Joy of Discovery in the Modern Era: Some Reflections and Directions

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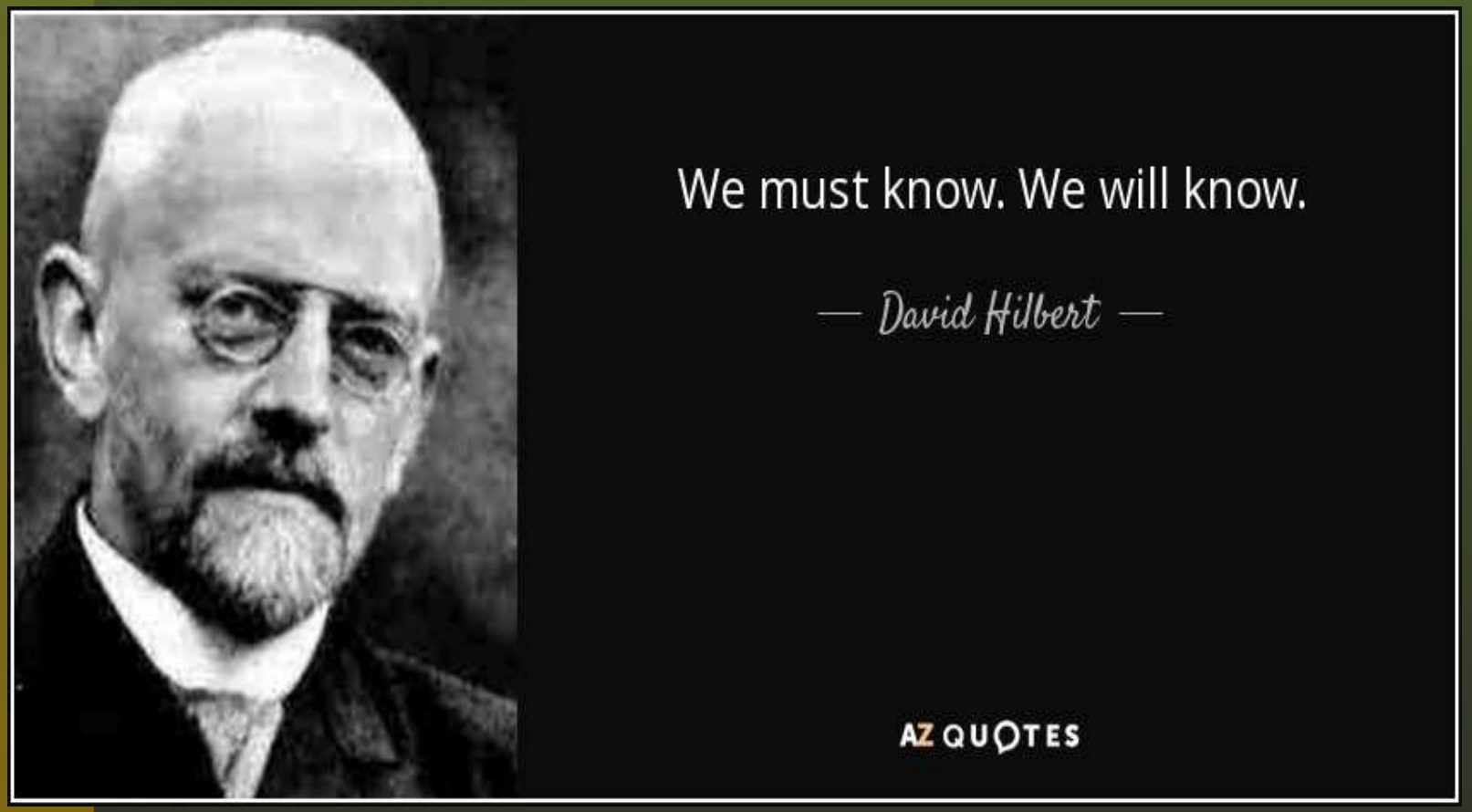
Dedication

To the Memory of all those martyrs to knowledge, young and old.

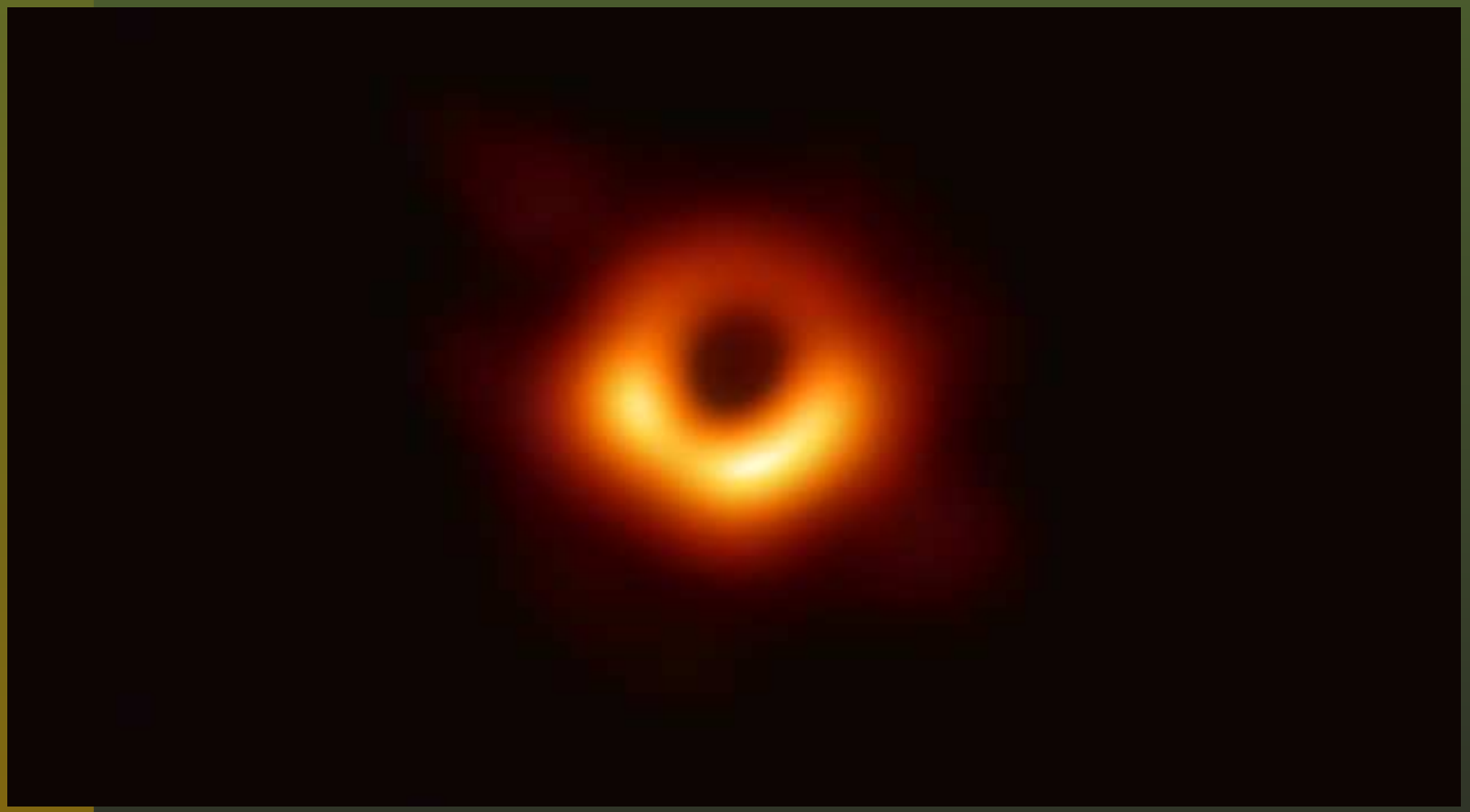
Your sacrifices will not be forgotten.

David Hilbert



Who is this?

Black Hole Horizon, EHT



LIGO Discovery

- “Ladies and gentlement, we have detected gravitational waves. We did it!” Professor David Reitze
- Prediction of the General Theory of Relativity made about a 100 years ago
- Gravitational waves from binary black hole system of 36 and 29 solar masses (total of 65 solar masses): a new black hole of 62 solar masses 1.4 billion years ago
- More recently a neutron star merger has also been seen
- NANOgrav: consistent with supermassive blackholes
- INDIGO for better triangulation
- Comes after vast improvements in technology, laser interferometry
- Nobel Prize to Kip Thorne, Rainer Weiss and Barry Barish

Founders of GR and black holes



Albert Einstein, Karl Schwarzschild and Roy Patrick Kerr

KS (static black hole), RPK(spinning black hole)

Some observations

- “I was sitting in a chair in the patent office at Bern when all of sudden a thought occurred to me: If a person falls freely he will not feel his own weight. I was startled. This simple thought made a deep impression on me. It impelled me toward a theory of gravitation.” – Albert Einstein

Some observations

- "In some strange way, any new fact or insight that I may have found has not seemed to me as a "discovery" of mine, but rather something that had always been there and that I had chanced to pick up." – Subrahmanyan Chandrasekhar

S. Chandrasekhar

Scales in the Universe

Orders of Magnitude

Higgs Discovery

- "As a layman I would say: 'I think we have it'. Would you agree?"(words of Dr. Rolf-Dieter Heuer, Director General of CERN on July 4, 2012, reporting the discovery of the CMS and ATLAS collaborations of the Large Hadron Collider (LHC))
- Existence of particle predicted by Peter Higgs in 1964
- Higgs mechanism due to François Englert, Robert Brout, Higgs, (also Gerald Guralnik, Carl Hagen and Thomas Kibble)
- Took several generations of dedicated experiments at ever increasing energies to find it
- Massive amounts of technology, superconducting magnets, control of beams

What I do

- Elementary particle physics and field theory
- Standard Model and high precision
- Example: contribution of pion pairs to the muon anomalous magnetic moment
- Mathematical methods to improve Feynman diagram computations
- Renormalization theory and applications
- Planning for the future and physics at future accelerators

A recent paper

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Multiple Series Representations of N-fold Mellin-Barnes Integrals

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Mellin-Barnes (MB) integrals are well-known objects appearing in many branches of mathematics and physics, ranging from hypergeometric functions theory to quantum field theory, solid-state physics, asymptotic theory, etc. Although MB integrals have been studied for more than one century, until now there has been no systematic computational technique of the multiple series representations of N-fold MB integrals for $N > 2$. Relying on a simple geometrical analysis based on conic hulls, we show here a solution to this important problem. Our method can be applied to resonant (i.e., logarithmic) and nonresonant cases and, depending on the form of the MB integrand, it gives rise to convergent series representations or diverging asymptotic ones. When convergent series are obtained, the method also allows, in general, the determination of a single "master series" for each series representation, which considerably simplifies convergence studies and/or numerical checks. We provide, along with this Letter, a *Mathematica* implementation of our technique with examples of applications. Among them, we present the first evaluation of the hexagon and double box conformal Feynman integrals with unit propagator powers.

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Introduction.—N-fold Mellin-Barnes (MB) integrals are defined as

$$I(x_1,x_2,\ldots,x_N) = \int_{-i\infty}^{+i\infty}\frac{dz_1}{2\pi i}\cdots\int_{-i\infty}^{+i\infty}\frac{dz_N\prod_{i=1}^{k}\Gamma^{a_i}[s_i(\mathbf{z})+g_i]}{2\pi i\prod_{j=1}^{l}\Gamma^{b_j}[t_j(\mathbf{z})+h_j]}x_1^{z_1}\ldots x_N^{z_N}, \tag{1}$$

where a_i, b_j, k, l, and N are positive integers (with $k \geq N$ after possible cancellations due to the denominator), $\mathbf{z} = (z_1,\ldots,z_N)$ and where we have defined $s_i(\mathbf{z}) \doteq \mathbf{e}_i\cdot\mathbf{z}$ and $t_j(\mathbf{z}) \doteq \mathbf{f}_j\cdot\mathbf{z}$ for a later purpose. The vectors $\mathbf{e}_i$, $\mathbf{f}_j$ and the scalars g_i, h_j are reals, while $\mathbf{x} = (x_1,\ldots,x_N)$ can be complex, and the contours of integration, which avoid the poles of the gamma functions that belong to the numerator of the MB integrand, have to be specified. In the present Letter, we focus on the common situation where the set of poles of each of these gamma functions is not split in different subsets by the contours.

The importance of MB integrals cannot be overstated, as they appear in domains as diverse as hypergeometric functions theory [1–3], electromagnetic wave propagation in turbulence [4], asymptotics [5], quantum field theory (QFT) [6], etc. In QFT, which is of particular interest for the authors, an impressive array of publications of the last decades may be mentioned (see [6] for a complementary list). Early studies can be found in [7–9], followed by classical works [10–23] highlighting the relevance of MB integrals in QFT. These motivated the automatization of some of the computational steps of the MB technique [24–28]. Numerous applications were guided by the needs of particle physics phenomenology, e.g., [29–40] but also by more formal motivations [41–54]. Recently, MB integrals and the Mellin transform entered the conformal bootstrap, see e.g., [55,56] and references therein. Other recent and diverse applications exist as, for instance, in option pricing [57], detector physics [58] or Ruderman-Kittel-Kasuya-Yosida interaction in condensed matter [59].

Even though MB integrals have been thoroughly studied for several decades in theoretical physics, and in fact for

Robert Wilson on basic science

Only from a long-range point of view, of a developing technology. Otherwise, it has to do with: Are we good painters, good sculptors, great poets? I mean all the things that we really venerate and honor in our country and are patriotic about. In that sense, this new knowledge has all to do with honor and country but it has nothing to do directly with defending our country except to help make it worth defending.

Nature of Scientific Discovery

- Some areas require massive team work and technology for fundamental discoveries
- But also fantastic discoveries such as high T_c superconductors, graphene, topological insulators etc., and many are table top
- Made possible by advances in technology, single-mindedness and perseverence
- What is the role of the individual?
- Let us take some examples and see what do the protagonists say about their calling? What is the key message of all this?
- Struggle between reductionism and the need for synthesis

Examples from Mathematics

- One famous example is that of Pierre Fermat (1637) who gave his ‘Last theorem’ (FLT), which is that there are no whole number solutions for $x^n + y^n = z^n$, except when $n = 2$ (Pythagoras)
- Settled about 25 years ago by Andrew Wiles (childhood dream)
- This proof came from a long chain of ideas associated with a conjecture of Y. Taniyama and G. Shimura post-World War II
- Andrew Wiles proved a version of the conjecture which was sufficient in 1994 in a culmination of a decade of research work
- Wiles describes his ‘Eureka’ moment in a BBC interview

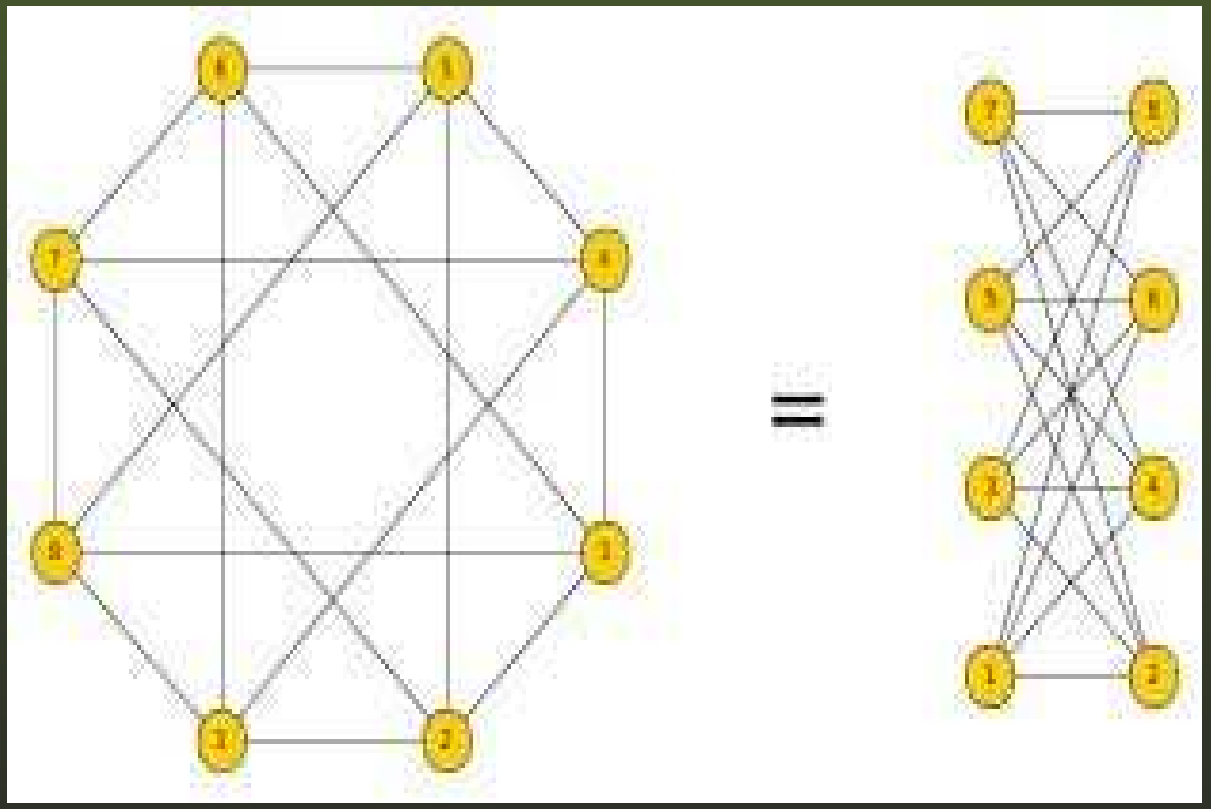
Examples from Mathematics

- Another recent example is the proof of the Poincaré conjecture
- Properties of surfaces in 3 dimensions and their ‘connectedness’
- Solved by Grigori Perelman using methods developed by Richard Hamilton, using ideas coming from physics community
- Perelman posts proofs on the internet and disappears from mathematics
- Refuses the Fields Medal and declines Clay Mathematics Prize worth 1 million dollars
- Adds to the romance of mathematics!

Grigori Perelman

Examples from Theoretical CS

- Kadison-Singer and Ramanujan Graphs



Kadison-Singer problem

- Quantum mechanics as a problem in linear algebra
- Questions of infinite-dimensional matrices
- Spielman and Kalai — problem in network sparsification while retaining properties of the original network
- Connecting the two through interlacing polynomials and Ramanujan graphs
- Solution found by Marcus, Spielman and Srivastava

Discoveries in other fields

- Theory of evolution
- Mendel's theory of genetics
- Continental drift
- Ice ages
- Pasteur's theory of microbes
- Shannon's information theory
- For biology, see 'The Eighth Day of Creation'

Science and mathematics by women

- Ada Lovelace(neé Byron) as founder of computer science
- Emmy Noether as founder of modern Algebra
- Marie Curie (Maria Sklodowska) and discovery of radium
- Cecelia Payne and the stuff the Universe is made of
- Julia Robinson and the solution to Hilbert's 10th problem – helped show that an algorithm for solutions to certain Diophantine solutions is undecidable

Science and mathematics by women

- Vera Rubin and the discovery of dark matter
- Jocelyn Bell and the discovery of the pulsar
- Rosalind Franklin and the path to the double-helix
- Late Miriam Mirzakhani and Maryna Viazovska
- Fabiola Gianotti, CERN's first woman Director-General
- Donna Strickland and Andrea Ghez, Physics Nobels
- Jennifer Doudna and Emmanuelle Charpentier, CRISPR, Medicine Nobel

Science in India before and after 1947

- Srinivasa Ramanujan and his fantastic mathematics
- C. V. Raman and discovery of the Raman effect
- M. N. Saha and his ionization equation
- S. N. Bose and the discovery of Bose statistics
- H. J. Bhabha and his scientific interests
- Vikram Sarabhai and his space mission
- A. K. Raychaudhuri and his equation
- G. N. Ramachandran and the triple helix

Science in India before and after 1947

- Birbal Sahni and paleobotany
- E. K. Janaki Ammal and cytogenetics
- Ram Prakash Bambah and Sarvadaman Chowla and Number Theory
- Harish-Chandra (1923-1983) as a towering figure
- E. C. George Sudarshan, Abdus Salam
- See books by G. Venkatraman on various Indian legends

Some recent and past romantic stories

- Bose, Parker and Shrikhande: disproved Euler conjecture regarding Latin squares
- IIT Madras group: Suhail Ahmad Rather and Arul Lakshminarayanan, co-workers from Poland – quantum solution to 36 entangled offiers of Euler
- (Partial?) solution of Hilbert's 12th Problem by Samit Dasgupta (Duke) and Mahesh Kakde of IISc with contributions from undergraduates

Psychology of creativity

- Jacques Hadamard and Hermann von Helmholtz: Preparation, Incubation, Illumination, and Verification.
- Example: By combining the formulae of Wien and Rayleigh, Planck announced in October 1900 a formula now known as Planck's radiation formula.
- Within two months Planck made a revolutionary deduction of his formula renouncing classical physics and introducing the quanta of energy, in order to reconcile the two classical limits.

Examples of Eureka moments

- Henri Poincaré (one of the last Universalists)

Henri Poincaré

- Describes his Eureka moment as to how when he was stepping onto an omnibus, the solution to a problem he was working on flashed on him

Examples of Eureka moments

- August Kekulé also talks of how he figured out the structure of benzene as a snake gobbling its own tail
- Max Planck on the discovery of Quantum Mechanics: “The outside world is something independent from man, something absolute, and the quest for the laws which apply to this absolute appeared to me as the most sublime scientific pursuit in life.”
- Scientists work very hard for this moment, which may or may not come!

Method of working

- For professionals, Nikhil Srivastava does not believe in being a "mega-workaholic."
- Short spurts of focused, high-quality work.
- Productivity in deeply thinking about one problem for weeks or months, and then "taking time off."
- "You should fail in different ways, but if you know you've tried everything and you're still failing in the same way, you may have to give up."

An example of science in the 21st century

- Venkataraman Ramakrishnan, ribosome discoveries, Nobel in Chemistry 2009
- 'It takes courage to tackle very hard problems in science'
- Does not want science to be converted into a contest
- Importance of multi-disciplinary training and taking ideas from many fields, and importance of hard work

Weinberg's 4 Golden Lessons

- No one knows everything, and you do not need to for starting research
- Aim for rough water
- Forgive yourself for wasting time
- Learn something about the history of science

Science and Communication

- Communication skills are essential
- Writing Research papers
- Era of multiple modes of communication, personal and electronic on a huge scale.
- Communication is ‘the imparting or exchanging of information by speaking, writing, or using some other medium.’
- Will be even more important in future.

Dialogue with other disciplines

- The creative process in other fields of human life
- How do writers, poets create?
- How do performing artists create?
- How do artists, painters, sculptors, those who work with new materials create?
- Role of computer aided creativity

Science in India into the present

- Post 1947 with science emerging as a profession
- Opportunities for present generations (IITs, IISERs, NISER, CMI, IISc, many universities)
- Practically no discourse on the process of creativity and discovery
- Exception is the primality testing settled by Agarwal, Kayal and Saxena
- Prof. Anil Kumar (IISc) on the Nuclear Overhauser effect in biological molecules

Examples of my Eureka moments

- Program I wrote was running, searching for the value of $\tan\beta$ and b-quark mass within acceptable range and the corresponding mass of the top-quark keeping mass of τ fixed (April 1991)
- Working out correlations for $e^+e^- \to \tau^+\tau^-$, found the correct formula while waiting for a bus at Panjrapol in Ahmedabad (1992)
- Figured out how to compute an integral while waiting for family to come to the car (2009)
- Figured out how to use helicity amplitudes as a pattern recognition problem (2011)

Advice for young people

- Follow your hearts
- Nothing is more fun than research and discovery
- Throw open the doors to a new tomorrow
- Require a lot of patience, like running (sprinting?) a marathon
- Thomas Alva Edison on genius as 99% perspiration and 1% inspiration
- See also Richard Hamming in speech entitled ‘You and your research’
- Luck is what happens when preparation meets opportunity – Seneca the younger (disputed)

Charles Darwin on his theory

“ There is grandeur in this view of life, with its several powers, having been originally breathed into a few forms or into one; and that, whilst this planet has gone cycling on according to the fixed law of gravity, from so simple a beginning endless forms most beautiful and most wonderful have been, and are being, evolved.”

Wir müssen wissen, Wir werden wissen

We must know, We will know