# Theshold Resummation:
# Drell-Yan and Higgs productions at the LHC

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with
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# Plan of my talk

- Inclusive reactions
- Soft and Collinear partons
- Framework to study Next to Soft logarithms
- Framework to Resum Next to Soft terms N space
- Conclusions

# Parton Model in QCD

Hadronic Cross section:

$$\sigma^A(\tau, m_A^2) = \sigma^{A,(0)}(\mu_R^2) \sum_{a,b=q,\bar{q},g} \int_\tau^1 dy\, \Phi_{ab}(y, \mu_F^2) \Delta_{ab}^A \left(\frac{\tau}{y}, m_A^2, \mu_R^2, \mu_F^2\right)$$

Partonic Flux:

$$\Phi_{ab}(y, \mu_F^2) = \int_y^1 \frac{dx}{x} f_a(x, \mu_F^2) f_b\left(\frac{y}{x}, \mu_F^2\right) ,$$

Partonic cross section:

Precision Measurements

Precise Results

PDFs

# Theoretical Issues

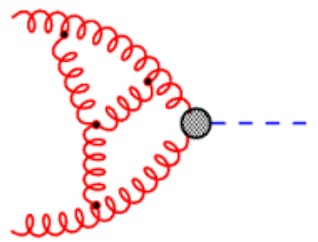
- UV Renormalisation Scale, Strong coupling

$$\alpha_s(\mu_R)$$

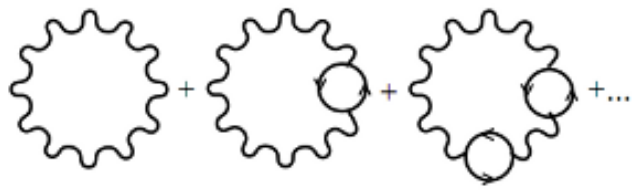
- Factorisation Scale and Parton Distribution Functions

$$f_a(x, \mu_F)$$

- Missing Higher Order corrections
- Stability of the perturbation theory



- Resummation Methods
- Hadronisation models



# Soft + Virtual + Hard

DY/Higgs production cross section:

$$\sigma(q^2,\tau) = \sigma_0(\mu_R^2) \int \frac{dz}{z} \Phi_{ab}\left(\frac{\tau}{z}, \mu_F^2\right) \Delta_{ab}(q^2, \mu_R^2, \mu_F^2, z)$$

Partonic Flux

Partonic cross section:

$$\Delta_{ab}(q^2,\mu_i^2,z) = \Delta_{ab}^{SV}(q^2,\mu_i^2,z) + \Delta_{ab}^{H}(q^2,\mu_i^2,z)$$

Soft + Virtual

Hard

$$\delta(1-z_i)$$

$$\left(\frac{\ln(1-z_i)}{(1-z_i)}\right)_+$$

Polylogs or HPLS $Li_n(g(z)), H(\vec{n}, h(z))$

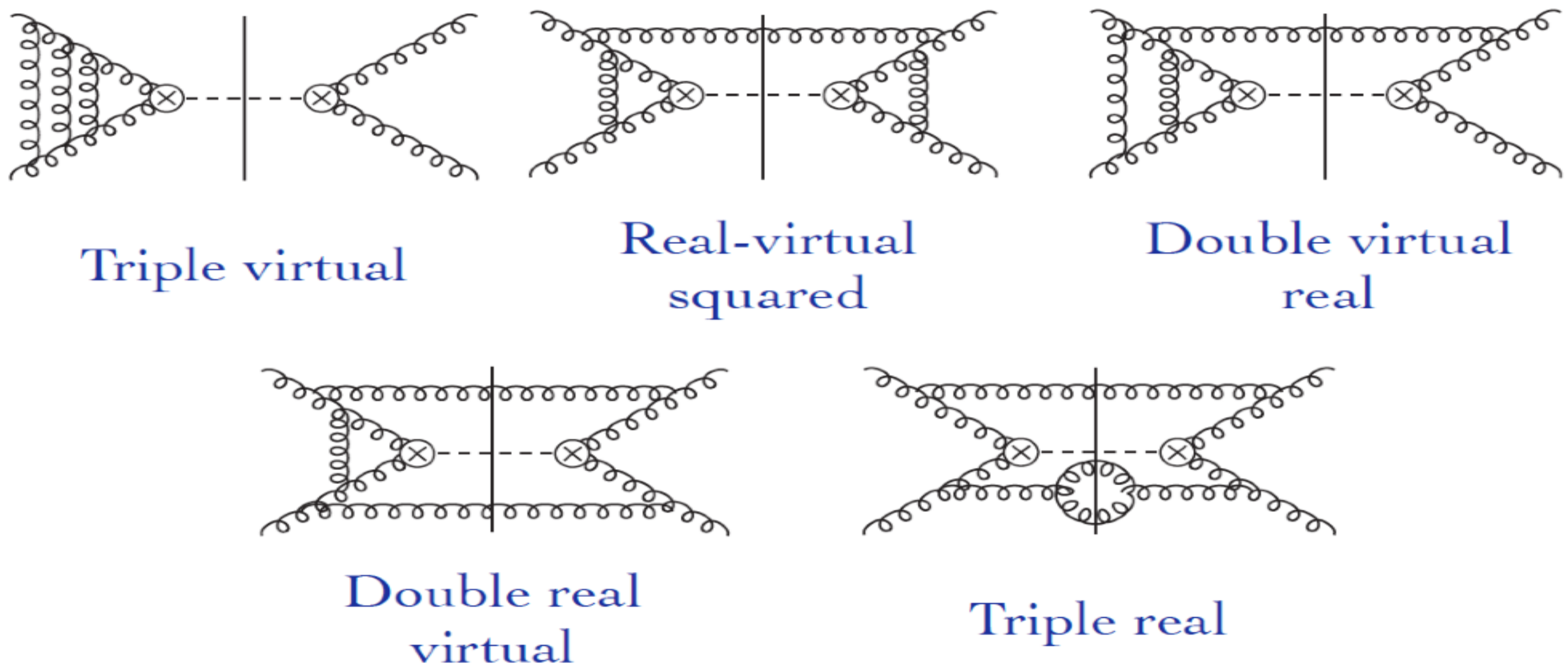
$$\log^k(1-z_i), \quad k = 0, \cdots \infty$$

$$(1-z_i)^k, \quad k = 1, \cdots \infty$$

# Perturbative Structure

## Coefficient function

$$\Delta(z) = \sum_{i=0}^{\infty} a_s(\mu_R^2)\Delta^{(i)}(\mu_R^2, z)$$




- Extreme threshold → $\delta(1-z_i)$
- Soft + Virtual (SV) → $\left(\frac{\ln(1-z_i)}{(1-z_i)}\right)_+$
- Next to SV → $\log^k(1-z_i), \quad k = 0, \cdots \infty$
- Beyond NSV (H) → $(1-z_i)^k, \quad k = 1, \cdots \infty$

# SV + Next to Soft + all that

Near threshold: $z = 1$

Hard part

$$\Delta_{ab}^{H}(z) = \Delta_{ab}^{NSV}(z) + \Delta_{ab}^{N^n SV}(z)$$

- Next to SV (NSV)

$$\Delta_{ab}^{NSV}(z) = \sum_{k=0}^{\infty} C_i^{NSV} \log^k(1-z)$$

- Next to next to....to soft (NnSV)

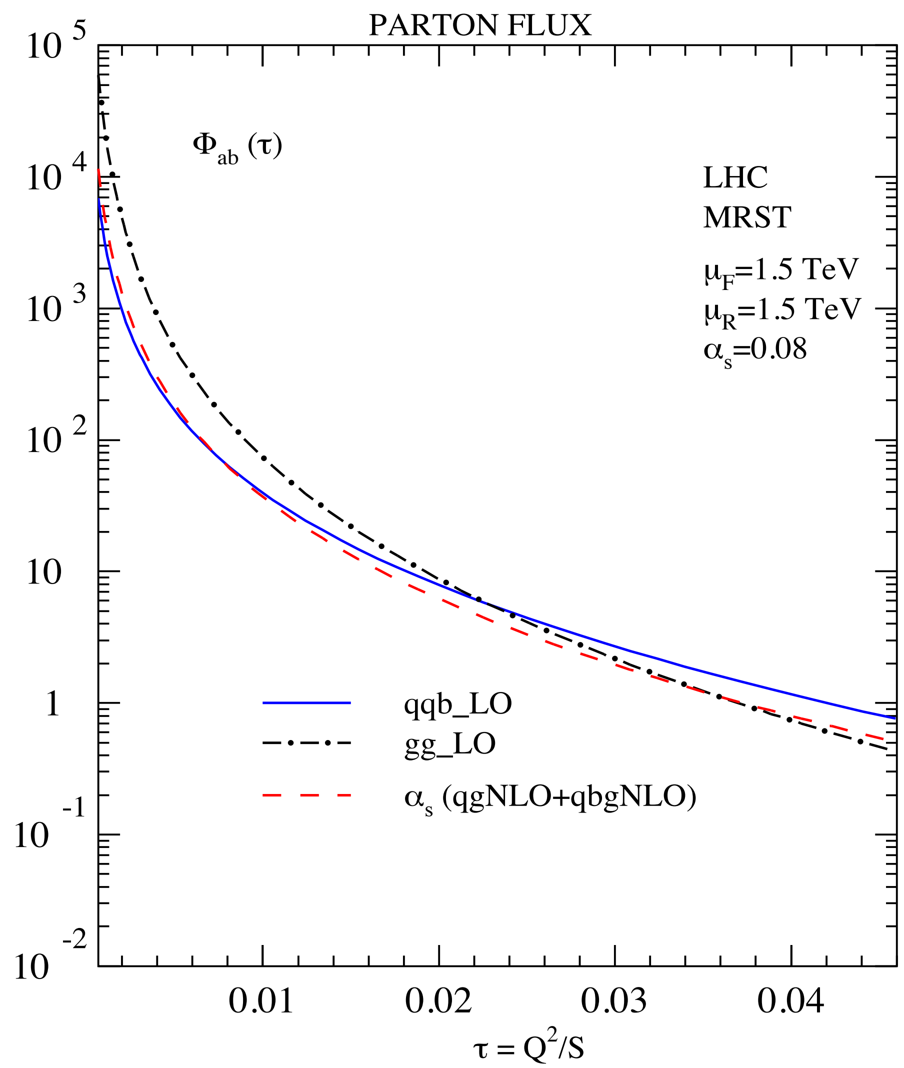
$$\Delta_{ab}^{N^n SV}(z) = \sum_{k=1}^{\infty} d_k (1-z)^k$$

# Why Threshold corrections?

partonic scaling variable $z = \frac{\tau}{x}$

*Catani et al, Harlander, Kilgore*

$$2S\, d\sigma^{P_1 P_2}(\tau, m_h) = \sum_{ab} \int_\tau^1 \frac{dx}{x} \Phi_{ab}(x)\; 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, m_h\right) \qquad \tau = \frac{m_h^2}{S}$$




Gluon flux is largest at LHC

- Parton flux $\Phi_{ab}(x)$ becomes large when $x \to x_{min} = \tau$
- Dominant contribution to Higgs production comes from the region when $x \to \tau$
- It is sufficient if we know the partonic cross section when $x \to \tau$
- $x \to \tau$ is called *soft limit*.
- Expand the partonic cross section around $x = \tau$.

$z \to 1$

- Dominantly come from virtual and soft gluon emission processes (SV)

# Mellin Moments and large N

Mellin Moment:

$$f_N = \int_0^1 dz\ z^{N-1} f(z)$$

Threshold limit $z \to 1$ in z-Space translates to

$N \to \infty$ in N-Space

$N \to \infty$ Taking into account SV and NSV terms

$$M_N\left[\left(\frac{\log(1-z)}{1-z}\right)_+\right] = \frac{\log^2 N}{N} - \frac{\log N}{2N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

$$M_N\left[\log^k(1-z)\right] = \frac{\log^k N}{N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

# N-Space structure

Mellin moment of CFs

$$\Delta_N^c = \int_0^1 dz\ z^{N-1} \Delta_c(z)$$

In N-Space $N \to \infty$ We can predict tower of log N s

$$\Delta_N^c = 1 + a_s \Big[ c_1^2 \log^2 N + c_1^1 \log N + c_1^0 + d_1^1 \frac{\log N}{N} + \mathcal{O}(1/N) \Big]$$

$$+ a_s^2 \Big[ c_2^4 \log^4 N + \cdots + c_2^0 + d_2^3 \frac{\log^3 N}{N} + \cdots + \mathcal{O}(1/N) \Big]$$

$$+ \cdots$$

$$+ a_s^n \Big[ c_n^{2n} \log^{2n} N + \cdots + d_n^{2n-1} \frac{\log^{2n-1} N}{N} + \cdots + \mathcal{O}(1/N) \Big]$$

$a_s \log N$ is of order `one` when $a_s$ is very small

Spoils the trucation of the series

# Sterman-Catani-Trentedue

Sterman (`87),Catani,Trentedue `89

**SUMMATION OF LARGE CORRECTIONS TO SHORT-DISTANCE HADRONIC CROSS SECTIONS**

George STERMAN*

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**RESUMMATION OF THE QCD PERTURBATIVE SERIES FOR HARD PROCESSES**

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$$\Delta_N(Q^2) \underset{N\to\infty}{=} \exp\left(2\int_0^1 \mathrm{d}x\, \frac{x^{N-1}-1}{1-x} \int_{Q^2}^{Q^2(1-x)} \frac{\mathrm{d}k^2}{k^2} A\left(\alpha_s\left((1-x)Q^2\right)\right) + \frac{3}{2}\frac{C_F}{\pi}\int_0^1 \mathrm{d}x\, \frac{x^{N-1}-1}{1-x}\alpha_s\left((1-x)Q^2\right)\right) + \mathrm{O}\left(\alpha_s(\alpha_s \ln N)^n\right). \qquad (3.25)$$

Exponentiation of S+V and

Predictions for Threshold logs to all orders

# SV + Next to Soft + all that

Near threshold: $z = 1$

Hard part

$$\Delta_{ab}^{H}(z) = \Delta_{ab}^{NSV}(z) + \Delta_{ab}^{N^n SV}(z)$$

- Next to SV (NSV)

$$\Delta_{ab}^{NSV}(z) = \sum_{k=0}^{\infty} C_i^{NSV} \log^k(1-z)$$

- Next to next to....to soft (NnSV)

$$\Delta_{ab}^{N^n SV}(z) = \sum_{k=1}^{\infty} d_k (1-z)^k$$

# Importance of beyond NSV

## Higgs production in the gluon fusion: N3LO

Anastasiou et al JHEP 03 (2015)

$$
\begin{aligned}
\eta^{(3)}_{gg}(z)\Big|_{(1-z)^0} &= -256\log^5(1-z) && (\to \quad 115.33\%)\\
&+959\log^4(1-z) && (\to \quad 101.07\%)\\
&+1254.029198\ldots\log^3(1-z) && (\to \quad -32.15\%)\\
&-11089.328274\ldots\log^2(1-z) && (\to \quad -89.41\%)\\
&+15738.441212\ldots\log(1-z) && (\to \quad -55.50\%)\\
&-5872.588877\ldots && (\to \quad -14.31\%)
\end{aligned}
$$

- The total SV contribution in z-space -> -2.25 % of the Born
- The total NSV contribution in z-space -> 25 % of the Born !

# Importance of beyond NSV

- The total SV contribution in Mellin N-space (conjugate space) -> 18 % of the Born
- The total NSV contribution in N-space -> 11 % of the Born !

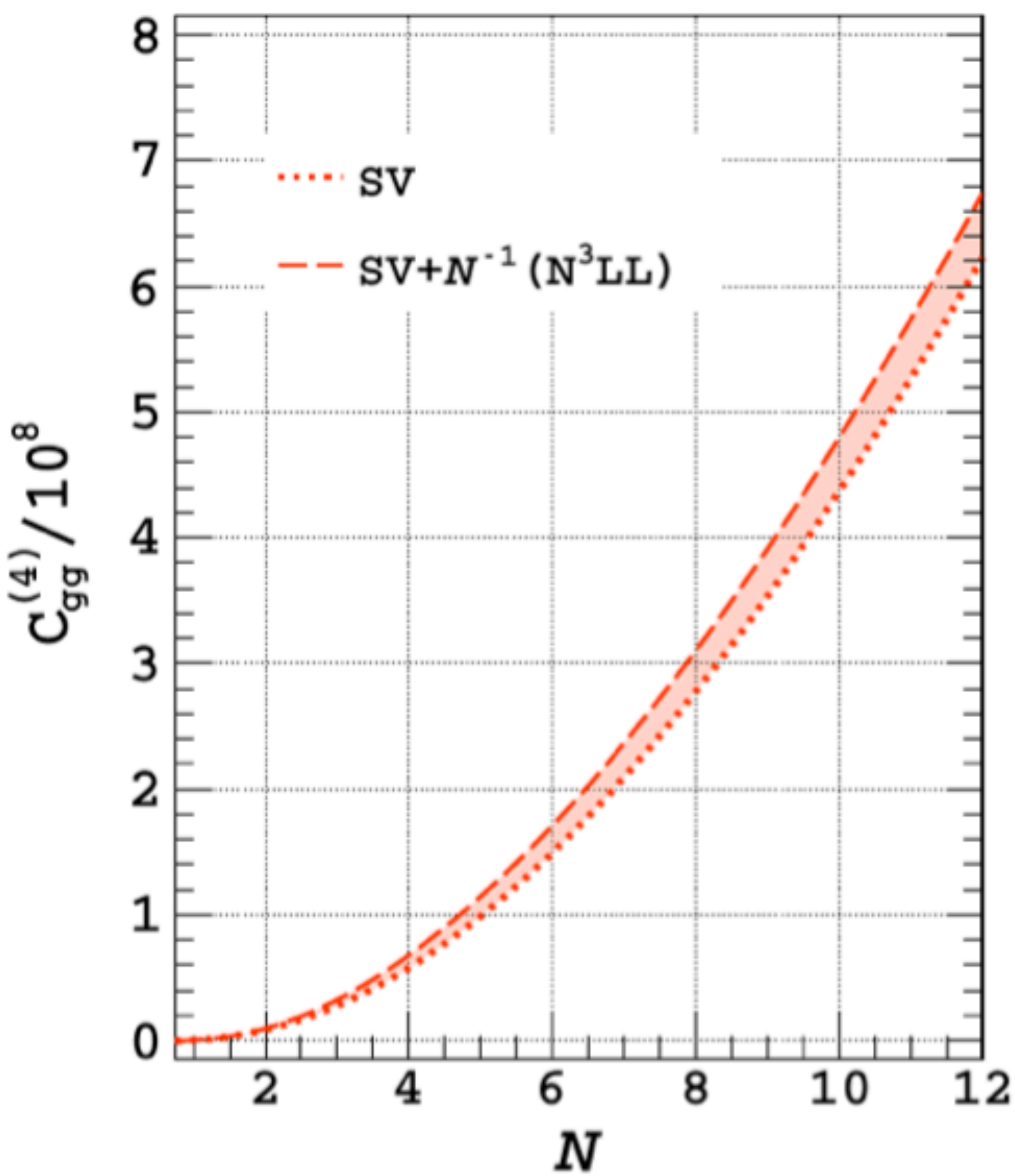
$$
\begin{aligned}
M\left[\eta_{gg}^{(3)}\right](N) \simeq\, & 36 \log^6 N && (\to 0.0013\%) \\
& + 170.679 \ldots \log^5 N && (\to 0.0226\%) \\
& + 744.849 \ldots \log^4 N && (\to 0.2570\%) \\
& + 1405.185 \ldots \log^3 N && (\to 1.0707\%) \\
& + 2676.129 \ldots \log^2 N && (\to 4.0200\%) \\
& + 1897.141 \ldots \log N && (\to 5.1293\%) \\
& + 1783.692 \ldots && (\to 8.0336\%) \\
& + 108 \frac{\log^5 N}{N} && (\to 0.0105\%) \\
& + 615.696 \ldots \frac{\log^4 N}{N} && (\to 0.1418\%) \\
& + 2036.407 \ldots \frac{\log^3 N}{N} && (\to 0.9718\%) \\
& + 3305.246 \ldots \frac{\log^2 N}{N} && (\to 2.9487\%) \\
& + 3459.105 \ldots \frac{\log N}{N} && (\to 5.2933\%) \\
& + 703.037 \ldots \frac{1}{N} && (\to 1.7137\%).
\end{aligned}
$$

# Approximate four-loop QCD correction to Higgs boson production

Phys.LettB.807 124446

Phys

G. Das[*a], S. Moch[†b] and A. Vogt[‡c]




Sizeable contribution from terms beyond SV (the NSV terms) due to the large coefficients

## Understanding the NSV sector is important because:

- Numerically sizeable
- Provide check of higher-order corrections
- Help to reduce the scale uncertainites
- Stabilize fixed-order calculations

**Revisiting parton evolution and the large-$x$ limit**

Yu. L. Dokshitzer[1,*], G. Marchesini[2,1] and G. P. Salam[1]
[1]LPTHE, Universities of Paris-VI and VII and CNRS, Paris, France
[2]University of Milano–Bicocca and INFN Sezione di Milano, Milan, Italy

## Dokshitzer-Marchesini-Salam (DMS) equation

$$\mu^2 \frac{\partial}{\partial \mu^2} \psi(x,\mu^2) = \int_x^1 \frac{dz}{z} \psi\left(\frac{x}{z}, z^\sigma \mu^2\right) \mathcal{P}\left(z, \alpha_s\left(\frac{\mu^2}{z}\right)\right) .$$

## Threshold Expansion (near x=1)

$$P(x) = \frac{A\,x}{(1-x)_+} + B\,\delta(1-x) + C\ln(1-x) + D + \mathcal{O}((1-x)\log^p(1-x)) . \qquad (13)$$

## Predictions for Ci and Di

$$C_1 = 0\,, \qquad C_2 = -\sigma A_1^2\,, \qquad C_3 = -2\sigma A_1 A_2\,, \qquad C_4 = -\sigma(A_2^2 + 2A_1 A_3)\,, \qquad \text{etc.} \quad (11b)$$

$$D_1 = 0, \quad D_2 = -A_1(\sigma B_1 + \beta_0)\,, \quad D_3 = -A_1(\sigma B_2 + \beta_1) - A_2(\sigma B_1 + \mathbf{2}\cdot\beta_0)\,, \quad \text{etc.} \quad (16)$$

Exact For DIS structure function:!

Not right!

On next-to-eikonal corrections to threshold resummation for the Drell-Yan and DIS cross sections

Eric Laenen[1] Lorenzo Magnea[2] Gerben Stavenga[3]

## Inspired by DMS equation

- For DIS structure function:

$$\ln\left[\widehat{F}_2(N)\right] = \mathcal{F}_{\rm DIS}\left(\alpha_s(Q^2)\right) + \int_0^1 dz\, z^{N-1}\left\{\frac{1}{1-z} B\left[\alpha_s\left(\frac{(1-z)Q^2}{z}\right)\right]\right.$$
$$\left. + \int_{Q^2}^{(1-z)Q^2/z} \frac{dq^2}{q^2} P_s\left[z, \alpha_s(q^2)\right] + \int_{(1-z)^2Q^2/z}^{(1-z)Q^2/z} \frac{dq^2}{q^2}\, \delta P\left[z, \alpha_s(q^2)\right]\right\}_+ . \qquad (37)$$

- For Drell-Yan process

$$\ln\left[\widehat{\omega}(N)\right] = \mathcal{F}_{\rm DY}\left(\alpha_s(Q^2)\right) + \int_0^1 dz\, z^{N-1}\left\{\frac{1}{1-z} D\left[\alpha_s\left(\frac{(1-z)^2Q^2}{z}\right)\right]\right.$$
$$\left. + 2\int_{Q^2}^{(1-z)^2Q^2/z} \frac{dq^2}{q^2} P_s\left[z, \alpha_s(q^2)\right]\right\}_+ , \qquad (31)$$

Two loop predictions are somewhat closer to exact results!

Journal of High Energy Physics

## On threshold resummation beyond leading 1 – *x* order

G. Grunberg[1] and V. Ravindran[2]

Published 21 October 2009 • Published under licence by IOP Publishing Ltd
Journal of High Energy Physics, Volume 2009, JHEP10(2009)
**Citation** G. Grunberg and V. Ravindran JHEP10(2009)055
**DOI** 10.1088/1126-6708/2009/10/055



Physical Evolution Equation:

$$\frac{d\mathcal{C}_2(x,Q^2,\mu_F^2)}{d\ln Q^2} = \int_x^1 \frac{dz}{z}\, K(x/z,Q^2)\,\mathcal{C}_2(z,Q^2,\mu_F^2)\ ,$$

Conjucturing the form of Kernel

$$K(x,Q^2) = \frac{1}{r}\,\mathcal{J}\left(W^2\right) + \frac{d\ln\left(\mathcal{F}(Q^2)\right)^2}{d\ln Q^2}\,\delta(1-x) + \mathcal{J}_0\left(W^2\right) + \mathcal{O}\left(r\right),$$

Approximate predictions!

**On Higgs-exchange DIS, physical evolution kernels and fourth-order splitting functions at large $x$**

G. Soar[a], S. Moch[b], J.A.M. Vermaseren[c] and A. Vogt[a]

Physical Evolution Kernel

$$\frac{d}{d\ln m_H^2}\mathcal{F}_{gg} \;=\; K_{gg}\otimes\mathcal{F}_{gg} \equiv \sum_{\ell=0}^{\infty} a_s^{\ell+1} K_{gg}^{(\ell)}\otimes\mathcal{F}_{gg}$$

From Exact 2nd and 3rd results:

$$K_{gg}^{(1)}\Big|_{N^{-1}} = -\,(8\,\beta_0 C_A + 32 C_A^2)\,\ln N + \mathcal{O}(1)\,,$$

$$K_{gg}^{(2)}\Big|_{N^{-1}} = -\,(16\,\beta_0^2 C_A + 112\,\beta_0 C_A^2)\,\ln^2 N + \mathcal{O}(\ln N)\,,$$

$$K_{gg}^{(3)}\Big|_{N^{-1}} = -\,\Big(32\,\beta_0^3 C_A + \xi_H^{(3)}\,\beta_0^2 C_A^2\Big)\,\ln^3 N + \mathcal{O}(\ln^2 N)\,,$$

Successful predictions for certain ``Next to SV , log(1-z) terms at 4th order in QCD for Splitting fn. and CFs

# Mass Factorisation of Partonic Cross section:

$$\frac{1}{z}\hat{\sigma}_{ab}(\varepsilon) = \sigma_0 \sum_{a'b'} \Gamma^T_{aa'}(\mu_F^2, \varepsilon) \otimes \left(\frac{1}{z}\Delta_{a'b'}(\mu_F^2, \varepsilon)\right) \otimes \Gamma_{b'b}(\mu_F^2, \varepsilon)$$

Infrared structure of

a) Partonic cross section
b) Mass factorization Kernels

can provide hints on the all order structure of SV and NSV terms!

Channels: 1. Diagonal q+qb -> H/DY+ X, g+g -> H+ X
2.Non-diagonal q + g - > H/DY+ X

# Factorisation

Factoring out the pure virtual contributions near z -> 1

$$\hat{\sigma}_{c\bar{c}}(z,\epsilon) = \left(Z_{c,UV}\right)^2 |\hat{F}_c(\epsilon)|^2 S_c(z,\epsilon)$$

UV renormalisation constant

Form Factor(virtual corrections)

Soft + next-to-soft corrections

UV finite mass-factorised partonic coefficient function for the diagonal channels :

$$\Delta_{c\bar{c}}(z,\epsilon,q^2\mu_R^2,\mu_F^2) = \left(\Gamma^T\right)^{-1} \otimes \left\{ \left(Z_{c,UV}\right)^2 |\hat{F}_c(Q^2,\epsilon)|^2 S_c(q^2,z,\epsilon) \right\} \otimes \left(\Gamma\right)^{-1}$$

# Factorisation: Diagonal

Begin with Mass factorised cross section

$$\frac{1}{z}\hat{\sigma}_{ab}(\varepsilon) = \sigma_0 \sum_{a'b'} \Gamma^T_{aa'}(\mu_F^2,\varepsilon) \otimes \left(\frac{1}{z}\Delta_{a'b'}(\mu_F^2,\varepsilon)\right) \otimes \Gamma_{b'b}(\mu_F^2,\varepsilon)$$

For Drell-Yan process:

Diagonal Channel:

$$\frac{\hat{\sigma}_{q\bar{q}}}{z\sigma_0} = \Gamma^T_{qq} \otimes \frac{\Delta_{qq}}{z} \otimes \Gamma_{q\bar{q}} + \Gamma^T_{qq} \otimes \frac{\Delta_{qg}}{z} \otimes \Gamma_{g\bar{q}} + \cdots$$

In the threshold limit z -> 1 , keeping only $\left(\frac{\ln(1-z_i)}{(1-z_i)}\right)_+$ $\delta(1-z_i)$ SV

$\log^k(1-z_i), \quad k = 0, \cdots \infty$ next to SV

$$\frac{\hat{\sigma}^{\rm sv+nsv}_{q\bar{q}}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta^{\rm sv+nsv}_{q\bar{q}} \otimes \Gamma_{\bar{q}\bar{q}} \,.$$

dropping

$(1-z_i)^k, \quad k = 1, \cdots \infty$

Remarkably Simple form !

# Factorisation: Off-diagonal

Begin with Mass factorised cross section

$$\frac{1}{z}\hat{\sigma}_{ab}(\varepsilon) = \sigma_0 \sum_{a'b'} \Gamma^T_{aa'}(\mu_F^2,\varepsilon) \otimes \left(\frac{1}{z}\Delta_{a'b'}(\mu_F^2,\varepsilon)\right) \otimes \Gamma_{b'b}(\mu_F^2,\varepsilon)$$

For Drell-Yan process:

Off-diagonal Channel:

$$\frac{\hat{\sigma}_{qg}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta_{qq} \otimes \Gamma_{qg} + \Gamma^T_{qq} \otimes \Delta_{qg} \otimes \Gamma_{gg} + \cdots$$

In the threshold limit z -> 1 , keeping only $\log^k(1-z_i), \quad k = 0, \cdots \infty$ next to SV

$$\frac{\hat{\sigma}^{\rm sv+nsv}_{qg}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta^{\rm sv+nsv}_{q\bar{q}} \otimes \Gamma_{\bar{q}g} + \Gamma^T_{qq} \otimes \Delta^{\rm nsv}_{qg} \otimes \Gamma_{gg} \,.$$

dropping $(1-z_i)^k, \quad k = 1, \cdots \infty$

Getting complicated due to Mixing of channels

# Master Formula !

$$\Delta_c(q^2, \mu_R^2, \mu_F^2, z) = \mathscr{C} \exp\Big(\Psi^c(q^2, \mu_R^2, \mu_F^2, z, \epsilon)\Big)\Big|_{\epsilon=0}$$

where,

Ravindran et al.

$$\mathscr{C} e^{f(z)} = \delta(1-z) + \frac{1}{1!} f(z) + \frac{1}{2!} f(z) \otimes f(z) + \ldots$$

$$\Psi^c\left(q^2, \mu_R^2, \mu_F^2, z, \varepsilon\right) = \left(\ln\left(Z_{UV,c}\left(\hat{a}_s, \mu^2, \mu_R^2, \varepsilon\right)\right)^2 + \ln\left|\hat{F}_c\left(\hat{a}_s, \mu^2, Q^2, \varepsilon\right)\right|^2\right)\delta(1-z)$$
$$+2\Phi^c\left(\hat{a}_s, \mu^2, q^2, z, \varepsilon\right) - 2\mathcal{C}\ln\Gamma_{cc}\left(\hat{a}_s, \mu^2, \mu_F^2, z, \varepsilon\right)$$

$$\mathcal{S}_c = \mathcal{C}\exp\left(2\Phi^c\right)$$

- $Z_{UV,c}(\mu_R^2)$
- $\Gamma_{cc}(\mu_F^2, z)$
- $\hat{F}_c(Q^2)$
- $\Phi^c(q^2, z)$

} Building Blocks !

# Renormalisation Group Equations

Renormalisation costant:

$$\log[Z_{c,UV}]^2 \qquad Z_{c,UV}(\hat{a}_s,\mu_R^2,\mu^2,\varepsilon)=\exp\left(\int_0^{\mu_R^2}\frac{d\lambda^2}{\lambda^2}\gamma_{c,UV}(\lambda^2,\varepsilon)\right) \qquad \alpha_s^3$$

Form factor

$$\log|\hat{F}^c|^2 \qquad \hat{F}^c(\hat{a}_s,Q^2,\mu^2,\varepsilon)=\exp\left(\int_0^{Q^2}\frac{d\lambda^2}{\lambda^2}\Gamma_{\hat{F},c}(\lambda^2,\epsilon)\right) \qquad \alpha_s^3$$

DGLAP Kernel

$$\log\Gamma_{cc} \qquad \Gamma_{cc}(z,\mu_F^2,\varepsilon)=\mathcal{C}\exp\left(\frac{1}{2}\int_0^{\mu_F^2}\frac{d\lambda^2}{\lambda^2}P_{cc}(\lambda^2,z,\varepsilon)\right) \qquad \alpha_s^3$$

Soft Function

$$\Phi^c$$

Unknown

# Solution to K+G/Sudakov Equation

Solution to K+G equation for $\Phi^c(\hat{a}_s, q^2, \mu^2, \varepsilon)$

$$\Phi^c = \Phi^c_A + \Phi^c_B$$

SV

$$\Phi^c_A = \sum_{i=1}^{\infty} \hat{a}^i_s \left(\frac{q^2(1-z)^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S^i_\varepsilon \left(\frac{i\varepsilon}{1-z}\right) \hat{\phi}^{c(i)}_{SV}(\varepsilon)\,,$$

Matrix element

Phase Space

NSV

$$\Phi^c_B = \sum_{i=1}^{\infty} \hat{a}^i_s \left(\frac{q^2(1-z)^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S^i_\varepsilon \varphi^{(i)}_c(z,\varepsilon)$$

$\Phi^c$ = IR Divergent part + Finite part

IR Div cancels with rest -> Finite part = $\Phi^c_f(z)$

# Our Results:

SV+ NSV Coefficient function for Diagonal Channels c = q, Qb, b,Bb,g

$$\Delta_c(z) = \Delta^{SV}_{c\bar{c}}(q^2, \mu_R^2, \mu_F^2, z) + \Delta^{NSV}_{c\bar{c}}(q^2, \mu_R^2, \mu_F^2, z)$$

Main Results of our work are

- Exponentiation of SV+NSV terms to all orders z-space

$$\Delta_c(z) = \mathcal{C} \exp\left(\Psi^c(q^2, \mu_R^2, \mu_F^2, z, \varepsilon)\right)\Big|_{\varepsilon=0},$$

- Resummation of SV+NSV terms to all orders N-space

$$\Delta_N^c(q^2) = C_0^c(q^2)\ e^{\Psi^c_{sv,N}(q^2,\omega)+\Psi^c_{nsv,N}(q^2,\omega)}$$

We use

- Factorisation of universal IR configuration
- Renormalisation Group Invariance

Mellin Convolution in z-space:

$$\mathcal{C}e^{f(z)} = \delta(1-z) + \frac{1}{1!}f(z) + \frac{1}{2!}f(z)\otimes f(z) + \cdots .$$

# Integral Representation

Integral representation:

$$\Delta_c(q^2, z) = C_0^c(q^2) \ \mathcal{C} \exp\left(2\Psi_{\mathcal{D}}^c(q^2, z)\right),$$

Exponent:

$$\Psi_{\mathcal{D}}^c(q^2, z) = \frac{1}{2}\int_{\mu_F^2}^{q^2(1-z)^2} \frac{d\lambda^2}{\lambda^2} P'_{cc}(a_s(\lambda^2), z) + \mathcal{Q}^c(a_s(q^2(1-z)^2), z)$$

$C_0^c$ Process dependent constant

$P'_{cc}(z)$ Process independent (Universal):
SV+NSV part of Alatarell-Parisi splitting function

$\mathcal{Q}^c(z)$ Process dependent function

# Perturbative predictions

$$\Delta_c(z) = \mathcal{C} \exp\left(\Psi^c(q^2, \mu_R^2, \mu_F^2, z, \varepsilon)\right)\Bigg|_{\varepsilon=0}$$

$$= \sum_{k=0}^{\infty} \Delta^c_{\mathcal{D},k} \left(\frac{\log^k(1-z)}{1-z}\right)_+ + \Delta^c_\delta\, \delta(1-z)$$ SV

$$+ \sum_{k=0}^{\infty} \Delta^c_{L,k}\, \log^k(1-z)$$ NSV

Perturbative expansion: $$\Delta^c_{J,k} = \sum_{i=0}^{\infty} a_s^i \Delta^{c,(i)}_{J,k}$$

Predictive Power:

Lower order results decide ``Certain `` higher order SV and NSV terms to all orders

# All order perturbative predictions

All order exponentiation can predict  to all orders from lower orders:

$$\Delta_c(z) = \mathcal{C} \exp\left(\Psi^c(q^2, \mu_R^2, \mu_F^2, z, \varepsilon)\right)\Big|_{\varepsilon=0}$$
$$= \sum_{i=0}^{\infty} a_s^i \; \Delta_c^{(i)}(z)$$

$$\mathcal{D}_k = \left(\frac{\log^k(1-z)}{1-z}\right)_+$$

$$L_z = \log(1-z)$$

| GIVEN | | | | PREDICTIONS | | |
|---|---|---|---|---|---|---|
| $\Psi_c^{(1)}$ | $\Psi_c^{(2)}$ | $\Psi_c^{(3)}$ | $\Psi_c^{(n)}$ | $\Delta_c^{(2)}$ | $\Delta_c^{(3)}$ | $\Delta_c^{(i)}$ |
| $\mathcal{D}_0, \mathcal{D}_1, \delta$ $L_z^1, L_z^0$ | | | | $\mathcal{D}_3, \mathcal{D}_2$ $L_z^3$ | $\mathcal{D}_5, \mathcal{D}_4$ $L_z^5$ | $\mathcal{D}_{(2i-1)}, \mathcal{D}_{(2i-2)}$ $L_z^{(2i-1)}$ |
| | $\mathcal{D}_0, \mathcal{D}_1, \delta$ $L_z^2, L_z^1, L_z^0$ | | | | $\mathcal{D}_3, \mathcal{D}_2$ $L_z^4$ | $\mathcal{D}_{(2i-3)}, \mathcal{D}_{(2i-4)}$ $L_z^{(2i-2)}$ |
| | | $\mathcal{D}_0, \mathcal{D}_1, \delta$ $L_z^3, \cdots, L_z^0$ | | | | $\mathcal{D}_{(2i-5)}, \mathcal{D}_{(2i-6)}$ $L_z^{(2i-3)}$ |
| | | | $\mathcal{D}_0, \mathcal{D}_1, \delta$ $L_z^n, \cdots, L_z^0$ | | | $\mathcal{D}_{(2i-(2n-1))}, \mathcal{D}_{(2i-2n)}$ $L_z^{(2i-n)}$ |

# Third order predictions for Drell-Yan production

## Drell-Yan production

Duhr et al

$$\Delta_q(z) = \delta(1-z) + a_s \Delta_q^{(1)}(z) + a_s^2 \Delta_q^{(2)}(z) + a_s^3 \Delta_q^{(3)}(z) + \cdots$$

## 3rd order prediction: Using known results upto 2nd order

$$\Delta_q^{(3)} = \Delta_q^{SV,(3)}(z) + \sum_{k=0}^{5} C_k \ \log^k(1-z) + \cdots$$

N3LO predictions

| | $\log^5(1-z)$ | | $\log^4(1-z)$ | | $\log^3(1-z)$ | |
|---|---|---|---|---|---|---|
| $C_F^3$ | $-512$ | -512 | 1728 | 1728 | $2272 + 3072\ \zeta 2$ | $2272 + 3072\ \zeta_2$ |
| $C_F^2 n_f$ | 0 | 0 | $-\frac{1280}{9}$ | $-\frac{1280}{9}$ | $\frac{19456}{27}$ | $\frac{6464}{9} + \chi_1$ |
| $C_A C_F^2$ | 0 | 0 | $\frac{7040}{9}$ | $\frac{7040}{9}$ | $-\frac{111904}{27} + 512\ \zeta_2$ | $-\frac{37184}{9} + 512\zeta_2 + \chi_2$ |
| $C_F n_f^2$ | 0 | 0 | 0 | 0 | $-\frac{256}{27}$ | $-\frac{256}{27}$ |
| $C_A C_F n_f$ | 0 | 0 | 0 | 0 | $\frac{2816}{27}$ | $\frac{2816}{27}$ |
| $C_A^2 C_F$ | 0 | 0 | 0 | 0 | $-\frac{7744}{27}$ | $-\frac{7744}{27}$ |

# Third order predictions for Higgs production

Anastasiou et al

Higgs boson production g+g -> Higgs

$$\Delta_g(z) = \delta(1-z) + a_s \Delta_g^{(1)}(z) + a_s^2 \Delta_g^{(2)}(z) + a_s^3 \Delta_g^{(3)}(z) + \cdots$$

3rd order prediction: Using known results upto 2nd order

$$\Delta_g^{(3)} = \Delta_g^{SV,(3)}(z) + \sum_{k=0}^{5} C_k \, \log^k(1-z) + \cdots$$

N3LO predictions

| | $\log^5(1-z)$ | | $\log^4(1-z)$ | | $\log^3(1-z)$ | |
|---|---|---|---|---|---|---|
| $C_a^3$ | $-512$ | -512 | $\frac{22592}{9}$ | $\frac{22592}{9}$ | $-\frac{111008}{27} + 3584\ \zeta_2$ | $-\frac{111008}{27} + 3584\ \zeta_2$ |
| $C_a^2 n_f$ | 0 | 0 | $-\frac{1280}{9}$ | $-\frac{1280}{9}$ | $\frac{6560}{9}$ | $\frac{19616}{27} + \chi_1$ |
| $C_a n_f^2$ | 0 | 0 | 0 | 0 | $-\frac{256}{27}$ | $-\frac{256}{27}$ |

# Third order predictions for Higgs production

Higgs boson production in b+Bbar -> g

Duhr et al

$$\Delta_b(z) = \delta(1-z) + a_s \Delta_b^{(1)}(z) + a_s^2 \Delta_b^{(2)}(z) + a_s^3 \Delta_b^{(3)}(z) + \cdots$$

3rd order prediction: Using known results upto 2nd order

$$\Delta_b^{(3)} = \Delta_b^{SV,(3)}(z) + \sum_{k=0}^{5} C_k \ \log^k(1-z) + \cdots$$

N3LO predictions

| | $\log^5(1-z)$ | | $\log^4(1-z)$ | | $\log^3(1-z)$ | |
|---|---|---|---|---|---|---|
| $C_F^3$ | $-512$ | -512 | 1728 | 1728 | $2272 + 3072\ \zeta 2$ | $2272 + 3072\ \zeta_2$ |
| $C_F^2 n_f$ | 0 | 0 | $-\frac{1280}{9}$ | $-\frac{1280}{9}$ | $\frac{19456}{27}$ | $\frac{6464}{9} + \chi_1$ |
| $C_A C_F^2$ | 0 | 0 | $\frac{7040}{9}$ | $\frac{7040}{9}$ | $-\frac{111904}{27} + 512\ \zeta_2$ | $-\frac{37184}{9} + 512\zeta_2 + \chi_2$ |
| $C_F n_f^2$ | 0 | 0 | 0 | 0 | $-\frac{256}{27}$ | $-\frac{256}{27}$ |
| $C_A C_F n_f$ | 0 | 0 | 0 | 0 | $\frac{2816}{27}$ | $\frac{2816}{27}$ |
| $C_A^2 C_F$ | 0 | 0 | 0 | 0 | $-\frac{7744}{27}$ | $-\frac{7744}{27}$ |

# Fourth order prediction

## 4th order QCD prediction for Drell-Yan production

Vogt,Moch et al,
DeFlorian et al, Das et al

$$\Delta_q^{(4)} = \left(-\frac{4096}{3}C_F^4\right)\log^7(1-x) + \left(\frac{39424}{9}C_F^3C_A + \frac{19712}{3}C_F^4 - \frac{7168}{9}n_fC_F^3\right)$$
$$\times\log^6(1-x) + \left(-\frac{123904}{27}C_F^2C_A^2 - \frac{805376}{27}C_F^3C_A + 9088C_F^4 + \frac{45056}{27}n_fC_F^2C_A\right.$$
$$\left.+\frac{139520}{27}n_fC_F^3 - \frac{4096}{27}n_f^2C_F^2 + 3072\zeta_2C_F^3C_A + 20480\zeta_2C_F^4\right)\log^5(1-x)$$

## 4th order QCD prediction for Higgs production in gluon fusion

$$\Delta_g^{(4)} = \left(-\frac{4096}{3}C_A^4\right)\log^7(1-x) + \left(\frac{98560}{9}C_A^4 - \frac{7168}{9}n_fC_A^3\right)\log^6(1-x)$$
$$+\left(-\frac{298240}{9}C_A^4 + \frac{174208}{27}n_fC_A^3 - \frac{4096}{27}n_f^2C_A^2 + 23552\zeta_2C_A^4\right)\log^5(1-x).$$

# Logarithmic Accuracy

The towers of $\ln N$ that we sum over,

$$\Delta_N^c =$$

**LL**

$a_s \ln^2 N$

$a_s^2 \ln^4 N$

$\vdots$

$a_s^i \ln^{2i} N$

Exponents: $\tilde{g}_{0,0}, g_1$

**Only 1-loop info**

**NLL**

$a_s^2\{\ln^3 N, \ln^2 N\}$

$a_s^3\{\ln^5 N, \ln^4 N\}$

$\vdots$

$a_s^i\{\ln^{2i-1} N, \ln^{2i-2} N\}$

Exponents: $\tilde{g}_{0,1}, g_2$

**Only 2-loop info**

$\cdots$

**N$^n$LL**

$a_s^n\{\ln^{n+1} N, \ln^n N\}$

$\vdots$

$a_s^i\{\ln^{2i-n+1} N, \ln^{2i-n} N\}$

Exponents: $\tilde{g}_{0,n-1}, g_n$

**Only n-loop info**

# Resummation of NSV logs

## Resummed NSV

$$\Psi^c_{nsv,N} = \frac{1}{N} \sum_{n=0}^{\infty} \sum_{k=0}^{n} h_{nk}(w)\ a_s^n \log^k N$$

## Resumed Logarithms

$$\log^k(1-z) \to \frac{1}{N} \log^k N$$

LL → $a_s^i \frac{1}{N} \log^{2i-1} N$

NLL → $a_s^i \frac{1}{N} \log^{2i-2} N$

NnLL → $a_s^i \frac{1}{N} \log^{2i-n} N$

# Logarithmic Accuracy

The towers of $\ln N/N$ that we sum over,

$$\Delta^c_N =$$

$\overline{\mathrm{LL}}$:

$$a_s \frac{1}{N} \log N$$
$$a_s^2 \frac{1}{N} \log^3 N$$
$$a_s^3 \frac{1}{N} \log^5 N$$
$$\vdots$$
$$a_s^i \frac{1}{N} \log^{2i-1} N$$

$\overline{\mathrm{NLL}}$:

$$a_s^2 \frac{1}{N} \log^2 N$$
$$a_s^3 \frac{1}{N} \log^4 N$$
$$a_s^4 \frac{1}{N} \log^6 N$$
$$\vdots$$
$$a_s^i \frac{1}{N} \log^{2i-2} N$$

$\cdots$

$\overline{\mathrm{N^{n-1}LL}}$:

$$a_s^n \frac{1}{N} \log^n N$$
$$\vdots$$
$$a_s^i \frac{1}{N} \log^{2i-n} N$$

Exponents:

| Logarithmic Accuracy | Resummed Exponents |
|---|---|
| $\overline{\mathrm{LL}}$ | $\tilde{g}^g_{0,0}, g^g_1, \bar{g}^g_1, h^g_0$ |
| $\overline{\mathrm{NLL}}$ | $\tilde{g}^g_{0,1}, g^g_2, \bar{g}^g_2, h^g_1$ |
| $\overline{\mathrm{NNLL}}$ | $\tilde{g}^g_{0,2}, g^g_3, \bar{g}^g_3, h^g_2$ |

# Matched cross section

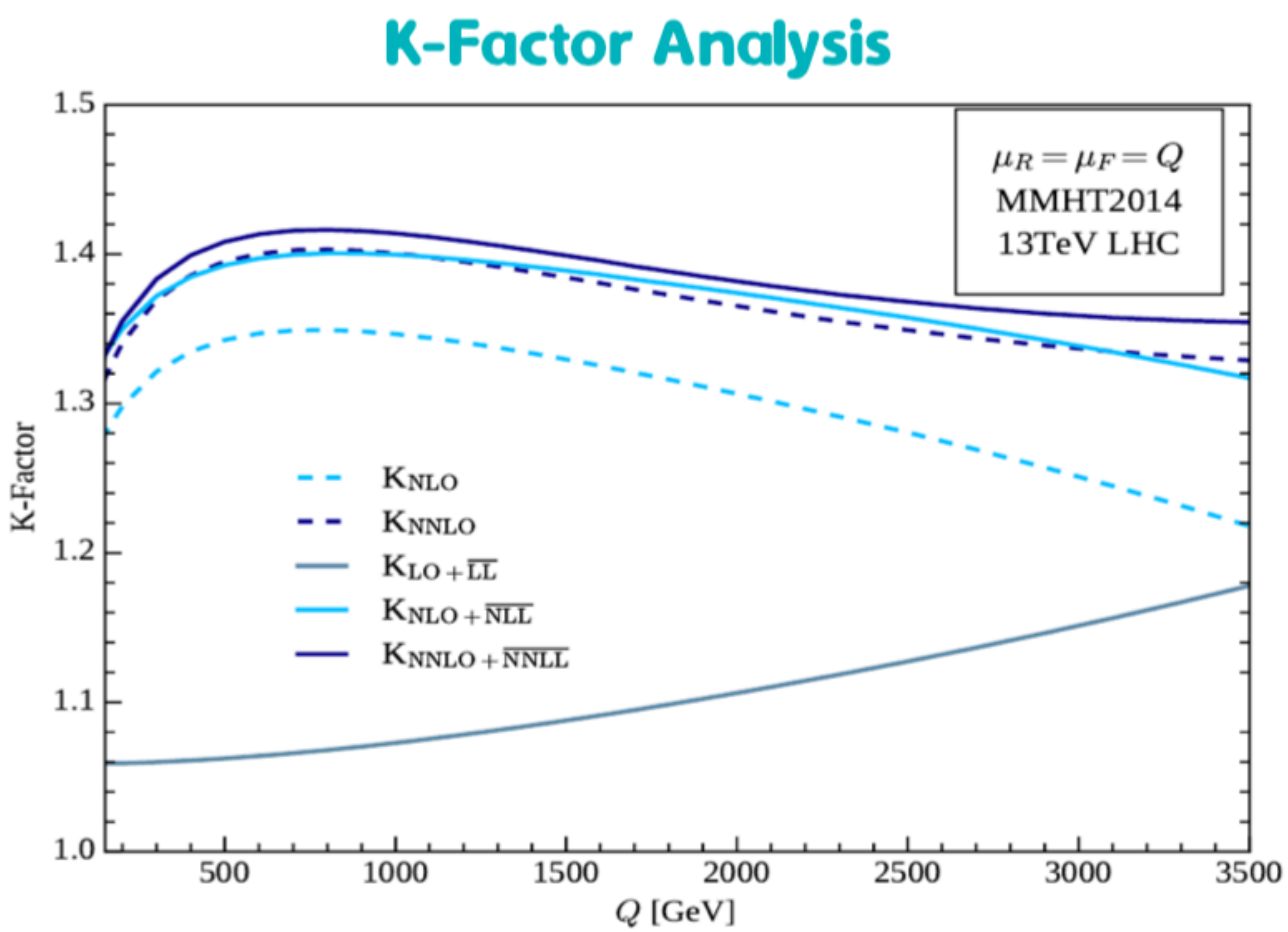
**Mellin Inversion of the resummed result and add it to fixed order results**

$$\sigma_N^{\mathrm{N^nLO}+\overline{\mathrm{N^nLL}}} = \sigma_N^{\mathrm{N^nLO}} + \sigma^{(0)} \sum_{ab\in\{q,\bar{q}\}} \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} (\tau)^{-N} \delta_{a\bar{b}} f_{a,N}(\mu_F^2) f_{b,N}(\mu_F^2) \times \left( \Delta_{q,N}\Big|_{\overline{\mathrm{N^nLL}}} - \Delta_{q,N}\Big|_{tr\ \mathrm{N^nLO}} \right).$$

The resummed results are matched to the fixed order result in order to avoid any double counting of threshold logarithms

The contour C in the Mellin inversion is chosen according to Minimal prescription

# K-factor for DY




$$K(Q) = \frac{\frac{d\sigma}{dQ}(\mu_R = \mu_F = Q)}{\frac{d\sigma^{LO}}{dQ}(\mu_R = \mu_F = Q)}$$

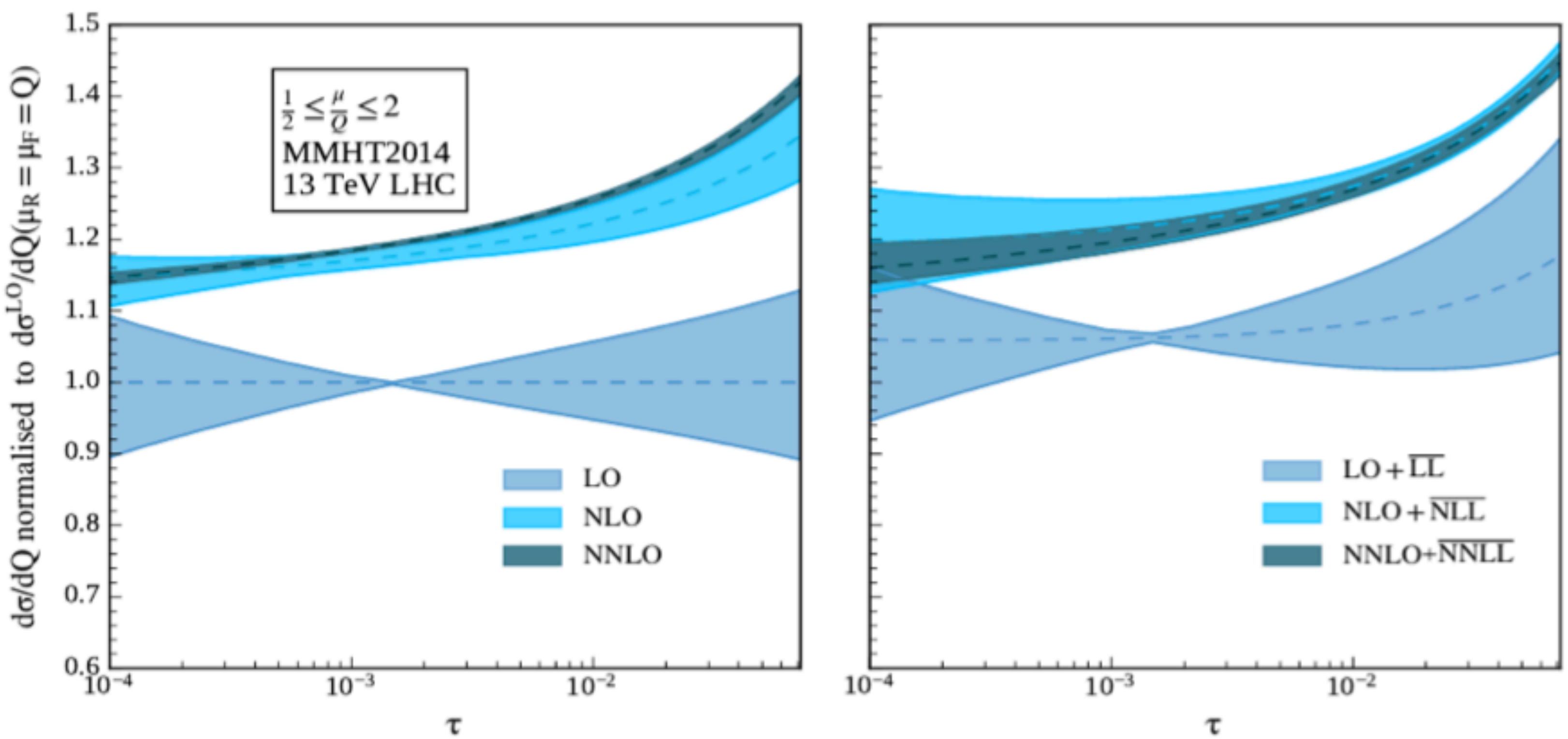
- Enhancement

| | Q=500 GeV | 2000 GeV |
|---|---|---|
| **LO-** $\rightarrow \mathrm{LO} + \overline{\mathrm{LL}}$ | **6.2 %** | **10.6%** |
| **NLO-** $\rightarrow \mathrm{NLO} + \overline{\mathrm{NLL}}$ | **3.7 % ,** | **5.2%** |
| **NNLO** $\rightarrow \mathrm{NNLO} + \overline{\mathrm{NNLL}}$ | **0.94 %** | **1.2%** |

- Resummed curves are closer
- They decreases as we go for higher order resummed contributions
- Perturbative convergence

| $\mu_R = \mu_F = Q(\mathrm{GeV})$ | $\mathrm{LO} + \overline{\mathrm{LL}}$ | NLO | $\mathrm{NLO} + \overline{\mathrm{NLL}}$ | NNLO | $\mathrm{NNLO} + \overline{\mathrm{NNLL}}$ |
|---|---|---|---|---|---|
| 500 | 1.0624 | 1.3425 | 1.3925 | 1.3950 | 1.4082 |
| 1000 | 1.0728 | 1.3464 | 1.3995 | 1.4004 | 1.4138 |
| 2000 | 1.1062 | 1.3064 | 1.3739 | 1.3652 | 1.3818 |

# Scale uncertainty for DY



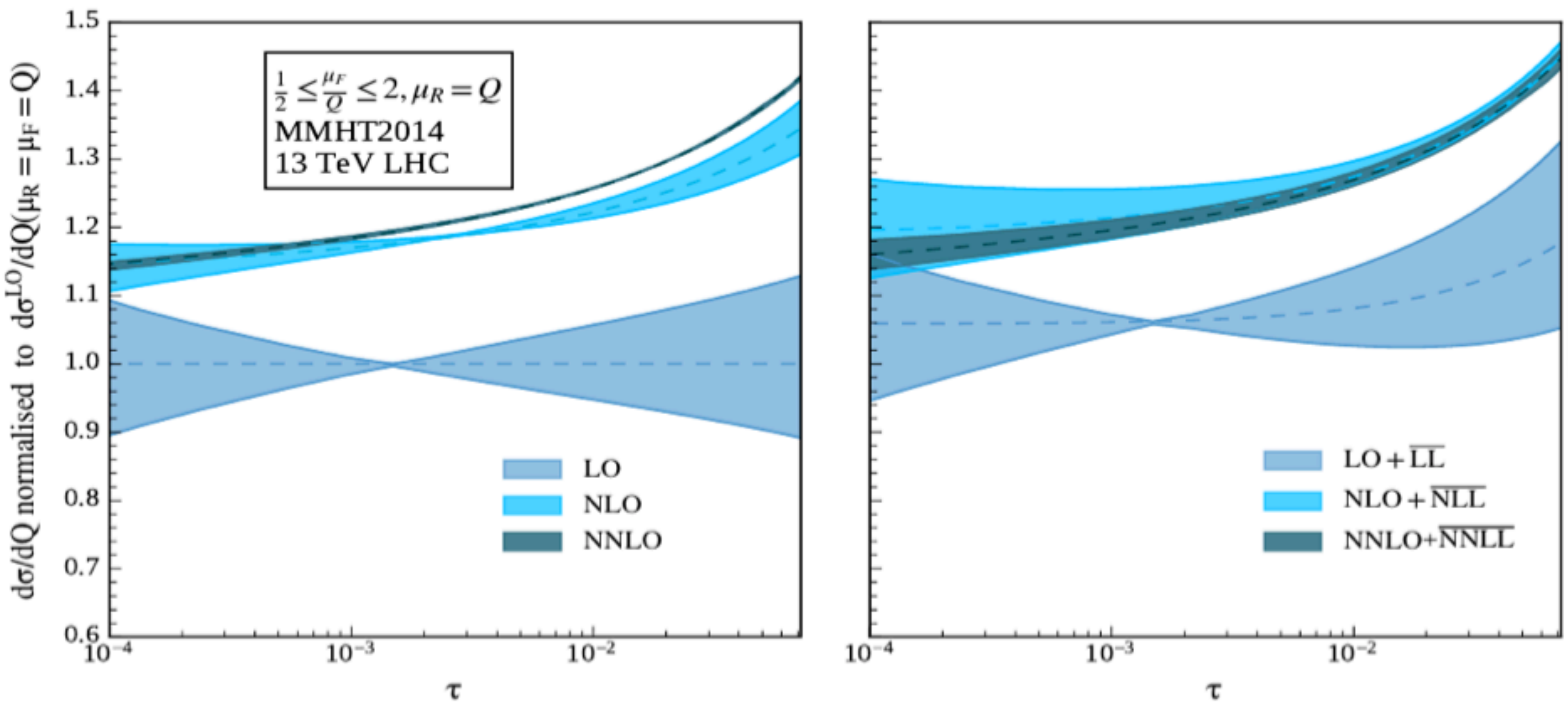

- Systematic reduction of the uncertainties at different logarithmic accuracies
- Improvement at the NLO+NLL than at the NNLO+NNLL in comparison to their respective F.O predictions

7-point var: μ = {μF , μR} is varied in the range [1/2Q, 2Q] keeping the ratio μR /μF not larger than 2 and smaller than 1/2.

| $Q$ | LO | LO+$\overline{\text{LL}}$ | NLO | NLO+$\overline{\text{NLL}}$ | NNLO | NNLO+$\overline{\text{NNLL}}$ |
|---|---|---|---|---|---|---|
| 1000 | $2.3476^{+4.10\%}_{-3.92\%}$ | $2.5184^{+4.49\%}_{-4.25\%}$ | $3.1609^{+1.79\%}_{-1.69\%}$ | $3.2857^{+2.08\%}_{-1.18\%}$ | $3.2876^{+0.20\%}_{-0.31\%}$ | $3.3191^{+1.13\%}_{-0.86\%}$ |
| 2000 | $0.0501^{+8.50\%}_{-7.46\%}$ | $0.0554^{+9.10\%}_{-7.91\%}$ | $0.0654^{+2.83\%}_{-2.98\%}$ | $0.0688^{+1.43\%}_{-1.23\%}$ | $0.0684^{+0.37\%}_{-0.62\%}$ | $0.0692^{+0.89\%}_{-0.78\%}$ |

Cross section in 10^−5 pb/GeV

# Factorisation scale uncertainty for DY




$NLO: 22\%$ from $q\bar{q}$

$-5\%$ from $qg$

$NNLO: 4.9\%$ from $q\bar{q}$

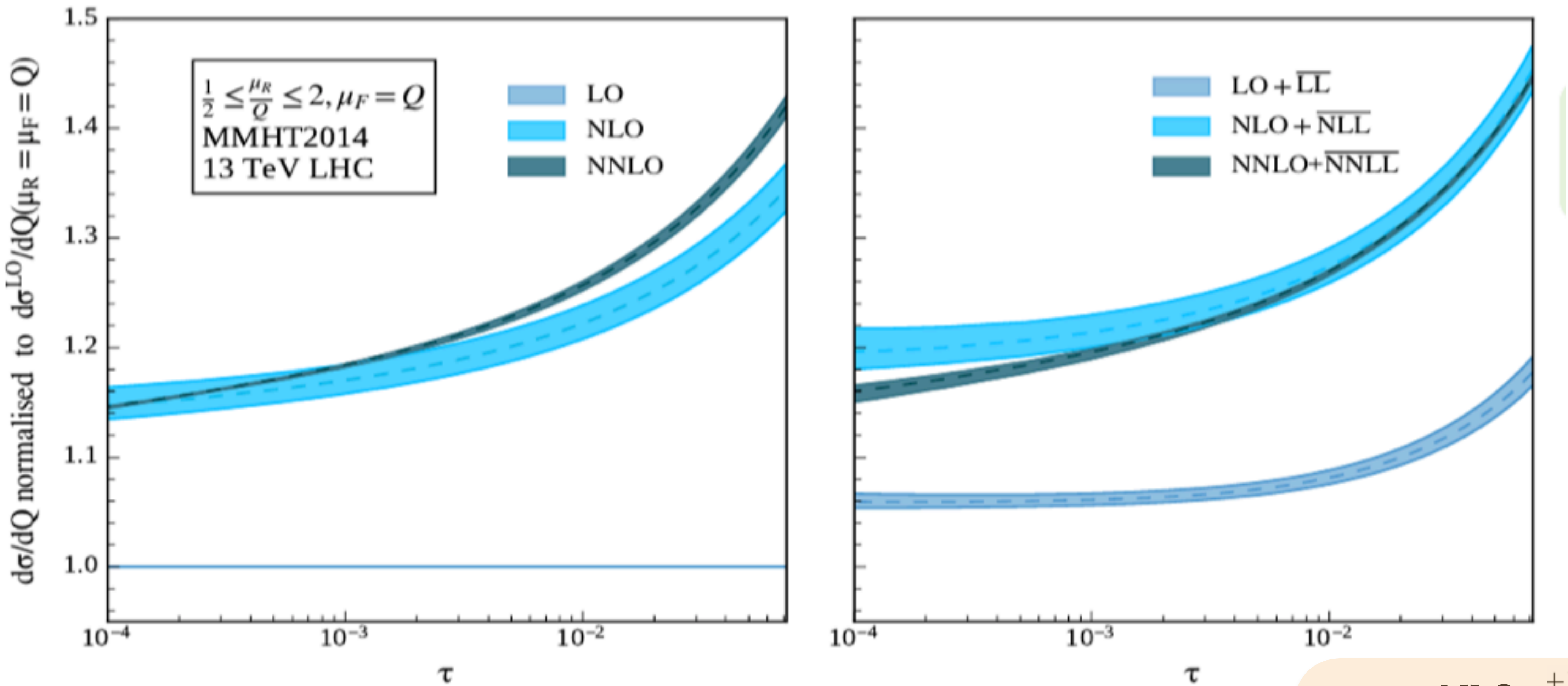
$-2.8\%$ from $qg$

- resummed bands look similar to that of 7-point bands --width of the 7-point bands mainly comes from the μF uncertainties

- NLO band gets improved with the inclusion of NLL, but NNLO band increses with the inclusion of NNLL

- Missing qg contribution increases the scale dependence

# Renormalisation scale uncertainty for DY



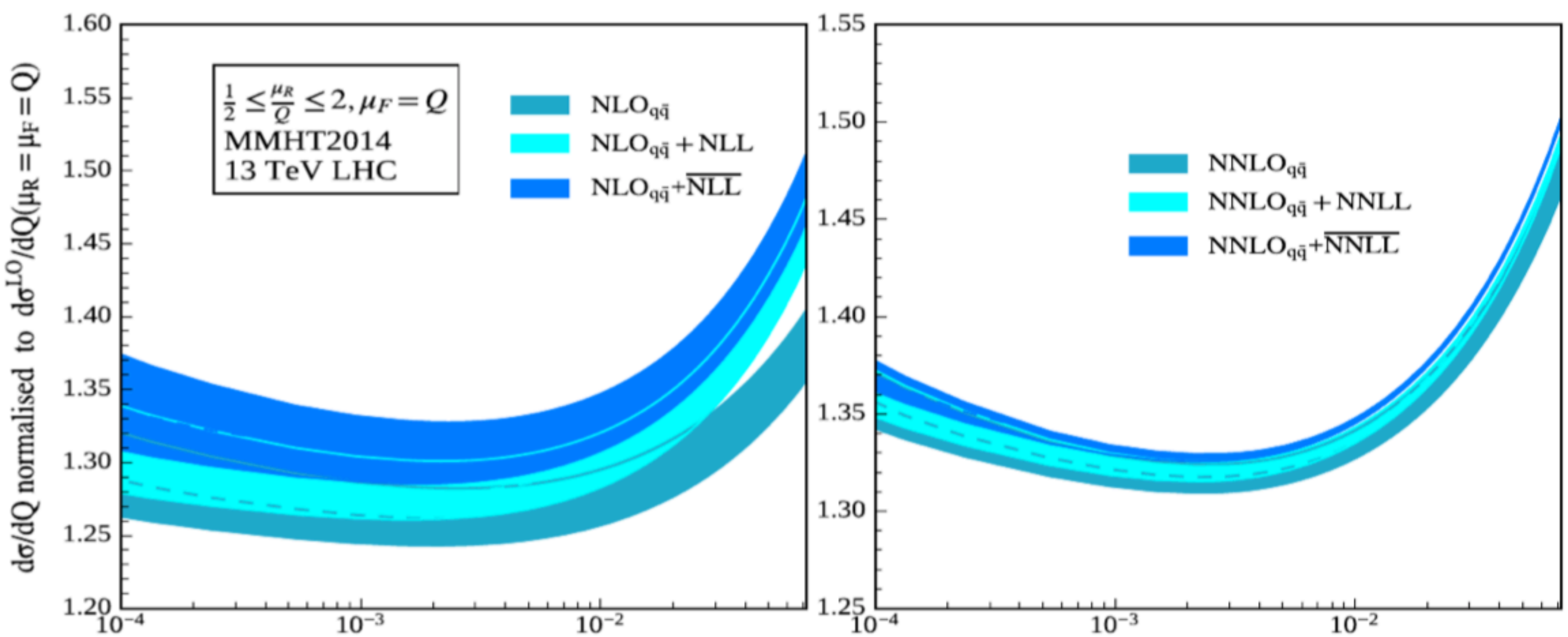

- NNLO+NNLL the error band becomes substantially thinner
- Each partonic channel is invariant under μ R variation and hence inclusion of More corrections within a channel is expected to reduce the uncertainity

- $\mathrm{NLO}: {}^{+1.46\%}_{-1.28\%}$
- $\mathrm{NLO} + \overline{\mathrm{NLL}}: {}^{+1.35\%}_{-1.23\%}$
- $\mathrm{NNLO}: {}^{+0.37\%}_{-0.46\%}$
- $\mathrm{NNLO} + \overline{\mathrm{NNLL}}: {}^{+0.02\%}_{-0.23\%}$

Scale dependence gets reduced significantly due to the inclusion NSV resummation

# Renormalisation scale uncertainty for q qb in DY

Uncertainities w.r.t renormalisation scale variation in the quark antiquark channel

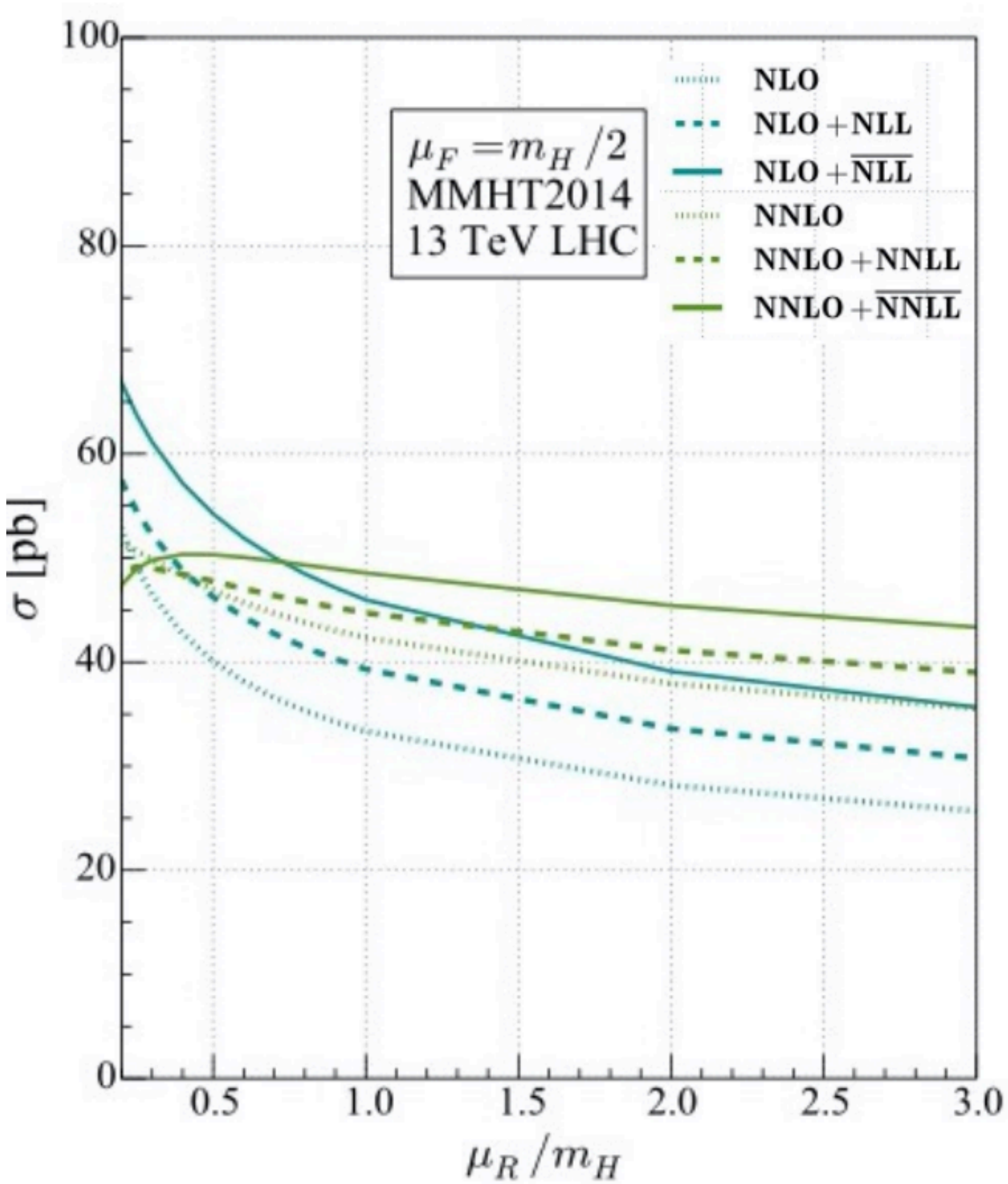



| $Q = \mu_R = \mu_F$ | $\text{NNLO}_{q\bar{q}}$ | $\text{NNLO}_{q\bar{q}} + \text{NNLL}$ | $\text{NNLO}_{q\bar{q}} + \overline{\text{NNLL}}$ |
|---|---|---|---|
| 1000 | $3.5260^{+0.49\%}_{-0.58\%}$ | $3.5376^{+0.25\%}_{-0.39\%}$ | $3.5576^{+0.006\%}_{-0.20\%}$ |
| 2000 | $0.0717^{+0.54\%}_{-0.62\%}$ | $0.0721^{+0.19\%}_{-0.33\%}$ | $0.0725^{+0.0\%}_{-0.15\%}$ |

Cross section in 10^−5 pb/GeV for $q\bar{q}$ channel

Interestingly, the behaviour of NNLO_qqb + NNLLbar is significantly improved from the corresponding SV results, NNLO_qqb + NNLL, for a wide range of Q.

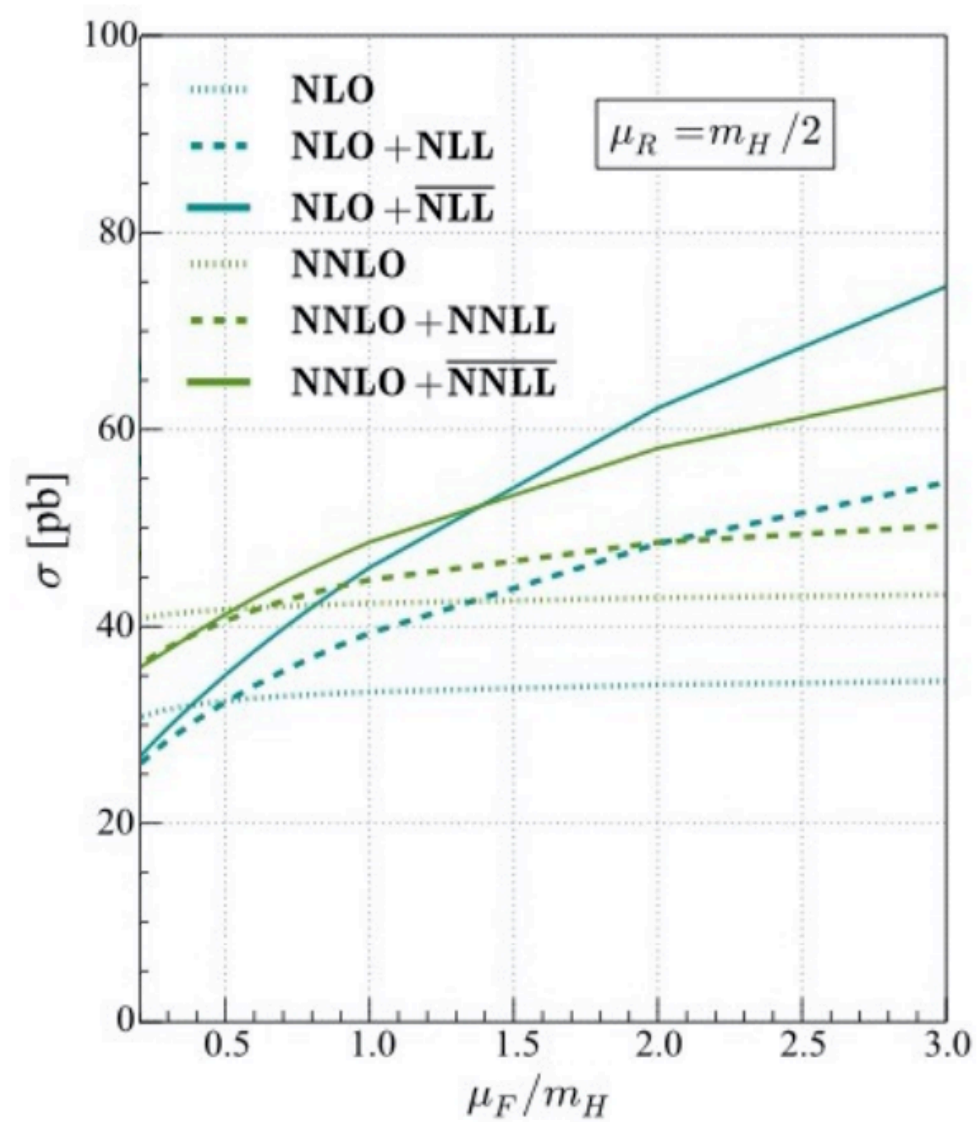
# Renormalisation scale uncertainty for Higgs

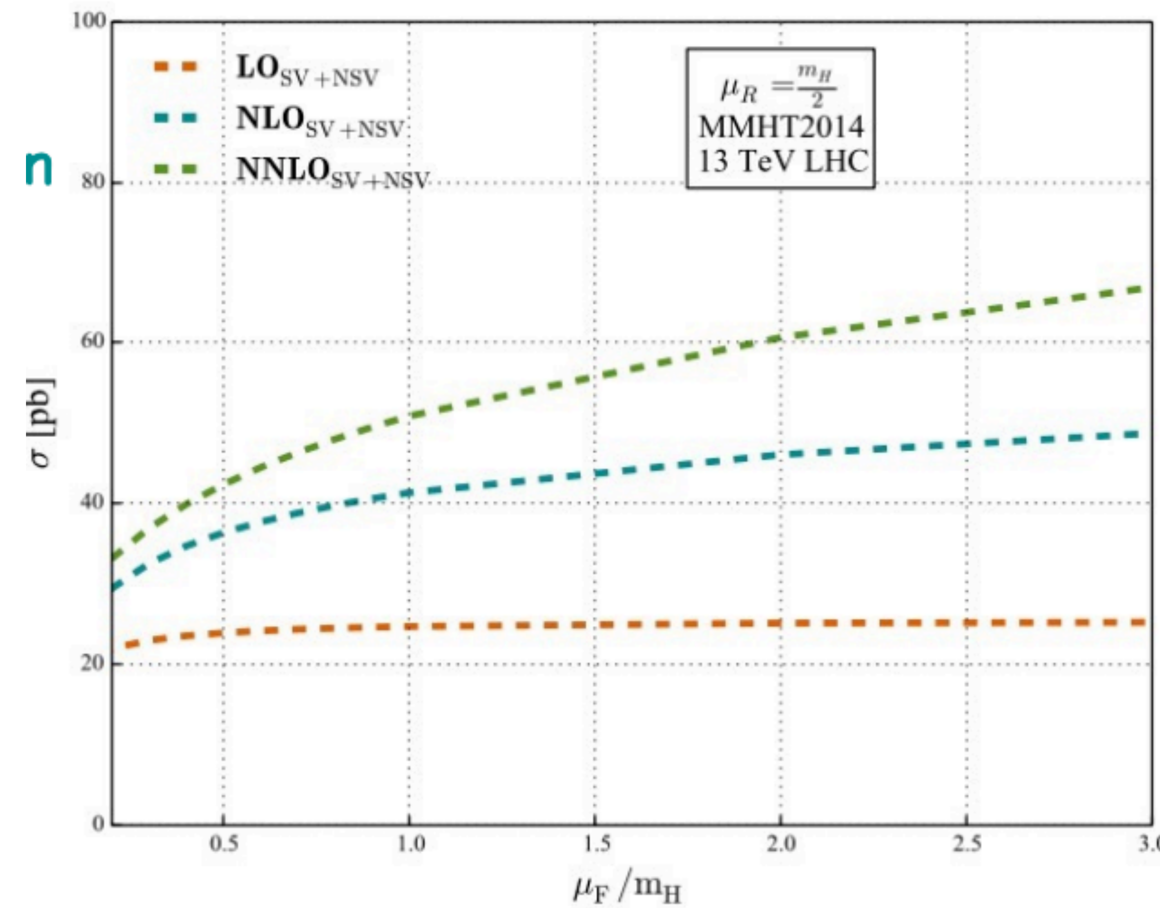



- Renormalisation scale variation is less for resummed results as compared to fixed order
- However, the scale variation is comparable for SV resumed and SV+NSV resumed at NLO accuracy
- No significant improvement by the inclusion of SV but Comprehensible improvement by NSV Res results at NNLO
- SV is dominant at NLO with 73.16% contribution
- NSV is dominant at NNLO with 58.9% contribution while SV is only 15.8%

| NLO | NLO+NLL | NLO+$\overline{\text{NLL}}$ | NNLO | NNLO+NNLL | NNLO+$\overline{\text{NNLL}}$ |
|---|---|---|---|---|---|
| $39.1681^{+9.09}_{-6.73}$ | $38.0142^{+7.06}_{-5.70}$ | $41.0325^{+7.06}_{-5.97}$ | $46.4304^{+4.11}_{-4.70}$ | $45.0904^{+4.32}_{-4.52}$ | $44.9685^{+2.94}_{-3.74}$ |

# Factorisation scale uncertainty for Higgs




- Fixed order truncated at SV+NSV contribution shows significant variations
- Factorization scale dependence due to NSV contribution cancels with beyond NSV terms
- Increase in dependence with the increase in order of accuracy μ
- % contribution of beyond NSV term increases with the order of accuracy

# Conclusions

- Studied Next to soft terms in Inclusive reactions
- Set up a Framework in z-space
- Framework to Resum Next to Soft terms N space
- Z-space prediction at third and fourth order
- N-space resummation to all orders for NSV
- Resummation in non-diagonal terms is still open problem