu v(p_1)\,, \qquad \mathbf{S}_2 : \bar{u}(p_3)\not{p}_2 u(\widetilde{P})\bar{u}(p_2)\not{n} v(p_1)\,,$$
$$\mathbf{S}_3 : \bar{u}(p_3)\not{p}_1 u(\widetilde{P})\bar{u}(p_2)\not{n} v(p_1)\,, \qquad \mathbf{S}_4 : \bar{u}(p_2)\gamma^\mu u(\widetilde{P})\bar{u}(p_3)\gamma_\mu v(p_1)\,,$$
$$\mathbf{S}_5 : \bar{u}(p_2)\not{p}_3 u(\widetilde{P})\bar{u}(p_3)\not{n} v(p_1)\,, \qquad \mathbf{S}_6 : \bar{u}(p_2)\not{p}_1 u(\widetilde{P})\bar{u}(p_3)\not{n} v(p_1)\,.$$



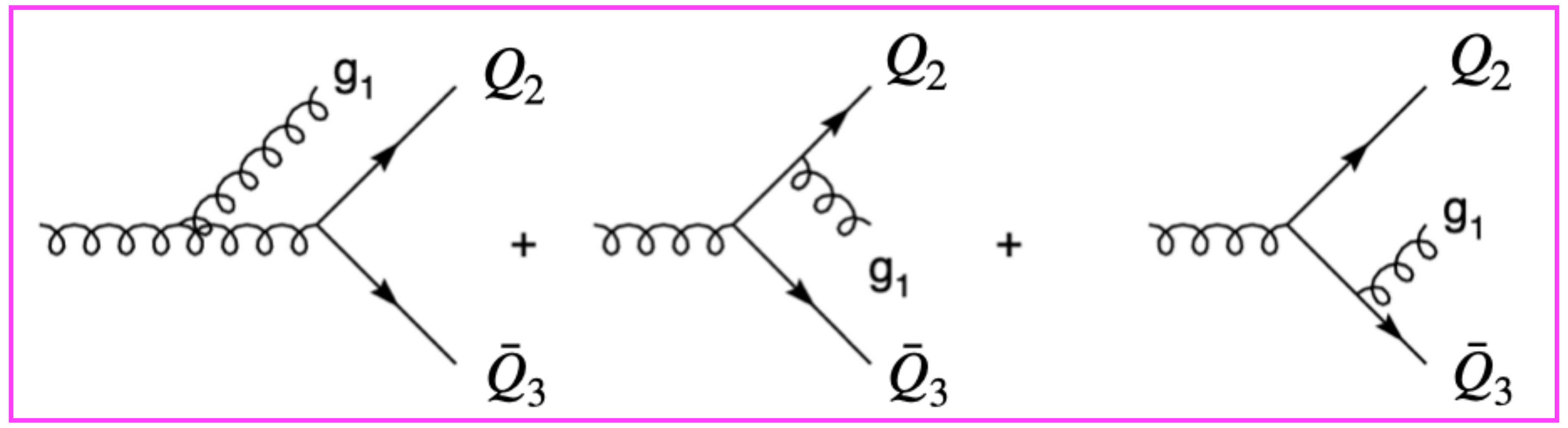
Splitting matrices:

$$\boldsymbol{Sp}^{(0)}_{\bar{Q}_1 Q_2 Q_3} = \frac{g_s^2 \mu_0^{2\epsilon}}{s_{123} - m_Q^2} \left[\frac{\mathbf{C}_1}{s_{12}} \left(-\mathbf{S}_1 + \frac{\mathbf{S}_2 + \mathbf{S}_3}{n \cdot p_{12}} \right) - \frac{\mathbf{C}_2}{s_{13}} \left(-\mathbf{S}_4 + \frac{\mathbf{S}_5 + \mathbf{S}_6}{n \cdot p_{13}} \right) \right]$$

$$\boldsymbol{Sp}^{(0)}_{\bar{Q}'_1 Q'_2 Q_3} = \frac{g_s^2 \mu_0^{2\epsilon}}{s_{123} - m_Q^2} \frac{\mathbf{C}_1}{s_{12}} \left(-\mathbf{S}_1 + \frac{\mathbf{S}_2 + \mathbf{S}_3}{n \cdot p_{12}} \right)$$

Splitting Matrices

$g \to g_1 Q_2 \bar{Q}_3$



Color structure: $\mathbf{C}_1 : (\boldsymbol{t}^{c}\boldsymbol{t}^{c_1})_{c_2c_3}$ $\quad$ $\mathbf{C}_2 : (\boldsymbol{t}^{c_1}\boldsymbol{t}^{c})_{c_2c_3}$

Spin structure:

$\mathbf{S}_1 : \bar{u}(p_2)\not{p}_1 \not{\varepsilon}(p_1)\not{\varepsilon}^*(\widetilde{P})v(p_3)\,,$ $\quad$ $\mathbf{S}_2 : \bar{u}(p_2)\not{\varepsilon}^*(\widetilde{P})v(p_3)\, p_2\cdot\varepsilon(p_1)\,,$

$\mathbf{S}_3 : \bar{u}(p_2)\not{\varepsilon}^*(\widetilde{P})v(p_3)\, p_3\cdot\varepsilon(p_1)\,,$ $\quad$ $\mathbf{S}_4 : \bar{u}(p_2)\not{\varepsilon}(p_1)v(p_3)\, p_1\cdot\varepsilon^*(\widetilde{P})\,,$

$\mathbf{S}_5 : \bar{u}(p_2)\not{p}_1 v(p_3)\, \varepsilon^*(\widetilde{P})\cdot\varepsilon(p_1)\,,$ $\quad$ $\mathbf{S}_6 : \bar{u}(p_2)\not{n} v(p_3)\, \varepsilon^*(\widetilde{P})\cdot\varepsilon(p_1)\,.$

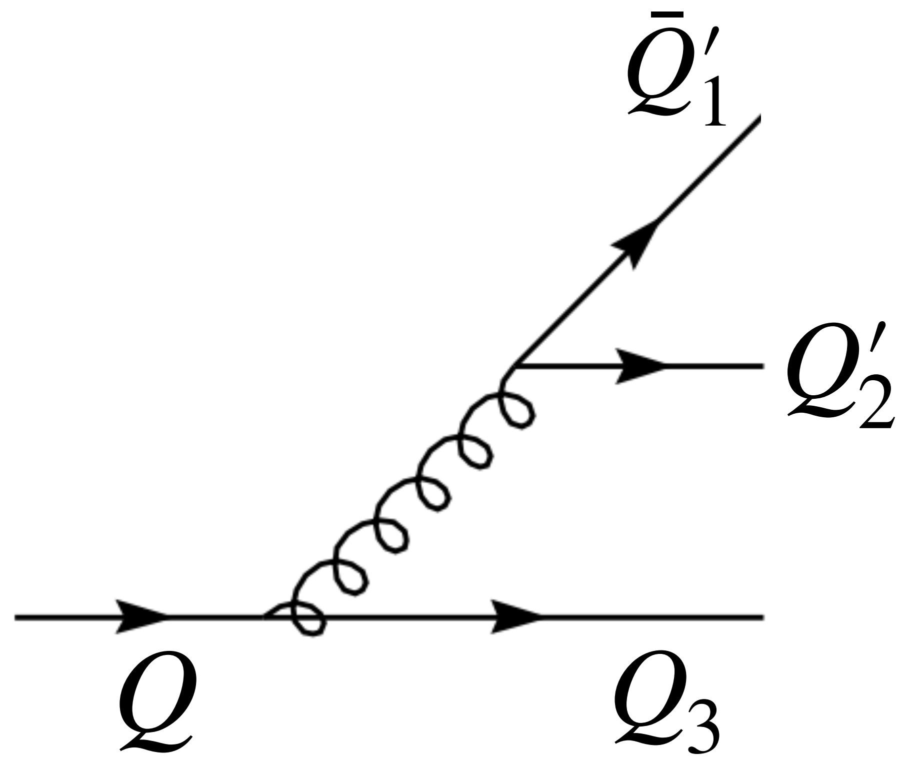
Splitting matrices:

$$\boldsymbol{Sp}^{(0)}_{g_1Q_2\bar{Q}_3} = \frac{g_s^2\mu_0^{2\epsilon}}{s_{123}}\left[\mathbf{C}_1\left(\frac{\mathbf{S}_1'+\mathbf{S}_2'}{\tilde{s}_{13}}\right)+\mathbf{C}_2\,\frac{\mathbf{S}_1'}{\tilde{s}_{12}}+(\mathbf{C}_1-\mathbf{C}_2)\left(\frac{\mathbf{S}_2'}{s_{23}}+\frac{s_{123}}{s_{23}}\frac{\mathbf{S}_6}{n\cdot p_{23}}\right)\right]$$

$$\mathbf{S}_1' = \mathbf{S}_1 - 2\,\mathbf{S}_2\,, \quad \mathbf{S}_2' = 2\,(\mathbf{S}_2+\mathbf{S}_3+\mathbf{S}_4-\mathbf{S}_5)$$

Splitting Kernels

$Q \to \bar{Q}'_1 Q'_2 Q_3$



$$t_{ij,k} = 2\frac{z_i \tilde{s}_{jk} - z_j \tilde{s}_{ik}}{z_i + z_j} + \frac{z_i - z_j}{z_i + z_j}\tilde{s}_{ij}$$

$$\begin{aligned}\langle \hat{P}^{(0)}_{\bar{Q}'_1 Q'_2 Q_3} \rangle = & C_F T_R \left\{ \frac{\tilde{s}_{12}\tilde{s}_{123}}{2 s_{12}^2} \left[-\frac{t_{12,3}^2}{\tilde{s}_{12}\tilde{s}_{123}} + \frac{4z_3 + (z_1 - z_2)^2}{1 - z_3} + (1-2\epsilon)\left(z_1 + z_2 - \frac{\tilde{s}_{12}}{\tilde{s}_{123}} \right) \right] \right. \\ & + \frac{2m_{Q'}^2}{s_{12}^2} \left[\frac{z_3 \tilde{s}_{123}}{(1-z_3)^2}(1 + 2z_3 - 3z_3^2 + 4z_1 z_2) - \frac{\tilde{s}_{23}}{1-z_3}(2 - 3z_1 - 5z_2 + z_1^2 + z_2^2) \right. \\ & \left. - \frac{\tilde{s}_{13}}{1-z_3}(2 - 5z_1 - 3z_2 + z_1^2 + z_2^2) - \epsilon\left(\tilde{s}_{123}(1-z_3) - \tilde{s}_{12}(1+z_3) \right) \right] - 2\frac{m_Q^2 \tilde{s}_{12}}{s_{12}^2} \\ & \left. + \frac{4m_{Q'}^4}{s_{12}^2} z_3 \left[\epsilon + \frac{2z_1 z_2}{(1-z_3)^2} + \frac{2z_3}{1-z_3} \right] - 4\frac{m_Q^2 m_{Q'}^2}{s_{12}^2} \right\}.\end{aligned}$$

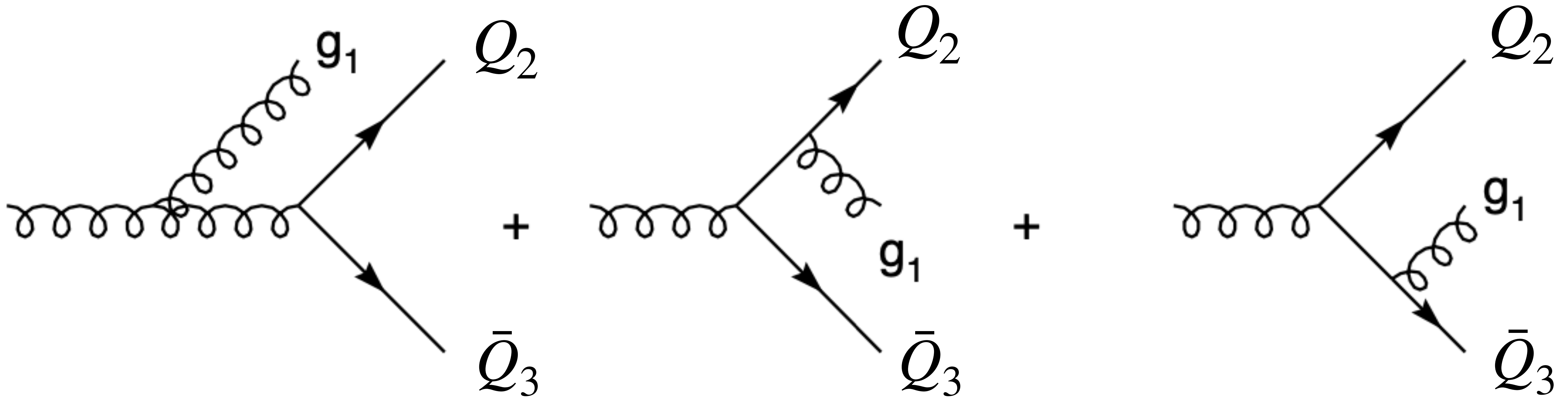
$s_{12} \to \tilde{s}_{12}$

Massless case

Confirmed by Craft, Gonzalez, Lee, Mecaj, Moult - 2310.06736

Splitting Kernels

$g \to g_1 Q_2 \bar{Q}_3$



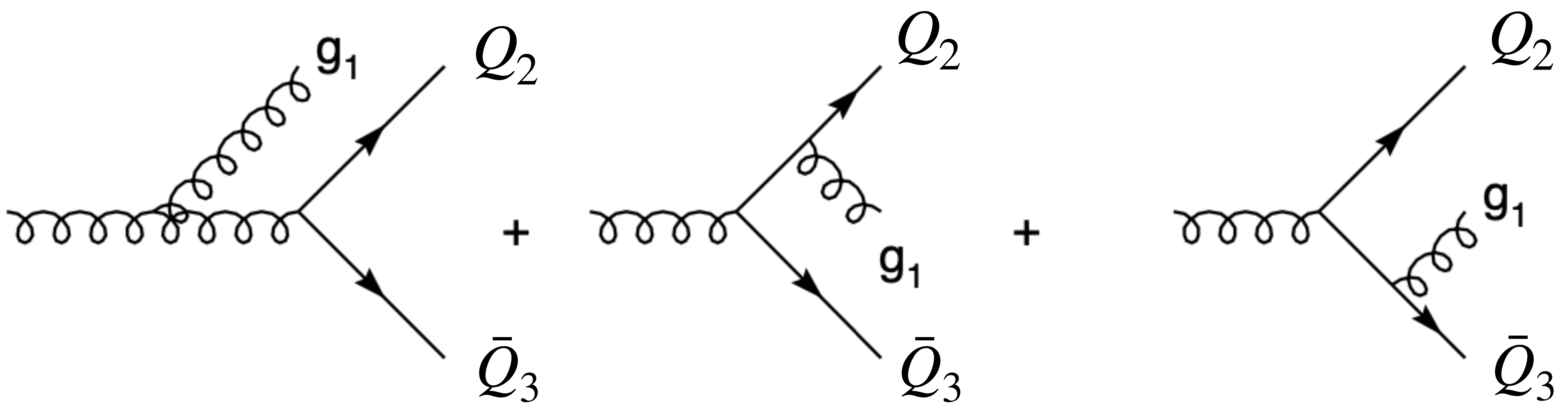
$$\hat{P}^{(0),\mu\nu}_{g_1 Q_2 \bar{Q}_3} = C_F T_R \hat{P}^{(0,\mathrm{ab}),\mu\nu}_{g_1 Q_2 \bar{Q}_3} + C_A T_R \hat{P}^{(0,\mathrm{nab}),\mu\nu}_{g_1 Q_2 \bar{Q}_3}$$

$$\hat{P}^{(0,\mathrm{ab}),\mu\nu}_{g_1 Q_2 \bar{Q}_3} = g^{\mu\nu}\Bigg[2 - \frac{2\tilde{s}_{123}\tilde{s}_{23} + (1-\epsilon)(\tilde{s}_{123} - \tilde{s}_{23})^2}{\tilde{s}_{12}\tilde{s}_{13}} + 2m_Q^2\left(\frac{\tilde{s}_{123}}{\tilde{s}_{12}^2} + \frac{\tilde{s}_{123}}{\tilde{s}_{13}^2} - \frac{2\tilde{s}_{23}}{\tilde{s}_{12}\tilde{s}_{13}}\right)$$
$$+ 4m_Q^4\left(\frac{1}{\tilde{s}_{12}^2} + \frac{1}{\tilde{s}_{13}^2}\right)\Bigg] + \frac{4\tilde{s}_{123}}{\tilde{s}_{12}\tilde{s}_{13}}\left[\tilde{k}_2^\mu \tilde{k}_3^\nu + \tilde{k}_3^\mu \tilde{k}_2^\nu - (1-\epsilon)\tilde{k}_1^\mu \tilde{k}_1^\nu\right]$$
$$+ 8m_Q^2\left[\frac{\tilde{k}_2^\mu \tilde{k}_2^\nu}{\tilde{s}_{13}^2} + \frac{\tilde{k}_3^\mu \tilde{k}_3^\nu}{\tilde{s}_{12}^2}\right] - \frac{8m_Q^2}{\tilde{s}_{12}\tilde{s}_{13}}(1-\epsilon)\tilde{k}_1^\mu \tilde{k}_1^\nu \,,$$

→ Massless case

Splitting Kernels

$g \to g_1 Q_2 \bar{Q}_3$



$$\langle \hat{P}^{(0)}_{g_1 Q_2 \bar{Q}_3} \rangle = C_F T_R \, \langle \hat{P}^{(0,\mathrm{ab})}_{g_1 Q_2 \bar{Q}_3} \rangle + C_A T_R \, \langle \hat{P}^{(0,\mathrm{nab})}_{g_1 Q_2 \bar{Q}_3} \rangle$$

$$\langle \hat{P}^{(0,\mathrm{ab})}_{g_1 Q_2 \bar{Q}_3} \rangle = \Bigg\{ \Bigg[\frac{s_{123}^2 \left(z_1^2 (1-\epsilon) + 2(1-z_2) z_3 - \epsilon \right)}{\tilde{s}_{12} \tilde{s}_{13} (1-\epsilon)} + \frac{2 s_{123} ((z_1+1)\epsilon + z_2 - 1)}{\tilde{s}_{12}(1-\epsilon)} + \frac{\tilde{s}_{13}(1-\epsilon)}{\tilde{s}_{12}} - \epsilon \Bigg]$$

$$+ \left[\frac{m_Q^2}{\tilde{s}_{12}} \left(\frac{2 s_{123}(2(1-z_3) z_3 + \epsilon - 1)}{\tilde{s}_{12}} + \frac{2 s_{123}(z_1 + 2 z_2 z_3 + \epsilon)}{\tilde{s}_{13}} - 4 \right) - \frac{4\, m_Q^4}{\tilde{s}_{13}} \left(\frac{1}{\tilde{s}_{12}} + \frac{1}{\tilde{s}_{13}} \right) \right] \frac{1}{1-\epsilon} \Bigg\} + (2 \leftrightarrow 3)\,,$$

$s_{123} \to \tilde{s}_{123}$ → Massless case

Summary & Future Outlook

* Singular elements coming from the QCD factorisation in the (quasi)collinear limit are key ingredients in the prediction of higher order contributions to jet observables

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Thank You