# Lecture II:
# Slicing methods & (some)Applications

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**Advanced School & Workshop on Multiloop Scattering Amplitudes**
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# Outline

Slicing methods

- toy-model example
- connection to resummation
- $q_T$-subtraction

Applications to NC Drell-Yan process

Remarks

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# Toy model @ NLO: phase space slicing

Consider a toy model of a NLO calculation with only one singular (soft) region

- $\hat{\mathcal{O}}$ is an infrared and collinear (IRC) observable, for example a bin of a well defined kinematical histogram with/or a collection of requirements (acceptance, jet algorithm, isolation)
- the expectation value for $\hat{\mathcal{O}}$ is obtained considering the differential cross section as probability distribution

SLICING: the art of splitting the phase space

$$\int_0^1 dx \frac{A + Cx}{x^{1+\epsilon}} F_{\hat{\mathcal{O}}}(x) = \int_0^\delta dx \frac{A + Cx}{x^{1+\epsilon}} F_{\hat{\mathcal{O}}}(x) + \int_\delta^1 dx \frac{A + Cx}{x^{1+\not\epsilon}} F_{\hat{\mathcal{O}}}(x)$$

power suppressed contribution, $p > 0$

$$= \int_0^\delta \frac{dx}{x^{1+\epsilon}} \left[ A F_{\hat{\mathcal{O}}}(0) + \mathcal{O}(\delta^p) \right] + \int_\delta^1 dx \frac{A + Cx}{x} F_{\hat{\mathcal{O}}}(x)$$

$$= A F_{\hat{\mathcal{O}}}(0) \frac{\delta^{-\epsilon}}{-\epsilon} + \int_\delta^1 dx \frac{A + Cx}{x} F_{\hat{\mathcal{O}}}(x) + \mathcal{O}(\delta^p)$$

$$= -\frac{A}{\epsilon} F_{\hat{\mathcal{O}}}(0) + [\log \delta] F_{\hat{\mathcal{O}}}(0) + \int_\delta^1 dx \frac{A + Cx}{x} F_{\hat{\mathcal{O}}}(x) + \mathcal{O}(\delta^p) + \not{\mathcal{O}}(\epsilon)$$

# Toy model @ NLO: phase space slicing

Consider a toy model of a NLO calculation with only one singular (soft) region

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- the expectation value for $\hat{\mathcal{O}}$ is obtained considering the differential cross section as probability distribution

SLICING: the art of splitting the phase space

$$< \hat{\mathcal{O}} > \; = \left(\frac{A}{\epsilon} + B\right) F_{\hat{\mathcal{O}}}(0) - \frac{A}{\epsilon} F_{\hat{\mathcal{O}}}(0) + [\log \delta] F_{\hat{\mathcal{O}}}(0) + \int_\delta^1 dx \frac{A + Cx}{x} F_{\hat{\mathcal{O}}}(x) + \mathcal{O}(\delta^p)$$

$$= B F_{\hat{\mathcal{O}}}(0) + [\log \delta] F_{\hat{\mathcal{O}}}(0) + \int_\delta^1 dx \frac{A + Cx}{x} F_{\hat{\mathcal{O}}}(x) + \mathcal{O}(\delta^p)$$

The explicit logarithmic term cancels the logarithmical behaviour of the integral at small $x$ → For IRC-safe observables, final result free from IRC divergences

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The formula is exact only in the limit $\delta \to 0$

**Residual power suppressed terms due to the expansion performed in the below-cut part region**

# Toy model @ NLO: phase space slicing

SLICING: the art of splitting the phase space

$$< \hat{\mathcal{O}} > \; = BF_{\hat{\mathcal{O}}}(0) + [\log \delta] F_{\hat{\mathcal{O}}}(0) + \int_{\delta}^{1} dx \frac{A + Cx}{x} F_{\hat{\mathcal{O}}}(x) + \mathcal{O}(\delta^p)$$

**Global cancellation** among large quantities can spoil the numerical accuracy of the final result:
**choose a relative large cut-off $\delta$**

# Toy model @ NLO: phase space slicing

SLICING: the art of splitting the phase space

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**Global cancellation** among large quantities can spoil the numerical accuracy of the final result:
**choose a relative large cut-off $\delta$**

**Power suppressed terms** must be kept under control in order to obtain a **unbiased** result:
**choose a relative small cut-off $\delta$**

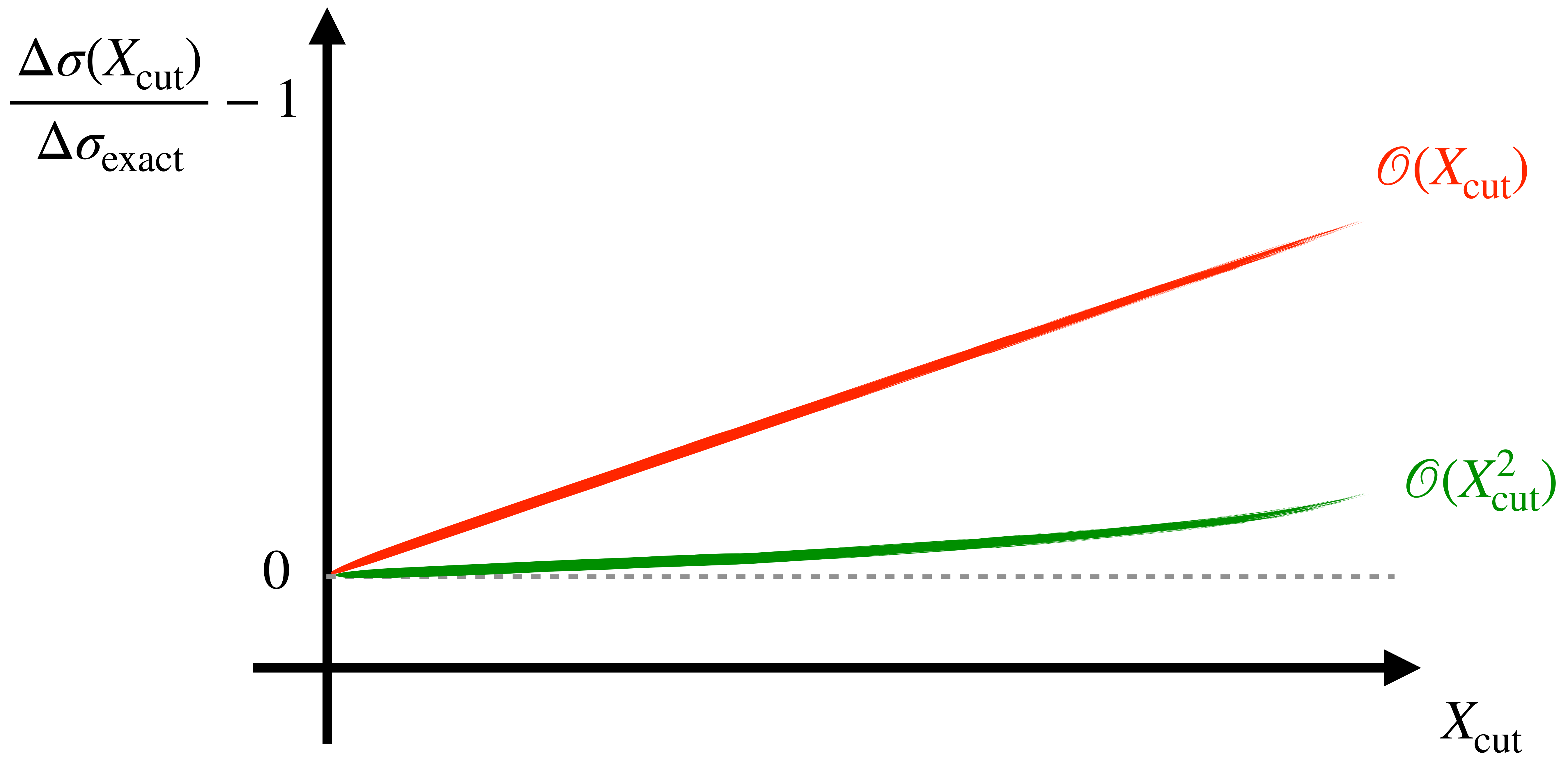
**Trade off** for choosing the cut-off

# Toy model @ NLO: phase space slicing

SLICING: the art of splitting the phase space

$$< \hat{\mathcal{O}} > \; = BF_{\hat{\mathcal{O}}}(0) + [\log \delta] F_{\hat{\mathcal{O}}}(0) + \int_\delta^1 dx \frac{A + Cx}{x} F_{\hat{\mathcal{O}}}(x) + \mathcal{O}(\delta^p)$$

Efficiency of the methods relies on how fast the calculation converge to the exact result in the limit $\delta \to 0$




Better performance for a larger **exponent $p$ of the power corrections!**

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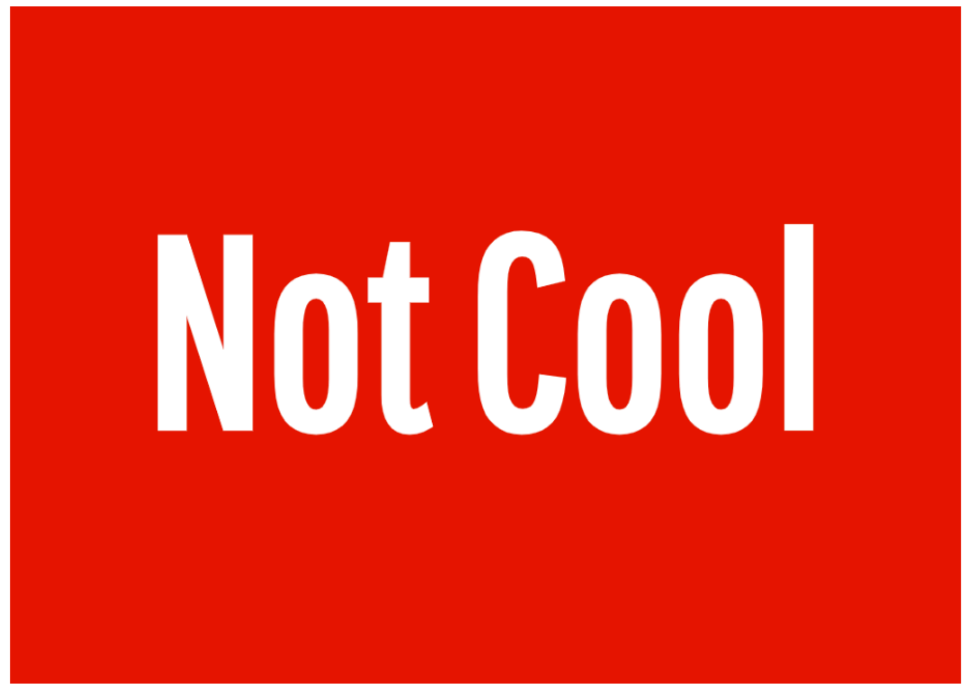
# Phase space slicing resurgence

**Slicing** $\int |\mathcal{M}|^2 F_J \, d\phi_d = \int_0^\delta \left[|\mathcal{M}|^2 F_J \, d\phi_d\right]_{\rm simp} + \int_\delta^1 |\mathcal{M}|^2 F_J \, d\phi_4 + \mathcal{O}(\delta)$

**Slicing** $\int |\mathcal{M}|^2 F_J \, d\phi_d = \int_0^\delta \left[|\mathcal{M}|^2 F_J \, d\phi_d\right]_{\rm simp} + \int_\delta^1 |\mathcal{M}|^2 F_J \, d\phi_4 + \mathcal{O}(\delta)$




Melnikov HP2 2022

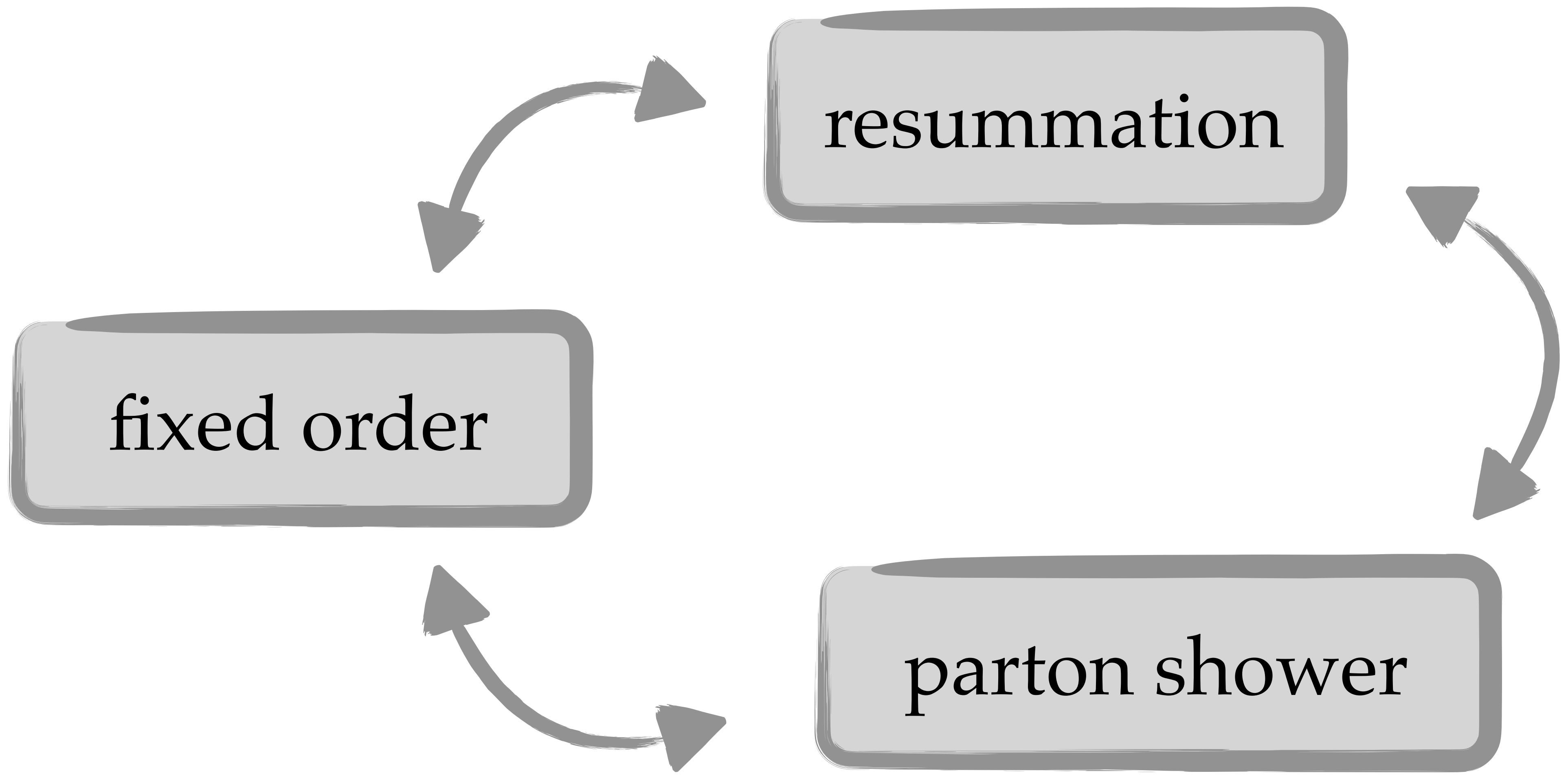

One of the main reasons for the slicing comeback is the increase in computing power available for such computations.

CONs

- large global cancellation of infrared logarithms
- residual power corrections in the slicing cut-off

PROs

- usually simpler (allowed to reach N$^3$LO for color singlet production)
- connection with **factorisation** theorems and **resummation**
- implications for higher-order matching (MiNNLO/GENEVA)

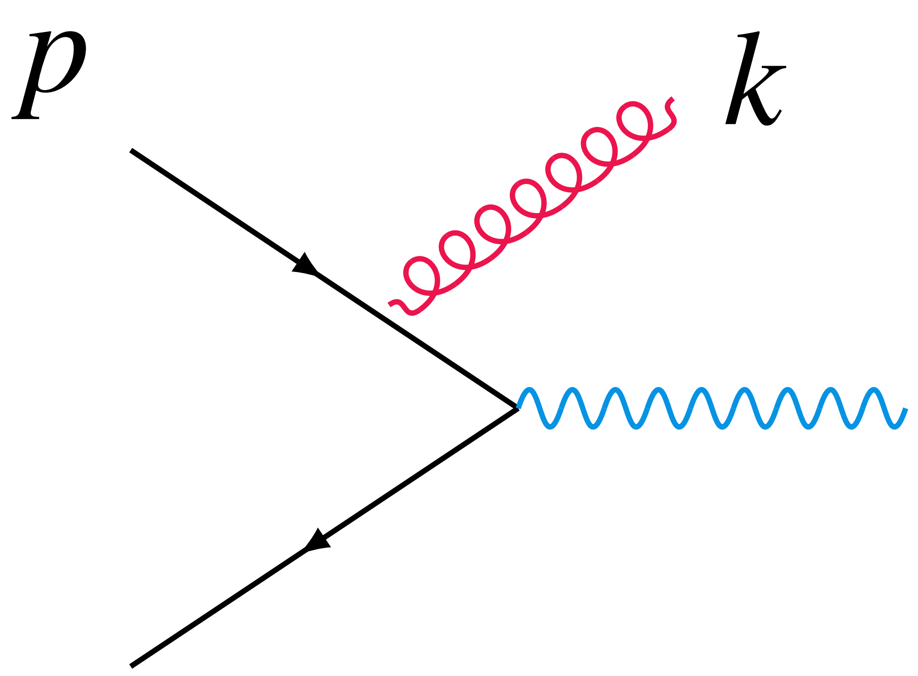


# Phase space slicing as a non-local subtraction

Introduce a **resolution variable** $X$ that discriminates a region with **1-resolved emission** from an completely **unresolved** region

$$d\sigma_{N^kLO} = \int d\sigma_{N^kLO}\Theta(X_{\text{cut}} - X) + \int d\sigma^R_{N^{k-1}LO}\Theta(X - X_{\text{cut}})$$

1. At NLO, the cross section is finite for $X > 0$
2. At N$^k$LO:
   - in the region $X > 0$, only N$^{k-1}$LO singularities
   - in the region $X = 0$, all genuine N$^k$LO unresolved limits




$$\frac{1}{2p\cdot k} = \frac{1}{\sqrt{s}}\frac{k^0 + k^3}{q_T^2}$$

For the production of color-less system $F$, its transverse momentum $q_T$ is a good resolution variable

# Phase space slicing as a non-local subtraction

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In the unresolved region, approximate the cross section by an expansion in the **soft-collinear limits**

Exploit factorisation theorems in EFT, resummation formulae in direct QCD

$$\int d\sigma_{N^kLO}\Theta(X_{\rm cut} - X) = \int d\sigma^{(\rm sing)}_{N^kLO}\Theta(X_{\rm cut} - X) + \mathcal{O}(X^p_{\rm cut}) = \int d\sigma^{(\rm sing)}_{N^kLO}\left[1 - \Theta(X - X_{\rm cut})\right] + \mathcal{O}(X^p_{\rm cut})$$

$$= \mathscr{H} \otimes d\sigma_{LO} - \int d\sigma^{CT}_{N^kLO}\Theta(X - X_{\rm cut}) + \mathcal{O}(X^p_{\rm cut})$$

**general structure of the singular component**

$$d\sigma^{(\rm sing)}_{N^kLO} = \mathscr{H} \otimes d\sigma_{\rm LO}\delta(X) + \sum_{\ell=0}^{2k-1} \Sigma_{\ell;k}\left[\frac{\Theta(X)\log^{\ell} X}{X}\right]_+ \otimes d\sigma_{\rm LO}$$

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Exploit factorisation theorems in EFT, resummation formulae in direct QCD

**Virtual correction** after subtraction of IR singularities and contribution of soft/collinear origin **(beam, soft, jet functions)**

$$d\sigma_{N^kLO} = \mathscr{H} \otimes d\sigma_{LO} + \int \left[d\sigma^R_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO}\right]_{X>X_{\rm cut}} + \mathcal{O}(X^p_{\rm cut})$$

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**Real contribution:** $N^{k-1}LO$ calculation, **divergent** in the limit $X_{\rm cut} \to 0$

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Complexity of the calculation reduced by one order!

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Exploit factorisation theorems in EFT, resummation formulae in direct QCD

**Counterterm** cancels the infrared behaviour of the real calculation in the limit $X_{\rm cut} \to 0$

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In the unresolved region, approximate the cross section by an expansion in the **soft-collinear limits**

Exploit factorisation theorems in EFT, resummation formulae in direct QCD

The cancellation is local in $X$ after integrating out all the other radiation variables:

non-local subtraction

**Counterterm** cancels the infrared behaviour of the real calculation in the limit $X_{\text{cut}} \to 0$

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Exploit factorisation theorems in EFT, resummation formulae in direct QCD

Missing **power corrections**
below the slicing cut-off

$$d\sigma_{N^kLO} = \mathscr{H} \otimes d\sigma_{LO} + \int \left[ d\sigma^{R}_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO} \right]_{X>X_{\text{cut}}} + \mathcal{O}(X^p_{\text{cut}})$$

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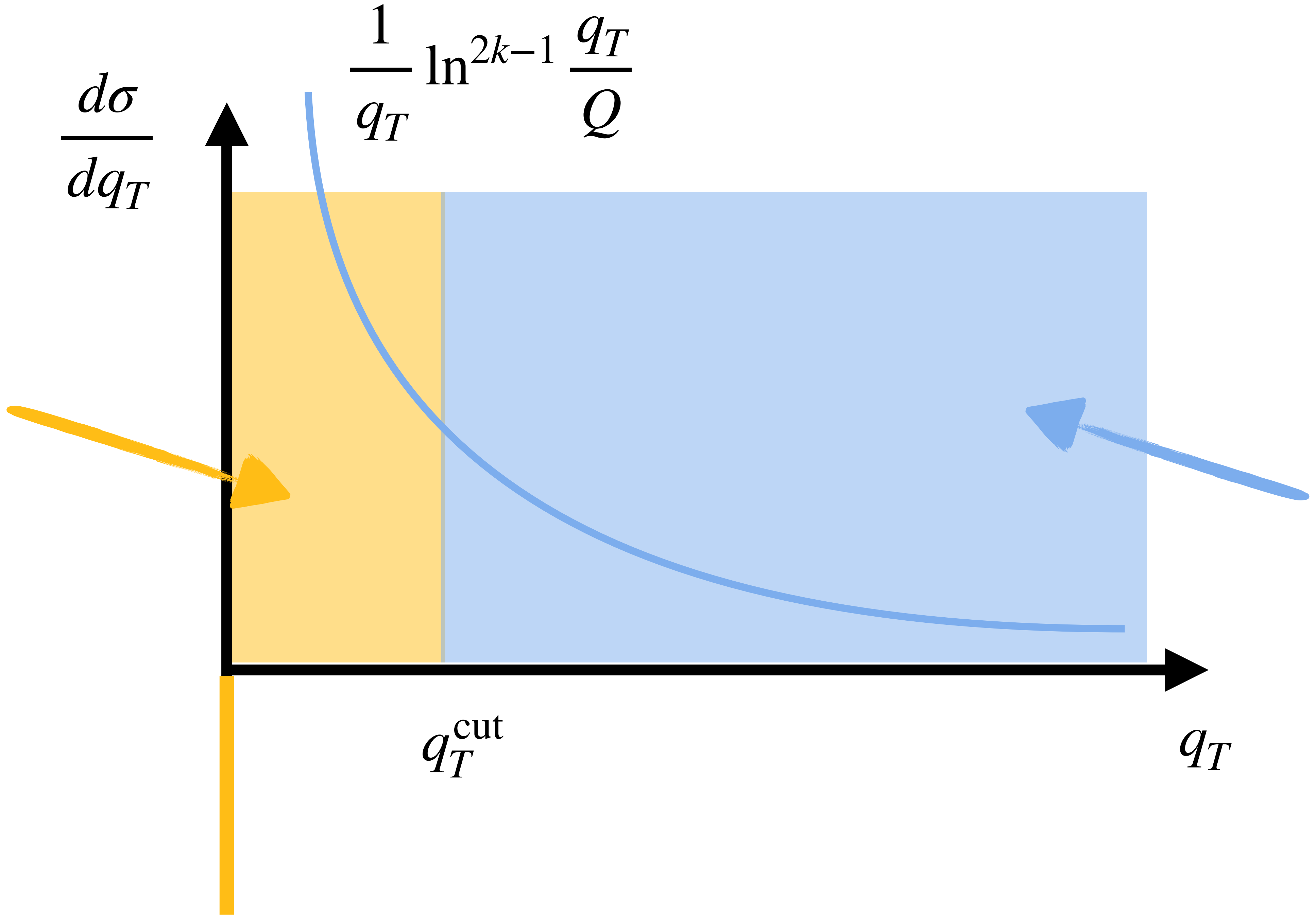
# $q_T$-subtraction formalism

[Catani, Grazzini, 2007]

Cross section for the production of a triggered final state $F$ at N$^k$LO

**All emission unresolved; approximate the cross section with its singular part in the soft and/or collinear limits**




**1 emission always resolved**

$F + j$ @ N$^{k-1}$LO

**complexity of the calculation reduced by one order!**

**$q_T$ resummation**

- **expand to fixed order**
- **$\mathcal{O}(\alpha_s^k)$ ingredient required**

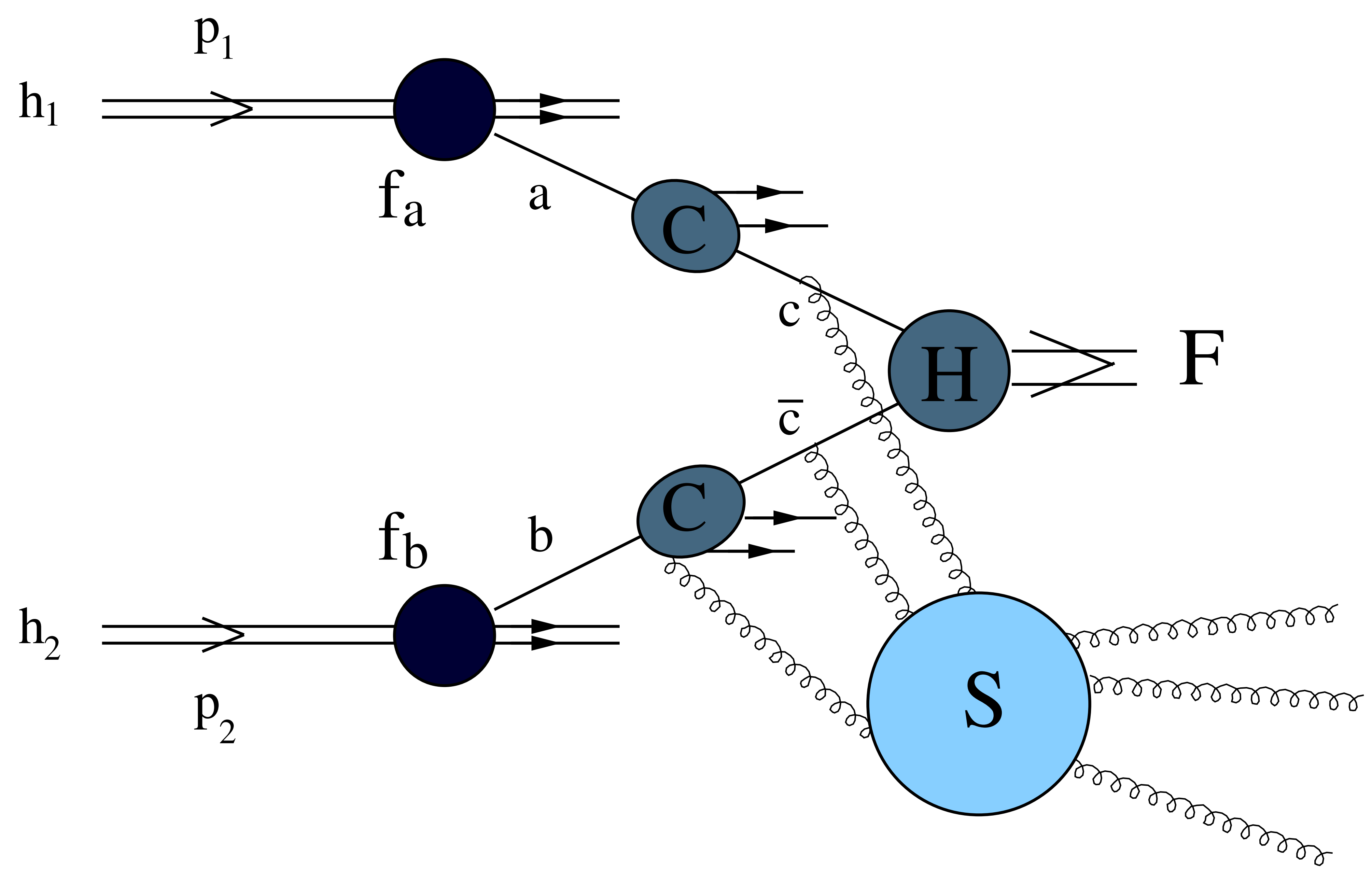
$$d\sigma_{N^kLO} = \mathcal{H} \otimes d\sigma_{LO} + \int \left[ d\sigma^R_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO} \right]_{q_T > q_T^{\text{cut}}} + \mathcal{O}\left( (q_T^{\text{cut}})^p \right)$$

# $q_T$-subtraction formalism: color-less final state

**color-less system** $F$**:** $(Q^2, Y, q_T)$

[Catani, de Florian, Grazzini, 2001]

$$\frac{d\sigma^{(sing)}}{dQ^2 dY dq_T d\Omega} = \frac{1}{S}\sum_c \frac{d\sigma^{(0)}_{c\bar{c},F}}{d\Omega}\int_0^\infty db \frac{b}{2} J_0(bq_T) S_c(Q,b) \sum_{a_1,a_2}\int_{x_1}^1 \frac{dz_1}{z_1}\int_{x_2}^1 \frac{dz_2}{z_2} [H^F C_1 C_2]_{c\bar{c};a_1a_2} f_{a_1/h_1}(x_1, b_0^2/b^2) f_{a_2/h_2}(x_2, b_0^2/b^2)$$



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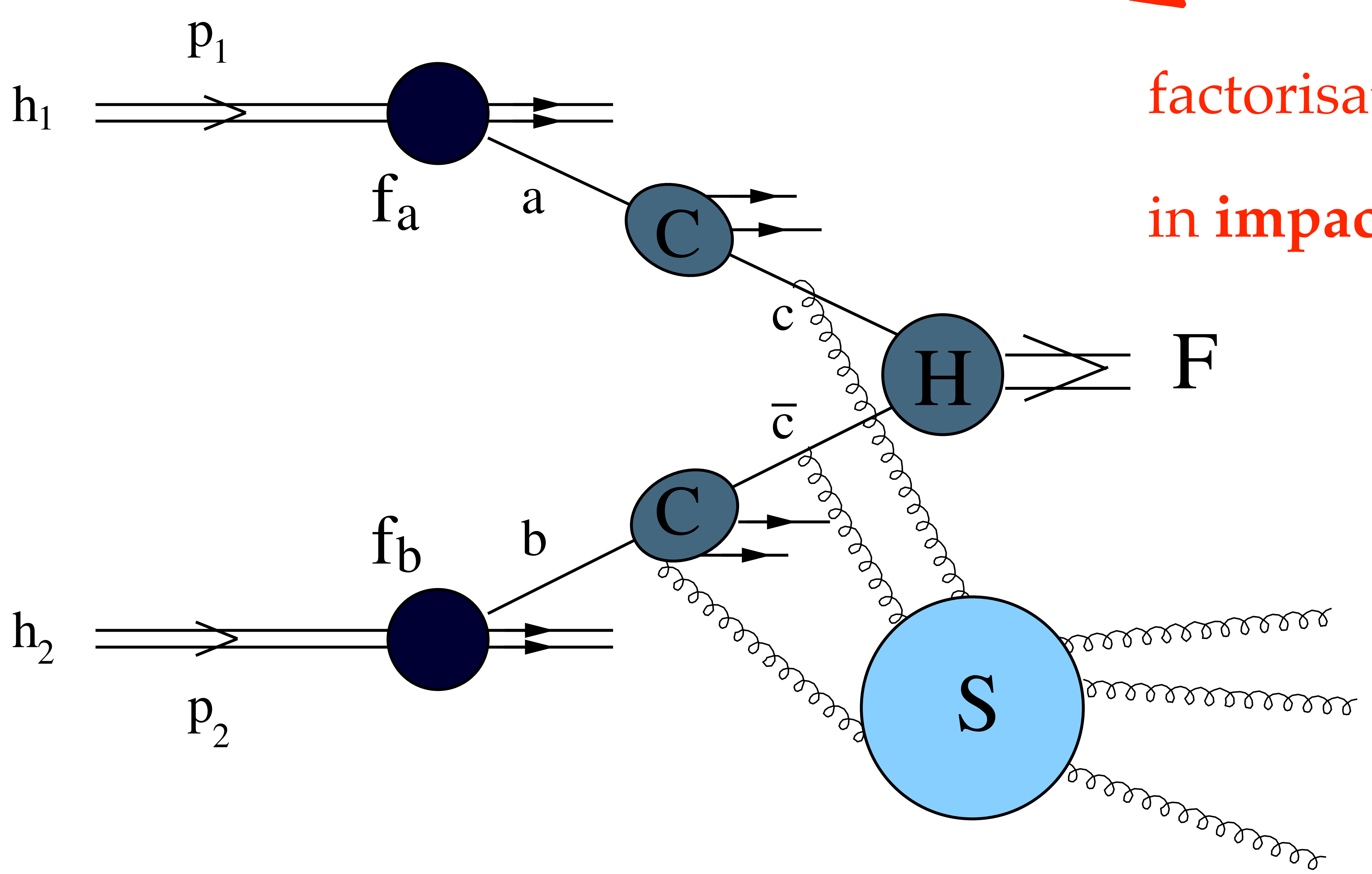


factorisation of the constraint $\delta^2\left(\mathbf{q}_T - \sum_i \mathbf{k}_{T,i}\right)$

in **impact parameter space**

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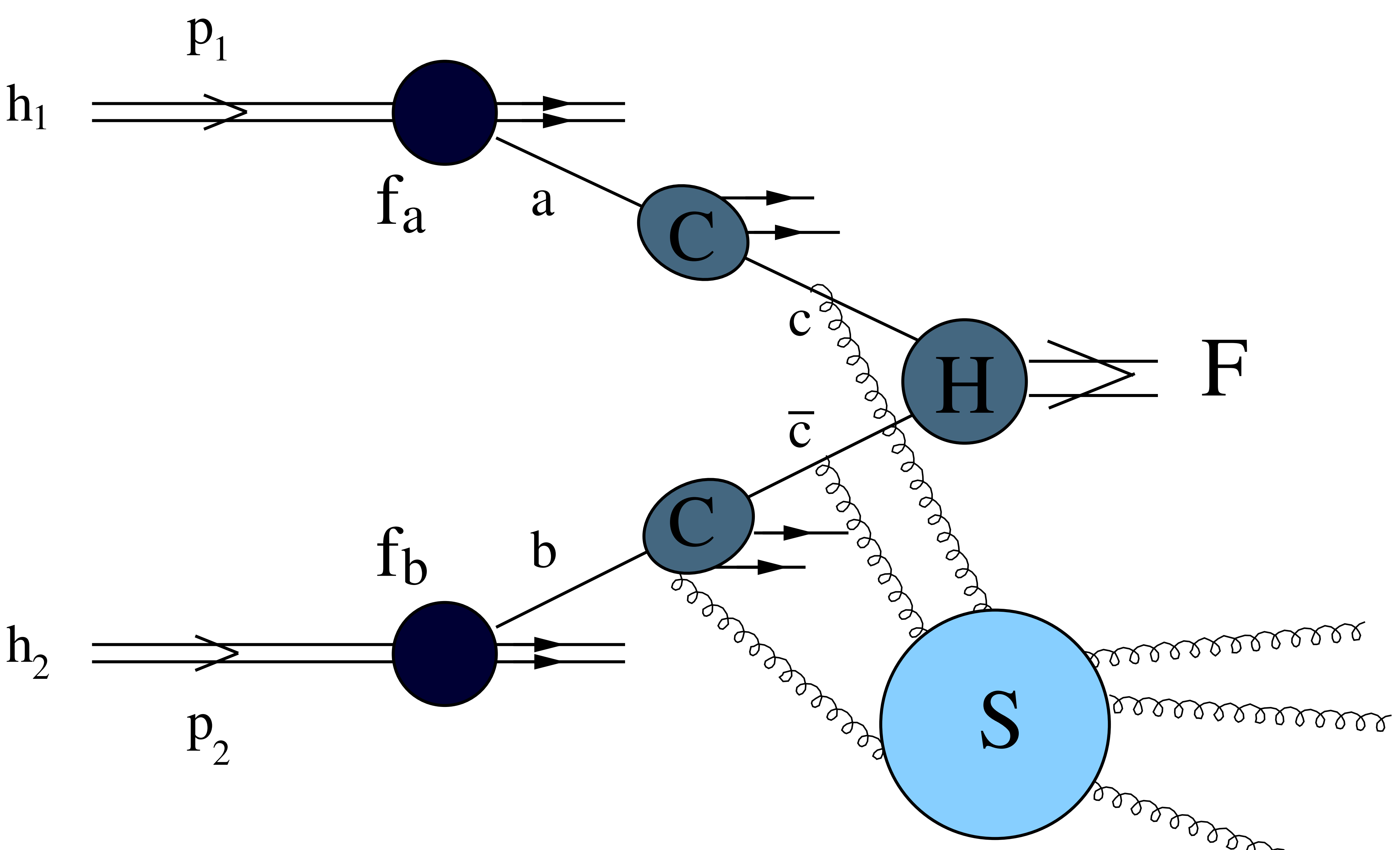


Universal **Sudakov Form Factor**:
exponentiation of soft-collinear emissions

$$S_c(Q,b) = \exp\left[-\int_{b_0^2/b^2}^{Q^2} dq^2 A_c\left(\alpha_S(q^2)\right) \ln\frac{Q^2}{q^2} + B_c\left(\alpha_S(q^2)\right)\right]$$

$A_c, B_c$ admits a perturbative expansion in $\alpha_S$

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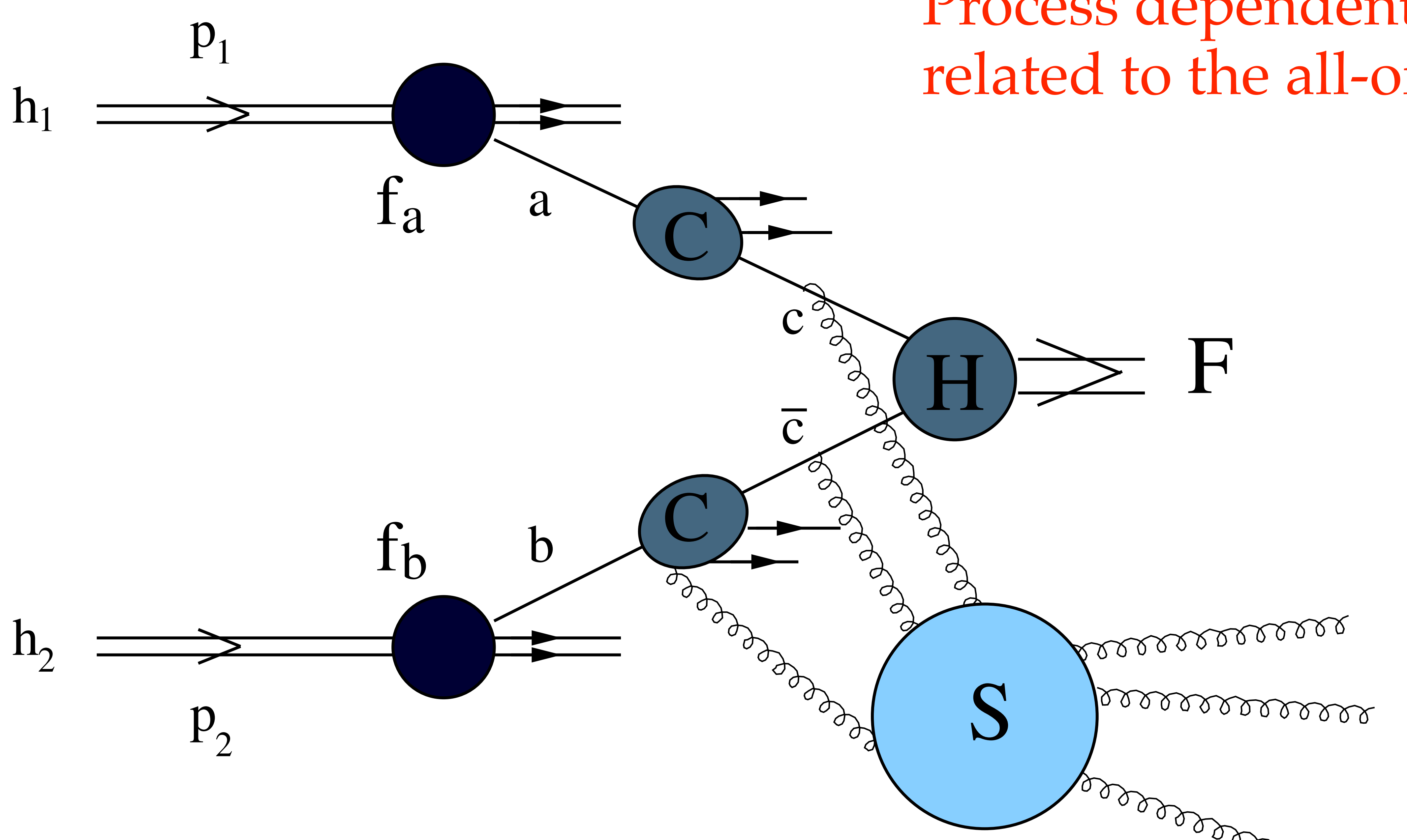
Process dependent **Hard-Virtual function**
related to the all-order elastic amplitude

Universal **collinear or beam function**



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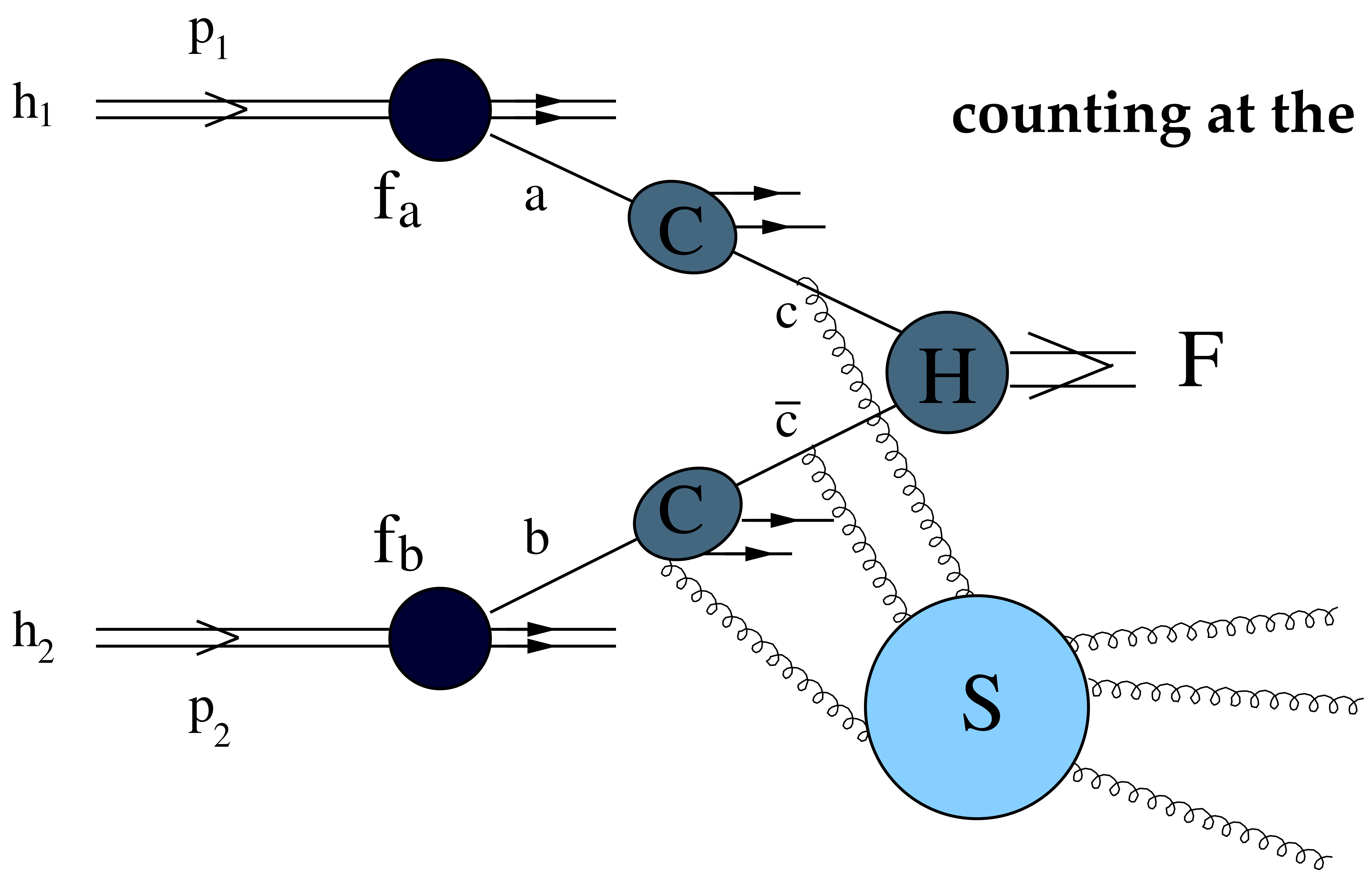


**expansion parameter:** $\alpha_S(Q) \times \ln\frac{Q^2 b^2}{b_0^2} = a_S L \sim 1$

**counting at the level of the exponent** $\sim \exp[Lg_1 + g_2 + \frac{\alpha_s}{\pi}g_3]$

| LL | NLL | NNLL | requires: |
|---|---|---|---|
| $\alpha_S L^2$ | $\alpha_S L$ | | $A_c^{(1)}$ |
| $\alpha_S^2 L^3$ | $\alpha_S^2 L^2$ | $\alpha_S^2 L$ | |
| $\vdots$ | $\vdots$ | $\vdots$ | |
| $\alpha_S^k L^{k+1}$ | $\alpha_S^k L^k$ | $\alpha_S^k L^{k-1}$ | |
| $\vdots$ | $\vdots$ | $\vdots$ | |

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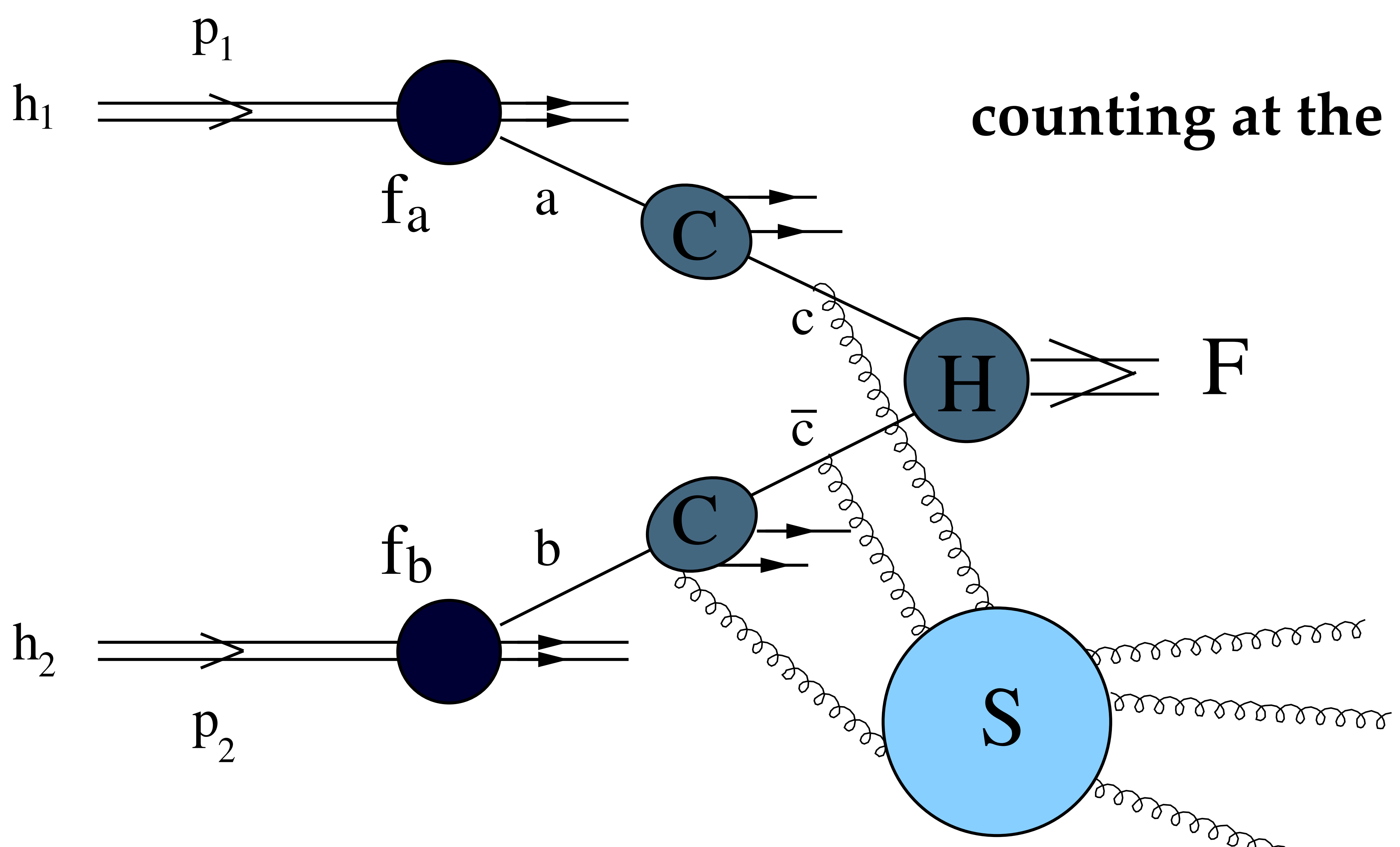
**expansion parameter:** $\alpha_S(Q) \times \ln \frac{Q^2 b^2}{b_0^2} = a_S L \sim 1$

**counting at the level of the exponent** $\sim \exp[L g_1 + g_2 + \frac{\alpha_s}{\pi} g_3]$




| LL | NLL | NNLL |
|---|---|---|
| $\alpha_S L^2$ | $\alpha_S L$ | |
| $\alpha_S^2 L^3$ | $\alpha_S^2 L^2$ | $\alpha_S^2 L$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| $\alpha_S^k L^{k+1}$ | $\alpha_S^k L^k$ | $\alpha_S^k L^{k-1}$ |
| $\vdots$ | $\vdots$ | $\vdots$ |

**requires:**

$A_c^{(1)}, A_c^{(2)}, B_c^{(1)}, C_c^{(1)}, H_F^{(1)}$

(plus **beta function** at 2loop and **collinear anomalous dimensions** at 1loop)

$q_T$ **subtraction @ NLO**

# $q_T$-subtraction formalism: color-less final state

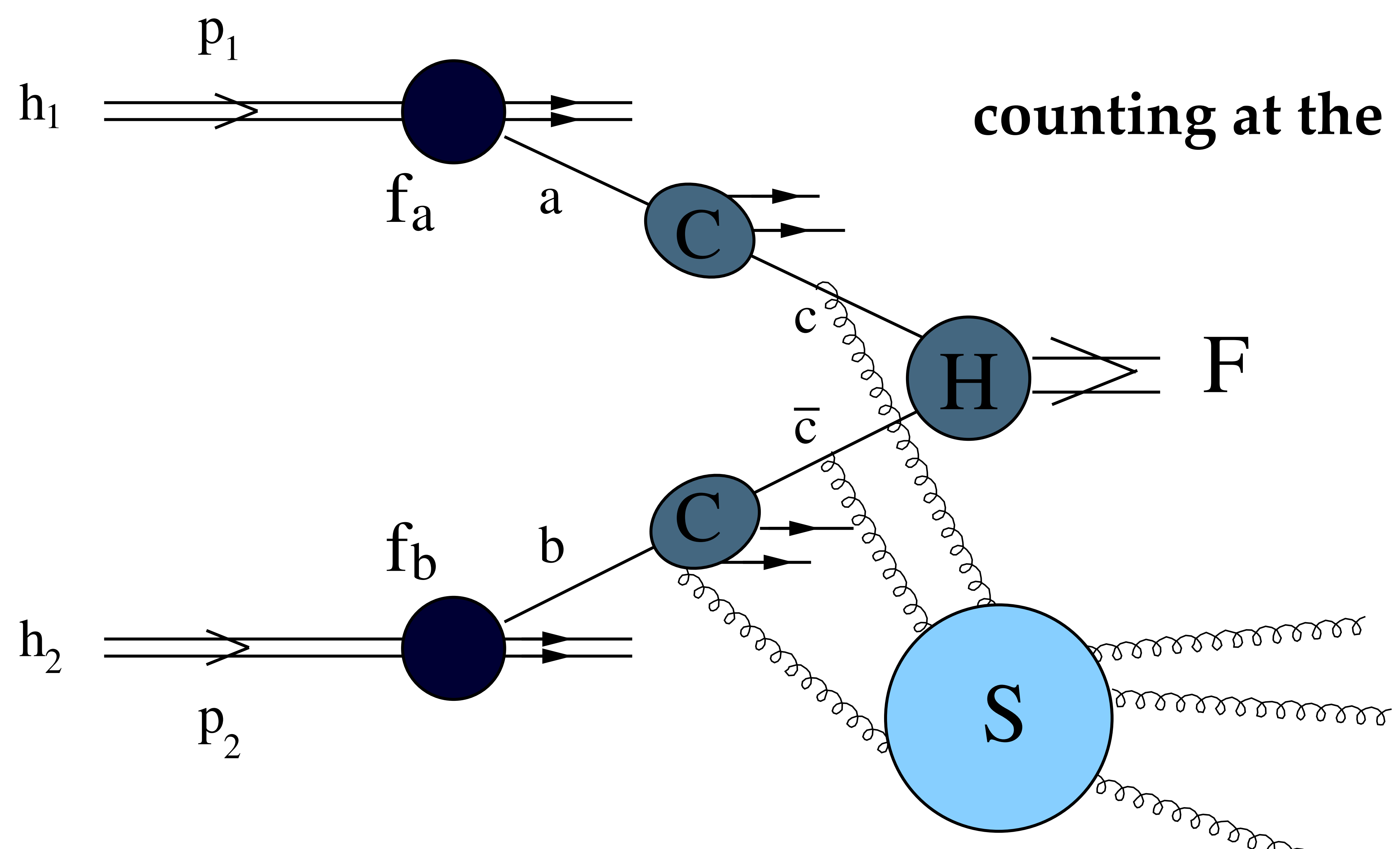
**color-less system** $F$**:** $(Q^2, Y, q_T)$

[Catani, de Florian, Grazzini, 2001]

$$\frac{d\sigma^{(sing)}}{dQ^2 dY dq_T d\Omega} = \frac{1}{S}\sum_c \frac{d\sigma^{(0)}_{c\bar{c},F}}{d\Omega}\int_0^\infty db\frac{b}{2}J_0(bq_T)S_c(Q,b)\sum_{a_1,a_2}\int_{x_1}^1\frac{dz_1}{z_1}\int_{x_2}^1\frac{dz_2}{z_2}[H^F C_1 C_2]_{c\bar{c};a_1a_2} f_{a_1/h_1}(x_1, b_0^2/b^2) f_{a_2/h_2}(x_2, b_0^2/b^2)$$

**expansion parameter:** $\alpha_S(Q)\times\ln\frac{Q^2b^2}{b_0^2} = a_S L \sim 1$

**counting at the level of the exponent** $\sim \exp[Lg_1 + g_2 + \frac{\alpha_s}{\pi}g_3]$




| LL | NLL | NNLL |
|---|---|---|
| $\alpha_S L^2$ | $\alpha_S L$ | |
| $\alpha_S^2 L^3$ | $\alpha_S^2 L^2$ | $\alpha_S^2 L$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| $\alpha_S^k L^{k+1}$ | $\alpha_S^k L^k$ | $\alpha_S^k L^{k-1}$ |
| $\vdots$ | $\vdots$ | $\vdots$ |

**requires:**

$A_c^{(1)}, A_c^{(2)}, B_c^{(1)}, C_c^{(1)}, H_F^{(1)},$

$A_c^{(3)}, B_c^{(2)}, C_c^{(2)}, H_F^{(2)}$

plus **beta function** at 2loop and **collinear anomalous dimensions** at 2loop)

**$q_T$ subtraction @ NNLO**

# $q_T$-subtraction formalism: extension to massive final state

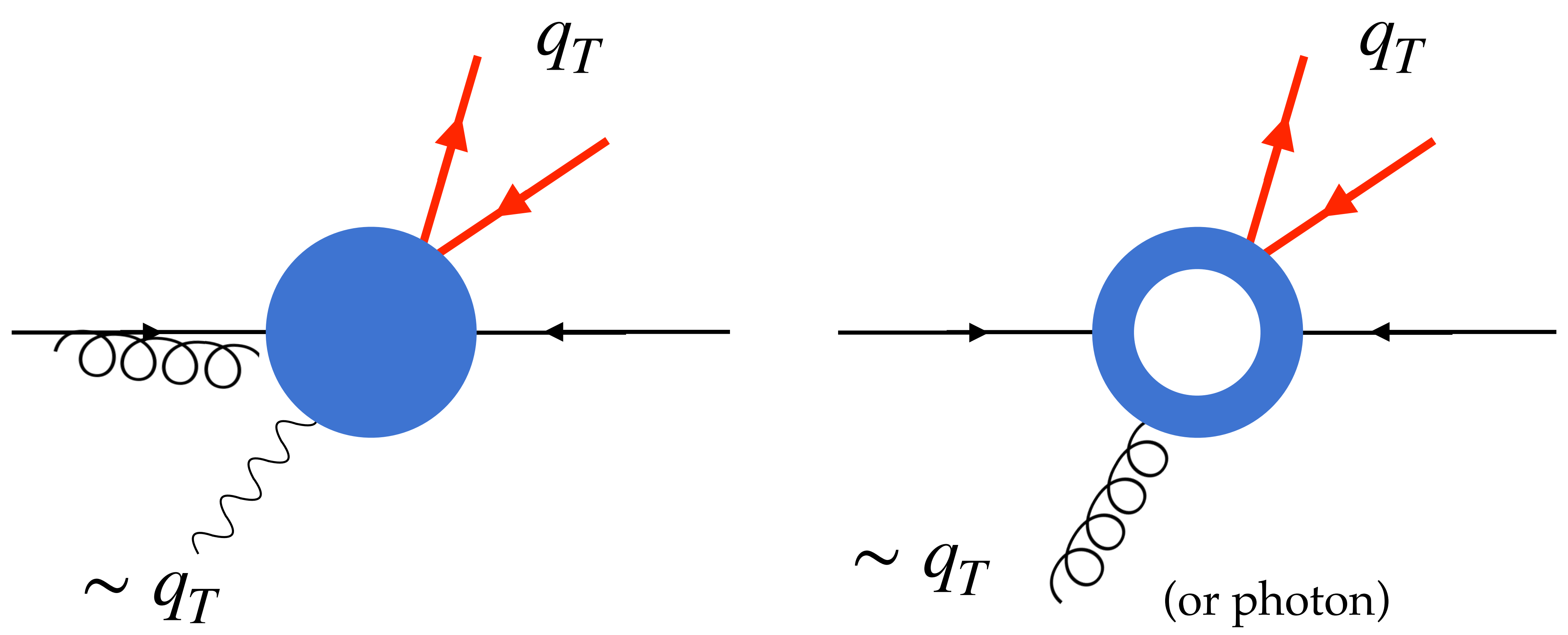
The transverse momentum of the final-state system controls the radiation emitted from initial-state partons

In the presence of **massless radiators** in the final state at LO, a different observable must be used, like $N$-jettiness or $k_T$-ness

Example: mixed QCDxQED corrections to Drell-Yan dilepton production $p + p \to \ell + \bar{\ell} + X$

**Initial-state radiation**

For $q_T > 0$ **one emission is always resolved**



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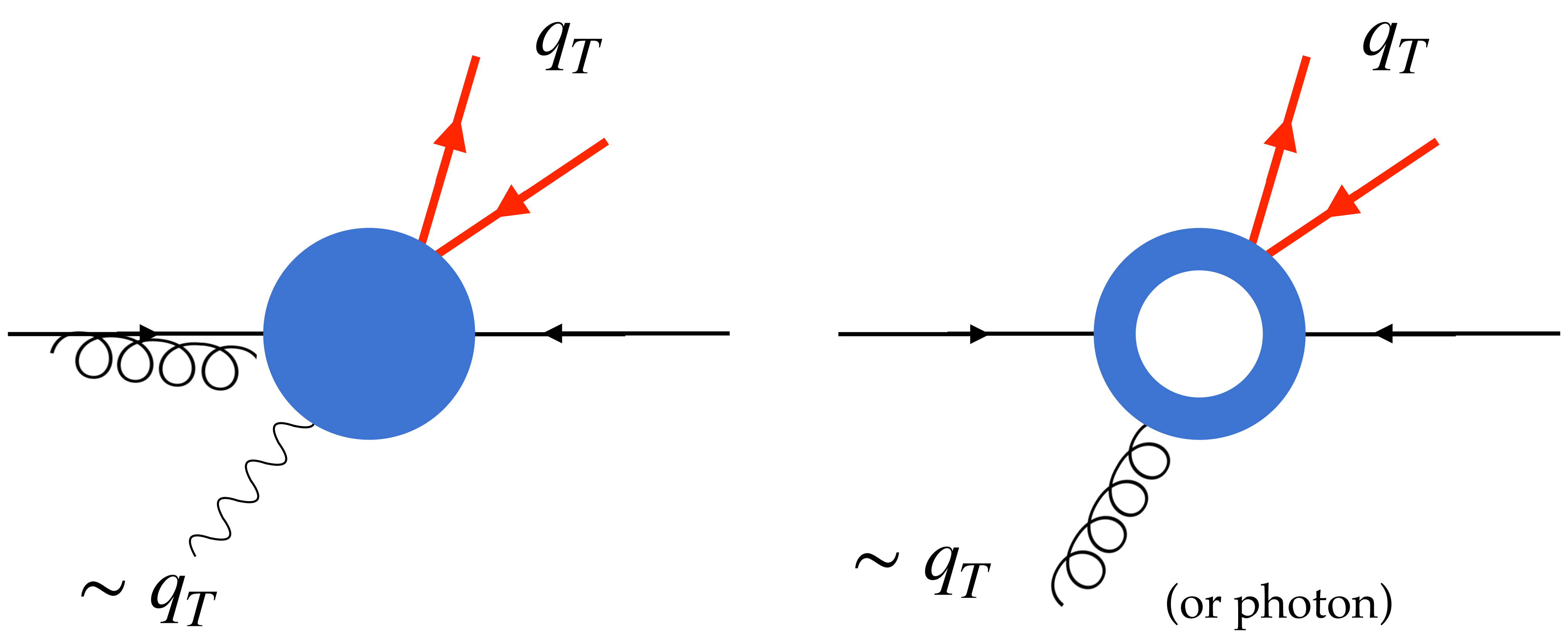
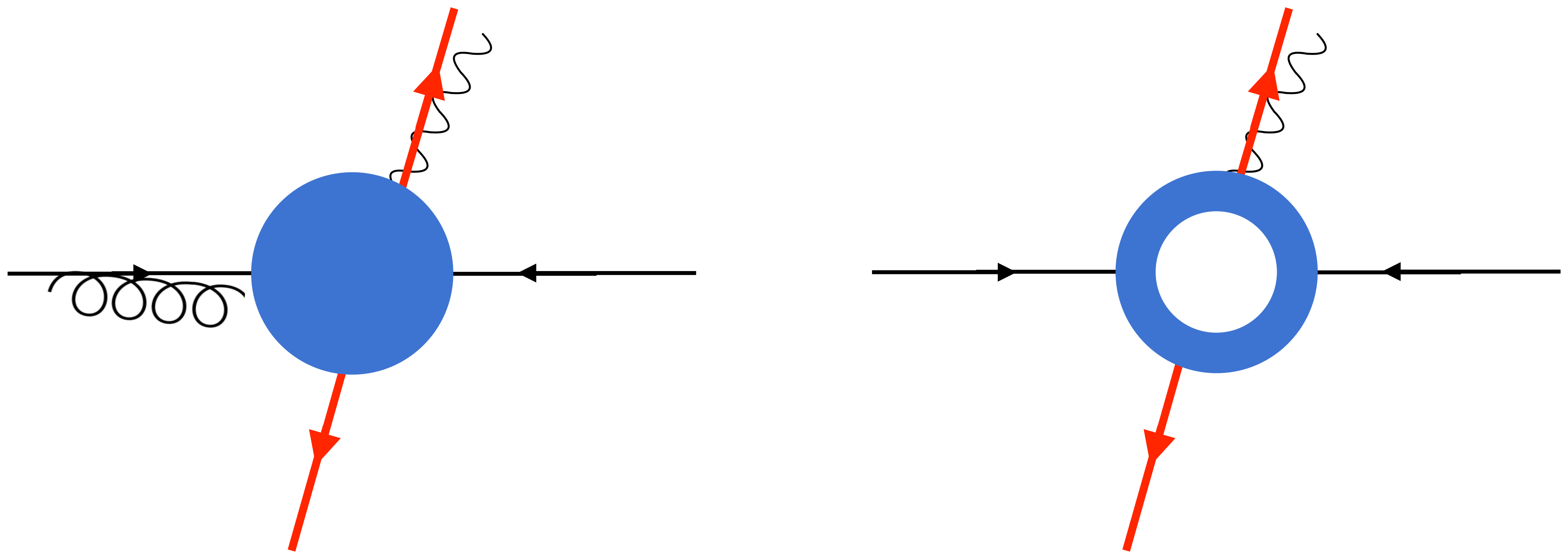



**Final-state (collinear) radiation**

There are configurations with $q_T > 0$ and **two unresolved emission if leptons are massless**



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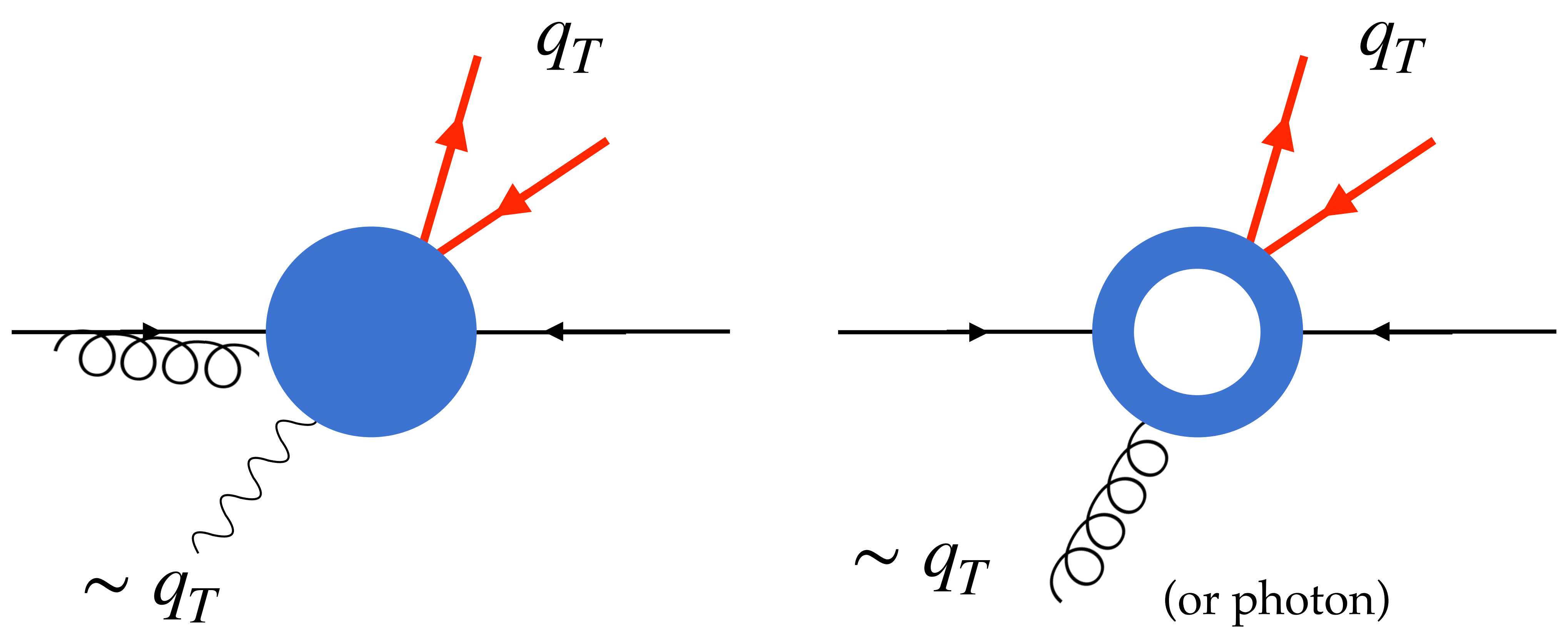


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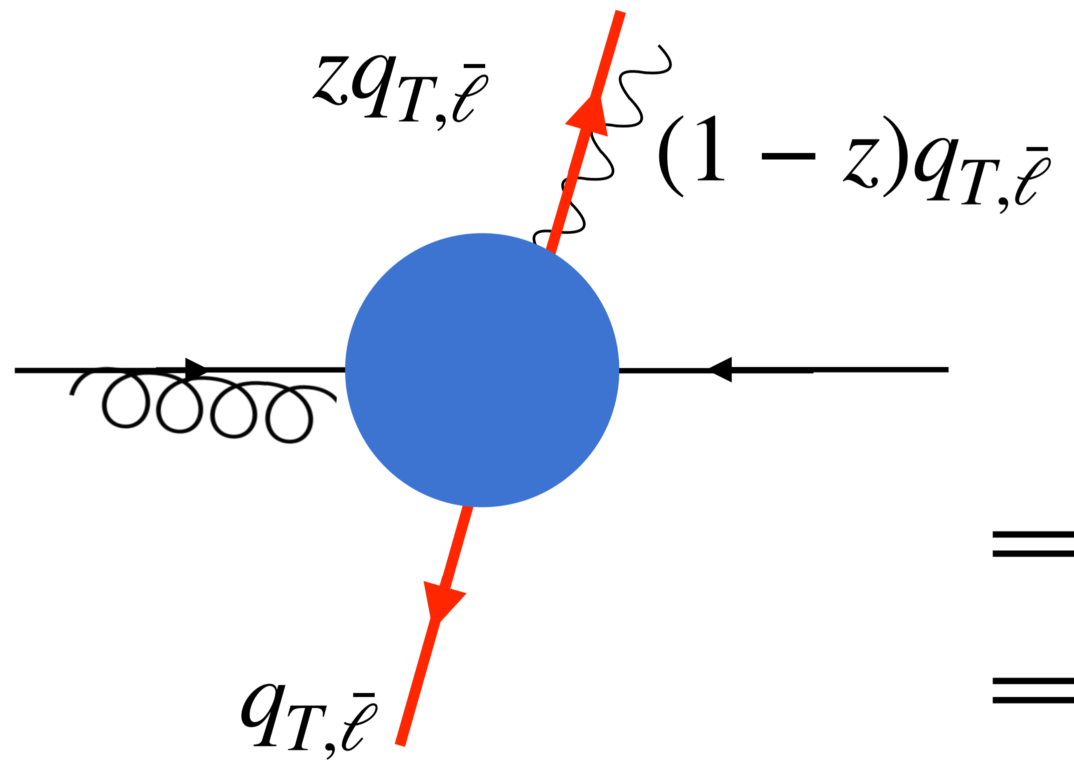


$$\mathbf{q}_{T,\bar{\ell}} + \mathbf{q}_{T,g} + \mathbf{q}_{T,\ell\gamma} = 0$$

$$\Longrightarrow -q_{T,\bar{\ell}} + zq_{T,\bar{\ell}} + (1-z)q_{T,\bar{\ell}} \sim 0$$

$$\Longrightarrow q_T = (1-z)q_{T,\bar{\ell}} > 0$$

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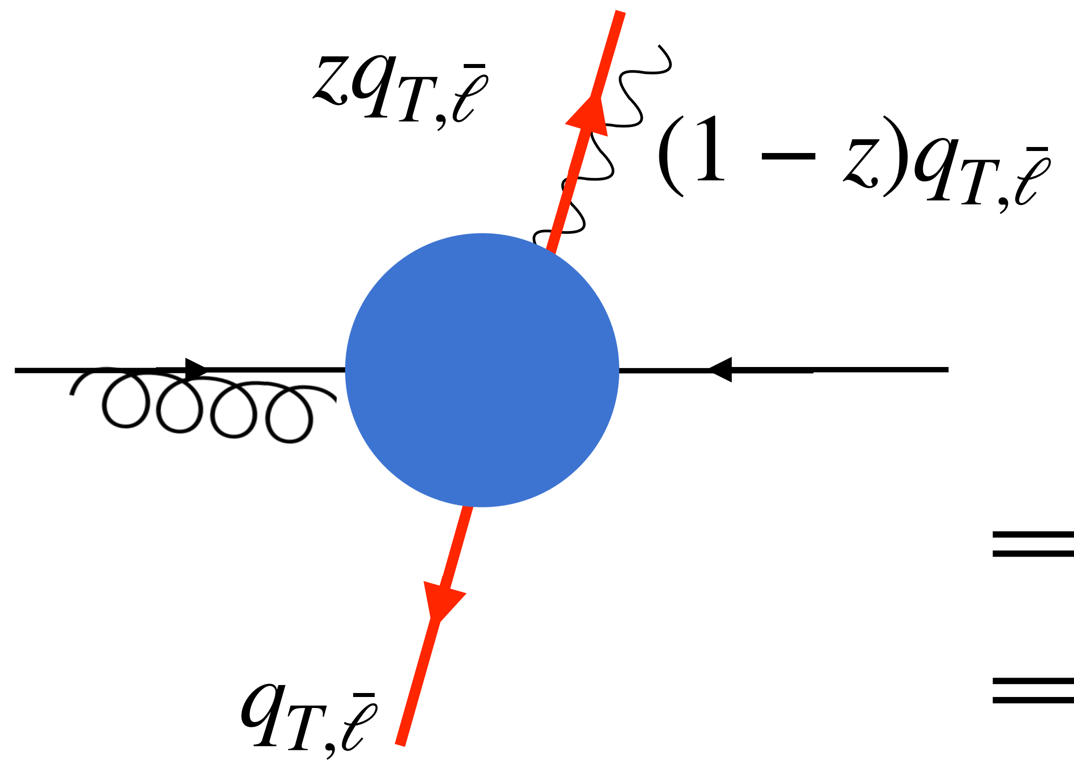
**Example:** mixed QCDxQED corrections to Drell-Yan dilepton production $p + p \to \ell + \bar{\ell} + X$

We need **massive** leptons to resolve/regulate the singular limits associated to a photon collinear to a final-state lepton (or coloured massive particles)!

Same reasoning applies to **heavy-quark** production

**Final-state (collinear) radiation**

There are configurations with $q_T > 0$ and **two unresolved emission if leptons are massless**

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# $q_T$-subtraction formalism: extension to massive final state

$$d\sigma_{NNLO} = \mathcal{H} \otimes d\sigma_{LO} + \int \left[ d\sigma^{R}_{NLO} - d\sigma^{CT}_{NNLO} \right]_{q_T > q_T^{\rm cut}} + \mathcal{O}\left( (q_T^{\rm cut})^p \right)$$

All ingredients for $F + j$ @ NLO available:

Required matrix elements implemented in **public one-loop provider** such as OpenLoops2 and Recola ✔

[Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller 2019]
[Actis, Denner, Hofer, Lang, Scharf, Uccirati, 2017]

**Efficient local subtraction scheme** available, for example dipole subtraction ✔

[Catani, Seymour, 1998] [Catani, Dittmaier, Seymour, Trocsanyi 2002]

Automatised implementation in the MATRIX framework, which relies on the efficient multi-channel Monte Carlo integrator MUNICH

[Grazzini, Kallweit, Wiesemann, 2017] [Kallweit in preparation]

# $q_T$-subtraction formalism: extension to massive final state

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$\mathcal{H}$ contains virtual correction after subtraction of IR singularities and contribution of soft/collinear origin

- Beam functions ✓
- Soft function

[Catani, Cieri, de Florian, Ferrera, Grazzini , 2012]
[Gehrmann, Luebbert, Yang, 2014]
[Echevarria, Scimemi, Vladimirov, 2016]
[Luo, Wang, Xu, Yang, Yang, Zhu, 2019]
[Ebert, Mistlberger, Vita]

Same beam function of color-singlet production

Computed up to N$^3$LO

# $q_T$-subtraction formalism: extension to massive final state

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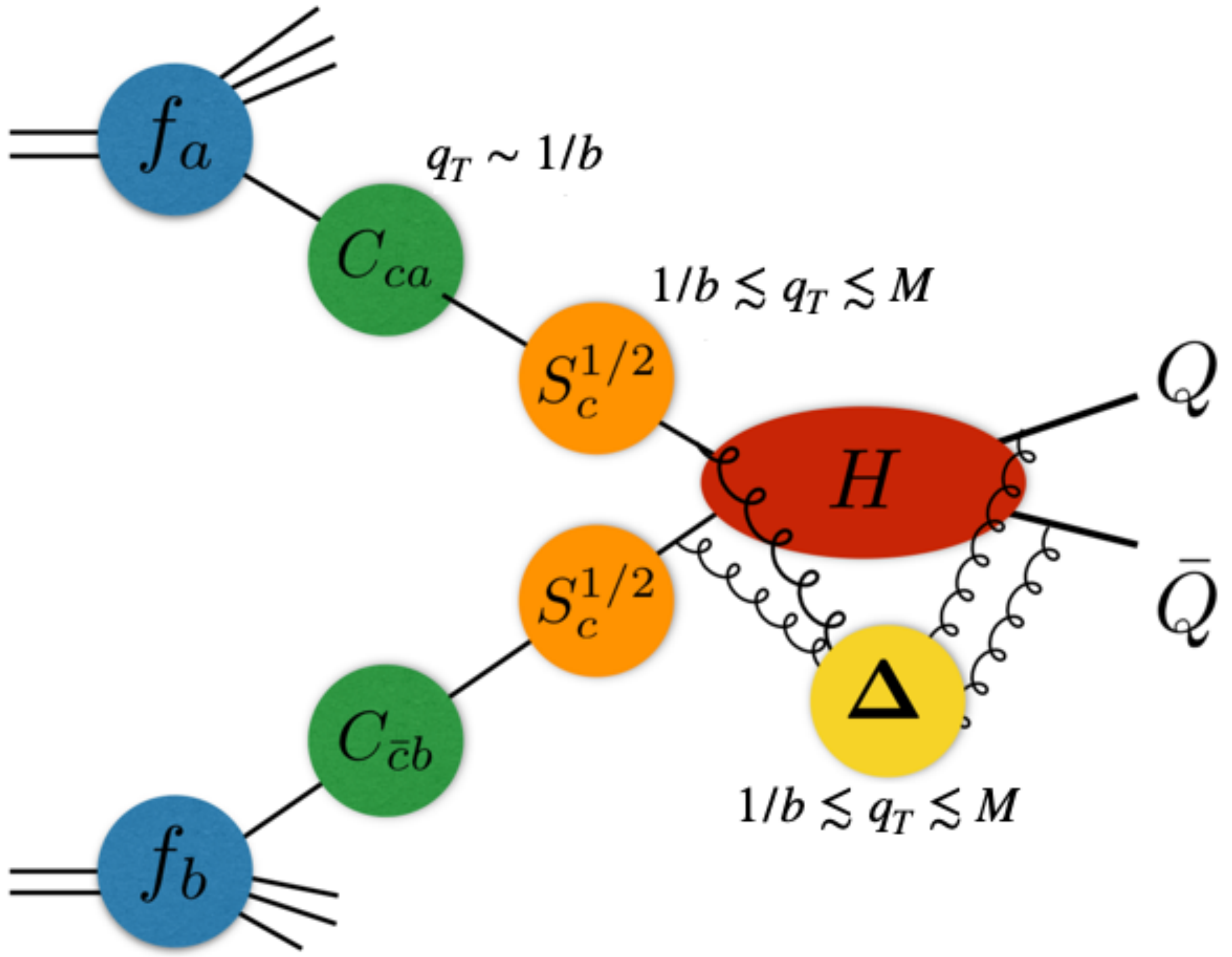


The resummation formula shows a **richer structure** because of additional soft singularities

- Soft logarithms controlled by the **transverse momentum anomalous dimension** $\boldsymbol{\Gamma}_t$ known up to NNLO [Mitov, Sterman, Sung, 2009], [Neubert, et al 2009]
- Hard coefficient gets a **non-trivial** colour structure (matrix in colour-space)
- Non trivial azimuthal correlations

# $q_T$-subtraction formalism: extension to massive final state

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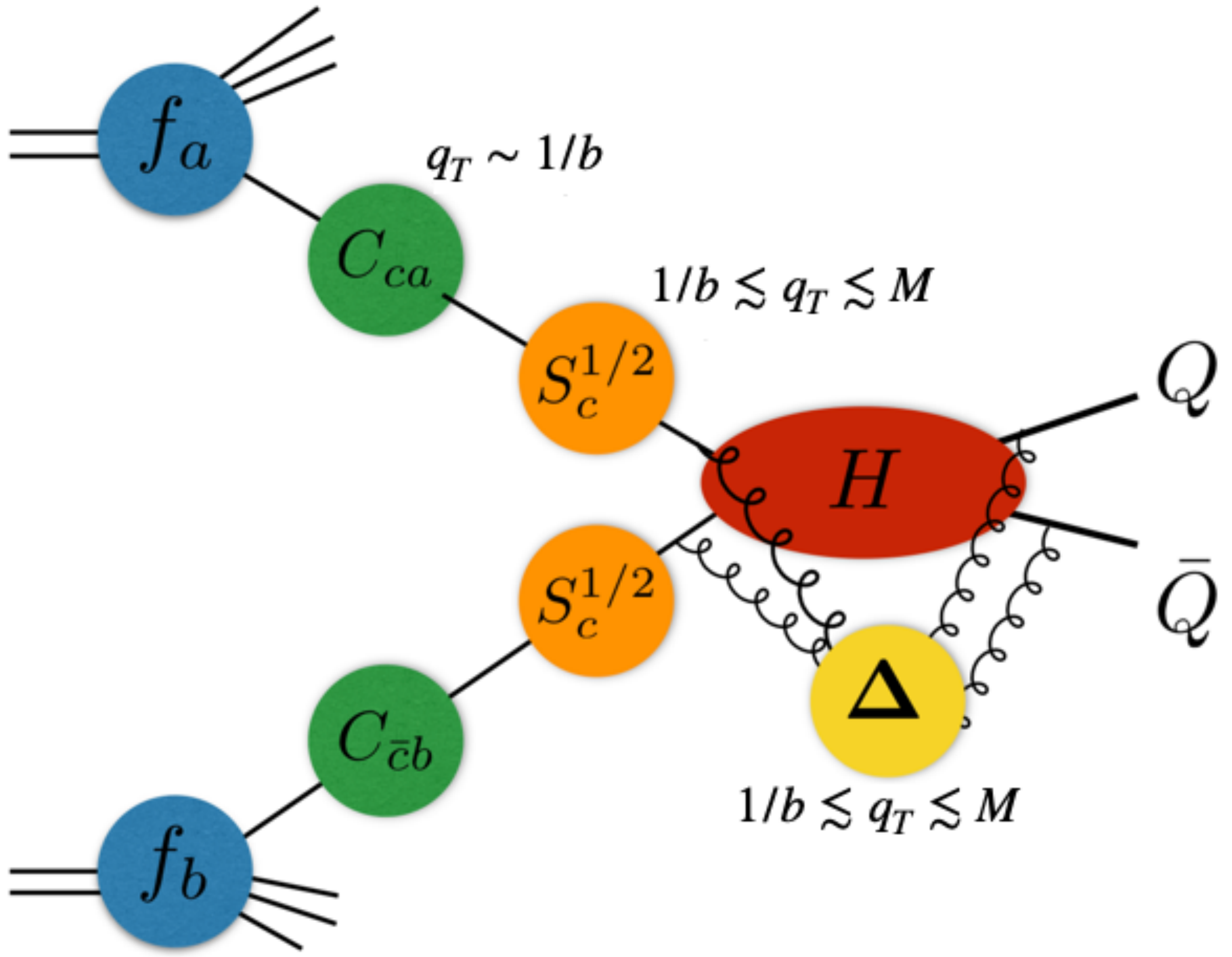



The resummation formula shows a **richer structure** because of additional soft singularities

Non trivial ingredient

- **Two-loop soft function** for heavy-quark (back-to-back Born kinematic) **[Catani, Devoto, Grazzini, Mazzitelli,2023]**
- Recently generalised to **arbitrary kinematics** **[Devoto, Mazzitelli in preparation]**

# $q_T$-subtraction formalism: recent calculations

$$d\sigma_{NNLO} = \mathscr{H} \otimes d\sigma_{LO} + \int \left[d\sigma^{R}_{NLO} - d\sigma^{CT}_{NNLO}\right]_{q_T > q_T^{\mathrm{cut}}} + \mathcal{O}\left((q_T^{\mathrm{cut}})^p\right)$$

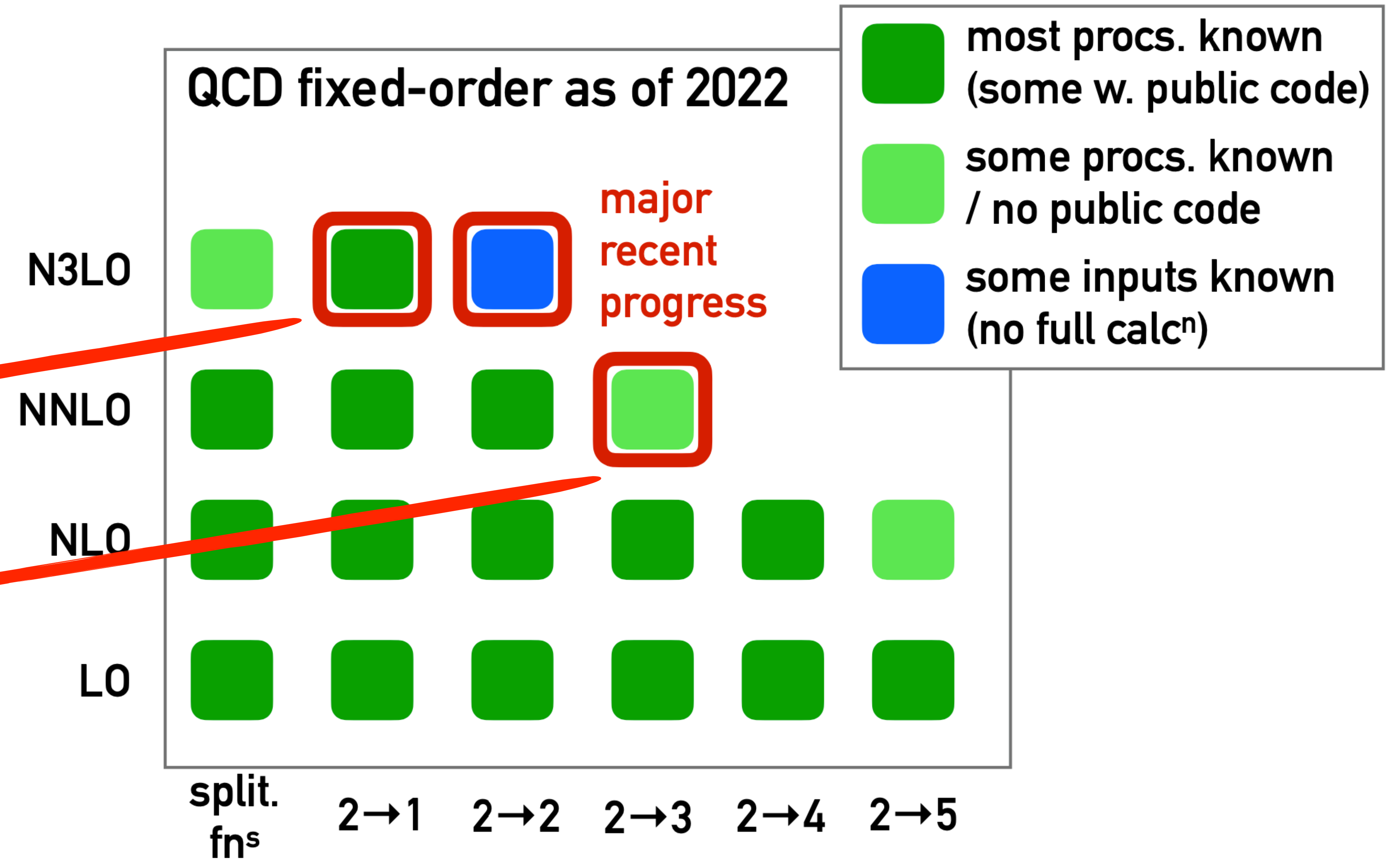
$\mathscr{H}$ contains virtual correction after subtraction of IR singularities and contribution of soft/collinear origin

Once the corresponding two-loop amplitudes are available, the framework allows one to calculate

- (N)NNLO QCD corrections for colour-singlet, heavy-quark, heavy-quark plus colour-singlet

DY

$Ht\bar{t}$, $Wt\bar{t}$, $Wb\bar{b}$
(with approximate two-loop amplitudes)



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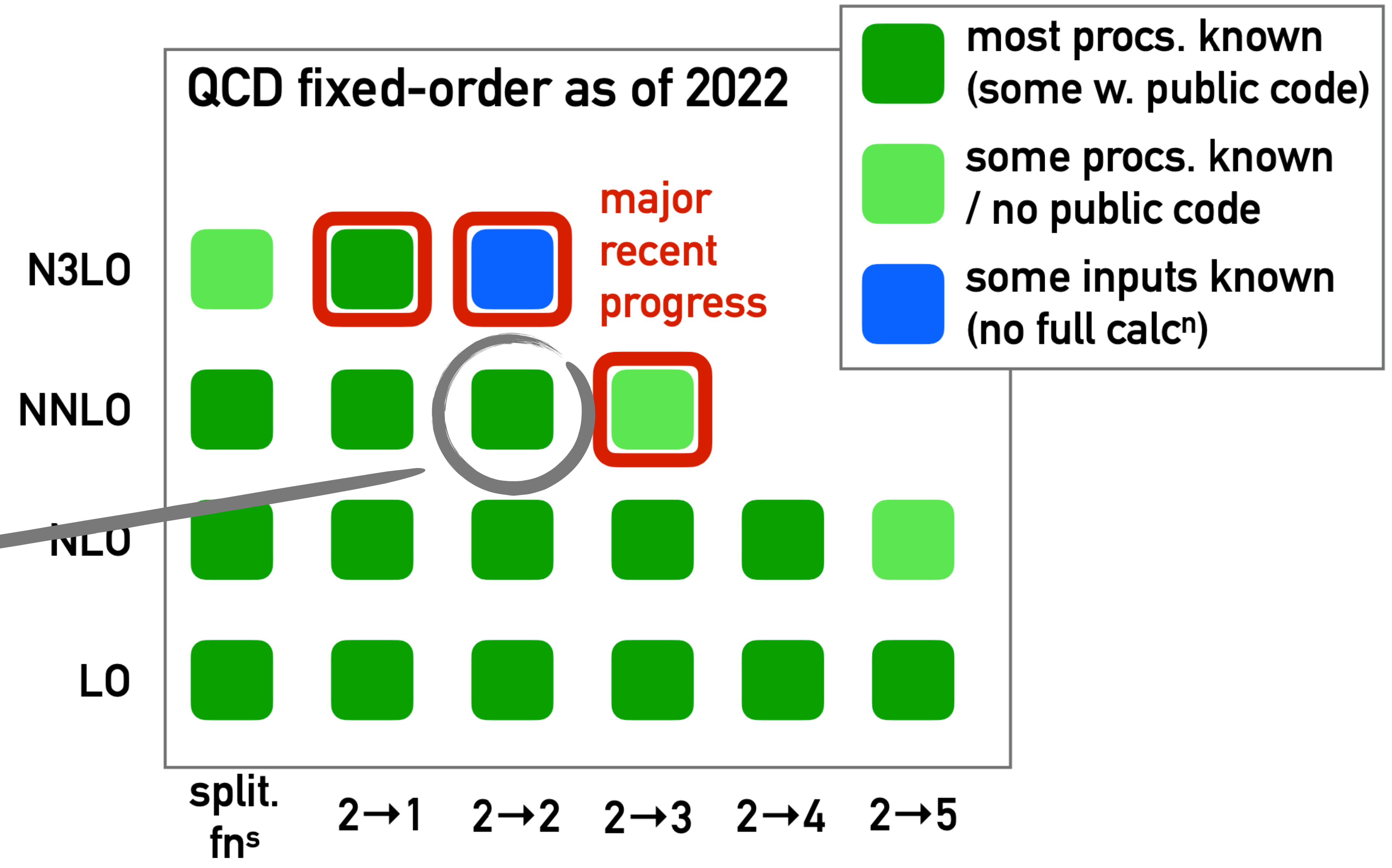
- (N)NNLO QCD corrections for colour-singlet, heavy-quark, heavy-quark plus colour-singlet
- **mixed QCDxEW corrections** to color singlet-production




NC DY @ $\mathcal{O}(\alpha\alpha_S)$, two-loop $2 \rightarrow 2$ but many scales!

# Outline

Slicing methods

- toy-model example
- connection to resummation
- $q_T$-subtraction

Applications to NC Drell-Yan process

Remarks

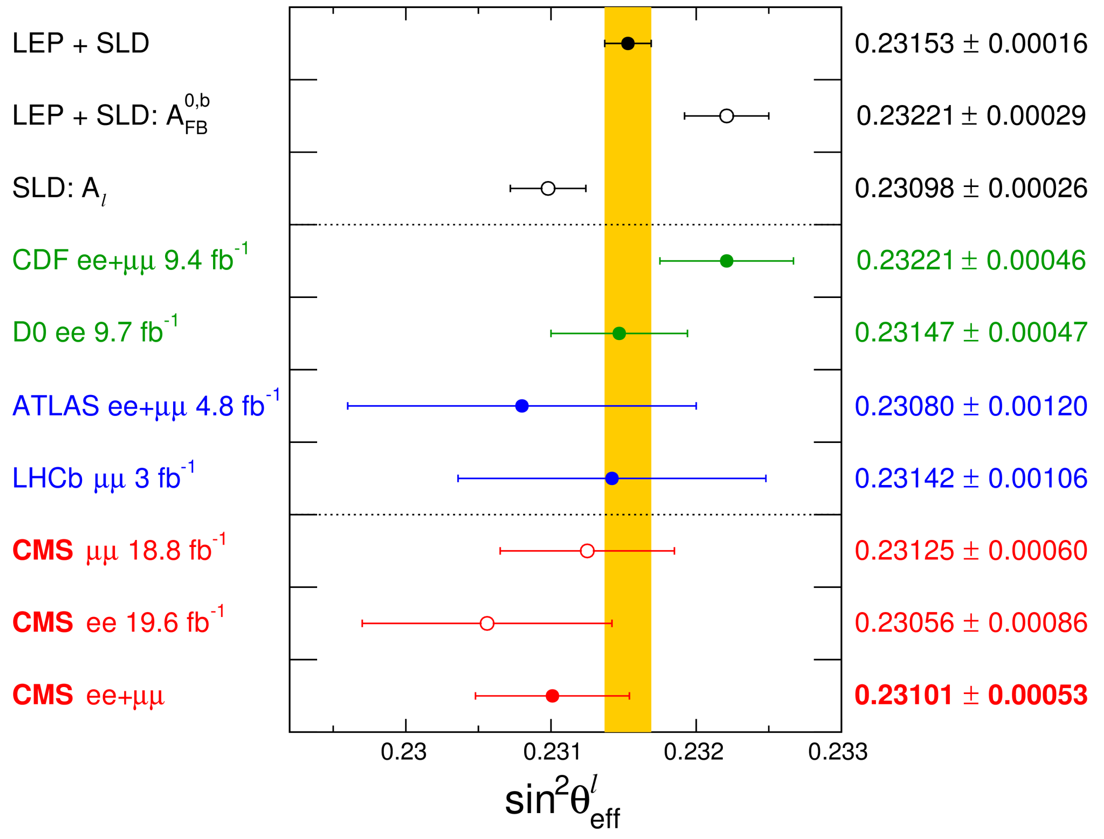
# State-of-art predictions for NC Drell-Yan: intro

**Resonant Region**

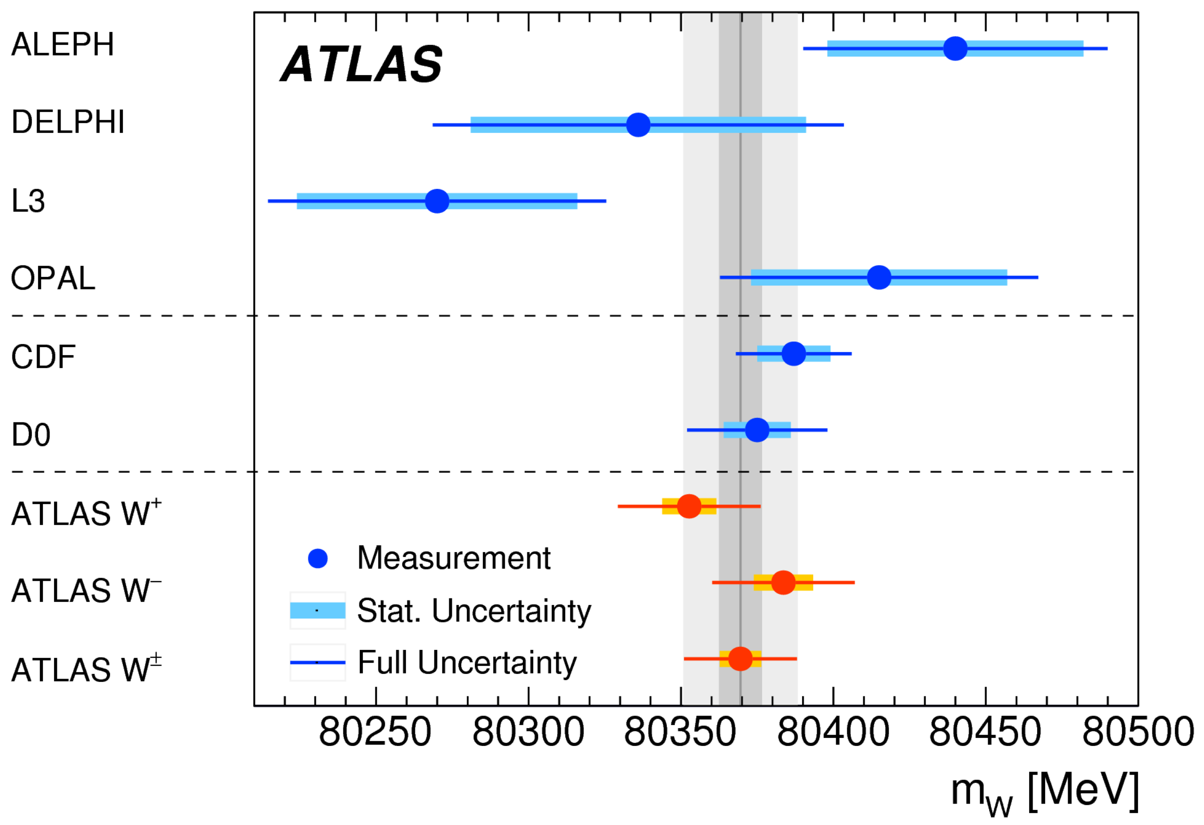
**LHC** Electro-Weak precision physics:

- extremely **precise** determination of **W mass** 80.354±0.007 GeV with expected **uncertainties** at the level of $\mathcal{O}(10\,\text{MeV})$ at the end of HL-LHC
- measurement of the effective mixing angle starts to **compete with LEP**: $\sin^2\theta^{\ell}_{\text{eff}} = 0.23101 \pm 0.00053$

CMS 1806.00863

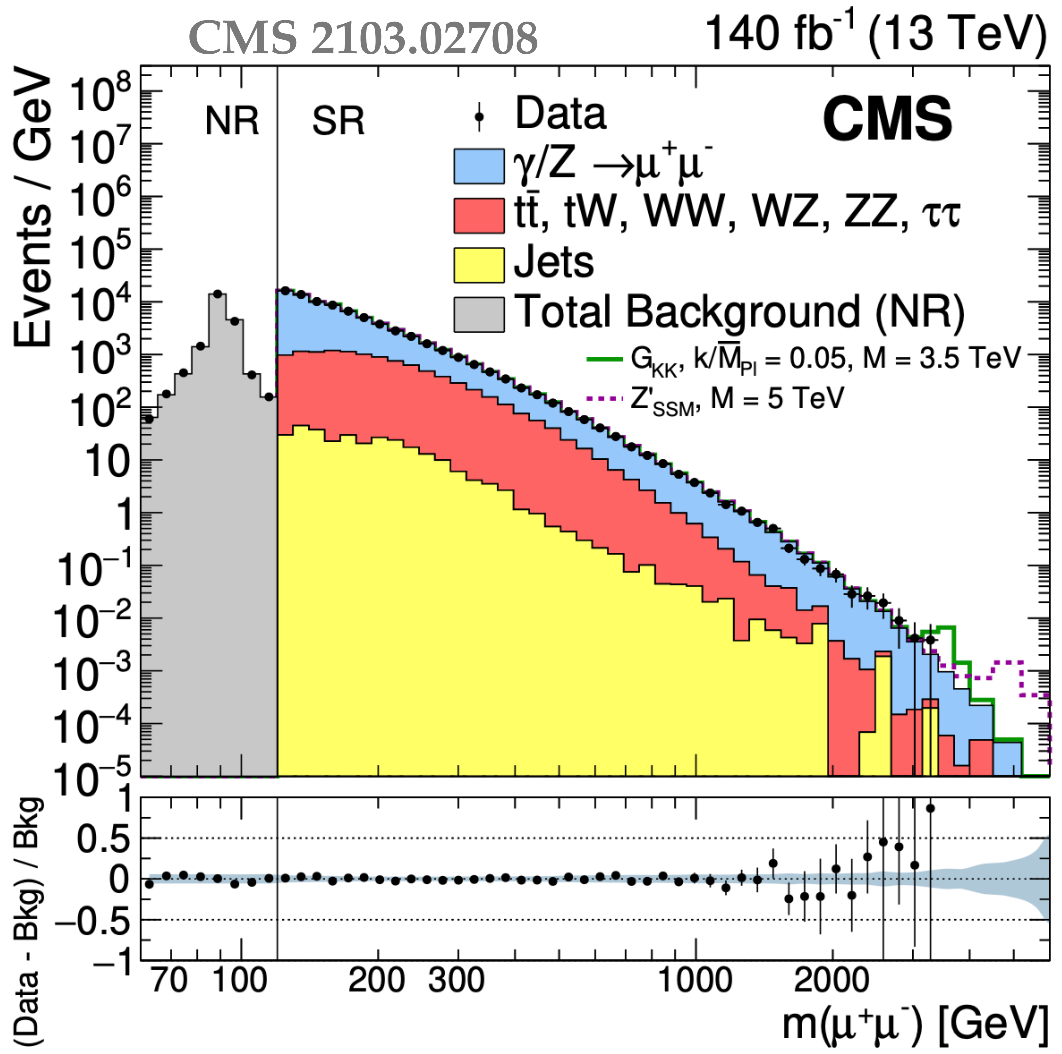



ATLAS 1701.07240




**Off-Shell Region**

CMS 2103.02708




| mass window [GeV] | stat. unc. 140fb$^{-1}$ | stat. unc. 3ab$^{-1}$ |
|---|---|---|
| **$600<m_{\mu\mu}<900$** | 1.4% | 0.2% |
| **$900<m_{\mu\mu}<1300$** | 3.2% | 0.6% |

- Modelling of the SM background **relevant** for new physics searches
- Measurement of the dilepton invariant mass spectrum **expected at $\mathcal{O}(1\%)$ at $m_{\ell\ell} \sim 1$ TeV**
- Requires control of the SM prediction at the $\mathcal{O}(0.5\%)$ level in the TeV

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$$\sigma = \int dx_1 dx_2 f_{a/h_1}(x_1,\mu_F) f_{b/h_2}(x_2,\mu_F) \hat{\sigma}_{ab}(\hat{s},\mu_R,\mu_F) + \mathcal{O}(\Lambda/Q)$$

$$\hat{\sigma}_{ab} = \hat{\sigma}^{(0,0)}_{ab} + \hat{\sigma}^{(1,0)}_{ab} + \hat{\sigma}^{(2,0)}_{ab} + \hat{\sigma}^{(3,0)}_{ab} + \ldots$$

QCD

☑ QCD corrections dominant effects. They are known up to

- **NNLO differential cross sections**
  [Anastasiou, Dixon, Melnikov, Petriello (2003)], [Melnikov, Petriello (2006)] [Catani, Cieri, Ferrera, de Florian, Grazzini (2009)] [Catani, Ferrera, Grazzini (2010)]
- **N$^3$LO inclusive cross sections and di-lepton rapidity distribution**
  [Duhr, Dulat, Mistlberger (2020)] [Chen, Gehrmann, Glover, Huss, Yang, and Zhu (2021)] [Duhr, Mistlberger (2021)]
- **N$^3$LO fiducial cross sections and distributions**
  [Camarda, Cieri, Ferrera (2021)], [Chen, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli (2022)]

# Power Corrections (PCs) in the Drell-Yan process

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  [Ebert, Tackmann, 2019],[Salam, Slade, 2021]
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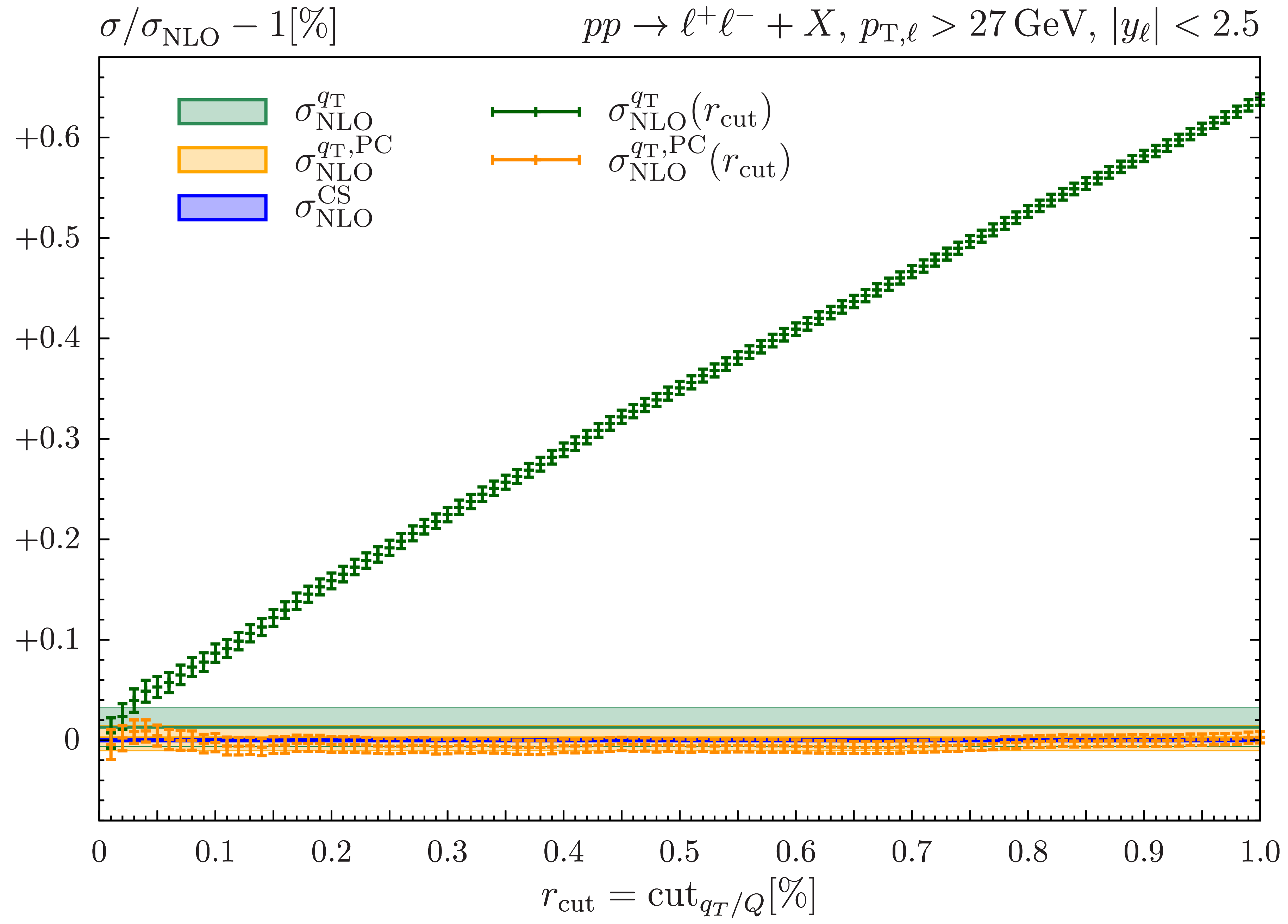


For example, **symmetric cuts** on the dilepton final state

$$p_{T,\ell} > p_{\rm cut}, p_{T,\bar{\ell}} > p_{\rm cut}$$

but also **asymmetric cuts on the transverse momenta of the hardest and softest lepton** lead to the same problem

**Remark:** this is connected to a more fundamental problem of the convergence of the perturbative espansion

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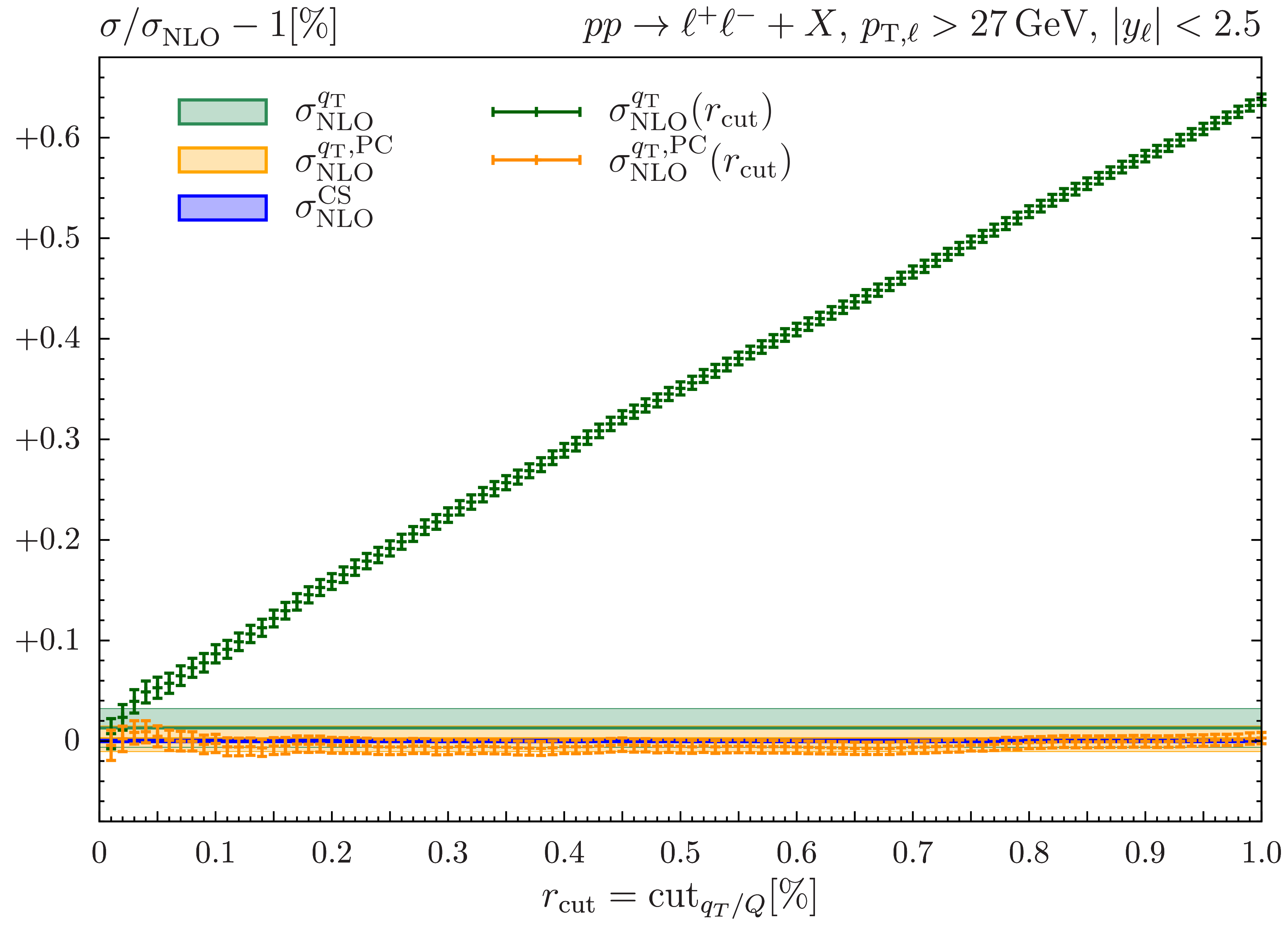
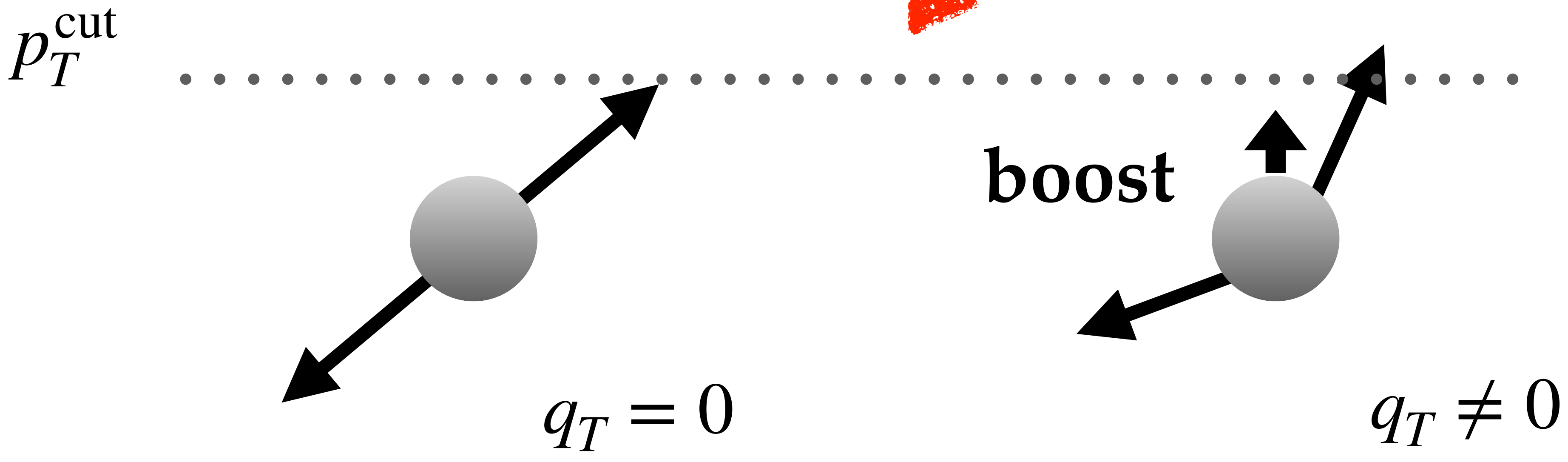



[**LB**, Kallweit, Rottoli, Wiesemann, 2021], [Camarda, Cieri, Ferrera, 2021]

Concerning the slicing method, there is a general solution for this class of fiducial power corrections through a modification of the subtraction formula:

$$\Delta\sigma^{\rm linPCs}(r_{\rm cut}) = \int d\Phi_{\rm F} \int_0^{r_{\rm cut}} dr' \left[ \frac{d\sigma^{\rm F}_{\rm CT}}{d\Phi_{\rm F}} \Theta_{\rm cuts}\left(\Phi^{\rm rec}_{\rm F}(\Phi_{\rm F}, r')\right) - \frac{d\sigma^{\rm F}_{\rm CT}}{d\Phi_{\rm F}} \Theta_{\rm cuts}(\Phi_{\rm F}) \right]$$



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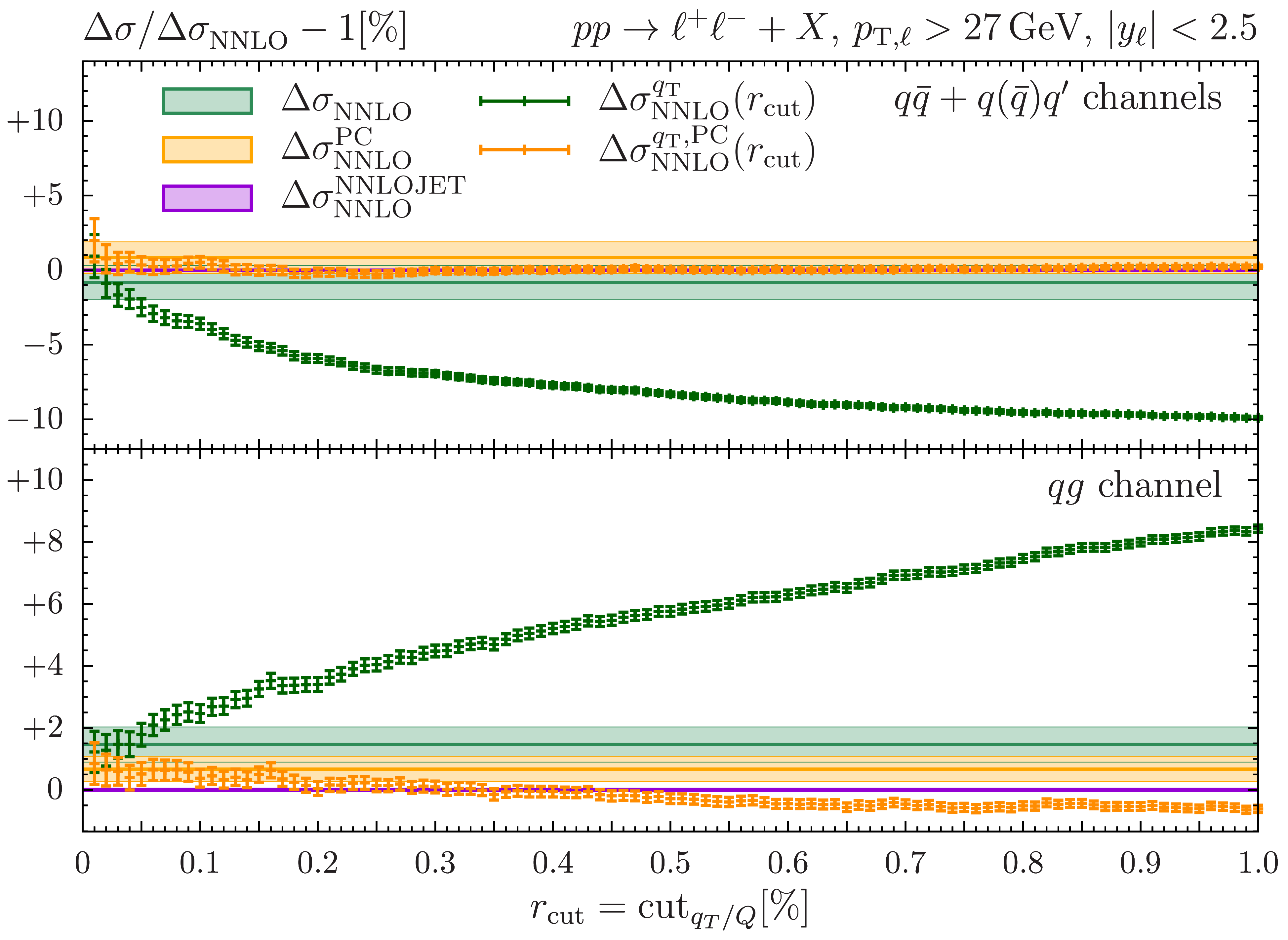


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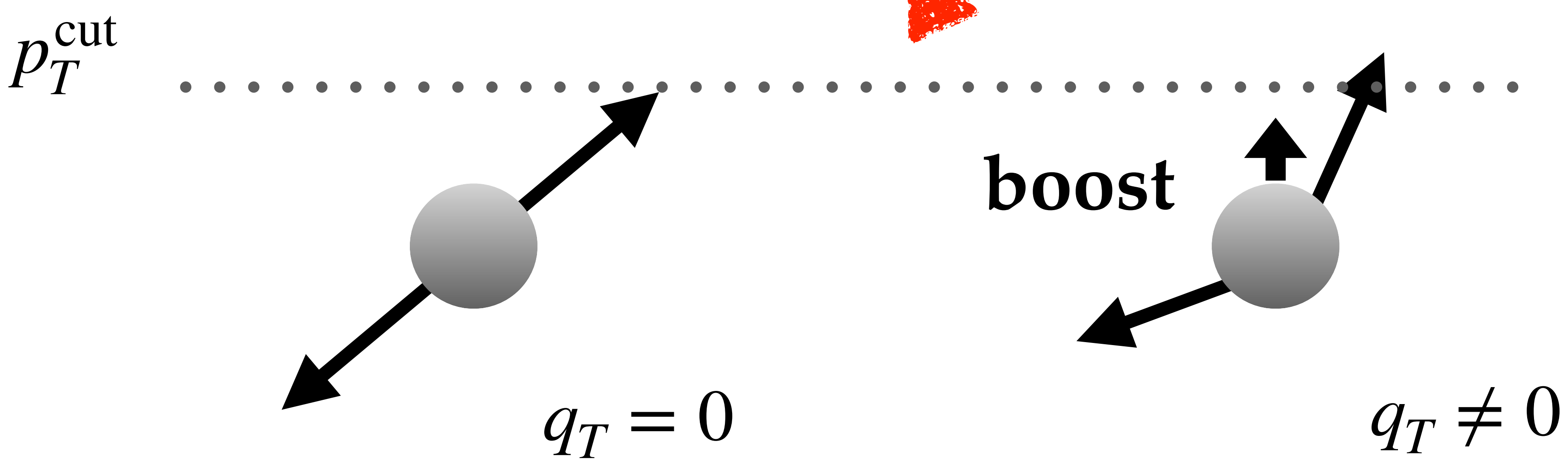

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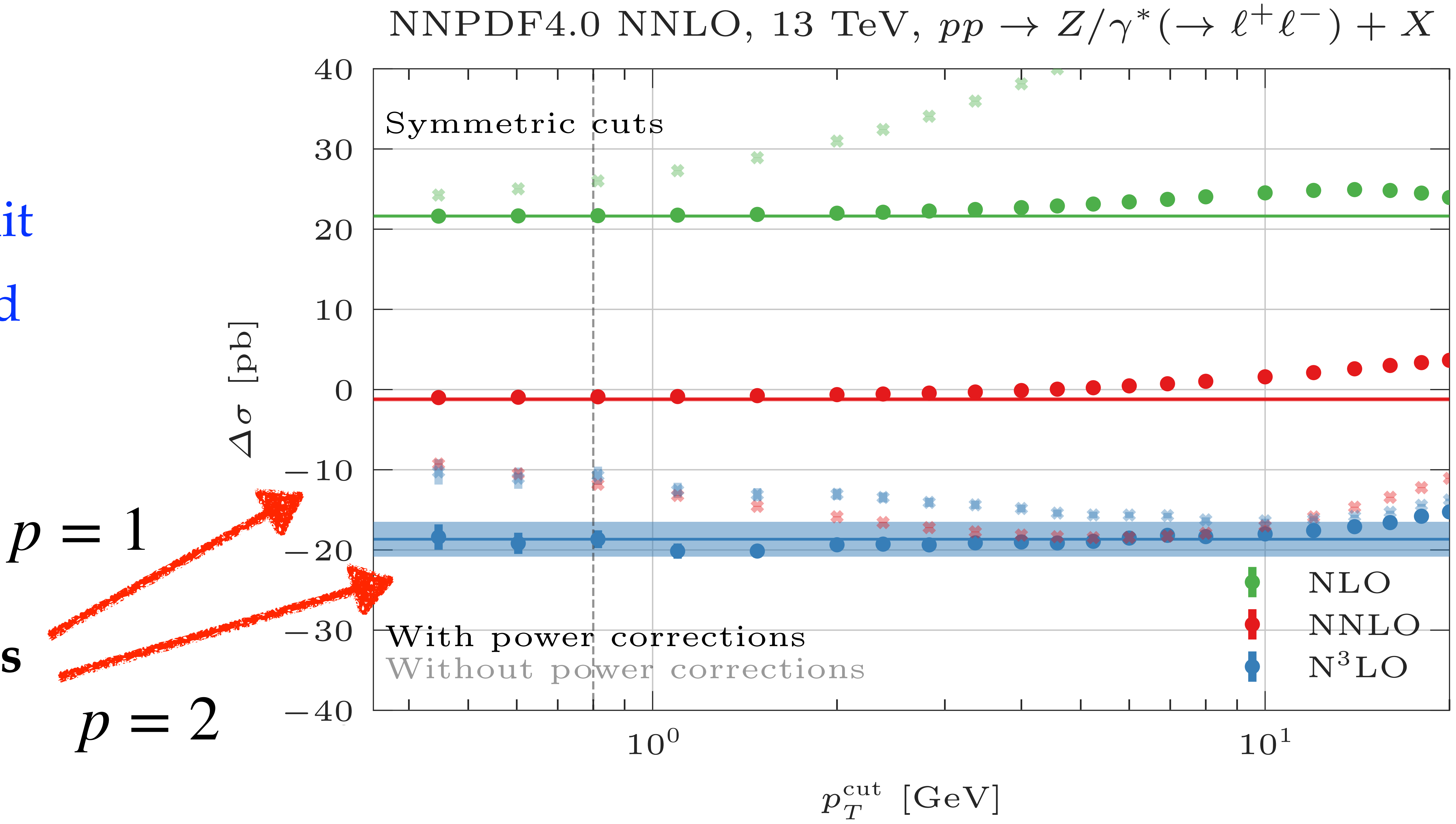
All the ingredients avaialble and efficiently computable
time load: $\mathcal{O}(\lesssim 1\%)$

[X. Chen, T. Gehrmann, E.W.N. Glover, A. Huss, P. Monni, E. Re, L. Rottoli, P. Torrielli, 2023]

Calculation of $Z + 1j$ @ NNLO pushed to its limit (performed with ANTENNA subtraction as implemented in the NNLOJET code)

- 2-loop amplitudes pushed in single unresolved limit
- 1-loop amplitudes pushed in the double unresolved limit

time load: $\mathcal{O}(99\%)$




Knowledge of linear power corrections **crucial for this application**

# Power Corrections (PCs) in the Drell-Yan process

$$d\sigma_{N^3LO} = \mathcal{H} \otimes d\sigma_{LO} + \int \left[ d\sigma^{R}_{NNLO} - d\sigma^{CT}_{N^3LO} \right]_{q_T > q_T^{\rm cut}} + \mathcal{O}\left( (q_T^{\rm cut})^p \right)$$

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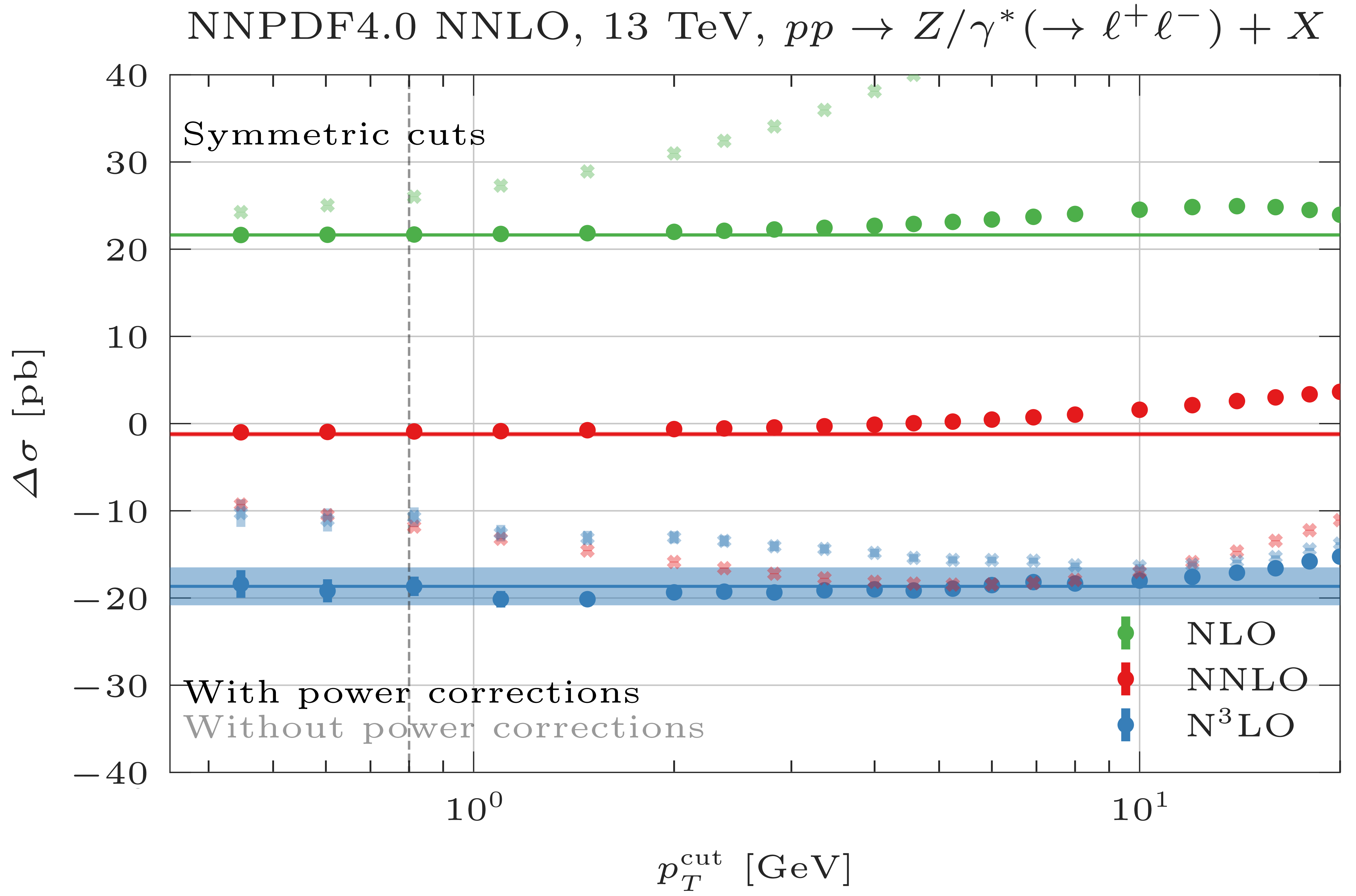
- 2-loop amplitudes pushed in single unresolved limit
- 1-loop amplitudes pushed in the double unresolved limit

time load: $\mathcal{O}(99\%)$

Overall running time: $\mathcal{O}(5M)$ CPU hours
to get results for single differential distributions

Experimentalists can measure triple-differential distributions:
Naive estimate $\mathcal{O}(100M)$ CPU hours

[X. Chen, T. Gehrmann, E.W.N. Glover, A. Huss, P. Monni, E. Re, L. Rottoli, P. Torrielli, 2023]



# State-of-art predictions for NC Drell-Yan: intro

→ very accurate SM predictions!

$$\sigma = \int dx_1 dx_2 f_{a/h_1}(x_1, \mu_F) f_{b/h_2}(x_2, \mu_F) \hat{\sigma}_{ab}(\hat{s}, \mu_R, \mu_F) + \mathcal{O}(\Lambda/Q)$$

$$\hat{\sigma}_{ab} = \hat{\sigma}^{(0,0)}_{ab} + \hat{\sigma}^{(1,0)}_{ab} + \hat{\sigma}^{(2,0)}_{ab} + \hat{\sigma}^{(3,0)}_{ab} + \dots \qquad \text{QCD}$$

$$+\hat{\sigma}^{(0,1)}_{ab} + \dots \qquad \text{EW}$$

☑ **NLO EW** corrections

- known since long
  [S. Dittmaier and M. Kramer (2002)], [Baur,Wackeroth (2004)], [Baur, Brein, Hollik, Schappacher, Wackeroth (2002)]
- nowadays **automatised** in different available generators
  [Les Houches 2017, 1803.07977]

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$$\begin{aligned} \hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0,0)} + \hat{\sigma}_{ab}^{(1,0)} + \hat{\sigma}_{ab}^{(2,0)} + \hat{\sigma}_{ab}^{(3,0)} + \ldots & \quad \text{QCD} \\ + \hat{\sigma}_{ab}^{(0,1)} + \ldots & \quad \text{EW} \\ + \hat{\sigma}_{ab}^{(1,1)} + \ldots & \quad \text{QCD-EW} \end{aligned}$$

**Mixed QCD-EW corrections**

- should **compete with N$^3$LO** according to the physical counting $\alpha \approx \alpha_S^2$ and represent the leading residual theoretical uncertainties due to truncation of the perturbative expansion
- is **highly desirable** in view of the expected precision target at HL-LCH, both in the resonant and and in the off-shell regions

→ from factorised ansatz, $\mathcal{O}(-2\%)$ at $m_{\ell\ell} = 1$ TeV

$$\frac{d\sigma}{dX} = \frac{d\sigma^{(1,0)}}{dX} \frac{d\sigma_{qq}^{(0,1)}}{dX} \left( \frac{d\sigma^{(0,0)}}{dX} \right)^{-1}$$

# MATRIX framework

**MATRIX** *[Grazzini, Kallweit, Wiesemann, 2018]*

**MUNICH** (by S. Kallweit)

- **efficient multichannel** phase space generation
- **bookkeeping** all subprocesses
- automatic implementation of dipole subtraction

**AMPLITUDES**

- 1-loop amplitudes: **OpenLoops, Recola** (Collier, CutTOols,...)
- 2-loop: dedicated 2-loop codes (VVamp, GiNac, TDHPL,...)

**SUBTRACTION SCHEME**

- @NLO: dipole (and $q_T$ subtraction)
- @NNLO: $q_T$ subtraction

MATRIX v2.0

- NNLO QCD differential predictions for many color singlet processes: $H$, $V$, $\gamma\gamma$, $V\gamma$, $VV$ for all leptonic decays
- combination with NLO EW for all leptonic $V$ and $VV$ processes
- loop-induced gluon fusion channel at NLO QCD for neutral $VV$ processes

NEW MATRIX v2.1 matrix.hepforge.org

- NNLO QCD for $t\bar{t}$ and $\gamma\gamma\gamma$ production
- **bin-wise extrapolation** and inclusion of QCD **fiducial power corrections** in 2-body kinematics

# MATRIX framework

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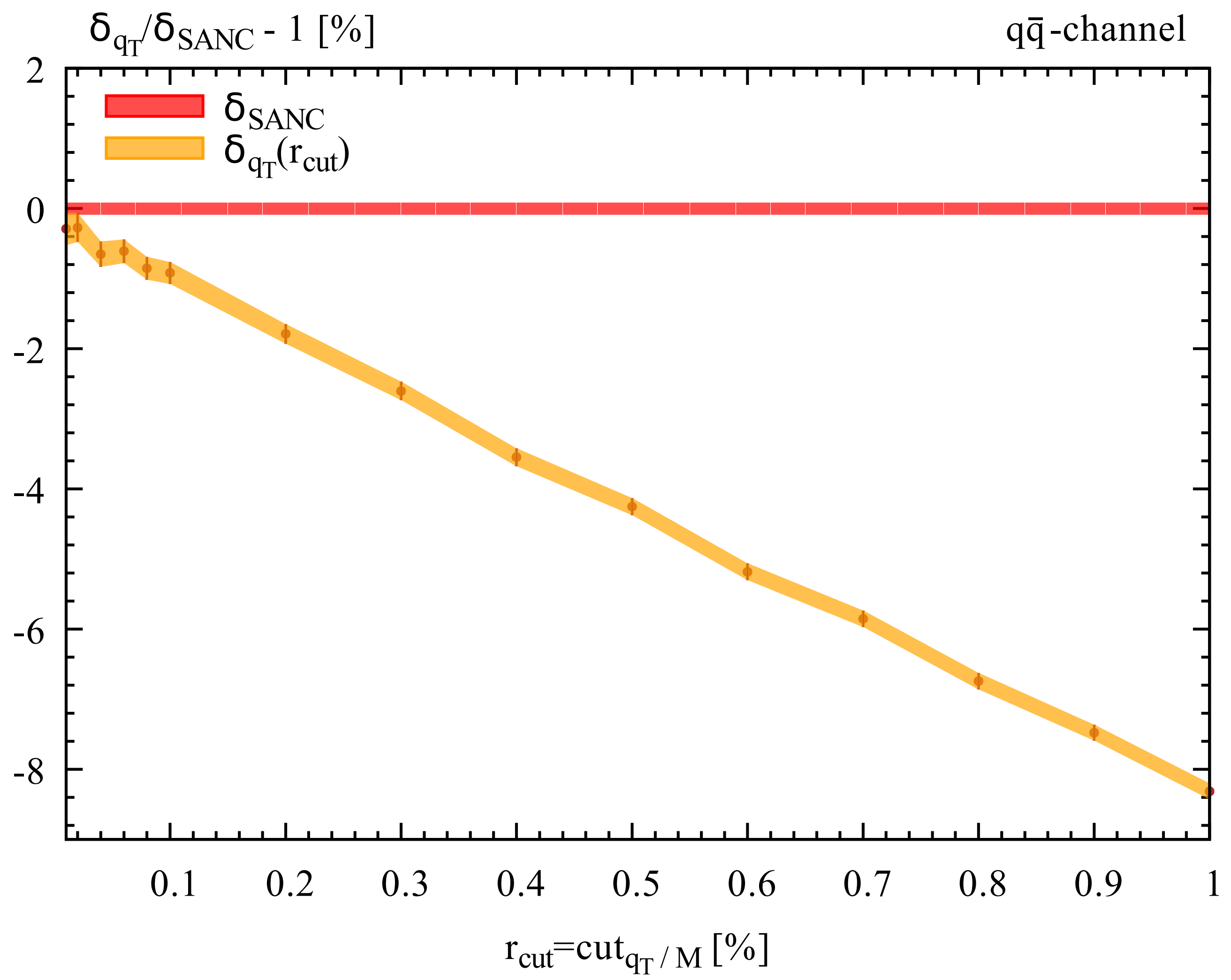
Mixed QCD-EW corrections for Drell-Yan available in a future release

# Mixed QCD-EW corrections: dependence on the cutoff

- EW corrections: **linear** ($m = 1$) power corrections due to final-state emission




**analytical** insight for **inclusive cross section** in pure QED

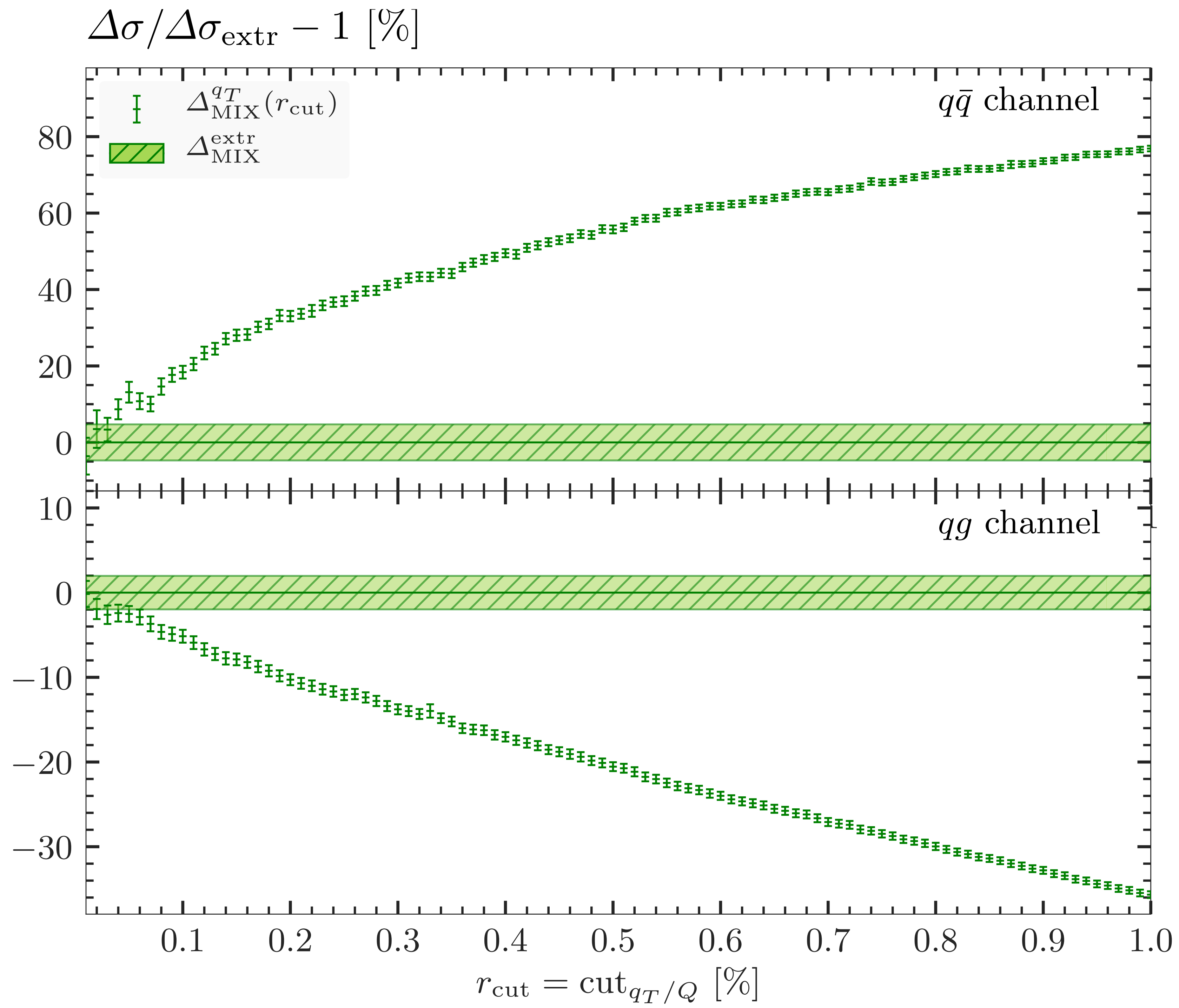
$$\sigma^{\rm NLP}(s; r_{\rm cut}) = -\frac{3\pi}{8}\frac{\alpha}{2\pi} r_{\rm cut}\left[\frac{6(5-\beta^2)}{3-\beta^2} + \frac{-47+8\beta^2+3\beta^4}{\beta(3-\beta^2)}\log\frac{1+\beta}{1-\beta}\right]\sigma_B(s)$$

$$\beta = \sqrt{1-\frac{4m^2}{s}}$$

[**LB**, Grazzini, Tramontano, 2019]

in general we have to rely on an extrapolation procedure!

- Mixed QCD-NLO EW: linear (m=1) + log enhancement
  - Rather **large** $r_{\rm cut}$ dependence on the mixed corrections
  - Control of the mixed corrections at $\mathcal{O}$(5-10%) which translates into **per mille level accuracy on the total cross section**
  - **Bin-wise extrapolation** for distributions



# Mixed QCD-EW corrections: Hard-Virtual coefficient

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)}$$

$\mathcal{O}(\alpha_s^m \alpha^n)$ term

$$d\sigma^{(1,1)} = \mathcal{H}^{(1,1)} \otimes d\sigma_{LO} + \left[ d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{\rm cut}}$$

- The **hard-collinear** coefficient brings in the **virtual corrections** and finite remainder that lives at $q_T = 0$, restoring the correct **normalisation**

$$\mathcal{H}^F = [\ H^F C_1 C_2]$$

**Process dependent hard-virtual functions**: universal relation with the all-order virtual amplitude
[Catani, Cieri, de Florian, Ferrera Grazzini (2013)]

$$|\tilde{\mathcal{M}}> = (1 - \tilde{I})|\mathcal{M}>$$

$$H^F \sim <\tilde{\mathcal{M}}|\tilde{\mathcal{M}}>$$

Process independent (universal) **collinear functions** known up N³LO in QCD
[Catani,Grazzini (2011)],
[Catani, Cieri, de Florian, Ferrera, Grazzini (2012)
[Luo, Yang, Zhu, Zhu (2019)]
[Ebert, Mistlberger, Vita (2020)]

computed with abelianisation

$$\mathcal{H}^{(m,n)} = H^{(m,n)} \delta(1 - z_1)\delta(1 - z_2) + \delta\mathcal{H}^{(m,n)}$$

IR subtracted amplitude

$$H^{(1,1)} \equiv \frac{2\mathrm{Re}\left(\mathcal{M}_{\rm fin}^{(1,1)} \mathcal{M}^{(0,0)*}\right)}{|\mathcal{M}^{(0,0)}|^2}$$

# Mixed QCD-EW corrections: Hard-Virtual coefficient

WORKFLOW

Feynman diagrams: QGRAF

Dirac Algebra and interference terms: FORM

IBPs: KIRA, LITERED, REDUZE2

Evaluation of MIs

UV renormalisation, subtraction of IR poles

Numerical grid

**Treatment of $\gamma_5$**: Naive anti-commuting $\gamma_5$ with reading point prescription and Larin prescription as an independent cross check

MIs with **up to one massive** boson exchange are evaluated analytically
[Bonciani, Di Vita, Matrolia, Schubert, 2016], [Heller, von Manteuffel, and Schabinger, 2020] [Hasan, Schubert, 2020], [R. Bonciani, A. Ferroglia, T. Gehrmann, D. Maitre, and C. Studerus, 2008], [R. Bonciani, A. Ferroglia, T. Gehrmann, D. Maitre, and C. Studerus, 2008], [P. Mastrolia, M. Passera, A. Primo, and U. Schubert, 2017]

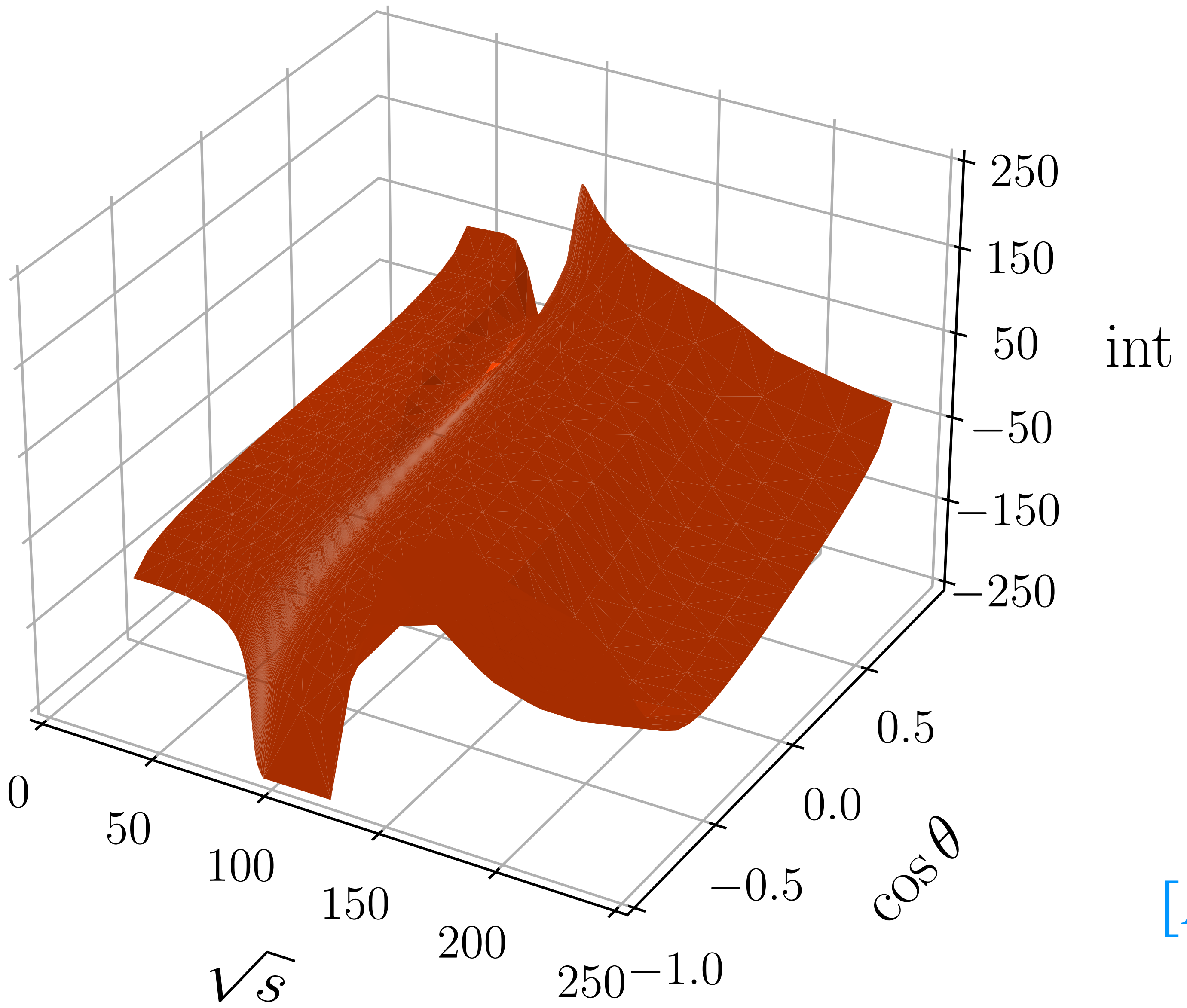
5 MIs with **two massive** bosons cannot be easily expressed in terms of GPls
Require an **alternative strategy** (see also [Heller, von Manteuffel, Schabinger (2019)])

Semi-analytical evaluation of tree-loop interference
[Armadillo, Bonciani, Devoto, Rana, Vicini 2022]

- Numerical resolution of differential equations for MIs via **series expansions,** inspired by DiffExp [Hidding (2006)] but extended for **complex masses**
- **Arbitrary number of significant digits** (with analytic boundary condition)
- The method is **general** (applicable to other processes)
- Numerical evaluation of amplitudes takes $\mathcal{O}$(10 min/point) per core

# Mixed QCD-EW corrections: Hard-Virtual coefficient

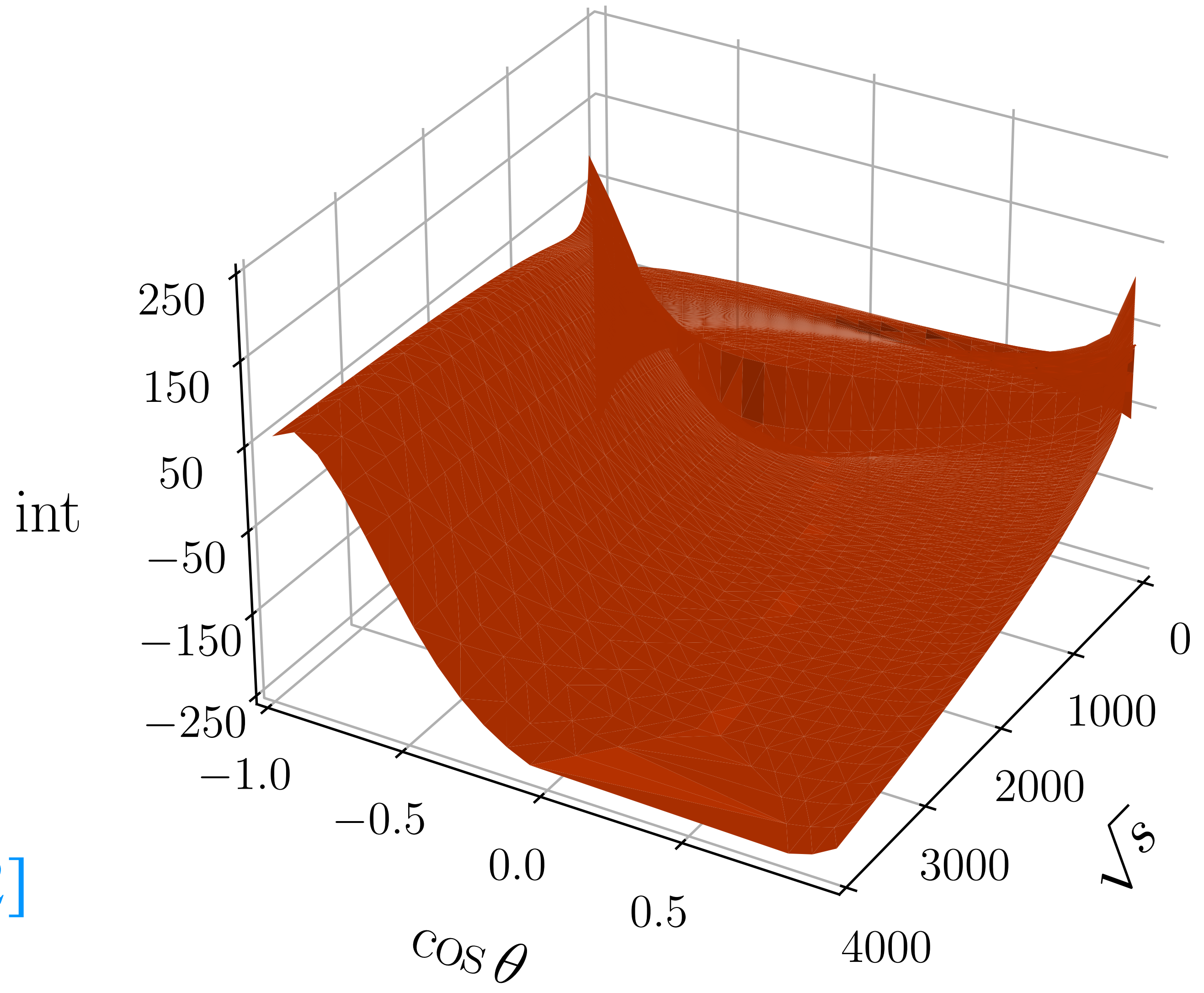
- *Validation*: several checks of the MIs performed with Fiesta and PySecDec, comparison with the PA in the resonant region
- *Evaluation:* preparation of **an optimised numerical grid** covering the physical $2 \to 2$ phase space relevant the LHC in $(s, \cos\theta)$ with GiNaC and SeaSide and **interpolation with cubic splines** for numerical integration
  - $\mathcal{O}(9\ \text{h})$ on a 32-cores machines for 3000 grid points
  - the **lepton mass is kept finite** wherever needed to regularise the final state collinear divergence; the logarithms of the lepton mass are subtracted from the numerical grid and added back analytically
  - the resulting UV- and IR-subtracted Hard-Virtual coefficient is a **smooth, slowly varying** function




$$H^{(1,1)} \equiv \frac{2\text{Re}\left(\mathcal{M}^{(1,1)}_{\text{fin}}\mathcal{M}^{(0,0)*}\right)}{|\mathcal{M}^{(0,0)}|^2}$$

in units $\frac{\alpha_s}{\pi}\frac{\alpha}{\pi}\sigma_{\text{LO}}$

[Armadillo, Bonciani, Devoto, Rana, Vicini 2022]



# Mixed QCD-EW corrections: pheno results

**First calculation** of complete mixed QCD-EW correction to Drell-Yan [LB, Bonciani, Grazzini, Kallweit, Rana, Tramontano,Vicini 2021]

**SETUP** (LHC @ $\sqrt{s} = 14$ TeV)

- `NNPDF31_nnlo_as_0118_luxqed`
- $p_{T,\mu} > 25$ GeV, $|y_\mu| < 2.5$, $m_{\mu^+\mu^-} > 50$ GeV
- massive muons (no photon lepton recombination)
- $G_\mu$ scheme, complex mass scheme
- fixed scale $\mu_F = \mu_R = m_Z$

| $\sigma$ [pb] | $\sigma_{\rm LO}$ | $\sigma^{(1,0)}$ | $\sigma^{(0,1)}$ | $\sigma^{(2,0)}$ | $\sigma^{(1,1)}$ |
|---|---|---|---|---|---|
| $q\bar{q}$ | 809.56(1) | 191.85(1) | −33.76(1) | 49.9(7) | −4.8(3) |
| $qg$ | — | −158.08(2) | — | −74.8(5) | 8.6(1) |
| $q(g)\gamma$ | — | — | −0.839(2) | — | 0.084(3) |
| $q(\bar{q})q'$ | — | — | — | 6.3(1) | 0.19(0) |
| $gg$ | — | — | — | 18.1(2) | — |
| $\gamma\gamma$ | 1.42(0) | — | −0.0117(4) | — | — |
| tot | 810.98(1) | 33.77(2) | −34.61(1) | −0.5(9) | 4.0(3) |
| $\sigma^{(m,n)}/\sigma_{\rm LO}$ | | +4.2 % | −4.3 % | ~ 0 % | +0.5 % |

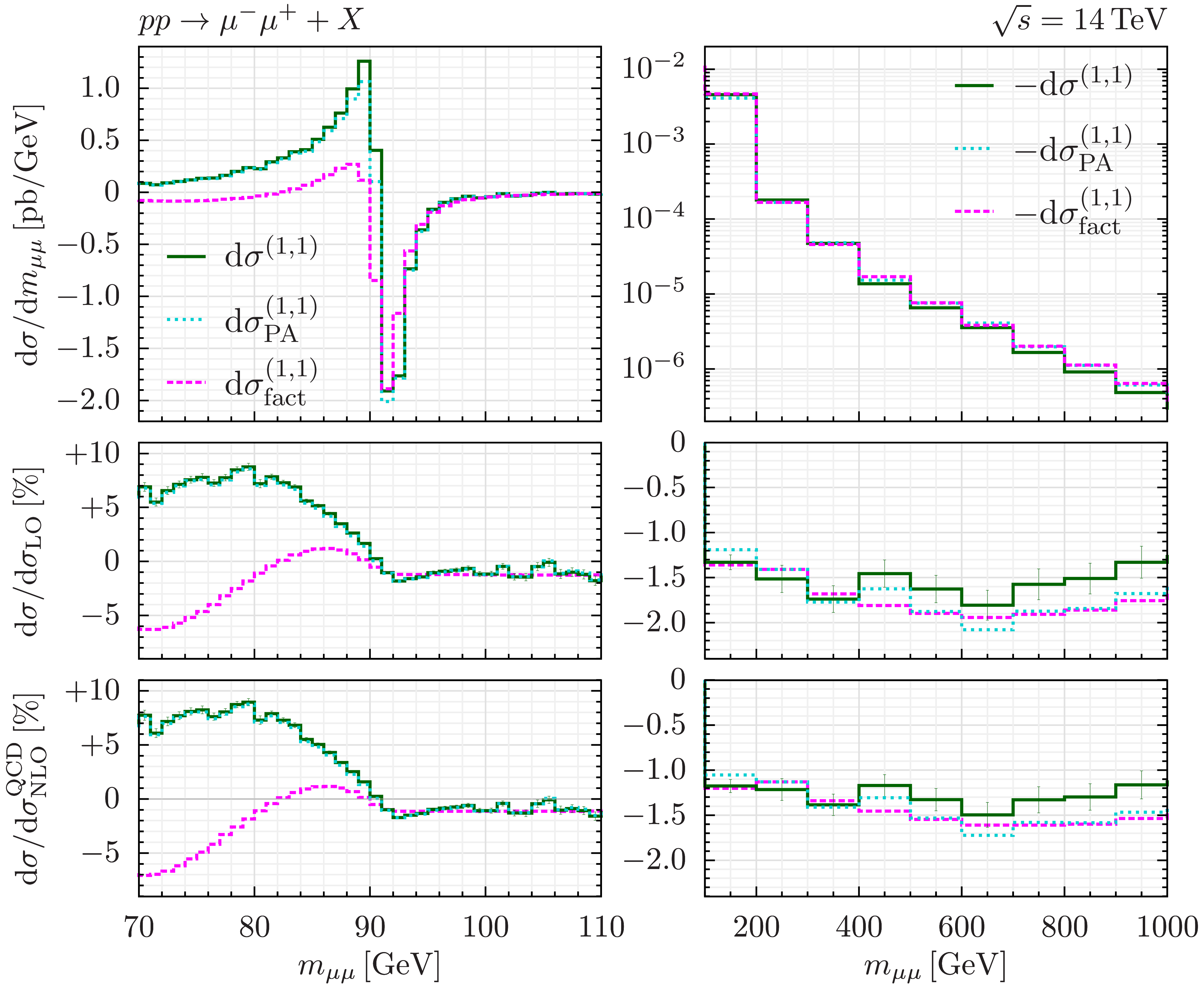
- NLO and NNLO QCD corrections show large cancellations among the partonic channels (especially between $q\bar{q}$ and $qg$)
- NLO QCD and NLO EW corrections are of the same order and opposite sign (accidental cancellation)
- Mixed QCD-EW corrections are dominated by the $qg$ channel and are larger than NNLO QCD (for the particular chosen setup)
- Photon induced processes rather suppressed

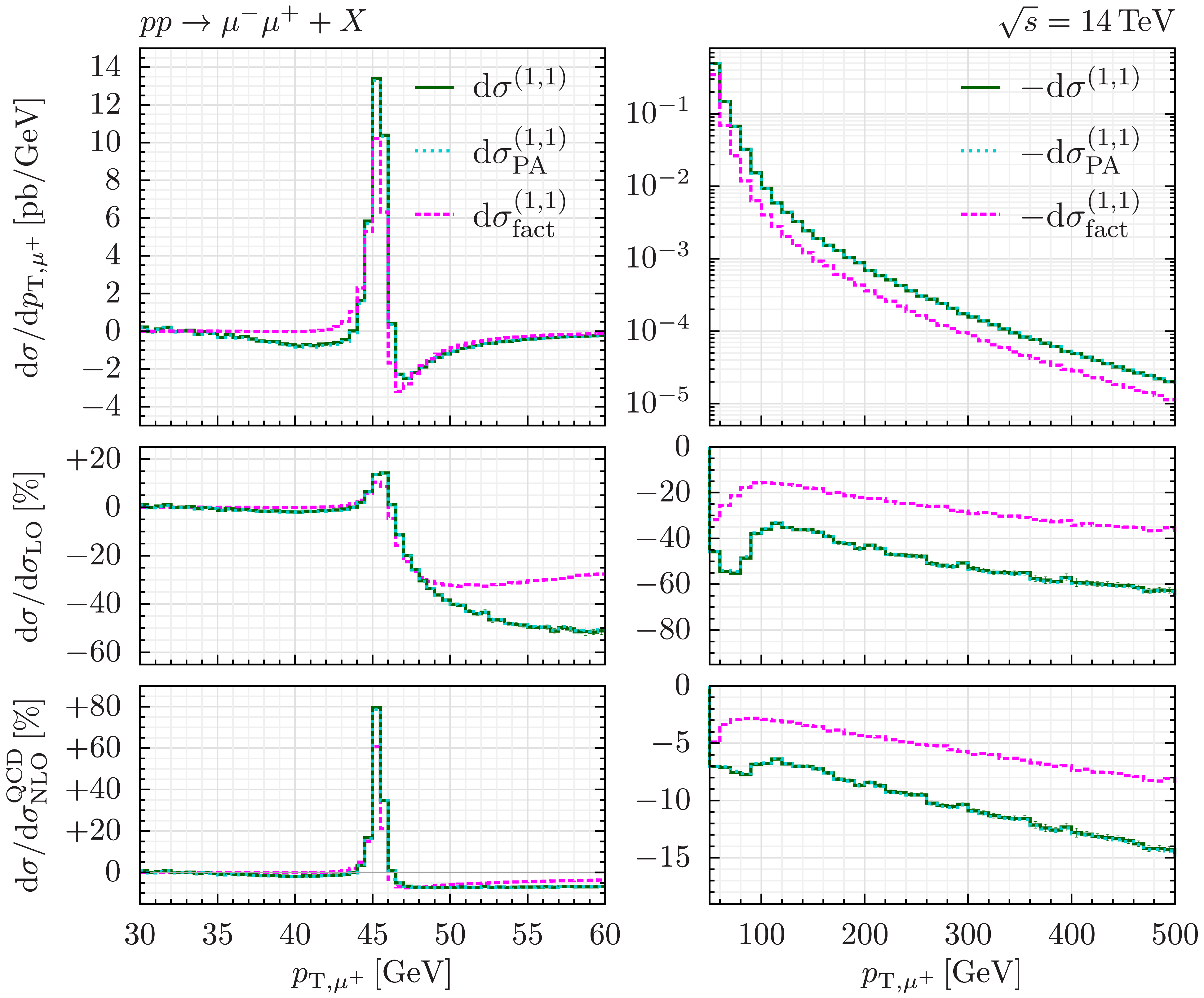
<u>Computational resources</u>

- $\mathcal{O}$(120k) core hours for NNLO QCD (reduced by a factor 2-3 by including fiducial PCs )
- $\mathcal{O}$(180k) core hours for mixed QCD-EW

# Mixed QCD-EW corrections: pheno results

**First calculation** of complete mixed QCD-EW correction to Drell-Yan [LB, Bonciani, Grazzini, Kallweit, Rana, Tramontano,Vicini 2021]




- Breakdown of naive QCD-QED factorisation below Z peak
- PA provides an excellent description **near the resonance**
- $\mathcal{O}(1\%)$ corrections at 1 TeV, PA slightly off

- Around resonance, breakdown of fixed-order
- Naive QCD-QED factorisation fails to describe the high-tail
- The high-tail is dominated by Z+1jet configurations, with Z almost on shell; qg channel by far dominant

# Concluding remarks

- **Jet processes:** main resolution variable is *N*-jettiness. [Stewart, Tackmann, Waalewijn, 2010]
  Ingredients known at NNLO up to two jets (in pp collisions)

- New resolution transverses-like variables for jet processes: $k_T$-ness, so far formulated only at NLO
  [LB, Grazzini, Haag, Rottoli, Savoini, 2022]

- Interconnection with parton shower in the context of matching higher fixed-order corrections to parton showers in formalism as MiNNLO and GENEVA
  [Monni, Nason, Re, Wiesemann,Zanderighi, 2022]
  [Alioli, Bauer, Berggren, Hornig, Tackmann, 2012]

# Backups

# Hard-Virtual coefficient: IR structure and finite amplitudes

$$\mathscr{M}^{(1,0)}_{\text{fin}} = \mathscr{M}^{(1,0)} + \frac{1}{2}\left(\frac{\alpha_s}{\pi}\right) C_F \left[\frac{1}{\epsilon^2} + \left(\frac{3}{2} + i\pi\right)\frac{1}{\epsilon} - \frac{\pi^2}{12}\right] \mathscr{M}^{(0)}$$

$$\mathscr{M}^{(0,1)}_{\text{fin}} = \mathscr{M}^{(0,1)} + \frac{1}{2}\left(\frac{\alpha}{\pi}\right) \left\{ \left[\frac{1}{\epsilon^2} + \left(\frac{3}{2} + i\pi\right)\frac{1}{\epsilon} - \frac{\pi^2}{12}\right] \frac{e_c^2 + e_{\bar{c}}^2}{2} - \frac{2\Gamma_t}{\epsilon} \right\} \mathscr{M}^{(0)}$$

$$\begin{aligned}
\mathscr{M}^{(1,1)}_{\text{fin}} = \; & \mathscr{M}^{(1,1)} - \left(\frac{\alpha_s}{\pi}\right)\left(\frac{\alpha}{\pi}\right) \left\{ \frac{1}{8\epsilon^4}(e_c^2 + e_{\bar{c}}^2) C_F + \frac{1}{2\epsilon^3} C_F \left[\left(\frac{3}{2} + i\pi\right)\frac{e_c^2 + e_{\bar{c}}^2}{2} - \Gamma_t\right] \right\} \mathscr{M}^{(0)} \\
& + \frac{1}{2\epsilon^2}\left\{ \left(\frac{\alpha}{\pi}\right)\frac{e_c^2 + e_{\bar{c}}^2}{2} \mathscr{M}^{(1,0)}_{\text{fin}} + C_F \left(\frac{\alpha_s}{\pi}\right) \mathscr{M}^{(0,1)}_{\text{fin}} \right. \\
& \qquad \left. + C_F \left(\frac{\alpha_s}{\pi}\right)\left(\frac{\alpha}{\pi}\right) \left[\left(\frac{7}{12}\pi^2 - \frac{9}{8} - \frac{3}{2} i\pi\right)\frac{e_c^2 + e_{\bar{c}}^2}{2} + \left(\frac{3}{2} + i\pi\right)\Gamma_t\right] \mathscr{M}^{(0)} \right\} \\
& + \frac{1}{2\epsilon}\left\{ \left(\frac{\alpha}{\pi}\right) \left[\left(\frac{3}{2} + i\pi\right)\frac{e_c^2 + e_{\bar{c}}^2}{2} - 2\Gamma_t\right] \mathscr{M}^{(1,0)}_{\text{fin}} + \left(\frac{\alpha_s}{\pi}\right) C_F \left[\frac{3}{2} + i\pi\right] \mathscr{M}^{(0,1)}_{\text{fin}} \right. \\
& \qquad \left. + \frac{1}{8} C_F \left(\frac{\alpha_s}{\pi}\right)\left(\frac{\alpha}{\pi}\right) \left[\left(\frac{3}{2} - \pi^2 + 24\zeta(3) + \frac{2}{3} i\pi^3\right)\frac{e_c^2 + e_{\bar{c}}^2}{2} - \frac{2}{3}\pi^2\Gamma_t\right] \mathscr{M}^{(0)} \right\}
\end{aligned}$$

# Factorise ansatz

We present our prediction for the $\mathcal{O}(\alpha_s\alpha)$ correction as

- absolute correction
- normalised correction with respect to the LO cross section
- normalised correction with respect to the NLO QCD cross section

We compare our results with the naive factorised ansantz given by the formula

$$\frac{d\sigma^{(1,1)}_{\text{fact}}}{dX} = \left(\frac{d\sigma^{(1,0)}}{dX}\right) \times \left(\frac{d\sigma^{(0,1)}_{q\bar{q}}}{dX}\right) \times \left(\frac{d\sigma_{\text{LO}}}{dX}\right)^{-1}$$

**Remark (especially for the transverse momentum distribution)**

A factorised approach is justified if the dominant sources of QCD and EW corrections factorise with respect to the hard W production subprocesses.

At NLO, gluon/photon initiated channels open up populating the tail of the $p_T$ spectrum, thus leading to large corrections (***giant K-factors***)

We do not include the **photon-induced** channels in the NLO-EW differential K-factor to avoid the multiplication of two giant K-factors of QCD and EW origin, which is not expected to work

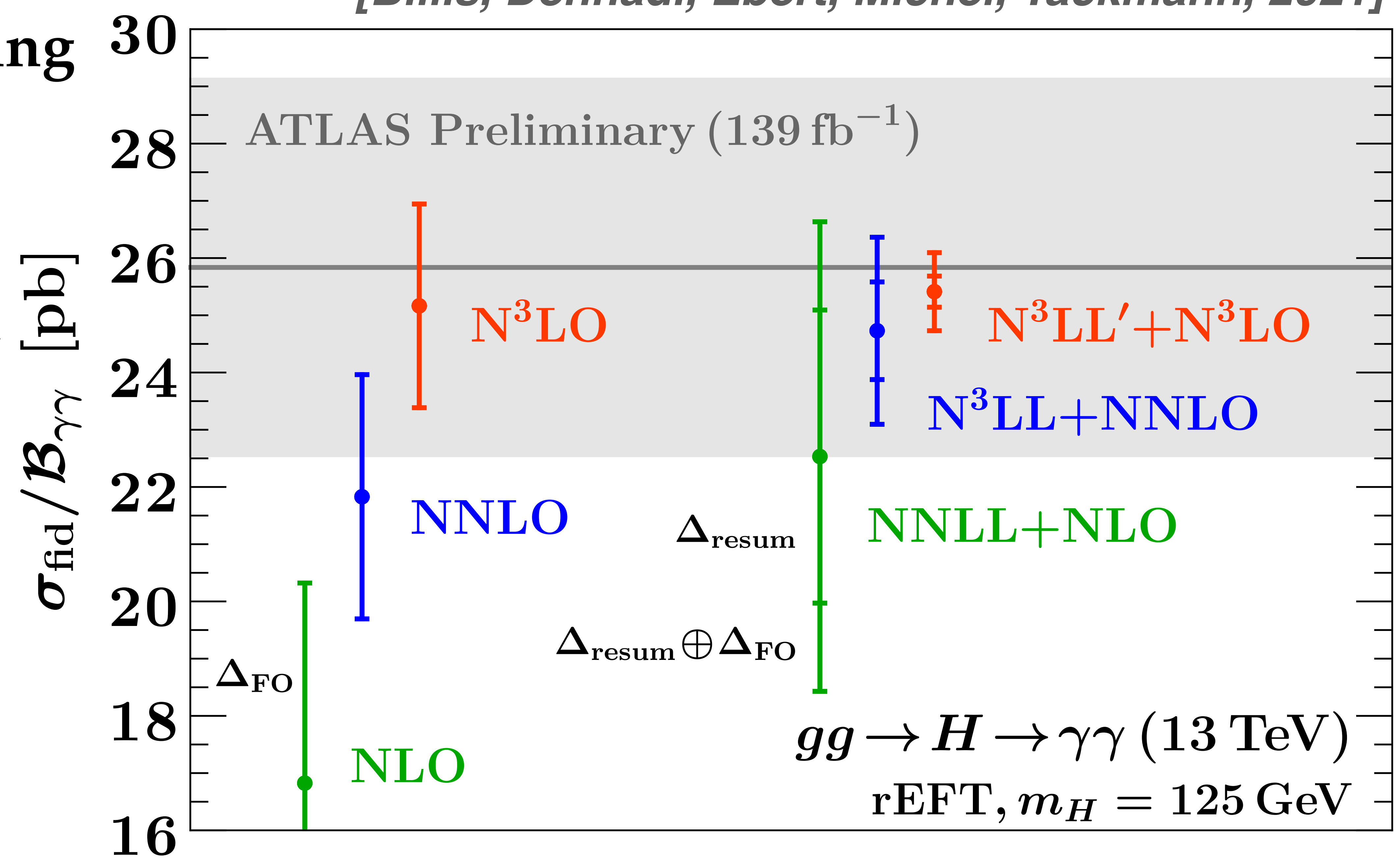
*[Lindert, Grazzini, Kallweit, Pozzorini, Wiesemann (2019)]*

# Convergence of the perturbative expansion in the presence of fiducial cuts

Fiducial cuts may challenge the convergence of the perturbative fixed-order series

- **Symmetric cuts** on the transverse momentum of the two-body decay products lead to an **enhanced sensitivity to soft radiation** when the two particles are back-to-back in the transverse plane
  *[Klaser, Kramer, 1996], [Harris, Owens, 1997], [Frixione, Ridolfi, 1997]*
- They lead to **linear power corrections** in the transverse momentum spectrum of the color singlet $q_T$
- The linear dependence in $q_T$ is related to a **factorial growth** of the coefficients in the perturbative series (with **alternating-sign** coefficient, hence Borel-summable) *[Salam, Slade, 2021]*
- The effect is larger for the case of the Higgs due to its **Casimir scaling**
- **Asymmetric cuts** on the transverse momentum of the hardest and the of the softest particle do not improve the situation.
- Symmetric cuts and asymmetric cuts are commonly used for Drell-Yan and Higgs analysis, respectively.

*[Billis, Dehnadi, Ebert, Michel, Tackmann, 2021]*




Viable resolution strategies

- improve the convergence by **resumming** the linear power corrections
- **alternative** choices of cuts *[Salam, Slade, 2021]*

# Restoring the quadratic dependence on $q_T$

In the standard $q_T$ subtraction master formula *[Catani, Grazzini, 2007]*

$$\sigma^{\mathrm{F}}_{\mathrm{(N)NLO}} = \int d\sigma^{\mathrm{F}}_{\mathrm{LO}} \otimes \mathscr{H} + \int \left[ d\sigma^{\mathrm{F+jet}}_{\mathrm{(N)LO}} - d\sigma^{\mathrm{F}}_{\mathrm{CT}} \right] \theta(q_T/Q - r_{\mathrm{cut}}) + \mathcal{O}\left(r^k_{\mathrm{cut}}\right)$$

the counterterm is given by a **pure LP expansion** of the $q_T$ spectrum

- above $r_{\mathrm{cut}}$ : all power corrections are exactly provided by the real matrix element (avoiding any double counting)
- below $r_{\mathrm{cut}}$ : all power corrections are missing

Formally, the residual dependence on the slicing parameter $r_{\mathrm{cut}}$ is given by the integral of the non-singular component of the real spectrum below the cut.

$$\int d\sigma^{\mathrm{F+jet,reg}}_{\mathrm{(N)LO}} \theta(r_{\mathrm{cut}} - q_T/Q)$$

For the case of **fiducial cuts**, the leading power correction is linear ($k = 1$). It can be **predicted by factorisation** and is **equivalent** to the $q_T$ recoil prescription

$$\int d\Phi_{\mathrm{F+jet}} \frac{d\sigma^{\mathrm{F+jet,reg}}_{\mathrm{(N)LO}}}{d\Phi_{\mathrm{F+jet}}} \theta(r_{\mathrm{cut}} - q_T/Q) \Theta_{\mathrm{cuts}}(\Phi_{\mathrm{F+jet}}) = \int d\Phi_{\mathrm{F}} \int_0^{r_{\mathrm{cut}}} dr' \left[ \frac{d\sigma^{\mathrm{F}}_{\mathrm{CT}}}{d\Phi_{\mathrm{F}}} \Theta_{\mathrm{cuts}}\left(\Phi^{\mathrm{rec}}_{\mathrm{F}}(\Phi_{\mathrm{F}}, r')\right) - \frac{d\sigma^{\mathrm{F}}_{\mathrm{CT}}}{d\Phi_{\mathrm{F}}} \Theta_{\mathrm{cuts}}(\Phi_{\mathrm{F}}) \right] + \mathcal{O}(r^2_{\mathrm{cut}})$$

where $\Theta_{\mathrm{cuts}}$ implements the fiducial cuts and $\Phi^{\mathrm{rec}}_{\mathrm{F}} = \Phi^{\mathrm{rec}}_{\mathrm{F}}(\Phi_{\mathrm{F}}, r')$ is the recoiled kinematics

# Restoring the quadratic dependence on $q_T$

**Improved $q_T$ subtraction master formula**

$$\sigma^{\mathrm{F}}_{\mathrm{(N)NLO}} = \int d\sigma^{\mathrm{F}}_{\mathrm{LO}} \otimes \mathscr{H} + \int \left[ d\sigma^{\mathrm{F+jet}}_{\mathrm{(N)LO}} - d\sigma^{\mathrm{F}}_{\mathrm{CT}} \right] \theta(q_T/Q - r_{\mathrm{cut}}) + \Delta\sigma^{\mathrm{linPCs}}(r_{\mathrm{cut}}) + \mathcal{O}\left(r_{\mathrm{cut}}^{k'}\right)$$

with the **linPC** term, $\Delta\sigma^{\mathrm{linPCs}}(r_{\mathrm{cut}})$, given by

$$\Delta\sigma^{\mathrm{linPCs}}(r_{\mathrm{cut}}) = \int d\Phi_{\mathrm{F}} \int_0^{r_{\mathrm{cut}}} dr' \left[ \frac{d\sigma^{\mathrm{F}}_{\mathrm{CT}}}{d\Phi_{\mathrm{F}}} \Theta_{\mathrm{cuts}}\left(\Phi^{\mathrm{rec}}_{\mathrm{F}}(\Phi_{\mathrm{F}}, r')\right) - \frac{d\sigma^{\mathrm{F}}_{\mathrm{CT}}}{d\Phi_{\mathrm{F}}} \Theta_{\mathrm{cuts}}(\Phi_{\mathrm{F}}) \right]$$

**Remarks on the linPC term**

- it affects the $q_T$ subtraction formula at the **power corrections** level only
- its formulation is **fully differential** with respect to the Born phase space
- it is **integrable** in 4 dimensions (local cancellation of infrared singularities)
- it is completely determined by the **knowledge of the counterterm** (can be easily **implemented** in any code implementing the $q_T$ subtraction method)

# Restoring the quadratic dependence on $q_T$

**Improved $q_T$ subtraction master formula**

$$\sigma^{\mathrm{F}}_{\mathrm{(N)NLO}} = \int d\sigma^{\mathrm{F}}_{\mathrm{LO}} \otimes \mathscr{H} + \int \left[ d\sigma^{\mathrm{F+jet}}_{\mathrm{(N)LO}} - d\sigma^{\mathrm{F}}_{\mathrm{CT}} \right] \theta(q_T/Q - r_{\mathrm{cut}}) + \Delta\sigma^{\mathrm{linPCs}}(r_{\mathrm{cut}}) + \mathcal{O}\left(r_{\mathrm{cut}}^{k'}\right)$$

with the **linPC** term, $\Delta\sigma^{\mathrm{linPCs}}(r_{\mathrm{cut}})$, given by

$$\Delta\sigma^{\mathrm{linPCs}}(r_{\mathrm{cut}}) = \int d\Phi_{\mathrm{F}} \int_0^{r_{\mathrm{cut}}} dr' \left[ \frac{d\sigma^{\mathrm{F}}_{\mathrm{CT}}}{d\Phi_{\mathrm{F}}} \Theta_{\mathrm{cuts}}\left(\Phi^{\mathrm{rec}}_{\mathrm{F}}(\Phi_{\mathrm{F}}, r')\right) - \frac{d\sigma^{\mathrm{F}}_{\mathrm{CT}}}{d\Phi_{\mathrm{F}}} \Theta_{\mathrm{cuts}}(\Phi_{\mathrm{F}}) \right]$$

**Remarks on the linPC term**

- for the case of fiducial cuts, we expect that its inclusion will **change the power correction scaling from linear to quadratic** ($k = 1$ to $k' = 2$)
- for other cases, we expect that its inclusion will not make the power correction scaling worse
- in principle, given its formulation, it can be applied to any process

# Origin of linear power corrections

**Kinematics** of the two-body decay [Ebert, Michel, Stewart, Tackmann, 2020], [Alekin, Kardos, Moch, Trocsanyi, 2021], [Salam, Slade, 2021]

$$q^\mu = (m_T \cosh Y, q_T, 0, m_T \sinh Y)$$

$$p_1^\mu = p_{T,1} \left( \cosh(Y + \Delta y), \cos\phi, \sin\phi, m_T \sinh(Y + \Delta y) \right)$$

$$p_1^\mu = q^\mu - p_1^\mu$$

in the small $q_T$ limit →

$$p_{T,1} = \frac{Q}{2\cosh \Delta y} \left[ 1 + \frac{q_T}{Q} \frac{\cos\phi}{\cosh \Delta Y} + \mathcal{O}\left(q_T^2/Q^2\right) \right]$$

$$p_{T,2} = p_{T,1} - q_T \cos\phi + \mathcal{O}\left(q_T^2/Q^2\right)$$

$$\eta_1 = Y + \Delta y$$

$$\eta_2 = Y - \Delta y - 2\frac{q_T}{Q} \cos\phi \sinh \Delta y + \mathcal{O}\left(q_T^2/Q^2\right)$$

The two-body decay **phase space** with cuts is given by

$$\Phi_{q \to p_1 + p_2}(q_T) = \frac{1}{8\pi^2} \int_0^{2\pi} d\phi \int d\Delta y \frac{p_{T,1}^2}{Q^2} \Theta_{\text{cuts}}(q_T, \phi, \Delta y; \text{cuts})$$

The integrand has a dependence on $q_T$ through the combinations $q_T^2$ and $q_T \cos\phi$. It follows that

presence of **linear fiducial power corrections** ↔ use of cuts **breaking the azimuthal symmetry**

# Origin of linear power corrections

**Symmetric cuts:** $p_{T,i} > p_T^{\rm cut}, \quad i = 1,2 \implies \min(p_{T,1}, p_{T,2}) > p_T^{\rm cut}$

two different integrands: **breaking of azimuthal symmetry**

$$\min(p_{T,1}, p_{T,2}) = \begin{cases} p_{T,1} & \cos\phi < 0 \\ p_{T,1} - q_T \cos\phi & \cos\phi > 0 \end{cases}$$

$$\Phi(q_T) - \Phi(0) = -\frac{1}{2\pi^2}\frac{q_T}{Q}\frac{p_T^{\rm cut}/Q}{\sqrt{1-(2p_T^{\rm cut})^2/Q^2}}\int_0^{\pi/2} d\phi \cos\phi$$

**Asymmetric cuts:** $p_T^{\rm hard} > p_T^{\rm cut,h}$ and $p_T^{\rm soft} > p_T^{\rm cut,s} \implies \min(p_{T,1}, p_{T,2}) > p_T^{\rm cut,s}$

Equivalent to the symmetric cuts case: **it does not solve the issue of the appearance of linear power corrections**!

**Staggered cuts:** $p_{T,1} > p_T^{\rm cut} + \delta p_T$ and $p_{T,2} > p_T^{\rm cut} \implies \min(p_{T,1} - \delta p_T, p_{T,2}) > p_T^{\rm cut}$

$$\min(p_{T,1} - \delta p_T, p_{T,2}) = \begin{cases} p_{T,1} - \delta p_T & \cos\phi < \delta p_T/q_T \\ p_{T,1} - q_T\cos\phi & \cos\phi > \delta p_T/q_T \end{cases}$$

In the region $q_T < \delta p_T$, the **quadratic dependence on $q_T$ is restored,** as numerically observed in *[Grazzini,Kallweit,Wiesemann, 2017]*

# Fiducial PCs and differential distributions