Probing Higgs boson properties via higher order effects

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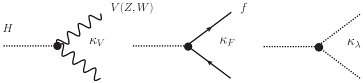
Higgs properties

mass, charge, spin, CP, couplings...

Precision calculations for Higgs *production and decay* play an important role in measuring these properties and probing new physics.

$$\boxed{\mathcal{L}_{\mathrm{Higgs}} = |\mathrm{D}_\mu \Phi|^2 - \sum_{\mathrm{f}} \mathrm{y_f} \bar{\mathrm{L}}_{\mathrm{f}} \Phi \mathrm{R_f} - \mathrm{V}(\Phi)}$$



$$g_{Hf\bar{f}} = \frac{m_f}{v}, \; g_{HVV} = \frac{2m_V^2}{v}, \; g_{HHVV} = \frac{2m_V^2}{v^2}, \; g_{HHH} = \frac{3m_H^2}{v}, \; g_{HHHH} = \frac{3m_H^2}{v^2}$$

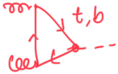
Couplings are proportional to the masses.

Measuring coupling requires a suitable parametrization.

$$\mathcal{L} = \kappa_Z \frac{m_Z^2}{v} Z_\mu Z^\mu H + \kappa_W \frac{2m_W^2}{v} W_\mu^+ W^{-\mu} H + \kappa_{VV} \frac{\alpha}{2\pi v} \left(\cos^2\theta_W Z_{\mu\nu} Z^{\mu\nu} + 2\, W_{\mu\nu}^+ W^{-\mu\nu}\right) H$$
$$+ \kappa_g \frac{\alpha_s}{12\pi v} G_{\mu\nu}^a G^{a\mu\nu} H + \kappa_\gamma \frac{2\alpha}{9\pi v} A_{\mu\nu} A^{\mu\nu} H + \kappa_{Z\gamma} \frac{\alpha}{\pi v} A_{\mu\nu} Z^{\mu\nu} H - \sum_F \kappa_F \frac{m_F}{v} \overline{F} F H + \kappa_3 \frac{m_H^2}{2v} H^3 + \ldots$$

In the standard model, $\kappa_i = 1$ for all i.

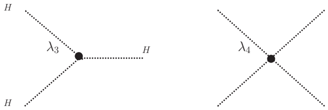
$$\mu_{\rm ggF} = 1.040\kappa_t^2 + 0.002\kappa_b^2 - 0.038\kappa_t\kappa_b, \quad \text{and} \quad \mu_{\rm VBF} = 0.73\kappa_W^2 + 0.27\kappa_Z^2$$



	LHC Run 1	ATLAS Run 2	CMS Run 2	HL-LHC (expected)
κ_γ	$0.87^{+0.14}_{-0.09}$	1.01 ± 0.06	1.10 ± 0.08	1.8%
κ_W	$0.87^{+0.13}_{-0.09}$	1.05 ± 0.06	1.02 ± 0.08	1.7%
κ_Z	-0.98 ± 0.10	0.99 ± 0.06	1.04 ± 0.07	1.5%
κ_g	$0.78^{+0.13}_{-0.10}$	0.95 ± 0.07	0.92 ± 0.08	2.5%
κ_t	$1.40^{+0.24}_{-0.21}$	0.94 ± 0.11	1.01 ± 0.11	3.4%
κ_b	$0.49^{+0.27}_{-0.15}$	0.89 ± 0.11	0.99 ± 0.16	3.7%
κ_τ	$0.84^{+0.15}_{-0.11}$	0.93 ± 0.07	0.92 ± 0.08	1.9%
κ_μ	–	$1.06^{+0.25}_{-0.30}$	1.12 ± 0.21	4.3%
$\kappa_{Z\gamma}$	–	$1.38^{+0.31}_{-0.36}$	1.65 ± 0.34	9.8%

Higgs self-couplings

$$\begin{aligned} V^{SM}(\Phi) &= -\mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2 \\ \text{EWSB} \Rightarrow V(H) &= \frac{1}{2}m_H^2 H^2 + \lambda_3 v H^3 + \frac{1}{4}\lambda_4 H^4. \end{aligned}$$



The mass and the self-couplings of the Higgs boson depend only on λ and $v = (\sqrt{2}G_\mu)^{-1/2}$,

$$m_H^2 = 2\lambda v^2;\ \lambda_3^{SM} = \lambda_4^{SM} = \lambda.$$

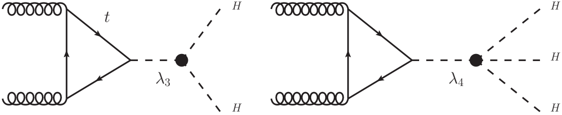
$$m_H = 125 \text{ GeV and } v \sim 246 \text{ GeV}, \Rightarrow \boxed{\lambda \simeq 0.13}.$$

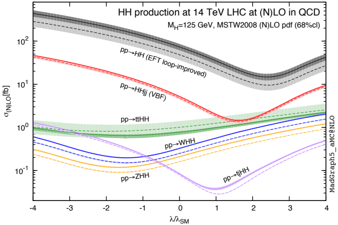
Presence of new physics at higher energy scales can contribute to the Higgs potential and modify the Higgs self-couplings.

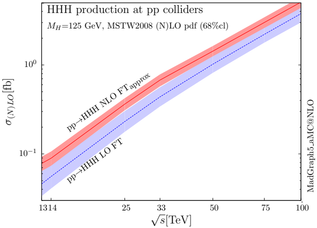
Independent measurements of λ_3 and λ_4 are crucial.

Direct determination of Higgs self-couplings

Information on λ_3 and λ_4 can be extracted by studying multi-Higgs production processes.







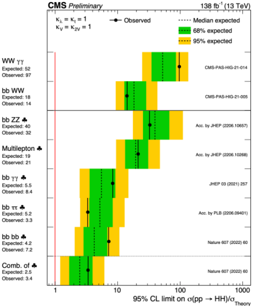
[Frederix et al. ‘14, 1408.6542]

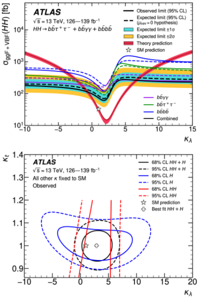
Very challenging due to small cross sections: ~33 fb (HH), ~ 0.1 fb (HHH)

Compare it with the single Higgs production ($gg \to H$) cross section: ~ 50 pb

Current and future experimental sensitivity ($\kappa_\lambda = \lambda_3/\lambda_3^{SM}$)

Non-observation of double Higgs production leads to an upper bound on the cross section.



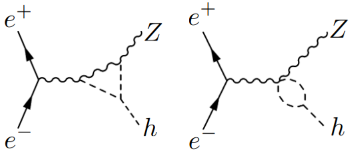


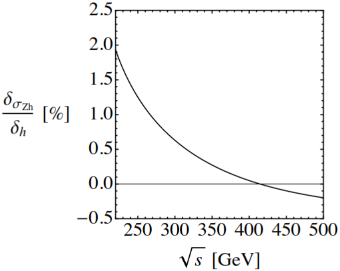
$$-0.6 < \kappa_\lambda < 6.6 \text{ (ATLAS)} \quad -1.2 < \kappa_\lambda < 6.5 \text{(CMS)}$$

Bounds are sensitive to κ_t value.

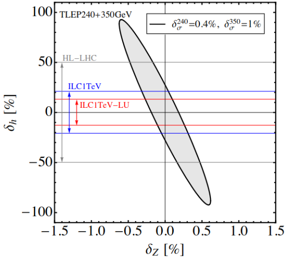
Indirect determination of λ_3 in single Higgs

Motivated by the study of McCullough'13 in $e^+e^- \to ZH$...





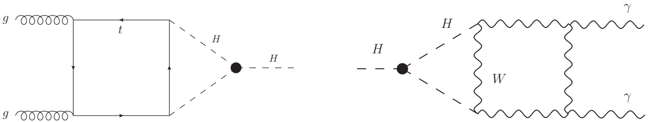
$$\delta_\sigma^{240} = 100\,(2\delta_Z + 0.014\delta_h)\,\%$$



For a direct probe of λ_3 via T-odd observables in $e^+e^- \to f\bar{f}H$, see Nakamura, AS 1812.01576

Indirect determination of λ_3 in single Higgs

Since the data on single Higgs is already there!



Gorbahn, Haisch: 1607.03773; Degrassi, Giardino, Maltoni, Pagani: 1607.04251; Bizon, Gorbahn, Haisch, Zanderighi: 1610.05771; Di Vita, Grojean, Panico, Riembau, Vantalon: 1704.01953; Maltoni, Pagani, AS, Zhao: 1709.08649

Master formula: *Anomalous trilinear coupling* ($\kappa_3 = \lambda_3/\lambda_3^{\mathrm{SM}}$)

$$\boxed{\Sigma_{\mathrm{NLO}}^{\mathrm{BSM}} = Z_H^{\mathrm{BSM}}[\Sigma_{\mathrm{LO}}(1 + \kappa_3 C_1 + \delta Z_H) + \Delta_{\mathrm{NLO}}^{\mathrm{SM}}]}$$

$$Z_H^{\mathrm{BSM}} = \frac{1}{1-(\kappa_3^2 - 1)\delta Z_H}, \delta Z_H = -1.536 \times 10^{-3}$$

Z_H^{BSM} arises from *wave function renormalization* and it is *universal* to all processes.

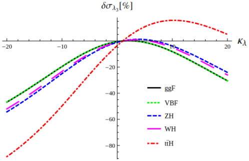
C_1 arises from the *interference between LO amplitude and λ_3-dependent virtual corrections*.

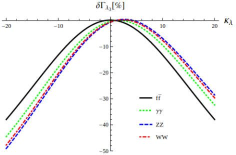
Degrassi, Giardino, Maltoni, Pagani '16; Maltoni, Pagani, AS, Zhao '17

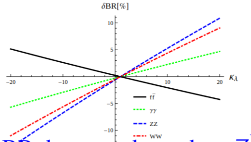
Channels	ggF	VBF	ZH	WH	$t\bar{t}H$	tHj	$H \to 4\ell$
$C_1(\%)$	0.66	0.63	1.19	1.03	3.52	0.91	0.82

$C_1^{\Gamma}[\%]$	$\gamma\gamma$	ZZ	WW	$f\bar{f}$	gg
on-shell H	0.49	0.83	0.73	0	0.66

$$\delta\Sigma_{\kappa_3} = \frac{\Sigma^{\rm BSM}_{\lambda_3} - \Sigma^{\rm SM}_{\lambda_3}}{\Sigma_{\rm LO}} = (Z_H^{\rm BSM} - 1)(1 + \delta Z_H) + (Z_H^{\rm BSM}\kappa_3 - 1)C_1,$$

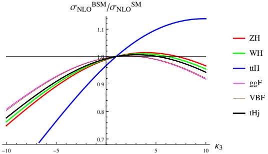






BR does not depend on $Z_H^{\rm BSM}$

The impact of full NLO EW corrections : more important for production channels



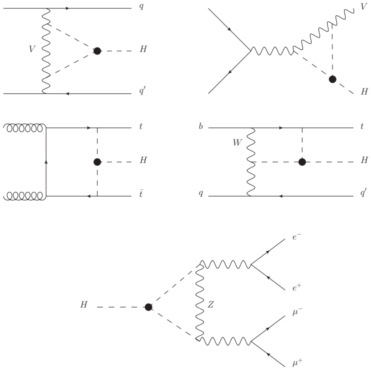
Channels	ggF	VBF	ZH	WH	ttH	tHj
$K_{\rm EW}$	1.049	0.932	0.947	0.93	1.014	0.95

The full EW effect at inclusive level in signal strength is neglegible.

Two MC public codes to calculate C_1 at differential level:

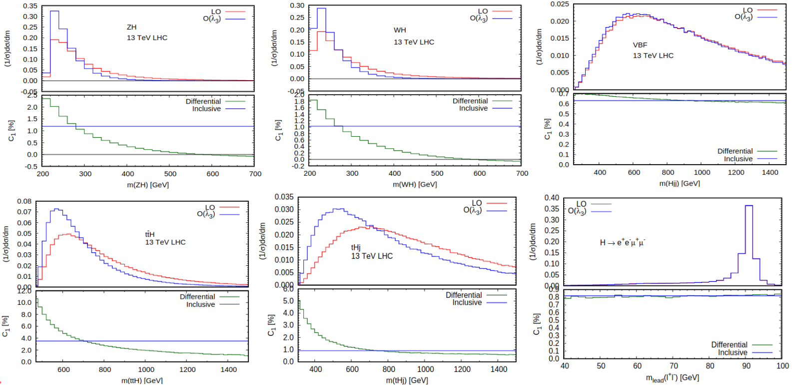
1. `trilinear-FF` 2. `trilinear-RW` (Recommended)

https://cp3.irmp.ucl.ac.be/projects/madgraph/wiki/HiggsSelfCoupling



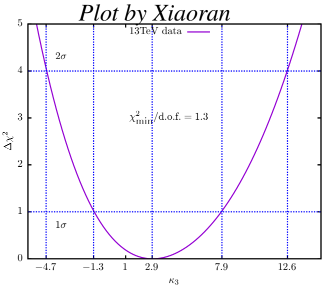
No provision for calculating differential C_1 in ggF channel.

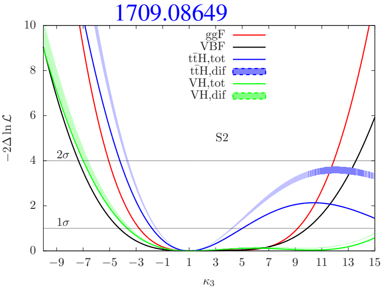
The kinematic dependence of C_1 is most significant in ttH.



$t\bar{t}H$

Current and future reach at the LHC





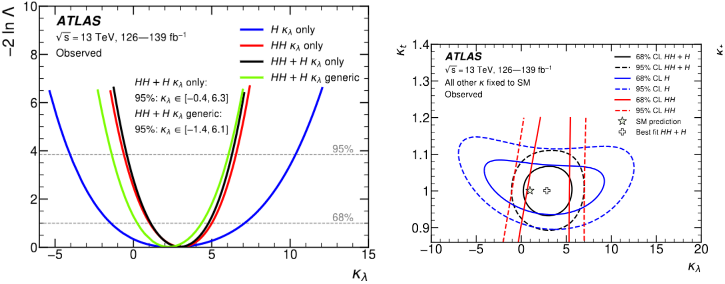
13 TeV:

$$\boxed{-4.7 < \kappa_3 < 12.6}$$

HL-LHC:

$$\boxed{-2 \lesssim \kappa_3 \lesssim 8}$$

Using the calculation of Maltoni, Pagani, AS, Zhao: 1709.08649 ATLAS (2211.01216) obtained



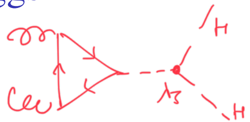
$$\boxed{-4 \lesssim \kappa_\lambda \lesssim 10.3}$$

Can we extend this strategy to double Higgs production?

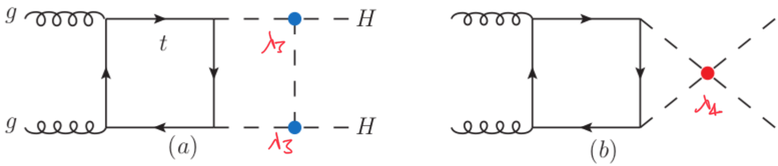
Maltoni, Pagani, Zhao: 1802.07616; Bizon, Haisch, Rottoli: 1810.04665; Borowka, Duhr, Maltoni, Pagani, AS, Zhao: 1811.12366

Indirect determination of λ_4 in double Higgs

At LO, the $gg \to HH$ amplitude is sensitive to only λ_3.



λ_4 affects $gg \to HH$ amplitude at two-loop level via NLO EW corrections.



EFT framework is necessary in order to vary cubic and quartic couplings independently in a consistent way.

$$V^{\mathrm{NP}}(\Phi) \equiv \sum_{n=3}^{\infty} \frac{c_{2n}}{\Lambda^{2n-4}} \left(\Phi^\dagger \Phi - \frac{1}{2} v^2\right)^n .$$

This also ensures gauge invariance and UV finiteness in our calculation.

NP Paramterization

$$V(H) = \frac{1}{2} m_H^2 H^2 + \lambda_3 v H^3 + \frac{1}{4} \lambda_4 H^4 + \lambda_5 \frac{H^5}{v} + O(H^6),$$

$$\kappa_3 \equiv \frac{\lambda_3}{\lambda_3^{\mathrm{SM}}} = 1 + \frac{c_6 v^2}{\lambda \Lambda^2} \equiv 1 + \bar{c}_6,$$

$$\kappa_4 \equiv \frac{\lambda_4}{\lambda_4^{\mathrm{SM}}} = 1 + \frac{6 c_6 v^2}{\lambda \Lambda^2} + \frac{4 c_8 v^4}{\lambda \Lambda^4} \equiv 1 + 6\bar{c}_6 + \bar{c}_8 .$$

We can trade κ_3 and κ_4 with parameters $\bar{c}_6$ and $\bar{c}_8$.

$$\bar{c}_6 \equiv \frac{c_6 v^2}{\lambda \Lambda^2} = \kappa_3 - 1 ,$$

$$\bar{c}_8 \equiv \frac{4 c_8 v^4}{\lambda \Lambda^4} = \kappa_4 - 1 - 6(\kappa_3 - 1) .$$

The Phenomenological quantity of interest

Inclusive/differential cross section

$$\boxed{\sigma^{\rm pheno}_{\rm NLO} = \sigma_{\rm LO} + \Delta\sigma_{\bar{c}_6} + \Delta\sigma_{\bar{c}_8}\ ,}$$

EFT insertion at one-loop :

$$\sigma_{\rm LO} = \sigma_0 + \sigma_1 \bar{c}_6 + \sigma_2 \bar{c}_6^2 ,$$

EFT insertions at two-loop :

$$\begin{aligned} \Delta\sigma_{\bar{c}_6} &= \bar{c}_6^2 \left[\sigma_{30}\bar{c}_6 + \sigma_{40}\bar{c}_6^2\right] + \tilde{\sigma}_{20}\bar{c}_6^2 , \\ \Delta\sigma_{\bar{c}_8} &= \bar{c}_8 \left[\sigma_{01} + \sigma_{11}\bar{c}_6 + \sigma_{21}\bar{c}_6^2\right] , \end{aligned}$$

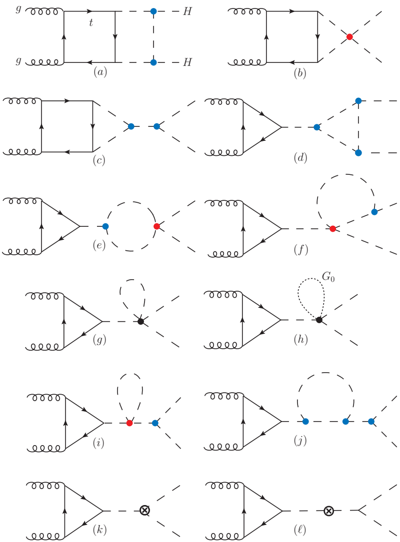
Ignored the SM EW corrections, kept highest powers of $\bar{c}_6$ in $\Delta\sigma_{\bar{c}_6}$.

$\Delta\sigma_{\bar{c}_8}$ *most relevant, it solely induces the sensitivity on* $\bar{c}_8$.

Assume that higher order QCD corrections factorize from two-loop EW effects.

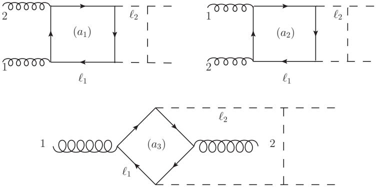
Relevant two-loop topologies

Non-factorizable, factorizable and counterterms:

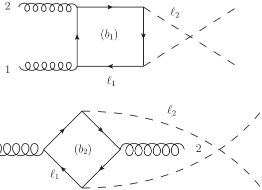


Non-factorizable contributions

The most challenging part of the calculation:



Double Box



Box-Triangle

$$
\begin{aligned}
\mathcal{M}_a &= 2(\mathcal{M}_{a_1} + \mathcal{M}_{a_2} + \mathcal{M}_{a_3}), \\
\mathcal{M}_b &= 2\mathcal{M}_{b_1} + \mathcal{M}_{b_2}, \\
\mathcal{M}_c &= \mathcal{M}_b \times \frac{6v^2}{\lambda_4} \frac{\lambda_3^2}{s - m_H^2},
\end{aligned}
$$

Projection to spin-0 and spin-2 form factors

For $gg \to HH$ amplitude

$$\mathcal{M}^{\mu_1\mu_2}\epsilon_{1,\mu_1}\epsilon_{2,\mu_2} = \delta^{c_1c_2}\mathcal{A}_0^{\mu_1\mu_2}\epsilon_{1,\mu_1}\epsilon_{2,\mu_2}F_0 + \delta^{c_1c_2}\mathcal{A}_2^{\mu_1\mu_2}\epsilon_{1,\mu_1}\epsilon_{2,\mu_2}F_2 \,.$$

$$\mathcal{A}_0^{\mu_1\mu_2} = \sqrt{\frac{2}{d-2}}\left(g^{\mu_1\mu_2} - \frac{p_1^{\mu_2}p_2^{\mu_1}}{p_1\cdot p_2}\right),$$

$$\mathcal{A}_2^{\mu_1\mu_2} = \sqrt{\frac{d-2}{2(d-3)}}\Bigg(-\frac{d-4}{d-2}\left[g^{\mu_1\mu_2} - \frac{p_1^{\mu_2}p_2^{\mu_1}}{p_1\cdot p_2}\right] + g^{\mu_1\mu_2} + \frac{(p_3\cdot p_3)p_1^{\mu_2}p_2^{\mu_1} + (2p_1\cdot p_2)p_3^{\mu_1}p_3^{\mu_2} - (2p_1\cdot p_3)p_2^{\mu_1}p_3^{\mu_2} - (2p_2\cdot p_3)p_3^{\mu_1}p_1^{\mu_2}}{p_T^2(p_1\cdot p_2)}\Bigg),$$

$$\mathcal{A}_i.\mathcal{A}_i = 1; \quad \mathcal{A}_0.\mathcal{A}_2 = 0$$

$\to F_{0,a}$, $F_{0,b}$, $F_{0,c}$ and $F_{2,a}$

The box-triangle amplitudes depend only on the spin-0 form factor.

Tools: `QGRAF`, `FORM`

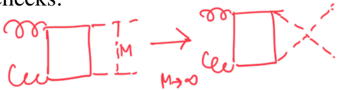
Numerical evaluation of form factors

The form factors contain two-loop integrals. They are computed using pySecDec.

Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk: 1703.09692, 1712.05755

Correctness of the calculation is ensured by various checks:

- UV finiteness of the form factors
- The large m_t limit of box-triangle amplitude
- Reduction of double-box into box triangle in heavy propagator limit



For phenomenological predictions at colliders, the form factors are required to be computed for many phase space points which can become very time consuming.

We build grids for form factors which can be interpolated for an efficient phase space integration: *1D grid ($\sqrt{s}$) for spin-0 FF and 2D grid ($\sqrt{s}, \theta$) for spin-2 FF*

Effect on inclusive cross section

Borowka, Duhr, Maltoni, Pagani, AS, Zhao: 1811.12366

For α_s, $\mu_R = \mu_F = \frac{1}{2} m(HH)$ while $\mu_{\text{EFT}} = 2m_H$.

One-loop:

$\sqrt{s}$ [TeV]	σ_0 [fb]	σ_1 [fb]	σ_2 [fb]
14	19.49 -	-15.59 (-80.0%)	5.414 (27.8%)
100	790.8 -	-556.8 (-70.5%)	170.8 (21.6%)

Two-loop:

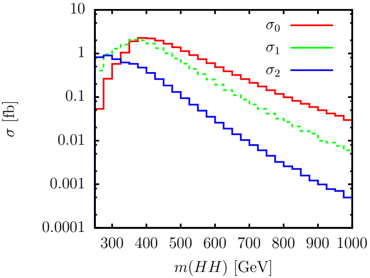
$\sqrt{s}$ [TeV]	$\tilde{\sigma}_{20}$ [fb]	σ_{30} [fb]	σ_{40} [fb]	σ_{01} [fb]	σ_{11} [fb]	σ_{21} [fb]
14	0.7112 (3.6%)	-0.5427 (-2.8%)	0.0620 (0.3%)	0.3514 (1.8%)	-0.0464 (-0.2%)	-0.1433 (-0.7%)
100	24.55 (3.1%)	-16.53 (-2.1%)	1.663 (0.2%)	12.932 (1.6%)	-0.88 (-0.1%)	-4.411 (-0.6%)

Cross sections grow considerably with energy. The contributions (numbers in brackets) from $\bar{c}_6$ and $\bar{c}_8$ slowly decrease wrt the SM LO prediction.

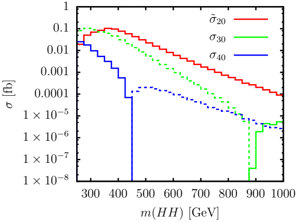
Effect on differential cross section

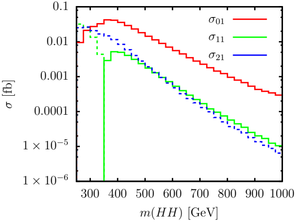
Borowka, Duhr, Maltoni, Pagani, AS, Zhao: 1811.12366

One-loop



Two-loop



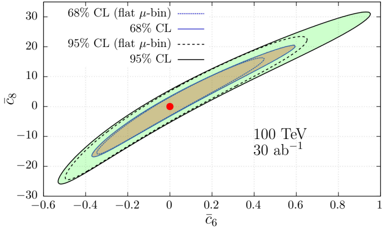


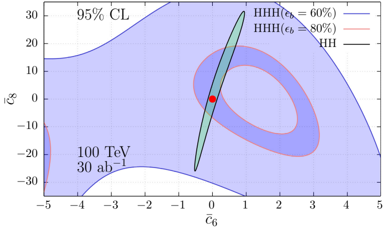
The dashed lines show absolute values of -ve contributions.

Projections for 100 TeV pp collider

Borowka, Duhr, Maltoni, Pagani, AS, Zhao: 1811.12366

Based on M_{HH}





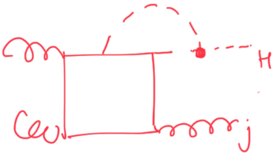
For $\kappa_3 = 1$, at 95% CL

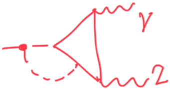
$-6 \lesssim \kappa_4 \lesssim 18$ [Direct from $HHH(4b2\gamma)$]

$-4.2 \lesssim \kappa_4 \lesssim 6.7$ [Indirect from $HH(2b2\gamma)$]

At 100 TeV pp collider, the HH channel would be more sensitive to independent variation in self-couplings than HHH channel.

Recent developments...





Gorban, Haisch 1902.05480; Gao et al 2302.04160: $\mathcal{O}(\lambda)$ correction to $p_T(H)$ in $H + j$ production

Degrassi, Vitti 2302.04160: λ in $H \to \gamma Z$

Haisch, Koole 2201.09711: λ in off-shell Higgs production $gg \to h^* \to ZZ$